Conversion between the local magnitude (M_L) and the moment magnitude (M_w) for earthquakes in the Croatian Earthquake Catalogue

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Based on 153 earthquakes (1959–2020) listed in the Croatian Earthquake Catalogue, a conversion relation was obtained between the local magnitude $M_{L,CR}$ and the corresponding moment magnitude M_w as reported by the global and regional agencies. As errors were present in both variables the York regression was used. The best fit line is given by: $M_{wL} = (-0.106 \pm 0.122) + (1.002 \pm 0.027) M_{L,CR}$ (coefficient of determination $R^2 = 0.90$). The earthquakes considered occurred in Croatia and the neighbouring regions, and their local magnitudes $M_{L,CR}$ ranged between 3.5 and 6.5. Residual analysis suggests that an artificial positive magnitude shift of up to 0.3 magnitude units may have occurred in the early 1980s, when Wiechert mechanical seismographs were replaced by the instruments with velocity proportional recordings without proper recalibration of the magnitude formula. The slope of the regression close to 1.0 indicates that on the average the faults' aspect ratio (width/length) is about 1/2.

Keywords: magnitude conversion, moment magnitude, local magnitude, Croatia

Magnitudes mentioned in text:

M_w – Moment magnitude, directly computed

M_{rep} – Original magnitude as reported in the catalogue (mostly $M_{rep} = M_L$, M_S, M_d, m_b, or M_m)

M_{wp} – $M_{wp} = f(M_{rep})$; proxy for M_w (*e.g.* $M_{wp} = M_{wL}$, M_{wS}, M_{wd}, M_{wb} or M_{wm})

M_{ww} – Moment magnitude from inversion of the W-phase

M_{wc} – Centroid moment magnitude from inversion of long-period surface waves

M_{wr} – Regional moment magnitude

M_L – Local magnitude

M_S – Surface wave magnitude

M$_d$ – Duration magnitude
m$_b$ – Body wave magnitude
M$_m$ – Macroseismic magnitude
M$_{L,CR}$ – Local magnitude reported in the Croatian Earthquake Catalogue
M$_2$ – Magnitude from other sources reported in the Croatian Earthquake Catalogue (mostly M$_L$)
M$_{cat}$ – The average of M$_{L,CR}$ and M$_2$. If one of them is missing, it equals the other one.
M$_{Lm}$ – Proxy for M$_L$ based on macroseismic data (epicentral intensity and depth of focus)

1. Introduction

The use of the most of modern ground motion prediction equations (GMPE) nowadays requires moment magnitude (M$_w$) as the input independent variable. This is why it has become a standard practice in earthquake hazard estimation studies to convert routinely reported magnitudes (M$_{rep}$) in the earthquake catalogues (mostly M$_{rep}$ = M$_L$, M$_S$, M$_d$, m$_b$, or M$_m$) to M$_w$-proxy, M$_{wp}$ (*e.g.* M$_{wp}$ = M$_{wL}$, M$_{wS}$, M$_{wd}$, M$_{wb}$ or M$_{wm}$) via empirical conversion formulas. However, there are some caveats to this approach which are often ignored:

a) saturation of M$_{rep}$ results in unrealistic conversion for large magnitudes, thus causing underestimation of M$_{wp}$;

b) in principle, magnitudes like M$_L$ are more representative of the strong-motion amplitudes than M$_w$ which is measured at very long periods and is representative of the total seismic energy released;

c) as the two magnitudes sample very different parts of the seismic spectrum, their ratio will often be indicative of the properties of a particular seismic source. Conversion regressions will average out this variability.

d) unless the functional relationship between M$_{rep}$ and M$_w$ is linear for the whole magnitude range of interest, if the Gutenberg-Richter magnitude distribution holds for M$_{rep}$ it does not hold for M$_{wp}$ (and *vice versa*);

e) lack of reported M$_w$ for small events prevents their use in definition of earthquake rates, sometimes well above the completeness threshold of the respective catalogue;

f) if only a relatively small number of reported pairs (M$_w$, M$_{rep}$) exists (*e.g.* in regions with low seismicity in the last decades), the conversion regression coefficients will have large confidence intervals, thus decreasing the reliability of results, and

g) often unreported standard errors for individual values of M$_{rep}$ and M$_w$ introduce additional uncertainty into the regression outcome.

