Connecting Mathematics and Music in Preschool Education

UDK: 373.2 : [51 + 78]
Review paper (Pregledni rad)

* **Vesna Svalina, Ph.D., Assistant Professor,**
Josipa Vukelić, univ. bacc. paed.,
Osijek, Croatia
vesna.svalina@gmail.com
josipa.vukelic@hotmail.com

Abstract

In this paper, we wanted to explore the essential components of mathematics and music and determine the possibility of integrating them into preschool education. Basic mathematical concepts, on which the development of each child's intellectual ability depends, are formed in the preschool age. By combining components of elements from mathematics and music, we can see their connection in terms of symmetry, values and measurements, and pattern recognition.

Through various musical activities, children can acquire certain skills that precede the learning of mathematical operations. Thus, we can practice our comparison making mathematical skill with children by comparing the long and short tones, the treble and the deep tones, the loud and quiet sounds, and the mathematical skill of counting by performing suitable music games, rhymes, and songs in which numbers are mentioned.

Counting in rhymes and songs helps a child in learning both the notion of numbers and mathematical operations such as addition and subtraction. Games that combine music and mathematics usually use music as a driving force for a productive and dynamic educational environment. Rhythm and melody help in the process of mathematical thinking as children receive infor-

* Corresponding Author: Vesna Svalina, vesna.svalina@gmail.com, Personal website: www.vesna-svalina.net

mation directly and as a whole. That is why it is important to connect music and mathematics in preschool education as often as possible.
Keywords: early childhood education, games, mathematics, music, preschool children, songs

Introduction

The connection between music and mathematics has been debated since ancient times. In ancient Greece (around 500 BC), Pythagoras and his disciples were at the forefront of this and they believed that a number is at the basis of everything and that everything can be understood by numbers and their proportions. Pythagoras introduced the notion of a musical interval in the music theory, which represents simple mathematical proportions that are obtained by vibrating strings whose lengths are positioned in certain mathematical proportions (Plavša, 1981).

While music was exclusively considered to be a form of science in ancient Greece, in the Middle Ages it was regarded as both a form of science and an art form. The study of music is associated with the study of arithmetic, geometry and astronomy. In medieval times, the prevailing view was that if geometry gave birth to astronomy, then arithmetic is the mother of music (Chailley, 2006).

In the 16th century, music theorists continued to argue for Pythagoras' sonic numbers and the claim that perfect chords were actually intervals on the musical scale expressed by the ratios of numbers 1, 2, 3, and 4. However, there is still a long way to go. Italian music theorist Gioseffo Zarlino determined the relationship between the individual stages of the scale on the basis of acoustic measurements and calculations, proportions of small and large thirds, pure quarters and quintiles in different parts of an octave, he distinguished two basic types of chords – a major and minor third (partition into the major and minor scale) and proposed that the octave be divided into twelve equal parts (Partch, 1974).

Nowadays music is no longer viewed as a branch of science but as a branch of art, but the debate about the connection between music and mathematics continues. Music theorists and scientists have found that there is a correlation in terms of rhythm and measure, in terms of scale formation and musical instrument tuning. Tone colour is associated with the distribution of aliquot tone strengths (Andreis, 1967), and the dissonance is a consequence of the shocks generated by close aliquot frequencies (Plomp, and Levelt, 1965).

Throughout history, composers were fascinated by mathematics and numbers, and there have been mathematicians who love music and are actively involved in music. Johann Sebastian Bach was a composer whose musical works show unusual numbers and symmetries that can be mathematically expressed. The symbolism of numbers is expressed even in his last name – BACH which is expressed as number 14: B = 2 + A = 1 + C = 3 + H = 8. The sum of these numbers is 14. Number 14 is also incorporated into his other works through the number of notes in the beat or the number of beats in the composition. Thus, we have 14 canons of *Goldberg Variations* and 14 *contrapunctus* in *Die Kunst der Fuge* (Currie 1974; Rumsey 1997; Tatlow, 2015).

On the other hand, Albert Einstein, a prominent German theoretical physicist, said that he often thinks about music and views his life in music categories. Einstein emphasized that if he were not a physicist, he would probably be a musician. He spent much of his free time playing music and occasionally performed as a violinist at local concerts. Music was a type of pleasure for Einstein but also an inspiration for his scientific work (Foster, 2005; White, 2005).

