

A Non-cooperative Game-Theoretic Framework for Sponsoring Content in the Internet Market

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Abstract—Data traffic demand over the Internet is increasing rapidly, and it is changing the pricing model between internet service providers (ISPs), content providers (CPs) and end users. One recent pricing proposal is sponsored data plan, i.e., when CP negotiates with the ISP on behalf of the users to remove the network subscription fees so as to attract more users and increase the number of advertisements. As such, a key challenge is how to provide proper sponsorship in the situation of complex interactions among the telecommunication actors, namely, the advertisers, the content provider, and users. To answer those questions, we explore the potential economic impacts of this new pricing model by modeling the interplay among the advertiser, users, and the CPs in a game theoretic framework. The CP may have either a subscription revenue model (charging end-users) or an advertisement revenue model (charging advertisers). In this work, we design and analyze the interaction among CPs having an advertisement revenue as a non-cooperative game, where each CP determines the proportion of data to sponsor and a level of credibility of content. In turn, the end-users demand for the content of a CP depends not only on their strategies but also upon those proposed by all of its competitors. Through rigorous mathematical analysis, we prove the existence and uniqueness of the Nash equilibrium. Based on the analysis of the game properties, we propose an iterative algorithm, which guarantees to converge to the Nash equilibrium point in a distributed manner. Numerical investigation shows the convergence of a proposed algorithm to the Nash equilibrium point and corroborates the fact that sponsoring content may improve the CPs outcome.

Index Terms—Pricing, Credibility of content, Nash equilibrium, Sponsoring, Game theory.

I. INTRODUCTION

THE proliferation of social media, such as Facebook, Netflix, and Youtube, has increased the demand for content traffic, CPs are economically dependent on the amount of traffic consumed by end-users: more volume of traffic consumed conducted to more advertisement and then revenue. Therefore, one of the important challenges for CPs is how to incentive end-users for more access to the contents, and thus achieve a higher demand. For this reason, CPs are turning to new types of content pricing "sponsored data plan", sponsored data plan allows a CP to pay an ISP access fees, and thereby the end-users can access content from the CP through the ISP for free. In 2014, AT&T launched sponsored data plan [1],

where the CP (e.g., Youtube, Twitter) can pay the access price of end-users to their content through the ISP. Examples in the USA are Netflix or Bing-On with T-Mobile, DIRECTV, etc.

Between the CPs and end-users, the ISP provide network access. In a traditional model, the ISP charges the end-users for content access. One effective way to reduce end-users access cost is the so-called sponsored data plan [2]-[4], which has been employed by many CPs worldwide. With the sponsored content, the CPs cooperate with the ISP, and the CP will pay the network access price to motivate end-users to join the network [5], [6]. Therefore, the sponsored data plan causes a positive cycle for the ISPs, the CPs, and end-users, i.e., a win-win situation [7]. The CP sponsored content can motivate more end-users to have a higher demand for their content. Accordingly, the higher demand gives more payoff of the CP. For this reason, the academia and the industry have a remarkable interest to investigate the sponsored data plan.

With the remarkable interests from academia and industry, the sponsored data plan has attracted many researchers to investigate and innovate better schemes. Recent studies emphasize the impact of sponsored data plans on the competition among different CPs [8]-[10]. A study [10] on the competition between one big CP and one small CP observes that sponsored data plans favor big CPs in certain situations. In [11], the authors analyzed the effect of sponsored data plan, when the ISPs compete for end-users and CPs compete for a share of the bandwidth usage by the customers. The authors in [7] developed a new model to study the competition among CPs under sponsored data plans.

The sponsored data plan has been extensively investigated during the past few years, most of these studies, however, are concerned with a CP sponsored data plan with a subscription revenue model; little work-study the completion between CPs having advertisement based revenue model with the credibility of content. However, to the best of our knowledge, none of the existing works explores the great benefit of a CP has an advertising revenue model with a sponsored data plan. The present paper moves toward this less explored direction. The main objective of this paper is to study the competition between CPs having an advertisement revenue, where each CP choose the number of content to sponsor and the level of the credibility of content. Specifically, we aim to study how the advertisement revenue model will affect the CP's data sponsoring strategy as well as the credibility of content level and the CP payoff.

Game theory [12] has been widely used in wireless networks for modeling and analyzing the competitive interactions among

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different network entities (e.g., [13], [14]). The game theory formulated interaction with main incentive structures. It is a mathematical study of the theory and methods of the competitive nature of the phenomenon. The game theory considers the game to predict individual behavior and actual behavior, and to study their optimization strategy.

