DOI: 10.31534/engmod.2020.3-4.ri.05v
Original scientific paper
Received: 11.08.2020.

Mathematical modelling for multiproduct EPQ problem featuring delayed differentiation, expedited rate, and scrap

Singa Wang Chiu[(1)], Jia-Ning Lin[(2)], Yunsen Wang[(3)], Hong-Dar Lin[(2)*]

[(1)] *Department of Business Administration, Chaoyang University of Technology, Taichung 413, TAIWAN*

[(2)] *Department of Industrial Engineering & Management, Chaoyang University of Technology, TAIWAN*
e-mail: hdlin@cyut.edu.tw

[(3)] *School of Accounting, Southwestern University of Finance and Economics, Chengdu, CHINA. 611130*

[(3)] *Department of Accounting & Information Systems, Rutgers Business School, Newark, NJ 07102, USA*

SUMMARY

The client requirements of present-day markets emphasize product quality, variety, and rapid response. To gain competitive advantages in marketplaces and meet customer needs, manufacturers today seek the most economical and fastest fabrication schemes and strategies to produce their various goods, especially when commonality exists within these multiple end products. Inspired by the above viewpoints, this study uses a mathematical modelling approach for solving a multiproduct economic production quantity (EPQ) problem featuring scrap, delayed differentiation, and expedited rate on the fabrication of the common part. We build a two-stage multiproduct fabrication scheme. Stage one uses an accelerated rate to produce all necessary common parts for multi-item to shorten its uptime, while stage two fabricates finished products sequentially using a rotation cycle rule. Inevitable random scraps produced in both stages are identified and removed to achieve the anticipated quality. We determined the optimal cost-minimization operating cycle length and used a numerical example to show our model's capability and to explore collective and individual impacts of scrap, expedited-rate, and postponement strategies on various performances of the studied problem (such as uptime of common part, utilization, rotation cycle time, total system cost, and individual cost contributor, etc.) Our model can offer an optimization solution and in-depth managerial insights for fabrication and operations planning in a wide variety of present-day industries, such as automotive, household goods, clothing, etc.

KEY WORDS: *multiproduct manufacturing problem; rotation cycle; delayed differentiation; expedited rate; economic production quantity; random scrap.*

1. INTRODUCTION

This study builds a decision support model for solving a multiproduct economic production quantity (EPQ) problem that features scrap, delayed differentiation and expediting fabrication of the common parts. We aim at helping manufacturing firms meet client needs of product quality, variety, and rapid response and gain competitive advantages in marketplaces. Taft [1] proposed the EPQ model to help manufacturers in planning their batch production. A mathematical modelling approach was used to decide the cost-minimization batch quantity by merely balancing the fabrication setup and stock holding costs in a perfect fabrication condition. In the real manufacturing environments, the inevitable random defective items produced have to be screened and removed/reworked to achieve the finished batch's quality. Studies concerning imperfect quality in fabrication are surveyed as follows. Richter [2] investigated an economic order quantity (EOQ) model considering flexible setup numbers, scrap, and repairable items. Two separate parameters were used to control the system's fabrication and rework modes under a finite period. Both constant and variable scrap rates were assumed to determine the cost-minimized setup numbers and the relationship between the scrap rate and optimal system cost. Chelbi and Rezg [3] investigated a flawed manufacturing-inventory system conditioned by a minimum required stock level and stochastic machine failures. The authors assumed that the machine was under fixed-time preventive maintenance or a corrected action at equipment failure. A buffer stock level was piled up to prevent stock-out occurrence. The authors derived the cost-minimization operating policy based on these separate consideration parameters. Hajej et al. [4] explored a fabrication-maintenance-shipment problem that features stochastic demand, returns of deteriorating products, and service-level constraint. The authors developed a model to characterize the problem and used simulation to study the shipment policy's impact on planning the fabrication-maintenance-shipment. Other studies [5-12] addressed the impact of different imperfect fabrication processes and their consequent actions on production-inventory planning and management.

To meet client requirements of product variety and fast response, modern manufacturers often seek the most economical and quickest fabrication scheme, such as delayed differentiation to produce their various goods, especially when commonality exists within these multiple end products. Aviv and Federgruen [13] studied a two-stage multiproduct fabrication system. The first stage for producing common products was limited, and client demand had a seasonally fluctuating and random nature. The authors developed both close-to-optimal and lower-bound heuristics for the problem. They explored and compared diverse delayed differentiation strategies by focusing on relationships among benefits, service levels, and capacities/inventory investments. Graman and Sanders [14] studied the trade-off between costs of forecast accuracy enhancement and postponement capacity under a minimum service level and a lower stock level. The authors used numerical illustrations to investigate the relationship and interaction among the above system parameters to gain managerial in-sight and facilitate postponement decision-making. Berrade et al. [15] studied a single component reliability problem featuring a delay time in replacement. The authors developed a model to investigate the cost-effective conditions for postponement in maintenance concerning inspection frequency and system reliability. They revealed that delay time in maintenance and defect arrival time both have significant cost-effectiveness of maintenance. Other studies [16-21] explored the influence of various postponement strategies on production-inventory systems, supply-chain management, and business operations.

To respond rapidly to customer orders or smooth fabrication schedules, production managers often evaluate diverse, effective alternatives to reduce production uptime. One of these options is to accelerate the manufacturing rate or increase throughput. Khouja [22] incorporated flexible fabrication rates into an economic lot scheduling problem. The author considered the manufacturing rate as a decision variable. Each item's unit cost was a function of its manufacturing rate. Numerical examples demonstrated the author's proposed algorithm in solving the problem. Sharma [23] examined the relationship of variable demand rate and flexible production rate in a manufacturing system. The author formulated the problem incorporating a decrease and increase in demand levels and their consequent production rate changes to explore the issue explicitly. AlDurgam et al. [24] studied a vendor-manufacturer coordinated system featuring flexible fabrication rate, uncertain demand, and finished products' multi-shipment. Their study aimed at finding the optimal fabrication rate that kept the supply chain's total fabrication and delivery costs at a minimum. Numeral illustrations demonstrated their results. Other studies [25-30] investigated the effect of different flexible rates and overtime strategies on manufacturing-inventory systems and supply-chain management. Since few prior works concentrated on examining the collective and individual impact of delayed differentiation, expedited rate, and random scrap on the EPQ-based system, our aim is to fill the gap.

2. PROBLEM STATEMENT AND ASSUMPTION

A mathematical modelling approach is used to explore a multiproduct economic production quantity (EPQ) problem featuring product commonality and delayed differentiation strategy, expedited rate, and scrap. In the multiproduct fabrication planning phase, when the common part exists, the production managers have the option to design a two-stage delayed differentiation fabrication scheme to reduce the response times of end products or leverage the common part's cost with its volume. Inspired by this concept, we consider a manufacturing system that makes L distinct end products using a two-stage postponement scheme to meet their annual demand λ_i (where $i = 1, 2, \ldots, L$). The first stage makes all required common parts at a rate of $P_{1,0}$ per year, while the second stage produces L distinct end products at a rate of $P_{1,i}$ per year. The completion rate γ (compared with the finished product) of the common part is a known constant, and both $P_{1,0}$ and $P_{1,i}$ depend on γ. For instance, if $\gamma = 50\%$, then both $P_{1,0}$ and $P_{1,i}$ grow into twice as much as the standard rate in a single-stage system for making end product i.

We adopt an accelerated rate $P_{T1,0}$ option to reduce the time-consuming common part's uptime. The following relationships between product rates and its consequent cost-relevant variables are assumed:

$$P_{T1,0} = \left(1+\alpha_{1,0}\right)P_{1,0} \tag{1}$$

$$K_{T0} = \left(1+\alpha_{2,0}\right)K_0 \tag{2}$$

$$C_{T0} = \left(1+\alpha_{3,0}\right)C_0 \tag{3}$$

where K_{T0} and C_{T0} are the setup and unit production costs associated with expedited rate $P_{T1,0}$, K_0 and C_0 denote the setup and unit costs connected with standard rate $P_{1,0}$, and $\alpha_{1,0}$, $\alpha_{2,0}$, and $\alpha_{3,0}$ are the linking parameters.