Many authors have written about the connection between mathematics and music and the possibilities of combining them in education (Cheek, 1999; Vaughn, 2000; Church, 2001; Shilling, 2002; Fauvel, Flood, and Wilson, 2006; Geist, and Geist, 2008; McDonel, 2015). The basics of mathematical thinking have been a part of the preschool curriculum in the world for many years, more precisely, a part of the indispensable educational paradigm for the purpose of developing and strengthening the logical-mathematical intelligence. Music, on the other hand, reaches young minds more directly – phenomenologically. Rhymes, songs that tell a story and sounds of different instruments develop the musical intelligence of children, but they also provide diversity in teaching.

Mathematics in Preschool Education

In order to be able to bring music and mathematics closer to preschool children, we need to identify what main areas of mathematics are relevant for their education. To this end, we will use the classification recommended by the *National Council of Teachers of Mathematics*[1]. This classification applies to preschool children until the second grade of primary school. Numbers and operations, algebra, geometry, and measurement and data analysis are the areas of competence related to problem solving, communication, reasoning, connecting and representation.

---

[1] *The National Council of Teachers of* Mathematics (NCTM) is the largest mathematical education organization in the world and operates in the US and Canada.

Numbers and Operations

Getting acquainted with numbers and their possible connections is the basis and the beginning of the formal mathematical education. Therefore, the field of numbers and operations is the main basis for learning other mathematical fields, as well as for learning other educational fields, especially music and language.

One of the natural abilities present in humans is the ability to perceive relative quantities, i.e. to recognize the difference in quantity within the domain of visual perception. Children, however, must learn conventional pairing and counting methods to detect these differences more reliably. In the third year of their life, children determine the equality of two groups of objects based on their immediate proximity, and in their fourth year they can identify pairs of these groups to confirm that they are equal (Clements, Sarama, and DiBiase, 2004). By counting, the children combine the word and the very concept of number to come up with more complex concepts; order and size.

Addition and subtraction in their basic form depend on the learned counting skill (Vlahović-Štetić, and Vizek Vidović, 1998). Children who learn to count accurately can clearly distinguish between the sequence of sizes and the possibility of increasing and decreasing them, but still have a difficulty with counting back, indicating that the subtraction process is more difficult to understand. Subtraction is best understood by visualizing tasks with objects on the table or with children in the room.

The next development step is assembly and disassembly. Preschoolers understand the terms "whole" and "part whole". The five lined objects clearly form a whole, and with their grouping the children notice the creation of separate units from within the initial unit. Therefore, they will quickly understand that the number 5 contains the numbers 3 and 2 within itself. Assembling and disassembling is understood as a combination of the visualization of numbers and the counting process, and as such enables children to disassemble and assemble units into all possible variants and is a precursor for more complex analytical thinking. When it comes to numbers, children show an extreme fascination with big numbers. As the learning process of counting begins in the sphere of counting to 10, the area of higher numbers and the system of tens and hundreds makes more rules for the potential assembly and disassembly.

Grouping is the process of combining objects into sets that have an equal number of components. A set of 20 items can be divided into 5 sets of 4 items. This process leads to an understanding of multiplication, segment counting and measuring. Because large numbers can be harder to understand and can be considered as a combination of two separate numbers, grouping numbers into tens helps with this task. In addition to counting by numbers, in this way children learn to segment numbers that are a part of a larger series and, as a result of a group exercise, manage to deal with double-digit addition operations.

Geometry

Geometry plays an important role in the development of a child's spatial orientation. Thinking about and understanding space and movement as a set or a whole has an influence on the movement of the child and the perception of space as their area of activity. The aspect of geometry that is the most evident in a child's daily life is a shape. The basic two-dimensional forms have been learned by the age of four years, and, before starting the first grade, children are also familiar with the basic three-dimensional forms. In addition to the basic shapes, children clearly notice the specifics of polygons or multifaceted three-dimensional shapes. They notice, as with number operations, that more complex shapes are composed of complex shapes such as triangles or squares. As early as the fourth year of life, children come up with ways to combine shapes to create new ones. At the age of five and six, they clearly recognize similar shapes of different sizes and differently placed shapes and can reproduce them in custom computer programs.