So far, the problem of defining M_w-proxy (M_{wL}) for the local magnitudes (M_L) in the Croatian Earthquake Catalogue (CEC henceforth; Herak et al., 1996, last update 2020) was dealt with in two regional studies. Duni et al. (2010) derived M_L–M_w relations for the catalogues of six countries from SE Europe, Croatia included. The dataset for Croatia included 34 events in the period 1977–2008, resulting in a relationship $M_{wL} = 0.165 + 0.979\ M_L$ (coefficient of determination $R^2 = 0.92$, standard error of regression $s_r = 0.167$). Likewise, Markušić et al. (2016) performed a similar analysis for five catalogues of this region while compiling the BSHAP catalogue (time period 510BC–2012). Croatian data subset consisted of 31 events ($4.2 \leq M_L \leq 6.3$), and yielded $M_{wL} = (-0.11 \pm 0.38) + (1.011 \pm 0.080)\ M_L$ ($R^2 = 0.852$, $s_r = 0.229$). Both studies used error-in-variables regression model, but without specification of how the different assumed errors in the two variables were dealt with.

In this note, I'll address the last two issues mentioned above (small data sets and handling of observational errors), as related to conversion of M_L to M_{wp} for earthquakes reported in the CEC.

2. Croatian Earthquake Catalogue (CEC)

CEC was the result of the first major revision (Herak et al., 1996) of the *Catalogue of Earthquakes in Croatia and the Neighbouring Regions* compiled at the Department of Geophysics, Zagreb (late 1970s–early 1990s) that initially mostly relied on the works by Kišpatić (*e.g.* 1891, 1892, 1894, 1895, 1905) and on the 'Balkan catalogue' produced within the UNDP/UNESCO project *Survey of the seismicity of the Balkan region* (Shebalin et al., 1974), and was later frequently supplemented or partially revised. For more detail and references, please see Herak et al. (1996).

CEC may be tentatively divided into five periods: BC–1908 (pre-instrumental time), 1908–1970s (mechanical registration, Wiechert seismographs in Zagreb, ZAG), 1970s–1982 (Wiechert seismographs in Zagreb and analogue electromagnetic seismographs), 1982–2000 (analogue electromagnetic seismographs), 2001–present (digital BB seismographs), and the magnitude calculation varied accordingly (see Appendix for more information). The transitions between these periods were mostly rather gradual, not sharp. The catalogue was supposed to cover all regions whose seismicity could significantly influence seismic hazard in Croatia – besides Croatia itself, it thus covered whole territories of Slovenia, Montenegro and Bosnia and Herzegovina, and also included earthquakes from parts of Austria, Hungary, Italy, and Serbia. Nowadays, the coverage is about the same, but the catalogue is the authoritative source only for events within or close to Croatian borders. The first version of CEC reported 4853 events until 1992. The current version lists data for over 135,000 earthquakes until and including 2019 (and the aftershock series of the M_w 5.3 Zagreb earthquake of 22 March 2020). The catalogue's format remained the same until recently, when

additional parameters (station gap, elliptical confidence regions) have been introduced. When available, CEC reports three magnitudes:

– $M_{L,CR}$, local magnitude based on the seismograms from the station ZAG until the year 2000; the median of the individual M_L from stations belonging to the Croatian seismological network (code CR, DOI: 10.7914/SN/CR) thereafter;

– M_2, a magnitude from other sources (nearest neighbouring network, bulletins of international centres like NEIC, ISC or EMSC). Whenever available this is also a local magnitude (in the vast majority of cases). It was introduced in order to provide a magnitude for events for which $M_{L,CR}$ could not be determined (missing records, clipped analogue seismograms, poor recording ...). The corresponding reference is also given in the catalogue; and

– M_{cat}, the average of $M_{L,CR}$ and M_2. If one of them is missing, it equals the other one. This magnitude was considered the reference CEC magnitude in a number of studies.

More on the evolution of magnitudes in CEC is presented in the Appendix.

3. Data

The M_w magnitudes corresponding to $M_{L,CR}$ magnitudes reported in the CEC were taken from (in the order of the amount of data used):

– United States Geological Survey (USGS, ANSS Comprehensive Earthquake Catalog, https://earthquake.usgs.gov/data/comcat/),

– Global Centroid-Moment-Tensor Project (GCMT, Dziewonski et al., 1981; Ekström et al., 2012; https://www.globalcmt.org/),

– Istituto Nationale di Geofisica e Volcanologia (European-Mediterranean RCMT Catalog, http://rcmt2.bo.ingv.it/, Pondrelli, 2002).