Further, there are random scrap portions x_0 and x_i associated with each manufacturing stage. Thus, the production rates $d_{T1,0}$ and $d_{1,i}$ of scrap common parts and end products are as follows:

$$d_{T1,0} = (x_0) P_{T1,0} \quad \textbf{(4)}$$

$$d_{1,i} = (x_i) P_{1,i} \quad \textbf{(5)}$$

All scrap items produced in both stages are identified and removed from the finished lot at a unit disposal cost of $C_{S,0}$, and $C_{S,i}$, respectively. Moreover, we assume the production equipment is *100%* reliable (otherwise, one should explicitly explore the unreliable matter). The following are definitions for the additional notation of this study:

Q_i = the lot size of end item i (where $i = 1, 2, ..., L$) in stage two,

K_i = setup cost for end item i,

C_i = unit production cost for end product i,

$h_{1,i}$ = unit holding cost for end product i,

$h_{4,i}$ = unit holding cost for the safety end product i,

S_i = setup time for end product i,

T_A = the rotation production cycle length - the decision variable,

$H_{1,i}$ = the inventory status of end product i when its uptime finishes,

$t_{1,i}$ = uptime of end product i in stage 2,

$t_{2,i}$ = depletion time of end product i,

t_i^* = sum of optimal uptimes of end products in stage two,

Q_0 = the lot size of common parts in stage one,

λ_0 = annual demand of common parts,

S_0 = common part's setup time

$h_{1,0}$ = common part's unit holding cost,

τ_i = the specific holding cost ratio for the different safety end product,

$h_{4,0}$ = unit holding cost for the safety common part,

$H_{1,0}$ = the inventory status of common parts when its uptime ends,

H_i = the inventory status of common parts when the uptime of production process of product i completes,

$t_{1,0}$ = uptime of common parts when an expedited rate option is implemented,

$t_{2,0}$ = the depletion time of the common parts,

t_0^* = the optimal uptime in stage one,

i_0 = the inventory holding cost linking parameter (i.e., $h_{1,i} = (i_0)C_i$),

$I(t)_i$ = the inventory status at time t of product i (where $i = 0, 1, 2, ..., L$),

$I_d(t)_i$ = the stock level of scrap items at time t of product i (where $i = 0, 1, 2, ..., L$),

$E[T_A]$ = the expected rotation production cycle time,

$TC(T_A)$ = the system cost per cycle,

$E[TC(T_A)]$ = the expected system cost per cycle,

$E[TCU(T_A)]$ = the expected system cost per unit time.

Figure 1 shows the stock status of the proposed multiproduct delayed differentiation EPQ model featuring expedited rate and scrap. It indicates that the common part's inventory level (in stage) accumulates to $H_{1,0}$ at the end of its uptime $t_{1,0}$, and in stage 2 the inventory level for each product i (where $i = 1, 2, \ldots, L$) grows to $H_{1,i}$ at the end of its uptime $t_{1,i}$, while the common part's level gradually declines (refer to H_i in Figure 1) during $t_{2,0}$. Figure 2 separately depicts the common part's inventory status during productions of each end product i.

The proposed model does not allow any stock-out situation, therefore, both of the following formulas must hold: (1) $P_{T1,0} - d_{T1,0} > 0$ and (2) $P_{1,i} - d_{1,i} - \lambda_i > 0$. Figure 3 illustrates the stock level of scrap items in this multiproduct delayed differentiation EPQ model. It shows that in stage one, the level of scrap common parts accumulates to $(d_{T1,0}\, t_{1,0})$ at the end of uptime $t_{1,0}$, and during stage two, the level of scrap end product i reaches $(d_{1,i}\, t_{1,i})$ at the end of uptime $t_{1,i}$.

3. FORMULATION AND SOLUTION PROCESS

3.1 DURING THE PRODUCTION OF END PRODUCTS IN STAGE 2

The following formulas are observed in line with the description, assumption, and illustrations of Figures 1 to 3, for $i = 1, 2, \ldots, L$:

$$Q_i = \frac{\lambda_i T_A}{1-x_i} \quad (6)$$

$$t_{1,i} = \frac{Q_i}{P_{1,i}} = \frac{H_{1,i}}{P_{1,i} - d_{1,i} - \lambda_i} \quad (7)$$

$$H_{1,i} = \left(P_{1,i} - d_{1,i} - \lambda_i\right) t_{1,i} \quad (8)$$

$$t_{2,i} = \frac{H_{1,i}}{\lambda_i} \quad (9)$$

$$T_A = t_{1,i} + t_{2,i} = \frac{Q_i\left(1-x_i\right)}{\lambda_i} \quad (10)$$

3.2 DURING THE PRODUCTION OF COMMON PARTS IN STAGE 1

The needed common parts in the start of stage 2 are as follows (in line with Eq. (6)):

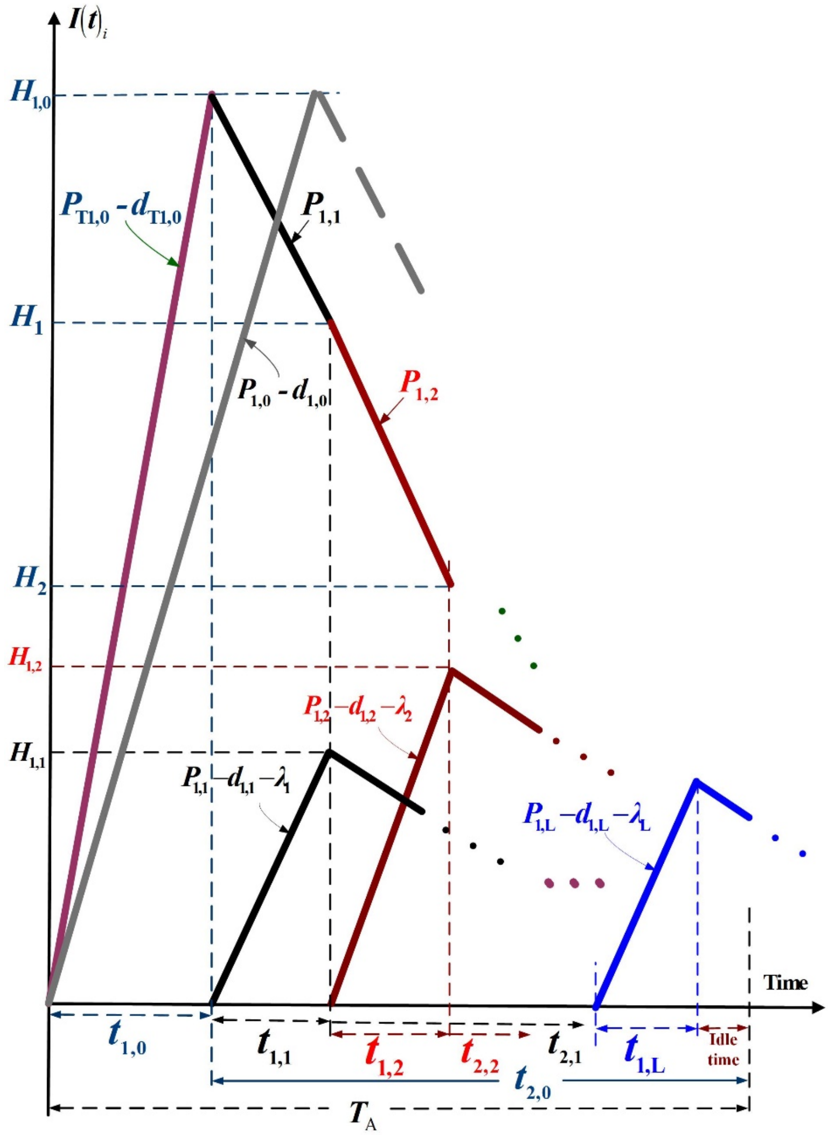

Fig. 1 *The stock status of the proposed multiproduct delayed differentiation EPQ model featuring expedited rate and scrap compared with the same system without adopting the expedited rate (in grey)*