In this paper, we investigate the complex interactions among the CPs under a sponsored content schema. More specifically, our contributions are summarized as follows:

- We present new models that include one important feature, the CP's revenue models with a sponsored data plan, the sponsored content sensitivity of users and the credibility of content.
- We include the credibility of content and sponsored content plan in competition between CPs for end-users and advertisers at the demand markets.
- We formulate the interactions among CPs as a non-cooperative game in terms of the credibility of content and number of sponsored content.
- We formally prove the existence and the uniqueness of the Nash equilibrium of the non-cooperative game among CPs, which means that there exists a stable state where all CP do not have an incentive to change their strategies. So, our model ensures the existence of an equilibrium for keeping the economy stable and achieving economic growth. We propose an iterative algorithm to achieve the Nash equilibrium.
- We conduct thorough numeric investigation to determine the effect of sponsored content on the strategies of CPs. In addition, the convergence of the proposed algorithm to the Nash equilibrium point is explored. We believe, based on results, that the proposed economic model is general and feasible, and this is applicable to modeling the competition among CPs content market with a sponsored content plan.

The rest of this paper is organized as follows. Section II discusses related work. In Section III, we describe the system model. We prove the existence and uniqueness of a Nash equilibrium point in Section IV. Then, we present a numerical investigation in Section V. In Section VI conclusions and future.

II. RELATED WORK

Game theory has been applied to a wide range of networking problems to study the interaction of players seeking to maximize their profit [15]. The authors [16] study models that involve CPs, CDN providers and end-users. The authors formulate interactions among CPs and among CDN as a non-cooperative game. In [17], the authors study competition between the CPs as a non-cooperative game. The authors in [18] studied competition between ISP in Information Centric Network using mathematical tools game theory. In [19], the authors modeled and studied competition between ISP where end-users are bounded rational. The authors [20] model and analyze ISPs best strategies in terms of quality offered to a big CP in a competitive context. They illustrate that a non-neutral behavior, differentiating traffic, is not leading to a desirable

situation. The authors in [21] formulated the interaction among ISPs as a non-cooperative game. Each ISP chooses the quality of service and the corresponding price.

The authors in [22] analyze a model with one CP several ISPs, where the CP can choose to sponsor a proportion of content and a level of advertisement. They show that sponsored data plan can be beneficial to the end-users and the ISPs. In [6], the authors consider a sponsored content market with a single ISP, a single CP, and a set of end-users. The interactions among three entities are modeled as two-stage Stackelberg game. The authors have analyzed two scenarios: a scenario where the ISP and the CP are in completion, and the scenario where the ISP and the CP cooperate for a common goal. The authors in [23], [24] investigate joint sponsored and caching content under the non-cooperative game. The interactions among ISP, CP, and end-users are modeled as a three-stage Stackelberg game. In [25], the authors model the service selection process between end-users as an evolutionary population game, and pricing-then-sponsoring process between the CP and the ISP as a non-cooperative game. The authors show that sponsoring can help improve the profit of both provider (CP and ISP) and the end-user quality of experience. The authors in [3] discussed the benefits of the sponsored data plan for the end-users, the CP and the ISP. A two-stage Stackelberg game was proposed by the authors in [4] to model the interaction between the CP and the ISP, and study the impact of the sponsored data plan on the Internet service market. In [26], the authors study the interactions among three entities under the sponsored data plan, namely, the ISP, the CP and end-users. They model the interactions as a hierarchical Stackelberg game, where the ISP and the CP act as the leaders determining the pricing and sponsoring strategies, respectively, and the end-users act as the followers deciding on their content demand. The authors in [27] studied how the edge caching affects the CPs sponsoring strategy as well as the end-users content demand. The authors in [28] explored the one-to-one interaction between the ISP and CP, and investigated the market dynamics using the bargaining game framework. The authors considered the problem of quality sponsored data in a non-neutral network. The authors in [8], [5] analyzed sponsored data plan from an economic point of view. They examine the implications of sponsored data plan on the CPs and the end-users and identify how the sponsored data influence the CP inequality. In [9], the authors studied the sponsorship competition among CPs in the Internet content market and demonstrated that the competitions improve the welfare of the ISP and the CP.

However, all of the above works studied the sponsored data plan without considering the complex interactions among CPs. In this paper, we focus on the competition between CPs, while taking the effects of advertisers, the number of sponsored content and transmission fee on CPs strategies.

III. MODEL

We consider an Internet market model that consists of tree economic entities, namely the advertisers, the CPs, and an arbitrary number of end-users who can switch from one CP to another. End-users can access the contents of the CP only

through the ISP network while CP provides the content to the end-users. The CP usually has two revenue models, the user subscription and the advertisement from the clicks of users. These two models, though sometimes coexisting with each other, in this work we study the obtain revenue from the advertisers. For the advertisement-based model, the CP attracts end-users clicks on online advertisements. Under the assumption that each CP can sponsor content, it can decide to sponsor either the entire or a portion of the requested content. Let N , the number of content that the CP sells. The sponsoring strategy adopted by each ISP is denoted by S that takes values in the interval $[0, N]$. Each CP advertises the end-users credibility of content c . Advertisers provide the revenue of CP and put an advertisement in content provided by the CP.