– International Seismological Centre (ISC-GEM catalogue, http://www.isc.ac.uk/iscgem/),

– Saint Louis University Moment Tensor Determinations (http://www.eas.slu.edu/eqc/eqcmt.html),

Only directly computed moment magnitudes were considered (*e.g.* M_w-proxies from the ISC-GEM catalogue were disregarded). If more than one magnitude M_w was found the order of preference was $M_{ww} \rightarrow M_{wc} \rightarrow M_{wr}$.

The area covered by the search is shown in Fig. 1 (to the NE of the blue line), and includes roughly the region comprised within CEC. A total of 153 events with reported M_w and $M_{L,CR}$ were found in the time period 1959–2020. Fig. 1 also shows individual differences ($M_w - M_{L,CR}$). The average difference is –0.080 magnitude units, with no apparent regional dependence. The distribution of events by time and magnitude is shown in Fig. 2.
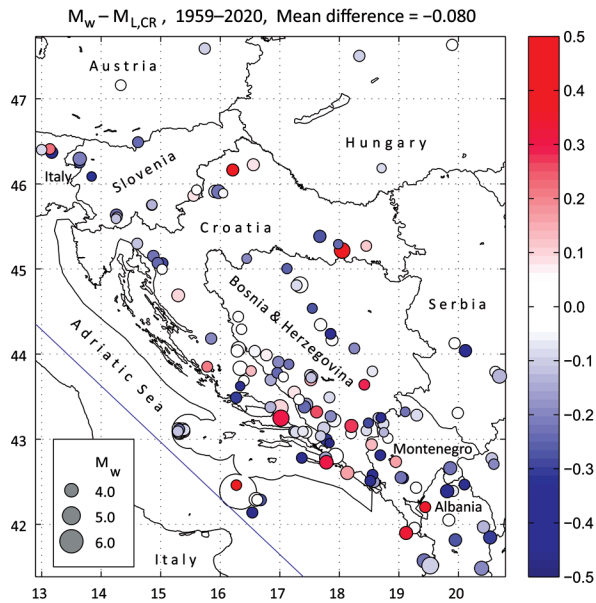
Figure 1. Epicentres of 153 earthquakes considered. Symbol size scales with M_w, and the colour indicates the difference between M_w and $M_{L,CR}$. The area shown roughly corresponds to the region covered by CEC. Events to the SW of the blue line were not considered.
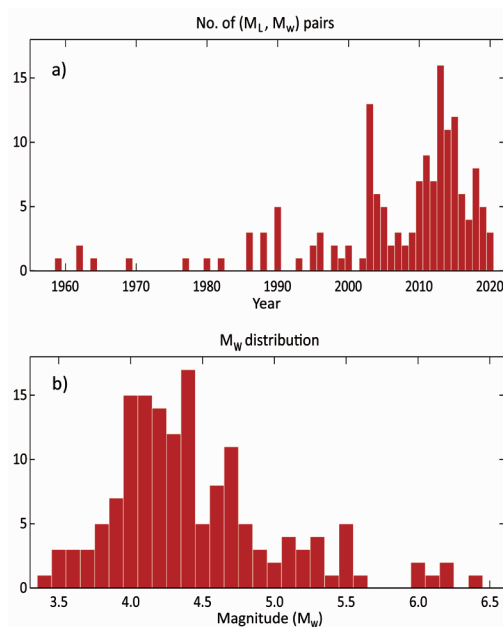
Figure 2. Distribution of considered earthquakes by time (*a*) and magnitude (*b*).

As both variables (M_w and $M_{L,CR}$) have errors, ordinary least squares regression cannot be applied (see *e.g.* Castellaro and Borman, 2007, or Castellaro et al., 2006). Instead, I use here the York regression (York et al., 2004), a general orthogonal regression (GOR) formulation that allows specification of individual standard errors in both variables, as well as the correlation coefficients of errors (Matlab program by T. Wiens, 2010). Lacking individual errors for most of the measurements, I have estimated the average M_w yearly standard errors using the whole ISC-GEM catalogue, which reports the standard error of all magnitudes, and used this as proxy for observed M_w in the dataset. For $M_{L,CR}$ an educated guess of standard error prior to the digital era was used depending on the number of stations in the Croatian network, the quality of their calibration and types of instruments. For the period after the year 2000, the standard error of each $M_{L,CR}$ was computed using all reported individual magnitudes from the CR-network, and averaged for each year. The graphs showing the proposed temporal evolution of standard errors of M_w and $M_{L,CR}$ is shown in Fig. 3.