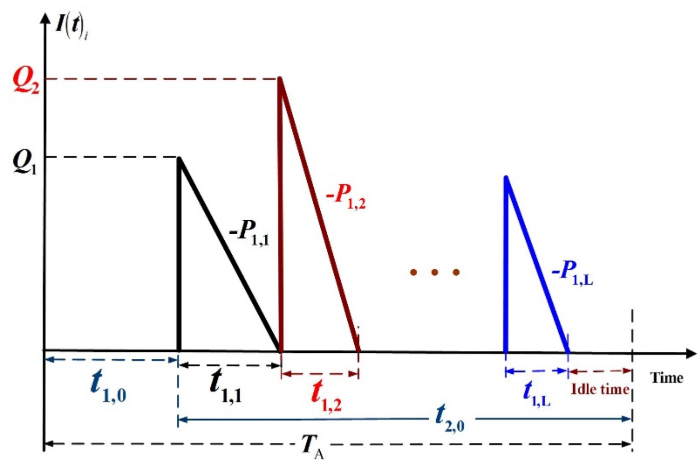

Fig. 2 *The common part's inventory status during the production of each end product i*

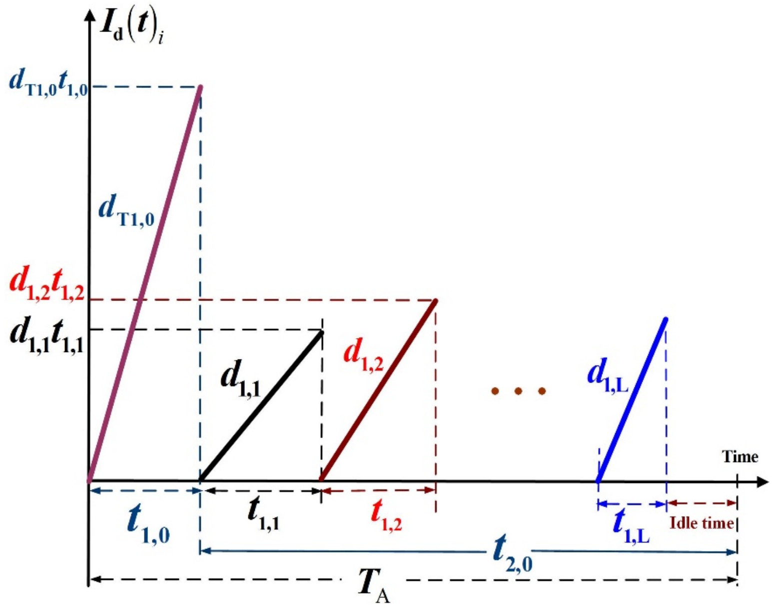

Fig. 3 *The stock level of scrap items in this multiproduct delayed differentiation EPQ model*

$$H_{1,0} = \sum_{i=1}^{L} Q_i = \sum_{i=1}^{L} \frac{\lambda_i T_A}{1-x_i} \tag{11}$$

The following additional equations can also be observed in relation to the description, assumption, and illustrations of Figures 1 to 3:

$$\lambda_0 = \frac{\sum_{i=1}^{L} Q_i}{T_A} \tag{12}$$

$$Q_0 = \frac{H_{1,0}}{1-x_0} \tag{13}$$

$$t_{1,0} = \frac{Q_0}{P_{T1,0}} \tag{14}$$

$$H_{1,0} = \left(P_{T1,0} - d_{T1,0}\right) t_{1,0} \tag{15}$$

$$H_1 = H_{1,0} - Q_1 \tag{16}$$

$$H_i = H_{(i-1)} - Q_i \text{ , for } i = 2, 3, \ldots, L \tag{17}$$

$$H_L = H_{(L-1)} - Q_L = 0 \tag{18}$$

$$T_A = t_{1,0} + t_{2,0} \tag{19}$$

3.3 THE COST FUNCTION OF THE PROBLEM AND SOLUTION DERIVATION

The system cost per cycle $TC(T_A)$ includes both stages' relevant costs as follows:

(i) common part's production variable, setup, disposal, and inventory holding costs in stage 1; (ii) the sum of L end products' production variable, setup, disposal, and inventory holding costs. Thus, $TC(T_A)$ becomes:

$$\begin{aligned} TC(T_A) &= C_{T0}Q_0 + K_{T0} + C_{S,0}(x_0Q_0) + h_{4,0}(x_0Q_0)T_A \\ &+ h_{1,0}\left[\frac{H_{1,0}t_{1,0}}{2} + \frac{d_{T1,0}t_{1,0}}{2}(t_{1,0}) + \sum_{i=1}^{L}\left[\frac{Q_i}{2}(t_{1,i}) + H_i(t_{1,i})\right]\right] \\ &+ \sum_{i=1}^{L}\left\{C_iQ_i + K_i + C_{S,i}(x_iQ_i) + h_{4,i}(x_iQ_i)T_A + h_{1,i}\left[\frac{H_{1,i}t_{1,i}}{2} + \frac{H_{1,i}}{2}(t_{2,i}) + \frac{d_{1,i}t_{1,i}}{2}(t_{1,i})\right]\right\} \end{aligned} \tag{20}$$

The expected system cost per unit time $E[TCU(T_A)]$ is derived as shown in Eq. (21), while Eq. (22) exhibits the optimal rotation production cycle length T_A* (for details, refer to Appendix A).

$$\begin{aligned} E\left[TCU(T_A)\right] &= \left\{ \begin{aligned} &\left[(1+\alpha_{3,0})C_0\right]\lambda_0 E_{00} + \frac{\left[(1+\alpha_{2,0})K_0\right]}{T_A} + C_{S,0}\lambda_0 E_{10} + \left(\frac{h_{1,0}}{2}\right)\left(T_A\lambda_0^2\right)E_{0P}E_{00}^2 \\ &+ h_{4,0}\lambda_0 T_A E_{10} + h_{1,0}\sum_{i=1}^{L}\left[\frac{\lambda_i^2 T_A}{2P_{1,i}}E_{0i}^2 - \frac{\lambda_i T_A}{P_{1,i}}E_{0i}\sum_{j=1}^{i}\lambda_j E_{0j}\right] + h_{1,0}\left[\left(\sum_{i=1}^{L}\lambda_i E_{0i}\right)\sum_{i=1}^{L}\left(\frac{\lambda_i T_A}{P_{1,i}}E_{0i}\right)\right] \end{aligned} \right\} \\ &+ \sum_{i=1}^{L}\left\{C_i\lambda_i(E_{0i}) + \frac{K_i}{T_A} + C_{S,i}\lambda_i(E_{1i}) + h_{4,i}\lambda_i T_A(E_{1i}) + \frac{h_{1,i}\lambda_i^2 T_A}{2}(E_{0i})^2 E_{iP}\right\} \end{aligned} \tag{21}$$

$$T_A^* = \sqrt{\frac{\left[(1+\alpha_{2,0})K_0\right] + \sum_{i=1}^{L}K_i}{\begin{aligned} &\left(\frac{h_{1,0}}{2}\right)(\lambda_0)^2 E_{0P}E_{00}^2 + h_{4,0}\lambda_0 E_{10} + \sum_{i=1}^{L}\left\{\frac{h_{1,i}\lambda_i^2}{2}(E_{0i})^2 E_{iP} + h_{4,i}\lambda_i(E_{1i})\right\} \\ &+ h_{1,0}\sum_{i=1}^{L}\left[\frac{\lambda_i^2}{2P_{1,i}}E_{0i}^2 - \frac{\lambda_i}{P_{1,i}}E_{0i}\sum_{j=1}^{i}\lambda_j E_{0j}\right] + h_{1,0}\left[\left(\sum_{i=1}^{L}\lambda_i E_{0i}\right)\cdot\sum_{i=1}^{L}\left(\frac{\lambda_i}{P_{1,i}}E_{0i}\right)\right] \end{aligned}}} \tag{22}$$

3.4 EFFECT OF SETUP TIMES ON T_A* AND A PREREQUISITE CONDITION FOR THE PROBLEM

One last point to be mentioned is that if the sum of setup times S_i is larger than the idle time in T_A* (refer to Figure 1), then T_{min} (Nahmias [31]) needs to be calculated. One must select the max (T_A*, T_{min}) as the final solution for the problem to ensure the rotation cycle length is long enough to accommodate the summation of setup plus production times of common parts and all end products.