A. Demand Model

1) *Subscription Model*: We consider that demand of content from end-users to CP is a linear function affected by strategies of all CP, as for example in ([29][30]) as follows:

$$D_f = d_f + \varsigma_f^f c_f + \chi_f^f S_f - \sum_{g=1, f \neq g}^F (\varsigma_f^g c_g + \chi_f^g S_g) \quad (1)$$

The parameter d_f expresses the potential demand of end-users. ς_f^g and χ_f^g they are two positive parameters representing respectively the responsiveness of CP_f to credibility of content c_g and number of sponsored content S_g of CP_g . For CP_f , the demand D_f is increasing in S_f , and decreasing in S_g . The analogous relationship holds in credibility of content, in this case D_f is increasing in c_f and decreasing in c_g .

Assumption 1 *The sensitivity ς verifies:*

$$\varsigma_f^f \geq \sum_{g=1, g \neq f}^F \varsigma_f^g, \quad \forall f = 1, \dots, F.$$

The sensitivity χ verifies:

$$\chi_f^f \geq \sum_{g=1, g \neq f}^F \chi_f^g, \quad \forall f = 1, \dots, F.$$

Assumption 1 implies that the influence of CP strategies on its observed demand is greater than the influence of the strategies of its opponent on their demand.

2) *Advertisement Model*: In this paragraph, we describe the economic interaction between advertisers and the CP_f . There are A advertisers, each advertiser interested in the CP_f has a fixed budget C_f in a given time interval (e.g., daily, weekly or monthly) and has a valuation τ_f to declare its maximum willingness to pay for each attention to CP_f . The valuation τ_f is a random variable in the range $[0, \bar{\tau}_f]$. Suppose that τ_f is characterized by probability density function $x(\tau_f)$ and cumulative distribution function (CDF) $X(\tau)$. We assume that the valuations of all advertisers are independent and identically distributed. Let p_f be the price of per attention charged by the CP_f . We denote by D_{a_f} the demand of attentions from advertisers to the CP_f . Therefore, D_{a_f} can be expressed as [31][32]:

$$D_{a_f} = \frac{AC_f}{p_f} \text{prob}(\tau_f \geq p_f) \quad (2)$$

Where $\text{prob}(\tau_f \geq p_f) = 1 - X(p_f)$ then:

$$D_{a_f} = \frac{AC_f}{p_f} (1 - X(p_f)) \quad (3)$$

When CP_f increase p_f , the attention of advertisers to CP_f will decrease. However, the demand of attentions from advertisers to the CP_f , is upper bounded by the demand of users CP_f . Therefore, D_{a_f} can expressed as [31]

$$D_{a_f} = \min\{D_f, \frac{AC_f}{p_f} (1 - X(p_f))\} \quad (4)$$

Suppose τ_f follows a uniform distribution in the range $[0, \bar{\tau}_f]$. Then, $X(p_f) = \frac{p_f}{\bar{\tau}_f}$ the demand becomes:

$$D_{a_f} = \frac{AC_f}{p_f} (1 - \frac{p_f}{\bar{\tau}_f}) \quad (5)$$

To find the optimal price we follow [31], the optimal price p is obtained when:

$$D_f = \frac{AC_f}{p_f} \left(1 - \frac{p_f}{\bar{\tau}_f}\right) \quad (6)$$

Then, the optimal price of per attention,

$$p_f = \frac{AC_f \bar{\tau}_f}{AC_f + D_f \bar{\tau}_f} \quad (7)$$

B. Utility Function

The utility of CP_f can be modeled as:

$$U_f = p_f D_{a_f} - p_u S_f D_f - p_t c_f D_f - \theta_f c_f \quad (8)$$

θ_f is the cost to produce a unit of credibility of content c_f . p_t denoted the price that the CP pays to the ISP when forwarding content from it. p_u is the sponsoring cost per unit of data. In the utility function, the first term $p_f D_{a_f}$ is the revenue of CP_f , the second term $p_u S_f D_f$ denotes the cost due to sponsorship, the third term $p_t c_f D_f$ is the transmission fee when CP_f forwards the request demand D_f with credibility c_f through ISP, and the fourth term $\theta_f c_f$ is the cost to produce the credibility of content c_f .

Credibility of content c_f of CP_f is a function of the quality of content (QoC) q_{c_f} and the quality service (QoS) q_{s_f} , which is written as follows [33][13]:

$$c_f = \lambda q_{s_f} + \mu q_{c_f} \quad (9)$$

where μ and λ are two positive constants. The QoC can be specified for a specific domain of content (e.g., video streaming [34]). QoS q_{s_i} is defined as the expected delay which is computed by the Kleinrock function [35].