The breaking points in the curve $\sigma(M_{L,CR})$ (red in Fig. 3) correspond to the years 1973 (beginning of introduction of electromagnetic seismographs), 1982
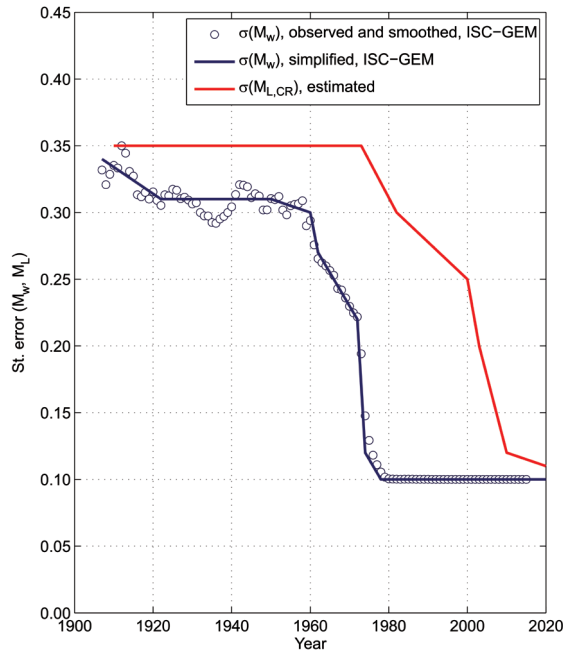

Figure 3. Standard errors (σ) of the two magnitudes considered. Blue circles are yearly averages of all M_w reported in the ISC-GEM catalogue. The blue line is the smoothed (simplified) course of $\sigma(M_w)$ used here as the $\sigma(M_w)$ proxy for the data set used. The red line is the same for the estimated $\sigma(M_{L,CR})$ (see text).

(end of mechanical recordings, more electromagnetic instruments), 2000 (intro-duction of digital instruments), and 2010 (rapid development of the network) (see section 2). The errors in M_w and $M_{L,CR}$ were assumed not to be correlated.

4. Results

The data and the results of application of the York regression to the pairs $(M_{L,CR}, M_w)$ as described above are shown in Fig. 4.

The conversion equation is:

$$M_{wL} = (-0.106 \pm 0.122) + (1.002 \pm 0.027)\, M_{L,CR} \tag{1}$$
$$R^2 = 0.897, \quad s_r = 0.187, \quad N_{dat} = 153.$$

The regression explains 89.7% of observed variance, with the regression standard error of $sr = 0.187$ magnitude units. This expression is very similar to the one obtained by Markušić et al. (2016), but it is applicable to a wider mag-nitude range, and has considerably smaller error in the regression coefficients due to almost five times more data. The plot of residuals $\Delta M_w = (M_w - M_{wL})$ (Fig. 5), however, reveals that all residuals prior to mid-1980s are positive (red in Fig. 5). The mean ΔM_w for the 8 earthquakes before 1982 is $\langle \Delta M_w \rangle = 0.27$,
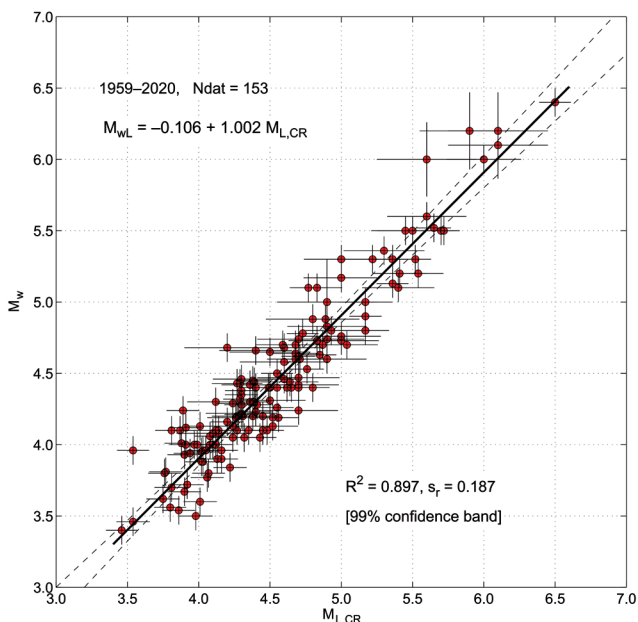