Besides, the following prerequisite condition (as shown in Eqs. (23) and (24)) must hold to ensure the machine has sufficient capacity to produce common parts and all end products in this specific problem (Nahmias, [31]). Furthermore, Eqs. (25) and (26) calculate system utilization and end product utilization, respectively.

$$\left(t_{1,0} + \sum_{i=1}^{L}t_{1,i}\right) < T_A \quad or \quad \left[Q_0\left(\frac{1}{P_{T1,0}}\right) + \sum_{i=1}^{L}Q_i\left(\frac{1}{P_{1,i}}\right)\right] < T_A \tag{23}$$

or:

$$\left[\left(\frac{\lambda_0}{\left[1-E\left[x_0\right]\right]}\right)\left(\frac{1}{P_{T1,0}}\right)+\sum_{i=1}^{L}\left(\frac{\lambda_i}{\left[1-E\left[x_i\right]\right]}\right)\left(\frac{1}{P_{1,i}}\right)\right]<1 \quad \textbf{(24)}$$

$$System\ utilization=\frac{\left(t_{1,0}+\sum_{i=1}^{L}t_{1,i}\right)}{T_A} \quad \textbf{(25)}$$

$$End\ product's\ utilization=\frac{\sum_{i=1}^{L}t_{1,i}}{T_A} \quad \textbf{(26)}$$

4. EXAMPLE AND DISCUSSION

We provide a simulated example below to demonstrate the applicability of our model. Suppose that a two-stage fabrication scheme featuring delayed differentiation is used to produce five distinct customized end products that share a common part. An expedited rate is used to speed up the manufacturing of common parts in stage 1. For each customized product, the annual demand and manufacturing rates along with cost parameters and defective/scrap rates are given in Tables 1 and 2. In comparison, the variables' values of the same problem using a single-stage scheme are displayed in Table B-1 (see Appendix B). In Table B-1, the assumption of holding cost $h_{1,i} = i_0C_i$ and $h_{4,i} = i_0C_i\tau_i$ (where τ_i is the specific holding cost ratio for different safety product).

Table 1 *The assumed variables' values in the stage 1 of this study*

λ_0	$P_{1,0}$	$\alpha_{1,0}$	$\alpha_{2,0}$	$\alpha_{3,0}$	$h_{1,0}$	$h_{4,0}$
17406	120000	0.5	0.1	0.25	$8	$1
K_0	C_0	x_0	γ	i_0	$C_{S,0}$	δ
$8500	$40	2.5%	0.5	0.2	$10	50%

Table 2 *The assumed variables' values in the stage 2 of this study*

Product i	λ_i	$P_{1,i}$	C_i	K_i	x_i	$C_{S,i}$	$h_{1,i}$	$h_{4,i}$
1	3000	112258	$40	$8500	2.5%	$10	$16	$3
2	3200	116066	$50	$9000	7.5%	$15	$18	$5
3	3400	120000	$60	$9500	12.5%	$20	$20	$7
4	3600	124068	$70	$10000	17.5%	$25	$22	$10
5	3800	128276	$80	$10500	22.5%	$30	$24	$13

4.1 CONVEXITY, SYSTEM COST, AND QUALITY COST ANALYSES

Applying equations (24) and (21), one obtains the optimal common cycle length $T_A* = 0.5149$ and $E[TCU(T_A*)] = \$2,299,453$. Figure 4 illustrates the collective influence of T_A and the average scrap rate on $E[TCU(T_A)]$ along with the convexity of $E[TCU(T_A)]$. It exposes that $E[TCU(T_A)]$ increases considerably as T_A deviates from T_A*, and as the mean scrap rate rises, $E[TCU(T_A)]$ boosts significantly. Table C-1 (see Appendix C) exhibits the influence of changes in $\alpha_{1,0}$ on various system performances.

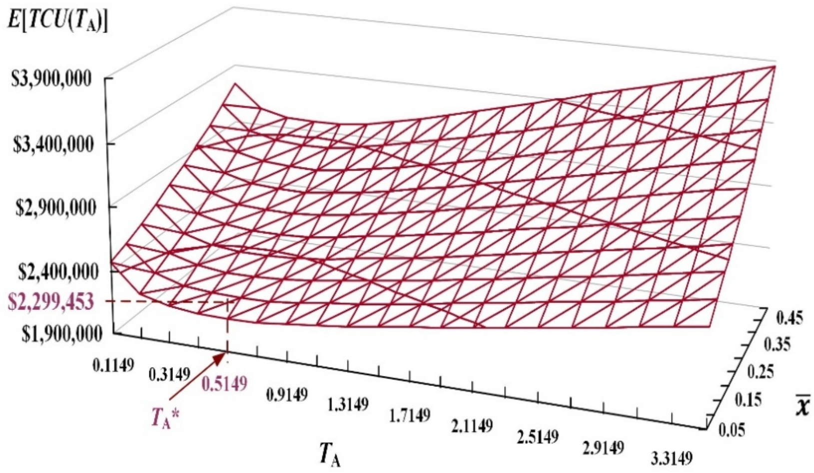

Fig. 4 *The joint influence of T_A and average scrap rate on E[TCU(T_A)]*

The optimal $E[TCU(T_A^*)]$ is explicitly analyzed for expedited ratio $\alpha_{1,0} = 0.5$ and the common part's completion rate $\gamma = 0.5$, and the result is depicted in Figure 5. It indicates that the quality cost in both stages adds up to *5.58%* of $E[TCU(T_A^*)]$. The expedited cost in stage one contributes *8.09%* to $E[TCU(T_A^*)]$ (see the last column of Table C-1), and the summation of holding, setup, and variable cost for end products in stage 2 contributes *53.56* to $E[TCU(T_A^*)]$.

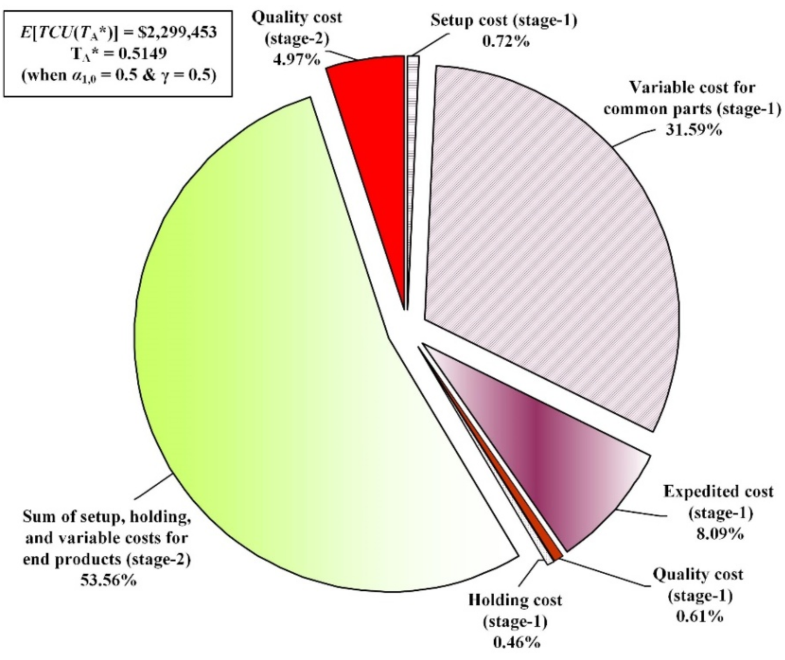

Fig. 5 *The breakup of E[TCU(T_A*)]*

The system quality is further analyzed and the outcome is depicted in Figure 6. It reveals that the extra variable costs to make up the scraps in both stages add up to *74.51%* of quality cost. The disposal costs in both stages contribute a total of *25.08%* to the quality cost. These are the two main contributors to system quality cost.