Submitting equation (7), (9) in equation (8), the CP_f utility become:

$$U_f = \frac{AC_f \bar{\tau}_f D_f}{AC_f + D_f \bar{\tau}_f} - p_u S_f D_f - p_t (\lambda q_{s_f} + \mu q_{c_f}) D_f - \theta_f (\lambda q_{s_f} + \mu q_{c_f}) \quad (10)$$

IV. GAME ANALYSIS

Let $G = [\mathcal{F}, \{\mathcal{S}_f, \mathcal{Q}_{s_f}, \mathcal{Q}_{c_f}\}, \{U_f(\cdot)\}]$ denote the non-cooperative QoC sponsoring QoS game (NQSQG), where $\mathcal{F} = \{1, \dots, F\}$ is the index set identifying the CPs, \mathcal{Q}_{s_f} is the QoS strategy set of CP_f , \mathcal{S}_f is the sponsoring strategy set of CP_f , \mathcal{Q}_{c_f} is the QoC strategy set of CP_f and $U_f(\cdot)$ is the utility function of CP_f defined in equation (10). We assume that the strategy spaces \mathcal{Q}_{s_f} , \mathcal{S}_f and \mathcal{Q}_{c_f} of each CP_f are compact and convex sets with maximum and minimum constraints. Thus, for each CP_f we consider as respective strategy spaces the closed intervals: $\mathcal{Q}_{s_f} = [q_{s_f}, \bar{q}_{s_f}]$, $\mathcal{S}_f = [S_f, \bar{S}_f]$ and $\mathcal{Q}_{c_f} = [q_{c_f}, \bar{q}_{c_f}]$. Let the QoS vector $q_s = (q_{s_1}, \dots, q_{s_F})^T \in Q_s^F = \mathcal{Q}_{s_1} \times \mathcal{Q}_{s_2} \times \dots \times \mathcal{Q}_{s_F}$, sponsoring vector $S = (S_1, \dots, S_F)^T \in \mathcal{S}^F = \mathcal{S}_1 \times \mathcal{S}_2 \times \dots \times \mathcal{S}_F$, QoC vector $q_c = (q_{c_1}, \dots, q_{c_F})^T \in Q_c^F = \mathcal{Q}_{c_1} \times \mathcal{Q}_{c_2} \times \dots \times \mathcal{Q}_{c_F}$.

A. Number of Sponsored Content Game

A NQSQG in number of sponsored content is defined for a fixed $\mathbf{q}_s \in \mathcal{Q}_s$, $\mathbf{q}_c \in \mathcal{Q}_c$ as $G(\mathbf{q}_s, \mathbf{q}_c) = [\mathcal{F}, \{\mathcal{S}_f\}, \{U_f(\mathbf{q}_s, \cdot, \mathbf{q}_c)\}]$.

Definition 1 A sponsoring vector $\mathbf{S}^* = (S_1^*, \dots, S_F^*)$ is a Nash equilibrium of the NQSQG $G(\mathbf{q}_s, \mathbf{q}_c)$ if:

$$\begin{aligned} \forall(f, S_f) &\in (\mathcal{F}, \mathcal{S}_f), \\ U_f(\mathbf{q}_s, S_f^*, \mathbf{S}_{-f}^*, \mathbf{q}_c) &\geq U_f(\mathbf{q}_s, S_f, \mathbf{S}_{-f}^*, \mathbf{q}_c) \end{aligned}$$

Theorem 1 For each $\mathbf{q}_s \in \mathcal{Q}_s$, $\mathbf{q}_c \in \mathcal{Q}_c$, the game $[\mathcal{F}, \{\mathcal{S}_f\}, \{U_f(\mathbf{q}_s, \cdot, \mathbf{q}_c)\}]$ admit a unique Nash Equilibrium.

Proof: To prove existence, we note that the strategy space \mathcal{S}_f is defined in the closed interval bounded by the minimum and maximum number of sponsored content. Thus, the strategy space \mathcal{S}_f is nonempty, convex, and compact subset of the Euclidean space \mathbb{R}^N . In addition, the second order derivative of the utility with respect to the number of sponsored content is negative as below :

$$-\frac{\partial^2 U_f}{\partial S_f^2} = -\frac{2(\chi_f^f)^2 A^2 C_f^2 \bar{\tau}_f^2}{(AC_f + D_f \bar{\tau}_f)^3} - 2\chi_f^f p_u \leq 0 \quad (11)$$

The second derivative of the utility function is negative, then the utility function is thus concave, which ensures existence of a Nash equilibrium point in the game $G(\mathbf{q}_s, \mathbf{q}_c)$.