Figure 4. Observed magnitudes $M_{L,CR}$ and M_w for the 153 earthquakes in the CEC. The corres-ponding estimated individual errors are given by the horizontal and vertical error bars, respectively. The full line is the regression (1), and the dashed lines bound the 99% confidence region for this line.
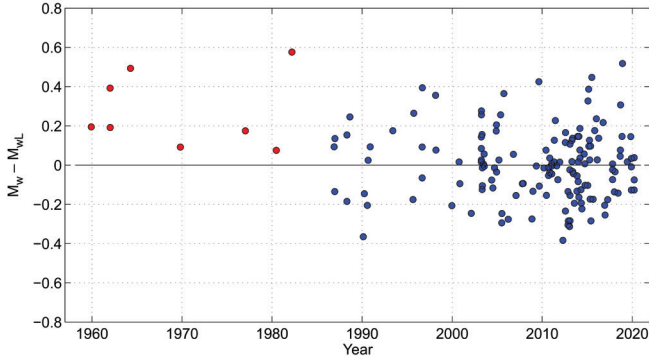
Figure 5. The residuals $\Delta M_w = (M_w - M_{wL})$ corresponding to the regression line (1). The ones belonging to the period prior to 1982 are shown in red.

whereas after 1982 it is $\langle \Delta M_w \rangle = 0.00$. The early 1980s is the time when the major change occurred in the CEC, as displacement Wiechert seismograms were replaced by the velocity-proportional recordings (see the section 2 above and the Appendix). Assuming no temporal inhomogeneity in the observed M_w, it thus appears that $M_{L,CR}$ before 1982 is underestimated with respect to the magnitudes of later earthquakes by about 0.2–0.3 magnitude units.

I therefore repeated the regression separately for data before and after 1982. Fig. 6 presents the regression results, and the proxy-M_w (M_{wL}) is given by:

$$M_{wL} = (+0.224 \pm 1.019) + (0.992 \pm 0.198)\, M_{L,CR}, \quad \text{for years 1959–1982}, \quad (2)$$
$$R^2 = 0.934, \quad s_r = 0.187, \quad N_{dat} = 8,$$

$$M_{wL} = (-0.062 \pm 0.123) + (0.992 \pm 0.028)\, M_{L,CR}, \quad \text{for years 1983–2020}. \quad (3)$$
$$R^2 = 0.888, \quad s_r = 0.176, \quad N_{dat} = 145.$$

The two regression lines are parallel to each other, with the one corresponding to the period 1959–1982 being shifted towards larger M_w by about 0.29 magnitude units. If M_w is taken as reference, the magnitudes $M_{L,CR}$ in the range $3.5 < M_{L,CR} < 6.5$ are underestimated by 0.17–0.20 magnitude units before 1982, and overestimated by 0.09–0.11 thereafter. For discussion on possible cause of this inhomogeneity in the CEC, please see the Appendix.

For the sake of completeness of analyses, let us consider also the magnitude M_{cat}. As noted above, this magnitude in CEC is the average of $M_{L,CR}$ and M_2. If one of them is missing, it equals the other one (see also the Appendix). As this magnitude was considered the reference CEC magnitude in a number of studies (*e.g.* probably also in Duni et al., 2010), it is worthwhile to check its relationship with M_w. Fig. 7 presents the data and regression:

$$M_{wcat} = (0.008 \pm 0.118) + (0.983 \pm 0.027)\, M_{cat} \quad (4)$$
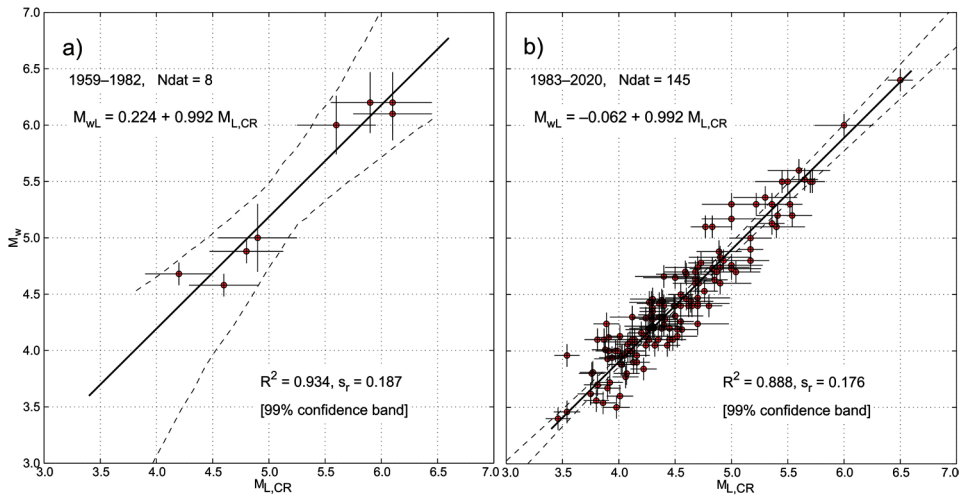$$R^2 = 0.894, \quad s_r = 0.190, \quad N_{dat} = 153.$$