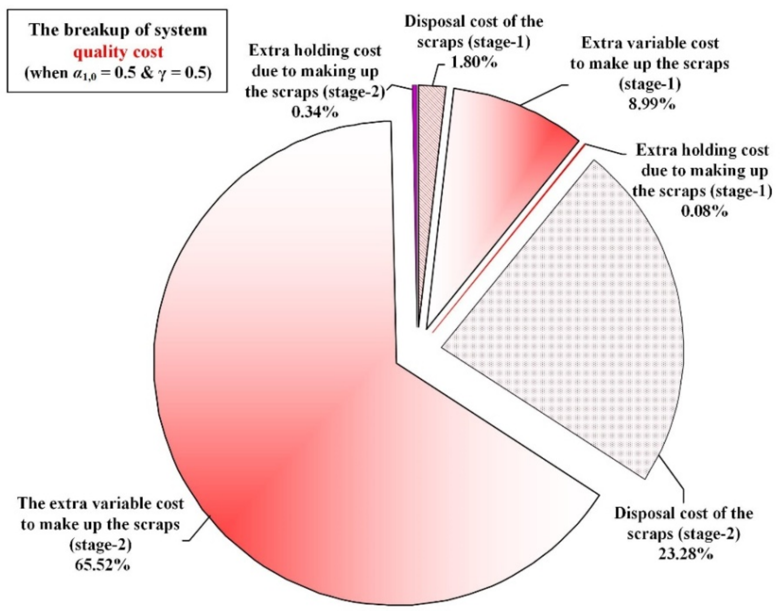

Fig. 6 *The breakup of system quality cost in E[TCU(T_A*)]*

4.2 THE EFFECT OF THE CHANGES IN EXPEDITED RATE ON OPTIMAL UPTIME T^*_0 AND UTILIZATION

Due to the expedited rate on the production of common parts, its effect on optimal uptime t^*_0 is investigated and displayed in Figure 7. It shows that at the expedited ratio $\alpha_{1,0} = 0.5$, the optimal uptime in stage one t^*_0 decreases *32.24%* (i.e., it declines from *0.0779* years to *0.0528* years; refer to Table C-1 in Appendix C).

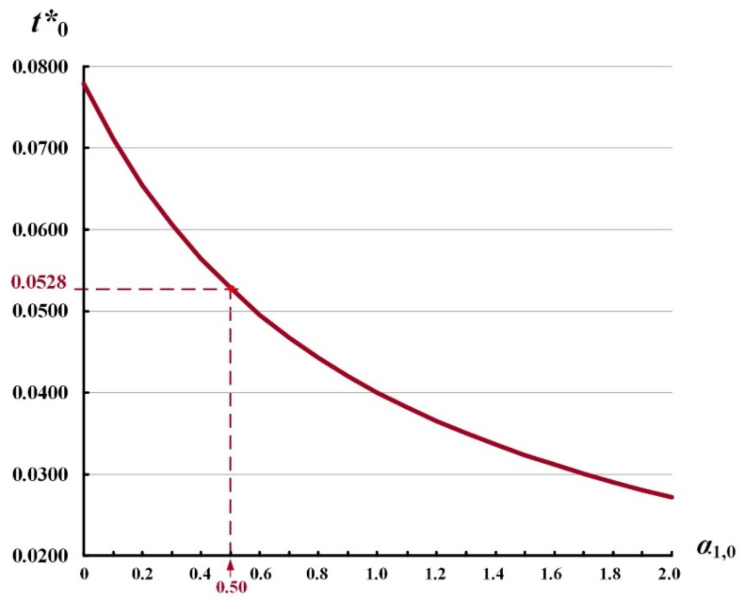

Fig. 7 *The effect of changes in expedited rates on the optimal uptime in stage 1*

The impact of differences in expedited rates on utilization is explored, and the outcome is illustrated in Figure 8. It reveals that at the expedited ratio $\alpha_{1,0} = 0.5$, the machine utilization drops from *0.3070* to *0.2536*, or a decline of *16.81%* (see Table C-1 in Appendix C).

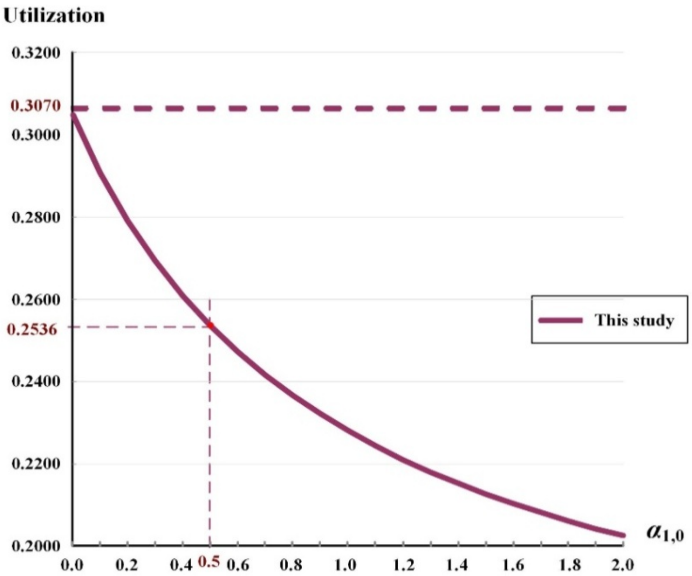

Fig. 8 *The impact of differences in expedited rates on utilization*

4.3 COMPARISON OF THE PROPOSED MODEL WITH OTHER CLOSELY RELATED MODELS

We compared the proposed model's machine utilization and system cost with that of the other closely related models and illustrated the results separately in Figures 9 and 10. It shows that our model's utilization decreases from *0.3048* to *0.2536* (see Table C-1), or a *16.80%* decline compared to a delayed differentiation model without expedited rate (see Figure 9); at the price increase of *8.71%* in total system cost (i.e., rising from *$2,115,234* to *$2,299,453*; refer to Table C-1 and Figure 10).

Moreover, our utilization drops from *0.3070* to *0.2536* or a *17.39%* decline compared to a single-stage fabrication model without expedited rate (see Figure 9); at the price increase of *4.91%* in total system cost (i.e., rising from *$2,191,924* to *$2,299,453*; refer to Figure 10).

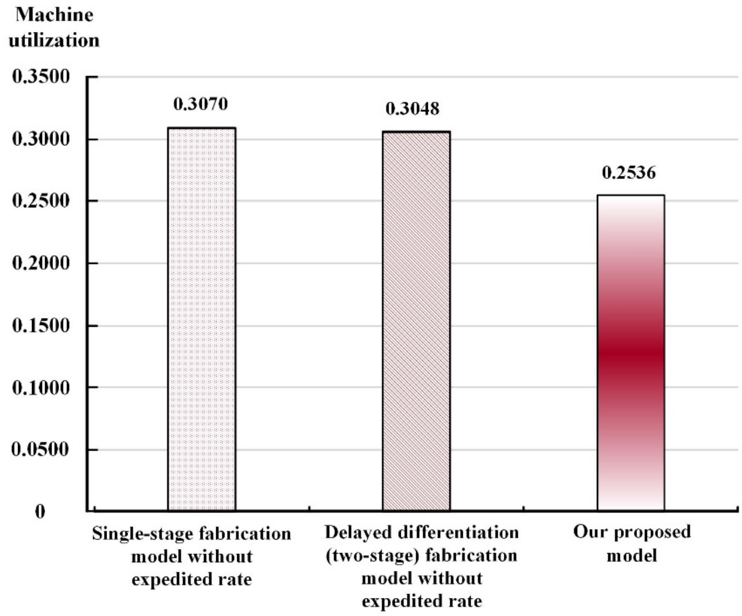

Fig. 9 *Comparison of our model's utilization with that of other closely related models*