We use the following proposition that holds for a concave game [36]: If a concave game satisfies the dominance solvability condition :

$$-\frac{\partial^2 U_f}{\partial S_f^2} \geq \sum_{g, g \neq f} \left| \frac{\partial^2 U_f}{\partial S_f \partial S_g} \right| \quad (12)$$

then the game $G(\mathbf{q}_s, \mathbf{q}_c)$ admits a unique Nash equilibrium point.

The mixed partial is written as:

$$\frac{\partial^2 U_f}{\partial S_f \partial S_g} = \frac{2\chi_f^f \chi_f^g A^2 C_f^2 \bar{\tau}_f^2}{(AC_f + D_f \bar{\tau}_f)^3} + \chi_f^g p_u \geq 0 \quad (13)$$

Then,

$$\begin{aligned} -\frac{\partial^2 U_f}{\partial S_f^2} - \sum_{g, g \neq f} \left| \frac{\partial^2 U_f}{\partial S_f \partial S_g} \right| &= \frac{2\chi_f^f A^2 C_f^2 \bar{\tau}_f^2}{(AC_f + D_f \bar{\tau}_f)^3} \\ &\times \left(\chi_f^f - \sum_{f, f \neq g} \chi_f^g \right) + (2\chi_f^f - \sum_{g, g \neq f} \chi_f^g) p_u \geq 0 \quad (14) \end{aligned}$$

Thus, the game $G(\mathbf{q}_s, \mathbf{q}_c)$ admit a unique Nash equilibrium point.

B. QoC Game

A NQSQG in QoC is defined for a fixed $\mathbf{q}_s \in \mathcal{Q}_s$, $\mathbf{S} \in \mathcal{S}$ as $G(\mathbf{q}_s, \mathbf{S}) = [\mathcal{F}, \{\mathcal{Q}_{c_f}\}, \{U_f(\mathbf{q}_s, \mathbf{S}, \cdot)\}]$.

Definition 2 A QoC vector $\mathbf{q}_c^* = (q_{c_1}^*, \dots, q_{c_F}^*)$ is a Nash equilibrium of the NQSQG $G(\mathbf{q}_s, \mathbf{S})$ if:

$$\begin{aligned} \forall(f, q_{c_f}) &\in (\mathcal{F}, \mathcal{Q}_{c_f}), \\ U_f(\mathbf{q}_s, \mathbf{S}, q_{c_f}^*, \mathbf{q}_{c_{-f}}^*) &\geq U_f(\mathbf{q}_s, \mathbf{S}, q_{c_f}, \mathbf{q}_{c_{-f}}^*) \end{aligned}$$

Theorem 2 For each $\mathbf{q}_s \in \mathcal{Q}_s$, $\mathbf{S} \in \mathcal{S}$, the game $[\mathcal{F}, \{\mathcal{Q}_{c_f}\}, \{U_f(\mathbf{q}_s, \mathbf{S}, \cdot)\}]$ admit a unique Nash Equilibrium.

Proof: To prove existence, we note that the strategy space \mathcal{Q}_{c_f} is defined in the closed interval bounded by the minimum and maximum QoCs. Thus, the joint strategy space \mathcal{Q}_{c_f} is nonempty, convex, and compact subset of the Euclidean space \mathbb{R}^N . In addition, the second order derivative of the utility with respect to the QoC is negative as below:

$$\frac{\partial^2 U_f}{\partial q_{c_f}^2} = -\frac{2\mu^2 (\varsigma_f^f)^2 A^2 C_f^2 \bar{\tau}_f^2}{(AC_f + D_f \bar{\tau}_f)^3} - 2\mu \varsigma_f^f p_t \leq 0 \quad (15)$$

The second derivative of the utility function is negative, then the utility function is thus concave, which ensures existence of a Nash equilibrium point in the game $G(\mathbf{q}_s, \mathbf{S})$.

We use the following proposition that holds for a concave game [36]: If a concave game satisfies the dominance solvability condition:

$$-\frac{\partial^2 U_f}{\partial q_{c_f}^2} \geq \sum_{g, g \neq f} \left| \frac{\partial^2 U_f}{\partial q_{c_f} \partial q_{c_g}} \right| \quad (16)$$

then the game $G(\mathbf{q}_s, \mathbf{S})$ admits a unique Nash equilibrium point.

The mixed partial is written as:

$$\frac{\partial^2 U_f}{\partial q_{c_f} \partial q_{c_g}} = \frac{2\mu^2 \varsigma_f^f \varsigma_f^g A^2 C_f^2 \bar{\tau}_f^2}{(AC_f + D_f \bar{\tau}_f)^3} + \mu^2 \varsigma_f^g p_t \geq 0 \quad (17)$$

Then,

$$\begin{aligned} -\frac{\partial^2 U_f}{\partial q_{c_f}^2} - \sum_{g, g \neq f} \left| \frac{\partial^2 U_f}{\partial q_{c_f} \partial q_{c_g}} \right| &= \frac{2\mu^2 \varsigma_f^f A^2 C_f^2 \bar{\tau}_f^2}{(AC_f + D_f \bar{\tau}_f)^3} \\ &\times \left(\varsigma_f^f - \sum_{f, f \neq g} \varsigma_f^g \right) + \mu^2 (\varsigma_f^f - \sum_{g, g \neq f} \varsigma_f^g) p_t \geq 0 \quad (18) \end{aligned}$$

Thus, the game $G(\mathbf{q}_s, \mathbf{S})$ admit a unique Nash equilibrium point.