A very important part of spatial orientation is the visualization of forms, i.e. the mental delineation of forms for the purpose of analyzing them. Through visualization, children map the space around them, determine the direction of their own and others' movement, and create reference points in the space. These mental images of two and three-dimensional shapes familiarize the children with the world around them, whether they are spaces or objects. They notice regularities and irregularities in relation to prototype examples of forms (Clements, 2001; Ho, 2003).

Measurement

The first type of measurement occurs by learning to count, it is upgraded by adopting concepts like length, weight, or height. By the third year of life, children will develop, to some extent, an understanding of these concepts. They have difficulty in noticing the differences in the weights of objects made of the same material by visual observation, and in some cases they are unable to compare the length of the two objects by placing them side by side from the starting point (Clements, Sarama, and DiBiase, 2004). At the age of four and five, the ability of children to compare objects by perceptual measurement is greatly improved.

The ability of the child to predict effects of his actions on certain objects in space depends on his understanding of the difference of scalar sizes. Although this understanding is seemingly simple and intuitive, it makes it possible to make further comparisons by using reference objects and tools. Measurement as a method of analysis and inference requires specific referents or units of measurement. These referents can be of any arbitrary size, such as the size of a child's palm or a standardized unit of measure. Children will learn the latter by using centimetre cubes or rulers, tools that have only recently become used in the teaching of early and late preschoolers. Of course, using such tools further develops fine motor skills and counting, and the outcome is the ability to measure accurately (Clements, Sarama, and DiBiase, 2004).

Algebra and Patterns

Unlike counting, geometry, and measurement, algebra is not as emphasized as an active field of teaching for preschoolers, and plays a more complementary role. Preschoolers' algebraic knowledge sums up to the pattern recognition skill that is of the utmost importance for their further intellectual development. The ability to notice patterns develops from an early age, and is observed in the child's environment by comparing, sorting and analyzing objects and behaviours of children and adults (Mason, Graham, and Johnston-Wilder, 2007; Kieran, 2018).

Data Analysis

Like algebra, data analysis is not at the forefront of developing a child's mathematical ability. Data analysis involves classifying, organizing, demonstrating and using information for the purpose of asking questions and providing answers. Children sort a group of items, such as buttons or toys, into groups according to the patterns they have observed in order to distinguish them from other groups. They clearly state the reasons for their sorting and ask questions about the purpose of certain differences. Children sort their conclusions by simple categories such as colour, size, shape, volume, and thus combine the ability to notice patterns and establish specific relationships of things (Selimović and Karić, 2011).

Music in Preschool Education

Musical skills of a preschooler are developed by singing, performing rhymes, (active) listening to music, performing rhythms and melodies on rhythmic and melodic percussion, and creative musical expression. At this age, children are introduced to musical concepts that relate to the expressive elements of a piece of music (rhythm, tempo, dynamics, tone, melody, and harmony) through an approach of course that is appropriate for preschoolers.

Tone

Tone is a basic element, a material used by the music arts. Unlike the sounds that surround us in nature, which arise from the improper vibration of elastic bodies, tone is born from the proper vibration of sound sources (Andreis, 1967). Scott points out that "the development of understanding of pitch and pitch relations is the key to approaching music in our culture" (Scott 1979, p. 87).

As a physically explained phenomenon, tone is poorly understood by children and is mostly identified with the term sound. Only when the tone is played individually, and in sequence with other tones, do children slowly perceive it as something separate and separable, but they practice it through the whole, i.e. songs and rhymes.

When attempting to produce high pitched sounds, children will often try to reach those heights by raising the chest with visible strain. Sam (1998) states that a child learns this height by listening and playing. First, the tone is understood by listening. The same tone will only be sung when the child is able to develop his vocal cords. Intonation receives greater attention at the age of five and beyond. Proper singing is learned using hearing, and tones are no longer abstract, and their duration is a clearly observed pattern (Vidulin, 2016).

Volume is also a very important component of a tone that children generally categorize in terms of comfort and discomfort. Strong or weak tones cause excessive or insufficient stimulus in the child, so a moderate pitch is best suited for preschool children (Sam, 1998). In addition to the volume, the colour of the tone is an enhancer of the aesthetic experience of music. Children notice very clearly the same songs and numbers played on different instruments or media, but also notice the colour differences they usually describe in relation to their own experiences related to this new source.