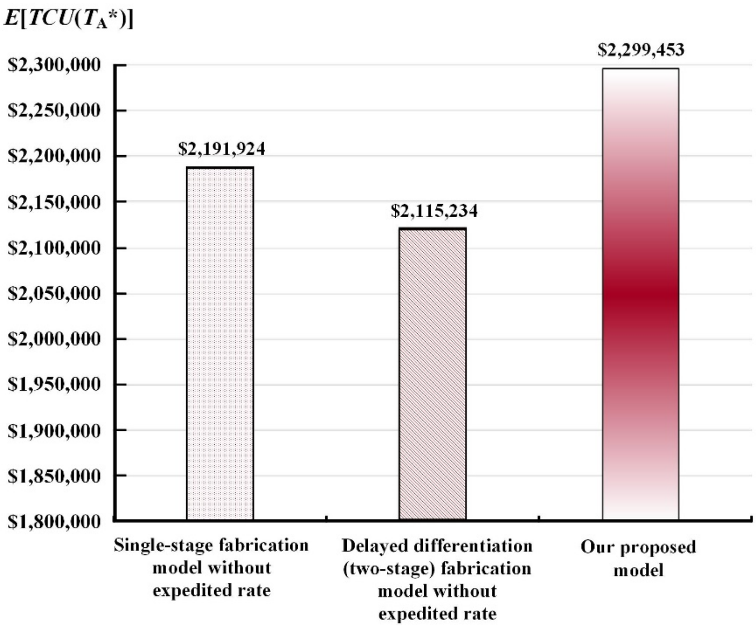

Fig. 10 *Comparison of our model's $E[TCU(T_A*)]$ with that of other closely related models*

4.4 DEMONSTRATING THE EXTRA ANALYTICAL CAPABILITIES OF OUR PROPOSED MODEL

Figure 11 demonstrates extra analytical capability our model in exploring common part's linear and nonlinear values (i.e., the factor δ) concerning its completion rate γ. For $\gamma = 0.5$ and $\delta = \gamma^1$ (as assumed in our example), it confirms $E[TCU(T_A*)] = \$2,299,453$. A nonlinear example of $\delta = \gamma^{1/3}$ (for $\gamma = 0.5$) results in $E[TCU(T_A*)] = \$2,345,991$.

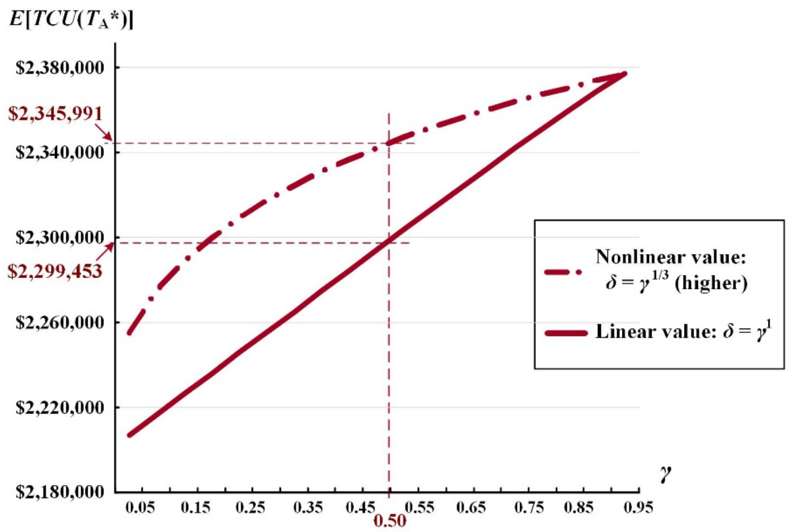

Fig. 11 *The effect of common part's linear and nonlinear values on E[TCU(T_A*)]*

Figure 12 further exhibits our model's capability in investigating machine utilization for each end product (in stage 2) concerning expedited ratio $\alpha_{1,0}$. It shows separate utilization for each end product. As $\alpha_{1,0}$ rises, these utilizations change little since the common part's expedited ratio $\alpha_{1,0}$ has little effect on the end product utilization in stage 2.

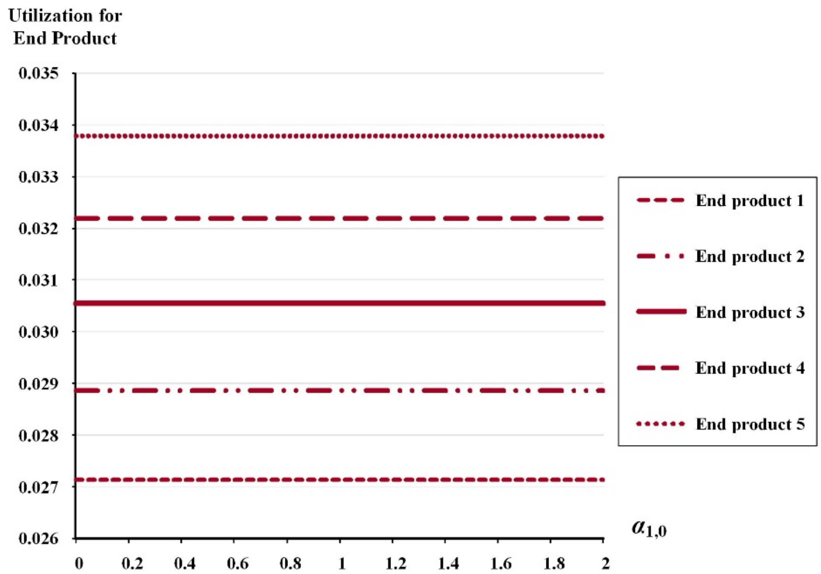

Fig. 12 *Machine utilization for each end product concerning expedited ratio $\alpha_{1,0}$*

Moreover, Figure 13 depicts the collective impact of differences in unit cost add-up percentage due to the expedited rate $\alpha_{3,0}$ and the common part's completion rate γ on $E[TCU(T_A*)]$. It exposes that as $\alpha_{3,0}$ increases, $E[TCU(T_A*)]$ rises, which upsurges radically when γ value is also high. As γ increases, $E[TCU(T_A*)]$ declines slightly when $\alpha_{3,0}$ value is relatively low (less than *0.125*), but $E[TCU(T_A*)]$ starts to surge as $\alpha_{3,0}$ value goes beyond *0.125*. When both values of γ and $\alpha_{3,0}$ are at a high level, $E[TCU(T_A*)]$ upsurges enormously.

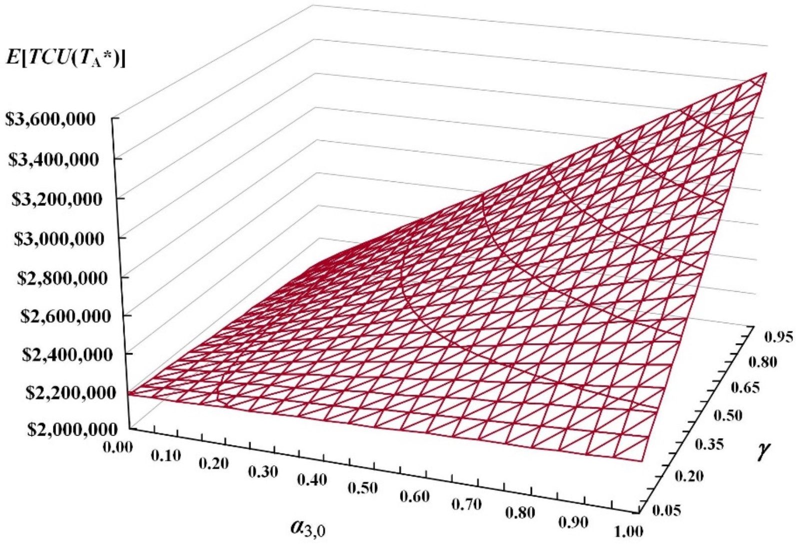

Fig. 13 *The collective impact of differences in $\alpha_{3,0}$ and γ on $E[TCU(T_A*)]$*

Figure 14 further explores the collective impact of differences in $\alpha_{3,0}$ and γ on the optimal rotation cycle length T_A*. It reveals that as $\alpha_{3,0}$ increases, T_A* has no changes since $\alpha_{3,0}$ has no effect on T_A* (see Eq. (22)). As γ increases, T_A* declines greatly.