C. QoS Game

A NQSQG in QoS is defined for a fixed $\mathbf{S} \in \mathcal{S}$, $\mathbf{q}_c \in \mathcal{Q}_c$ as $G(\mathbf{q}_c, \mathbf{S}) = [\mathcal{F}, \{\mathcal{Q}_{s_f}\}, \{U_f(\cdot, \mathbf{S}, \mathbf{q}_c)\}]$.

Definition 3 A QoS vector $\mathbf{q}_s^* = (q_{s_1}^*, \dots, q_{s_F}^*)$ is a Nash equilibrium of the NQSQG $G(\mathbf{q}_c, \mathbf{S})$ if:

$$\begin{aligned} \forall (f, q_{s_f}) \in (\mathcal{F}, \mathcal{Q}_{s_f}), \\ U_f(q_{s_f}^*, \mathbf{q}_{s_{-f}}^*, \mathbf{S}, \mathbf{q}_c) \geq U_f(q_{s_f}, \mathbf{q}_{s_{-f}}^*, \mathbf{S}, \mathbf{q}_c) \end{aligned}$$

Theorem 3 For each $\mathbf{S} \in \mathcal{S}$, $\mathbf{q}_c \in \mathcal{Q}_c$, the game $[\mathcal{F}, \{\mathcal{Q}_{s_f}\}, \{U_f(\cdot, \mathbf{S}, \mathbf{q}_c)\}]$ admit a unique Nash Equilibrium.

The QoS game has the same analysis as QoC game, just we replace μ by λ .

D. The Proposed Learning Algorithm

The section mentioned above shows clearly that the Nash equilibrium point is unique. Now, we devise an algorithm that converges to the Nash equilibrium in a distributed manner based on the best-response dynamic. A best response dynamics scheme contains a series of rounds, wherein each round, each CP observe the policy taken by its competitors in previous rounds and input them in its decision process to update its policy. In the first round, the choice of each CP is the best response based on its arbitrary belief about what the other players will choose.

Algorithm 1 summarizes the best response learning steps that each CP has to perform in order to find the Nash equilibrium point.

Algorithm 1 Best Response Algorithm

- 1: Initialize vectors $\mathbf{x}(0) = [x_1(0), \dots, x_F(0)]$ randomly;
 - 2: For each CP_f , $f \in \mathcal{F}$ at time instant t compute:
 - $x_f(t+1) = \underset{x_f \in \mathcal{X}_f}{\operatorname{argmax}} (U_f(\mathbf{x}(t)))$.
 - 3: If $\forall f \in \mathcal{F}$, $|x_f(t+1) - x_f(t)| < \epsilon$, then STOP.
 - 4: Else, $t \leftarrow t+1$ and go to step (2)
-

Such as:

- x refers to the vector of sponsored content S , vector q_s and vector q_c .
- \mathcal{X}_f refers to the policy profile number of sponsored content, QoS or QoC.

V. NUMERICAL INVESTIGATION

In this section, we perform the simulations to evaluate the strategy of CPs under competitive cases, where each CP sponsored a number of content. We consider a scenario with two homogeneous CP seeking to maximize their payoff.

Figures 1 and 2 illustrate the convergence to number of sponsored content and QoS. This figure shows the number of iterations needed for convergence to the Nash equilibrium point, it is clear that the speed of convergence is relatively high. The figures 1 and 2 demonstrates the existence and uniqueness of a Nash equilibrium point at which no CPs can profitably deviate given the strategies of another CP.

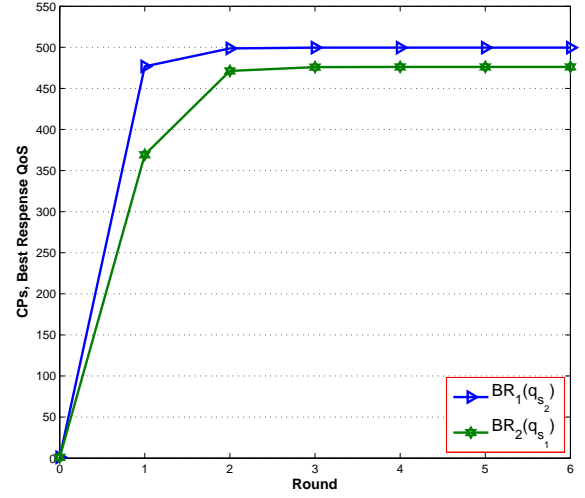


Fig. 1. Price game: Convergence to the QoS at Nash equilibrium.