Rhythm and Meter

Children have a special interactive encounter with music by using the body as an instrument or a percussion instrument (Orff Instrumentarium). By doing this, the children are introduced to the concept of rhythm and meter for the first time, but also to the creation of music, which gives us an insight into the motor skills of the child combined with hearing and musical experience. In addition to instruments, rhymes are used for learning rhythm and meter through a song. Rhymes teach children to follow a certain meter and to follow the difficult and easier measuring unit at a certain speed (Sam 1998). Rhyme learning begins in the youngest kindergarten groups and places less demands on the child when it comes to hearing development. Even before coming to kindergarten, children sway, swing, jump, shake, and generally move with the rhythm of music, and the infant responds to the rhythm with body and limb movements (Zentner, and Eerola, 2010). Rhythmicity in children is caused by a movement that allows quick learning of rhymes and the potential for new words and rhythmizing, i.e. new rhymes that can be composed with children. The phenomenon, which is generally applicable but also expressed in childhood, of two or more people repeating the same rhythm to harmonize their breathing, heart rate, brain waves, attention and movements, is also interesting. Such an observation once again confirms the phenomenological power of music and the influence it has on the emotional and psychophysical development of humans, since the harmonization of rhythm induces a sense of community, empathy and social cohesion.

Tempo and Dynamics

Slow, moderate, fast and very fast are the basis for marking the tempo of each composition, and there are several levels of subtle differences between them. Template tags that are understandable in preschool are usually slow and fast. This dichotomy is quickly learned, and at the age of five or six it is reasonable to expect that children will also be able to develop a sense of a moderate performance speed (Sam, 1998). Children notice very quickly that the speed of the performance changes the very character of the song, so a faster song will evoke an optimistic and joyful emotional state in them, while slower songs will have a relaxing or memorable impact (Vidulin, 2016).

As was already noted in the chapter on tone perception, the ideal volume is between a weak and a strong tone. Like tempo, the dynamics have basic labels that divide them into very quiet, quiet, medium-quiet, medium-strong, strong and very strong, and the ideal range for preschoolers would be between medium-quiet and medium-strong. The aesthetic experience of dynamics also influences the emotional changes in children.

Melody

A melody is a series of tones with a different pitch and duration that are performed consecutively and form a meaningful whole in terms of music content. From the example of melody formation, we can see that music lives in the connection and participation of different elements of rhythm, harmony, formal frame, dynamics and colour (Andreis, 1967).

Playing simple tunes in preschool requires a certain degree of rhythm and intonation. By listening to the melody, the child receives all the information without being aware of each individual element. However, playing a tune in preschool requires some adjustment to the components of the tune for ease of learning and teaching, and a sufficiently developed voice apparatus.

It is very important to distinguish between the quality of aesthetic standards and the ability to monitor melody structures when listening to a child's performance. Only after we have established that the child has learned the basics for following a tune, we can recognize the development of musical abilities of an individual child (Drexler, 1938).

Harmony

Harmony represents the chordal or vertical structure of a piece. The elements of harmony are chords that are contrary to the notion of melodic flow. Chords are created by the simultaneous sound of a series of tones that are placed one above the other. They are produced one after the other, and each of them is given a meaning in relation to the chords that precede them, i.e. the ones that follow (Andreis, 1967).

Making an example for this musical element for preschoolers is not an easy task since the concept of harmony is quite abstract and the terms of intervals like the third and the fifth are too specific, but still not impossible. The easiest way to show harmony is through a disharmonic rendition of already known songs. Children will easily notice that the composition is not performed in the "right" way or that it sounds "uncomfortable". If we change the tonality, children will recognize the similarity, and, in most cases, will not consider the transposed song a mistake. However, this does not mean that children will not consider the song a mistake if we play the song a third or a fourth below the original, since the song will be quite different from the original.

Harmony has been the focus of many researches, and results related to children have generally produced duplicate results. Costa-Giomi (1994) states that harmonic skills only emerge at the age of eight or nine, and the current practice of most teachers and the methodological literature are consistent with these conclusions. In contrast, some studies (Moog, 1976; Zimmerman, 1993; Welch, 2002) have found that preschoolers can distinguish between tone and chord, hear changes in chords, and distinguish harmony from disharmony (Berke, 2000).