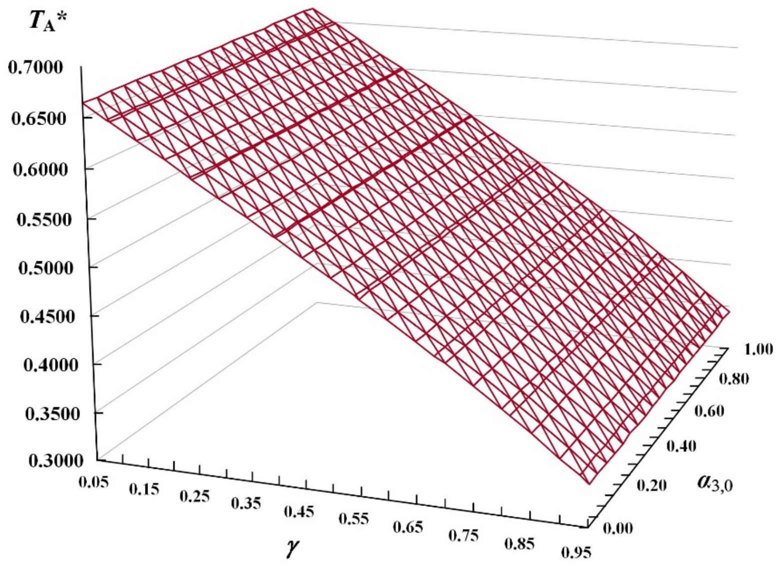

Fig. 14 *The impact of differences in $\alpha_{3,0}$ and γ on T_A**

5. CONCLUSIONS

To meet client requirements of product quality, variety, and fast response, modern manufacturers must seek the most economical and quickest fabrication scheme and strategy to produce various goods, especially when commonality exists within these multiple end products. To help manufacturing firms gain competitive advantages in marketplaces, we developed a decision support model for solving a multiproduct EPQ problem featuring scrap, delayed differentiation, and expedited rate on the fabrication of the common part. Through mathematical modelling, formulation, and optimization technique (see Figures 1 to 3 and Sections 2 to 3), we derived the optimal cost-minimization operating cycle length. A numerical example (refer to Section 4) is provided to show our model's capability and to explore collective and individual impacts of scrap, expedited-rate and postponement strategies on various performances of the studied problem (such as uptime of common part, utilization, rotation cycle time, total system cost, and individual cost contributor, etc.) Our model offers an optimization solution and in-depth managerial insights (see Figures 4 to 14) for fabrication and operations planning in a wide variety of present-day industries, such as automotive, household goods, clothing, etc. The future study topics worth investigating include (1) an explicit sensitivity study when the scrap rate is vast and (2) incorporating a discontinuous end-product delivery plan in the same context of our model.

6. APPENDIX

6.1 APPENDIX - A

Details of the derivations of Eq. (21).

The following steps are used to derive $E[TCU(T_A)]$: (i) Applying the expected values $E[x_0]$ and $E[x_i]$ to cope with the scrap rates' randomness. (ii) Substituting Q_i with T_A (for $i = 0, 1, 2, \ldots, L$; refer to Eqs (6) and (13)). (iii) Substituting Eqs. (1) to (19) in Eq. (20) and computing $E[TC(T_A)]$ / $E[T_A]$. $E[TCU(T_A)]$, as exhibited in Eq. (A-1), can then be obtained after additional derivation efforts.

$$E\left[TCU(T_A)\right]=\left\{\begin{aligned}&\left[(1+\alpha_{3,0})C_0\right]\left(\frac{\lambda_0}{1-E[x_0]}\right)+\frac{\left[(1+\alpha_{2,0})K_0\right]}{T_A}+C_{S,0}\left[E[x_0]\left(\frac{\lambda_0}{1-E[x_0]}\right)\right]+h_{4,0}E[x_0]\left(\frac{\lambda_0 T_A}{1-E[x_0]}\right)\\&+\left(\frac{h_{1,0}}{2}\right)\left(\frac{\lambda_0^2 T_A}{1-E[x_0]^2}\right)\left[\frac{1}{(1+\alpha_{1,0})P_{1,0}}\right]+h_{1,0}\sum_{i=1}^{L}\left[\frac{\lambda_i^2 T_A}{2P_{1,i}}\left(\frac{1}{1-E[x_i]}\right)^2\right]\\&+h_{1,0}\left(\sum_{i=1}^{L}\frac{\lambda_i}{(1-E[x_i])}\right)\cdot\sum_{i=1}^{L}\left[\frac{\lambda_i T_A}{P_{1,i}}\left(\frac{1}{1-E[x_i]}\right)\right]+h_{1,0}\sum_{i=1}^{L}\left[-\frac{\lambda_i T_A}{P_{1,i}}\left(\frac{1}{1-E[x_i]}\right)\sum_{j=1}^{i}\frac{\lambda_j}{(1-E[x_j])}\right]\end{aligned}\right\}$$
$$+\sum_{i=1}^{L}\left\{\begin{aligned}&C_i\lambda_i\frac{1}{(1-E[x_i])}+\frac{K_i}{T_A}+C_{S,i}\lambda_i\left(\frac{E[x_i]}{1-E[x_i]}\right)+h_{4,i}\lambda_i T_A\left(\frac{E[x_i]}{1-E[x_i]}\right)\\&+\left(\frac{h_{1,i}}{2}\right)\frac{\lambda_i^2 T_A}{(1-E[x_i])^2}\left(\frac{1-E[x_i](2-E[x_i])}{\lambda_i}+\frac{-1+2E[x_i]}{P_{1,i}}\right)\end{aligned}\right\}$$

(A-1)

Let $E_{00}, E_{10}, E_{0j}, E_{0P}, E_{iP}, E_{0i},$ *and* E_{1i} denote the following:

$$E_{00}=\frac{1}{\left(1-E\left[x_0\right]\right)};\ E_{10}=\frac{E\left[x_0\right]}{\left(1-E\left[x_0\right]\right)};\ E_{0j}=\frac{1}{\left(1-E\left[x_j\right]\right)}\ \ for\ \ j=1,2,\ldots,i;$$
$$E_{0P}=\left[\frac{1}{\left(1+\alpha_{1,0}\right)P_{1,0}}\right];\ E_{iP}=\left[\frac{1-E\left[x_i\right]\left(2-E\left[x_i\right]\right)}{\lambda_i}+\frac{-1+2E\left[x_i\right]}{P_{1,i}}\right]; \tag{A-2}$$
$$E_{0i}=\frac{1}{\left(1-E\left[x_i\right]\right)}\ \ for\ \ i=1,2,\ldots,L;\ \ E_{1i}=\frac{E\left[x_i\right]}{\left(1-E\left[x_i\right]\right)}\ \ for\ \ i=1,2,\ldots,L.$$

Substitute Eq. (A-2) in Eq. (A-1), $E[TCU(T_A)]$ is gained as follows:

$$E\left[TCU\left(T_A\right)\right]=\left\{\begin{array}{l}\left[\left(1+\alpha_{3,0}\right)C_0\right]\lambda_0E_{00}+\dfrac{\left[\left(1+\alpha_{2,0}\right)K_0\right]}{T_A}+C_{S,0}\lambda_0E_{10}+\left(\dfrac{h_{1,0}}{2}\right)\left(T_A\lambda_0^{\,2}\right)E_{0P}E_{00}^{\,2}\\ +h_{4,0}\lambda_0T_AE_{10}+h_{1,0}\displaystyle\sum_{i=1}^{L}\left[\frac{\lambda_i^{\,2}T_A}{2P_{1,i}}E_{0i}^{\,2}-\frac{\lambda_iT_A}{P_{1,i}}E_{0i}\sum_{j=1}^{i}\lambda_jE_{0j}\right]+h_{1,0}\left[\left(\sum_{i=1}^{L}\lambda_iE_{0i}\right)\sum_{i=1}^{L}\left(\frac{\lambda_iT_A}{P_{1,i}}E_{0i}\right)\right]\end{array}\right\}$$
$$+\sum_{i=1}^{L}\left\{C_i\lambda_i\left(E_{0i}\right)+\frac{K_i}{T_A}+C_{S,i}\lambda_i\left(E_{1i}\right)+h_{4,i}\lambda_iT_A\left(E_{1i}\right)+\frac{h_{1,i}\lambda_i^{\,2}T_A}{2}\left(E_{0i}\right)^2E_{iP}\right\} \tag{21}$$