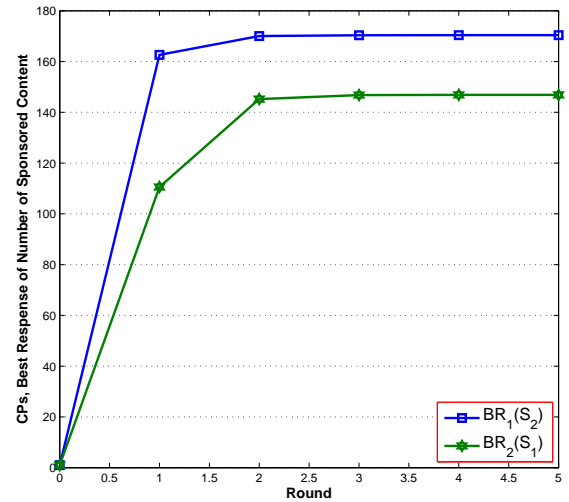


Fig. 2. Price game: Convergence to the number of sponsored content at Nash equilibrium.

Figures 3 shows the QoC as function of number of sponsored content S . The the QoC decrease as the number of sponsored content S get increases. The reason is that as the number of sponsored content S increases, the sponsored content demand of the end-users increases. Therefore, the transmission fee increases. Thus, the CP needs to slightly decrease its QoC to compensate the increase in the transit cost.

Figure 4 represents the impact of transmission price p_t on the number of sponsored content S of the two CPs. When the transmission price p_t keeps increasing the number of sponsored content S decreases. It comes from the fact that when the transmission price p_t is cheaper, CPs invest more to offer a large number of sponsored content for attracting more end-users. On the other hands, when the transmission price p_t is very expensive, CPs need to decrease the number of sponsored content to compensate the increase in the transmission price

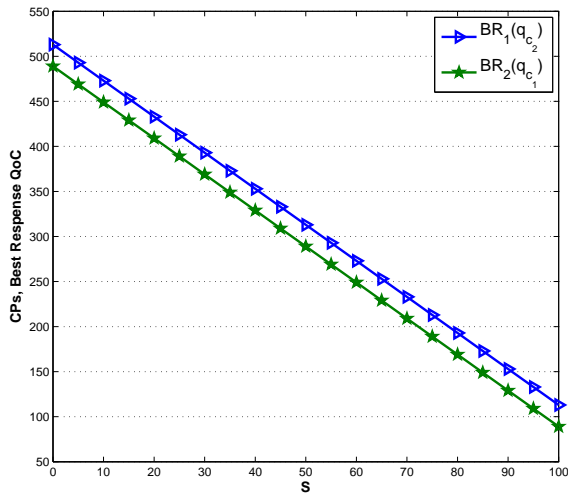


Fig. 3. Equilibrium QoS q_c as a function of number of sponsored content S .

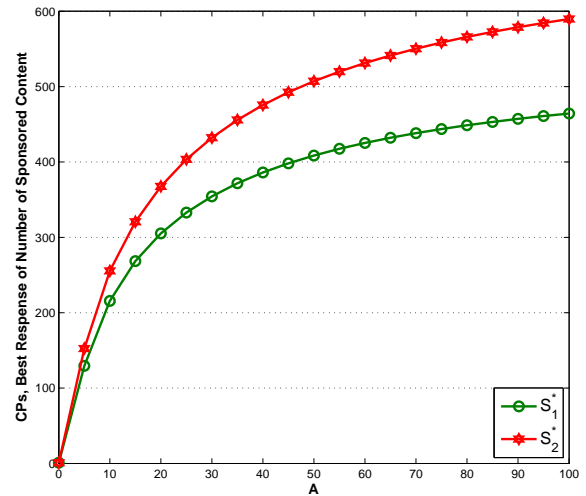


Fig. 5. Number of sponsored content at Nash equilibrium as a function of A .

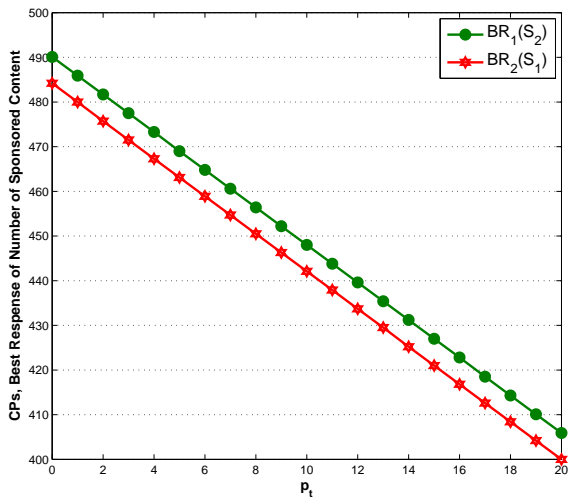


Fig. 4. Number of sponsored content at Nash equilibrium as a function of p_t .