Figure 6. Observed magnitudes $M_{L,CR}$ and M_w for the period before 1982 (*a*), and after 1982 (*b*). Symbols are the same as in Fig. 4.
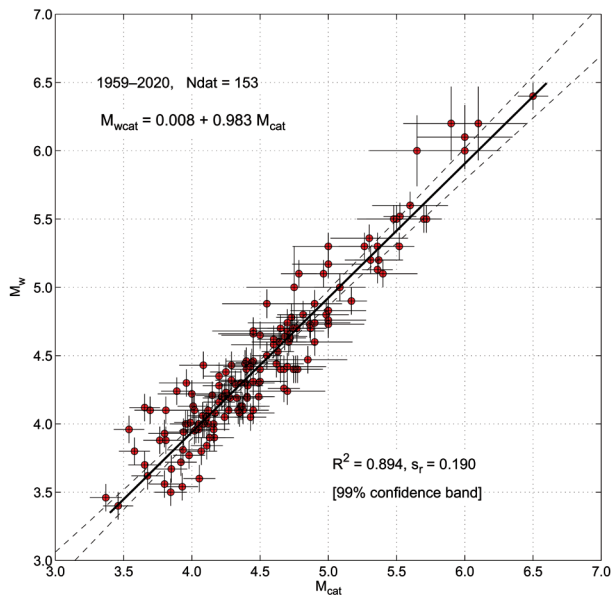

Figure 7. Observed magnitudes M_{cat} and M_w. Symbols are the same as in Fig. 4.

5. Discussion and conclusions

It has been shown that the local magnitudes $M_{L,CR}$ as reported in the Croatian Earthquake Catalogue closely correspond to M_w in the magnitude range 3.5–6.5.

On the average, $M_{L,CR}$ is about 0.1 magnitude units larger than M_w. It was also shown that the CEC-reported $M_{L,CR}$ before and after the early 1980s differ by about 0.3 magnitude units on the average, which might be explained by the (un)fortunate choice not to change the magnitude calibrating function after the major change in the equipment took place at that time. This issue should be looked into, and if verified, steps should be taken to homogenize the magnitudes in the catalogue.

As noted in the Introduction [point c)], there is no reason to assume that the scatter of points about the regression lines in Figs. 4, 6 and 7 is entirely due to measurement errors, as in reality the relationship between M_L and M_w depends on physical characteristics of the source. For instance, Mereu (2020) has theoretically shown that relationship $M_w = p\,M_L$ holds, where the slope $p = 2\,(1 + \beta)\,/\,3$, and β is the fault aspect ratio (width/length). As β may vary between 0 and 1, the slope p varies between 2/3 and 4/3, and variation of β for different events may alone account for much of the observed variance. The ratios $p = M_w/(M_{L,CR} - 0.106)$ ($M_{L,CR}$ is reduced by the intercept of the regression (1)) for individual earthquakes in our dataset range between 0.90 and 1.15, which implies maximum variation of the fault aspect ratio in the studied region to be $0.35 < \beta < 0.73$. β has a normal distribution with the mean of 0.51 and the standard deviation of 0.07.

The fact that on the average there is a linear (almost 1 : 1) relationship between $M_{L,CR}$ and M_w is quite fortunate, as it makes statistical analyses of the catalogue easier and more robust. For instance, Gutenberg-Richter distribution parameters can be fit using all reported $M_{L,CR}$ (even below the lower regression limit of $M_{L,CR} = 3.5$, but above the respective completeness threshold). The resulting $M_{L,CR}$ recurrence relations can then safely be converted to M_{wL} in the range of magnitudes of engineering interest, *i.e.* above $M_{wL} \approx 3.5$, and used in strong-motion simulations where a conversion to M_{wL} is required.