Then, applying the 1^{st} and 2^{nd} derivatives of $E[TCU(T_A)]$, we obtain the following results:

$$\frac{dE\left[TCU\left(T_A\right)\right]}{dT_A}=\left\{\begin{array}{l}-\dfrac{\left[\left(1+\alpha_{2,0}\right)K_0\right]}{T_A^{\,2}}+\left(\dfrac{h_{1,0}}{2}\right)\left(\lambda_0\right)^2E_{0P}E_{00}^{\,2}+h_{4,0}\lambda_0E_{10}\\ +h_{1,0}\displaystyle\sum_{i=1}^{L}\left[\frac{\lambda_i^{\,2}}{2P_{1,i}}E_{0i}^{\,2}-\frac{\lambda_i}{P_{1,i}}E_{0i}\sum_{j=1}^{i}\lambda_jE_{0j}\right]+h_{1,0}\left[\left(\sum_{i=1}^{L}\lambda_iE_{0i}\right)\sum_{i=1}^{L}\left(\frac{\lambda_i}{P_{1,i}}E_{0i}\right)\right]\end{array}\right\}$$
$$+\sum_{i=1}^{L}\left\{-\frac{K_i}{T_A^{\,2}}+h_{4,i}\lambda_i\left(E_{1i}\right)+\frac{h_{1,i}\lambda_i^{\,2}}{2}\left(E_{0i}\right)^2E_{iP}\right\} \tag{A-3}$$

$$\frac{d^2E\left[TCU\left(T_A\right)\right]}{dT_A^{\,2}}=\frac{2\left[\left(1+\alpha_{2,0}\right)K_0\right]}{T_A^{\,3}}+\sum_{i=1}^{L}\left(\frac{2K_i}{T_A^{\,3}}\right)>0 \tag{A-4}$$

From Eq. (A-4), since $\alpha_{2,0}$, K_0, K_i, and T_A are all positive, the convexity of $E[TCU(T_A)]$ is confirmed. Then, setting the 1st derivative of $E[TCU(T_A)]$ equal to zero, we can solve T_A* as follows:

$$T_A^{\,*}=\sqrt{\frac{\left[\left(1+\alpha_{2,0}\right)K_0\right]+\displaystyle\sum_{i=1}^{L}K_i}{\begin{array}{l}\left(\dfrac{h_{1,0}}{2}\right)\left(\lambda_0\right)^2E_{0P}E_{00}^{\,2}+h_{4,0}\lambda_0E_{10}+\displaystyle\sum_{i=1}^{L}\left\{\frac{h_{1,i}\lambda_i^{\,2}}{2}\left(E_{0i}\right)^2E_{iP}+h_{4,i}\lambda_i\left(E_{1i}\right)\right\}\\ +h_{1,0}\displaystyle\sum_{i=1}^{L}\left[\frac{\lambda_i^{\,2}}{2P_{1,i}}E_{0i}^{\,2}-\frac{\lambda_i}{P_{1,i}}E_{0i}\sum_{j=1}^{i}\lambda_jE_{0j}\right]+h_{1,0}\left[\left(\sum_{i=1}^{L}\lambda_iE_{0i}\right)\cdot\sum_{i=1}^{L}\left(\frac{\lambda_i}{P_{1,i}}E_{0i}\right)\right]\end{array}}} \tag{22}$$

6.2 APPENDIX - B

Table B-1 *Variables' values of the same problem using a single-stage fabrication scheme*

Product i	λ_i	C_i	$P_{1,i}$	x_i	$h_{1,i}$	$C_{S,i}$	K_i	τ_i	$h_{4,i}$
1	3000	\$80	58000	5%	\$16	\$20	\$17000	0.18	\$3
2	3200	\$90	59000	10%	\$18	\$25	\$17500	0.27	\$5
3	3400	\$100	60000	15%	\$20	\$30	\$18000	0.35	\$7
4	3600	\$110	61000	20%	\$22	\$35	\$18500	0.45	\$10
5	3800	\$120	62000	25%	\$24	\$40	\$19000	0.54	\$13

6.3 APPENDIX - C

Table C-1 *The influence of changes in $\alpha_{1,0}$ on various system performances*

$\alpha_{1,0}$	$P_{T1,0}/P_{1,0}$	T_A*	$E[TCU(T_A*)]$ (A)	(A) % increase	t_0* (B)	(B)% decline	t_0*+t_i* (C)	(C)% decline	Utilization (D)	(D) % decline	Expedited cost (E)	(E)/(A)%
0.0	1.00	0.5066	\$2,115,234	–	0.0779	–	0.1544	–	0.3048	–	\$0	0.00%
0.1	1.10	0.5086	\$2,151,943	1.74%	0.0711	-8.74%	0.1479	-4.21%	0.2909	-4.58%	\$37,231	1.73%
0.2	1.20	0.5104	\$2,188,736	3.47%	0.0654	-16.05%	0.1425	-7.73%	0.2792	-8.41%	\$74,461	3.40%
0.3	1.30	0.5120	\$2,225,594	5.22%	0.0606	-22.26%	0.1379	-10.70%	0.2694	-11.64%	\$111,688	5.02%
0.4	1.40	0.5135	\$2,262,503	6.96%	0.0564	-27.60%	0.1340	-13.24%	0.2609	-14.41%	\$148,913	6.58%
0.5	**1.50**	**0.5149**	**\$2,299,453**	**8.71%**	**0.0528**	**-32.24%**	0.1306	-15.45%	**0.2536**	**-16.81%**	**\$186,137**	**8.09%**
0.6	1.60	0.5163	\$2,336,436	10.46%	0.0496	-36.31%	0.1276	-17.37%	0.2472	-18.91%	\$223,359	9.56%
0.7	1.70	0.5176	\$2,373,446	12.21%	0.0468	-39.91%	0.1250	-19.06%	0.2415	-20.77%	\$260,580	10.98%
0.8	1.80	0.5188	\$2,410,479	13.96%	0.0443	-43.11%	0.1227	-20.55%	0.2365	-22.41%	\$297,799	12.35%
0.9	1.90	0.5200	\$2,447,532	15.71%	0.0421	-45.98%	0.1206	-21.89%	0.2320	-23.89%	\$335,018	13.69%
1.0	2.00	0.5211	\$2,484,600	17.46%	0.0401	-48.57%	0.1188	-23.08%	0.2280	-25.22%	\$372,235	14.98%
1.1	2.10	0.5222	\$2,521,682	19.22%	0.0382	-50.92%	0.1171	-24.16%	0.2243	-26.42%	\$409,450	16.24%
1.2	2.20	0.5233	\$2,558,776	20.97%	0.0366	-53.05%	0.1156	-25.13%	0.2210	-27.51%	\$446,665	17.46%
1.3	2.30	0.5243	\$2,595,881	22.72%	0.0350	-55.01%	0.1143	-26.01%	0.2179	-28.50%	\$483,879	18.64%
1.4	2.40	0.5253	\$2,632,994	24.48%	0.0337	-56.80%	0.1130	-26.82%	0.2152	-29.42%	\$521,092	19.79%
1.5	2.50	0.5263	\$2,670,115	26.23%	0.0324	-58.45%	0.1119	-27.55%	0.2126	-30.26%	\$558,303	20.91%
1.6	2.60	0.5273	\$2,707,243	27.99%	0.0312	-59.97%	0.1109	-28.22%	0.2102	-31.03%	\$595,514	22.00%
1.7	2.70	0.5283	\$2,744,377	29.74%	0.0301	-61.38%	0.1099	-28.84%	0.2080	-31.75%	\$632,723	23.06%
1.8	2.80	0.5292	\$2,781,517	31.50%	0.0291	-62.69%	0.1090	-29.41%	0.2060	-32.42%	\$669,932	24.09%
1.9	2.90	0.5301	\$2,818,661	33.26%	0.0281	-63.92%	0.1082	-29.93%	0.2041	-33.04%	\$707,140	25.09%
2.0	3.00	0.5311	\$2,855,809	35.01%	0.0272	-65.06%	0.1075	-30.42%	0.2024	-33.62%	\$744,346	26.06%

7. ACKNOWLEDGMENTS

The Authors truthfully thank the Ministry of Science and Technology of Taiwan for supporting this research (Grant No. MOST 107-2410-H-324-002).

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