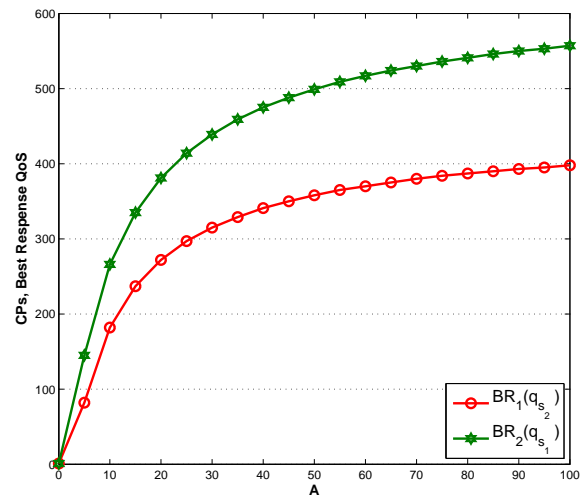


Fig. 6. Equilibrium QoS q_s as a function of A .

p_t .

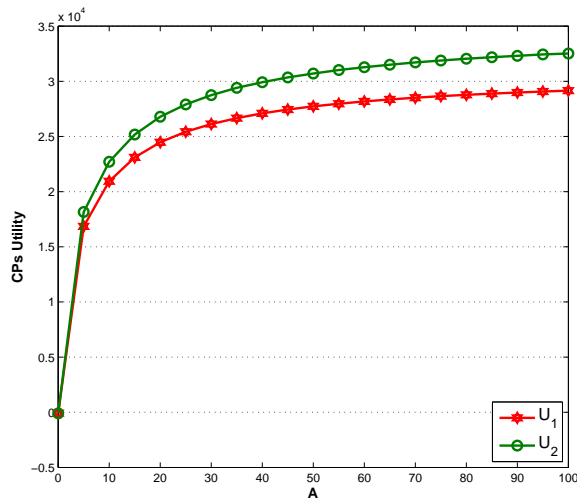
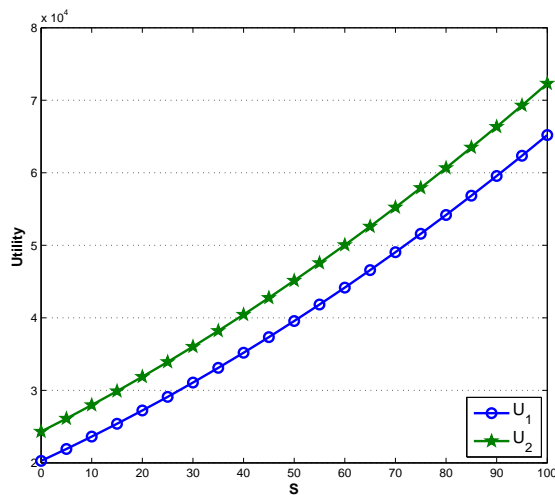
The impact of Number of advertisers A on the number of sponsored content, QoS and utility of the two CPs is illustrated in 5, 6 and 7. The figures show that the number of sponsored content, QoS and utility increases when A becomes larger. The reason is that as A increase, the revenue from advertisers increases too, which lead to an increase in the revenue of CPs. Therefore, the CPs invest for more to increase their QoS and the number of sponsored content in order to induce increased demand from the end-users.

We plot the expected utility of CPs as a function of the number of sponsored content S in Figure 8. Utility increase with respect to the number of sponsored content S . As S increases, the sponsored content demand of the end-users increases. Moreover, the increase of the sponsored content demand leads to the profit improvement of both CPs since

they can sell more content to the incoming end-users.

VI. CONCLUSION

In this paper, we developed an analytical framework for sponsoring content in the internet market that comprises of multiple content provider and end-users. We have formulated the interactions among the CPs as a non-cooperative game. Then, we have analytically analyzed the game. Furthermore, we proved the existence and uniqueness of the Nash equilibrium point in a competitive market under our proposed model. This result is significant because it implies that a stable solution with suitable economic incentives in sponsoring content is feasible in the internet paradigm. Additionally, we have proposed a distributed algorithm based on best response dynamic, which is ensured to converge to the Nash equilibrium point. Through the extensive simulation analysis, it has been verified that sponsoring content can improve the profit of CPs.

Fig. 7. Utility as a function of A .Fig. 8. Utility as a function of number of sponsored content S .

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