In view of the finding that $M_{L,CR}$ may systematically differ before and after the early 1980s, it is recommended to review the consistency of the magnitudes in CEC, and revise them as necessary.

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SAŽETAK

Pretvorba između lokalne magnitude (M$_L$) i momentne magnitude (M$_w$) za potrese u Hrvatskom katalogu potresa

Marijan Herak

Na temelju podataka o lokalnoj magnitudi (M$_{L,CR}$) iz Hrvatskog kataloga potresa, izvedena je konverzijska relacija između M$_{L,CR}$ i momentne magnitude Mw koju su za odabrane potrese javile svjetske i regionalne agencije. Odabrana je tzv. Yorkova regresija koja je prikladna u slučaju da su mjerne pogreške prisutne kod obje varijable. Najbolju prilagodbu postiže se relacijom M$_{wL}$ = (–0,106 ± 0,122) + (1,002 ± 0,027) M$_{L,CR}$ (uz koeficijent determinacije R^2 = 0,90). Odabrani potresi dogodili su se u Hrvatskoj i susjednim područjima, a imali su lokalne magnitude između 3,5 i 6,5. Analiza odstupanja ukazuje na mogućnost da se u katalogu početkom 1980-ih dogodio umjetni pozitivni skok u magnitudi M$_{L,CR}$ od ne više od 0,3 jedinice magnitude. Do njega je vjerojatno došlo pri zamjeni Wiechertovih seizmografa elektromagnetskima bez adekvatne korekcije magnitudne formule. Nagib pravca regresije vrlo blizak jedinici ukazuje da je prosječni omjer kraće i dulje stranice uzročnih rasjeda oko 1/2.

Ključne riječi: pretvorba magnituda, momentna magnituda, lokalna magnituda, Hrvatska

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Appendix – Magnitudes in the Croatian Earthquake Catalogue (CEC)

$M_{L,CR}$ for the pre-instrumental part of the catalogue (prior to 1908 when Mohorovičić installed the first Wiechert seismograph in Zagreb), as well as for the felt events for which magnitude could not be computed (clipped recordings, dislodged styluses, poor quality of seismograms...), is a macroseismic M_L-proxy (M_{Lm}) computed from the epicentral intensity using relations like the ones published by D. Herak et al. (1988), M. Herak (1989) or D. Herak (1995). In the first period of the instrumental era (1908 until mid 1980s), this magnitude is computed from the records of the three Wiechert seismographs operating at the station ZAG – two horizontals (80 kg and 1000 kg), and a vertical (1200 kg, installed in 1932). To the best of my recollection, the formula that was used to compute M_L was the one proposed by the Croatian team members (D. Cvijanović, B. Makjanić, D. Skoko) of the UNDP/UNESCO project *Survey of the Seismicity of the Balkan Region* (Shebalin et al.,1974):

$$M_{L,CR} = \log A_{max} + 2.094 \log \Delta° + 2.19, \qquad (A1)$$

(A – maximum ground displacement amplitude in μm, as recorded by the Wiechert instruments at ZAG; $\Delta°$ – epicentral distance in degrees). I know of no paper trace to the data or methods used to derive (A1). In the catalogue revision by Herak et al. (1996), the magnitudes were recomputed using the calibration function as proposed by the authors:

$$M_{L,CR} = \log A_{max} + 1.449 \log \Delta° + 2.554. \qquad (A2)$$

Expression (A2) was derived so that $M_{L,CR}$ on average agrees with the magnitudes M_2. The Wiechert seismographs operated until 1984 at their original place at Grič 3 in Zagreb, but in 1982, after the Geophysical Department moved to the new address (Horvatovac 95) the Sprengnether electromagnetic LP seismographs with ink recording became the official instruments.

By inertia of old habit, the equation (A1) continued to be used, only replacing A_{max} by the maximum of the velocity-proportional record, V_{max}, after it was realized that the resulting magnitudes remained comparable to those published elsewhere. This can make sense only when $A_{max} = V_{max}$, which is true only for periods of 2π s ≈ 6.3 s. The period of free oscillations of the Wiechert 1000-kg and 80-kg horizontals varied between about 4 and 10 s (Allegretti et al., 2000), so the flat part of the response curve mostly corresponded to periods lower than about 7–8 s (Fig. A1). For events in the magnitude range considered here, the periods corresponding to A_{max} as read from the seismograms were much lower – usually between 0.5 s for the lowest magnitudes and 3–5 s for the largest events. It is reasonable to expect that the bulk of data used to derive expression (A1) was related to events of magnitudes below 4.5, and that the corresponding predomi-
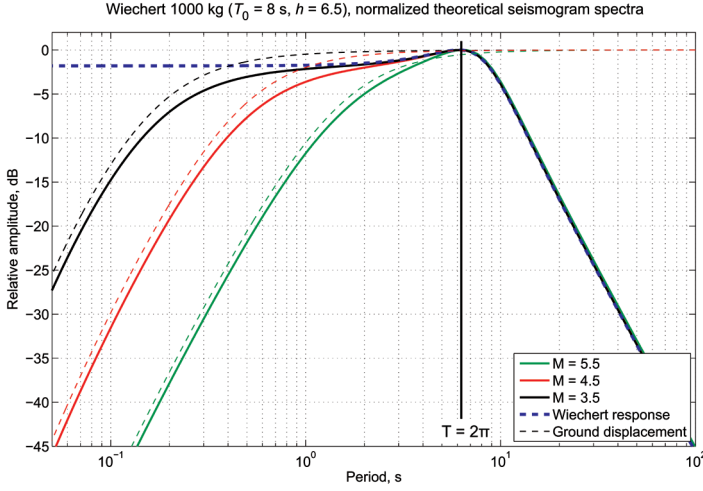
Figure A1. Theoretical Wiechert-seismogram spectra for earthquakes of magnitudes M = 3.5 (black), M = 4.5 (red) and M = 5.5 (green) computed assuming ω^2-model with the corner frequency magnitude dependence for S-waves after Havskov and Ottemöler (2010) with the stress-drop $\Delta\sigma$ = 100 bar, epicentral distance Δ = 300 km, frequency-dependent quality factor $Q(f)$ = 100 $f^{0.8}$, near-surface attenuation κ = 0.01, and the S-wave velocity of V_s = 3.5 km/s. All curves are normalized to their maximum. The 1000 kg horizontal Wiechert response (free period T_0 = 8 s, damping constant h = 6.5) is shown by the blue short-dashed line. The ground-motion spectral amplitudes are shown for the three magnitudes with long-dashed thin lines. The change of distance, $Q(f)$, κ, or V_s doesn't substantially change the figure.

nant periods range was about 0.5–3.0 s. This would imply that the amplitude read from the seismogram was up to a factor of 1.8 lower (–5 dB) than the amplitude at $T = 2\pi$ s, which translates to magnitude underestimation of up to 0.25 magnitude units with respect to the magnitude that would have been obtained at $T = 2\pi$ s. For the large part of the dataset this error should have been considerably smaller, so we may roughly put $A_{max} \approx A(T = 2\pi) \approx V_{max}$, or more precisely A_{max} is approximately equal or slightly less than V_{max}.

It is therefore plausible that expression (A1) (and consequently also A2) was on the average derived for amplitudes numerically similar (or a little smaller) to those of periods close to 6.3 s, which would explain that (A1) was apparently applicable to the Wiechert displacement seismograms as well as to the Sprengnether velocity-proportional records. The discussion above can also explain the observation (see Results) that M$_{L,CR}$ as reported in CEC after 1982 may be systematically larger than it was for the years prior to the early 1980s by up to about 0.25 magnitude units.

As the number of seismographs started to increase in the last two decades of the 20th century many small earthquakes recorded by other stations were not recorded at ZAG. Thus, the need occasionally arose to calibrate local magnitudes also for other instruments and other stations, most notably the HVAR station

(*e. g.* D. Herak et al., 1988) that was installed in the early 1970s. This was done with respect to the ZAG records of larger events, so the homogeneity of the catalogue was, hopefully, not compromised.

With the introduction of digital instruments at the turn of the centuries, the $M_{L,CR}$ in CEC was decoupled from the ZAG station, and is since 2001 defined as the median of all M_L magnitudes reported by the stations of the CR-network.

Expression (A1) continues to be used today. It is applied to both horizontal and the vertical components using the velocity amplitude defined as $V_{max} = (V_{max,H} + V_{max,Z})/2$ with $V_{max,H} = (V_{max,N} + V_{max,E})/2$. Here $V_{max,Z}$, $V_{max,N}$, and $V_{max,E}$ are the maximum amplitudes on the vertical, NS, and EW components, respectively. Magnitudes are computed on the high-pass filtered records ($f > 0.3$ Hz), only if the signal/noise ratio for P-waves exceeds 3.