Mutual Elements of Mathematics and Music

After determining the specifics of both areas (mathematics and music) we can show a cross-section of similar elements and their use in preschool education. The mathematical elements of counting, geometry, algebraic patterning, data analysis and measurement are contained, to some extent, in aspects of musical elements like tone, rhythm, melody, tempo and dynamics, and harmony.

The foundation for all the musical and mathematical elements listed so far, as well as all human functions, is in the cerebral activity. Therefore, the first link between music and mathematics is hidden in the functions of the brain area for mathematical and musical thinking and creativity. Nowadays, there is a general knowledge that the right cerebral hemisphere is a set of areas in charge of creative, artistic, spatial and holistic characteristics and actions, and the left dominates in structure, organization, analytical approach and logic. So, what was discovered by using brain mapping technology?

The "music brain" consists of complex and widespread neural systems, as well as locally specialized areas in the brain, and the results of initial studies were reduced to activities in the right hemisphere only. The most recent discoveries indicate hemisphere connecting as one of the characteristics specific to music processing and action (Hodges 2000). Musical potency does not stop at this point, since some studies have shown that engaging in music causes various changes in the human brain. Donald A. Hodges (2000) summarizes important findings from the study of music and cerebral activity, and states the following premises: 1) the human brain responds to and participates in music; 2) the "music brain" starts performing at birth and lingers throughout one's life: 3) continuous music practice from birth affects the organization of the "music brain"; 4) the local specialized areas of the music brain are: cognitive components, affective components and motor components; 5) the "music brain" is extremely resilient. The last premise is genuinely interesting as it relates to the persistence of high musical functions in spite of mental, emotional or degenerative difficulties.

The "mathematics brain", like the "music brain", is not limited for operation in only one hemisphere. However, mathematical functions are grouped in the parietal lobe and seemingly the "mobility" of these functions is lower than for the musical ones that occur in almost all parts of the brain (Cranmore, and Tunks, 2015).

It is also interesting that the activity of mathematical areas is stronger in children than adults when calculating. The reason for this is the understanding of these mathematical actions that are understood more generally and more abstractly by children than by adults, who have adopted these processes and can solve problems with ease and less activity (Rocha et al., 2005).

Since most tasks, whether related to mathematics or music, are intertwined with language, movement and emotion, it is impossible to ignore the interconnections of all these systems. One possible link between the mathematical and the musical brain is the correlation of geometric thinking and intense musical exercise. It has been observed that people who are actively practicing music have a better understanding of basic geometric systems (Spelke, 2008). MRI tests have shown that musicians are able to manipulate computation with fractions better than others because of an increase in their memory and an improvement in the abstraction of numerical quantity (Schmithorst, and Holland, 2004). The area where the intersection of mathematical and musical processing occurs is the prefrontal cortex, which is associated with executive actions and memory, as well as emotional reactions.

Perhaps the most common and widespread link between music and mathematics in preschool is noticing patterns. Music is a human's first contact with patterns, whether it is our mother's heartbeat, listening to music as a newborn, or learning songs in kindergarten, our brains notice musical patterns. Thus, music helps children perform mathematical tasks even when children do not experience these tasks as such. Music is a social, natural and developmentally appropriate way to accelerate the process of learning mathematics (Geist, Geist, and Kuznik, 2012). Melody and rhythm, as its components, are clear exercises that enhance the ability to recognize patterns. It is enough just to expose children to music, and we can already notice a cumulation of progress in terms of patterns. We can see this at the beginning of the harmony noticing process since children in preschool can only absorb it through passive listening.

As with symmetry, because of the impossibility of introducing the terminology of music theory, we cannot clearly visualize the values of tones, tempo, or dynamics to children, but we can relate them to the magnitudes they learned in mathematics and strive to bring these concepts auditorily. The children will clearly classify heights of two tones, especially if they are played in an ascending or descending sequence of a tone. Scales are great teaching materials for learning the values and proportions within music as children recognize the sequences they learned by counting, and connect different segments of rhythm and melody with addition.

Songs and Games with Elements of Mathematics and Music

Before a child can learn and apply mathematics, he or she must acquire certain skills that precede the learning of mathematical operations. Making comparisons is an important mathematical skill that we can practice through music activities. What kinds of comparisons does music include? Children can compare fast and slow beats, long and short tones, high and deep tones, and loud and quiet sounds.

To compare loud and quiet sounds, Voglar (1980) suggests playing the *Little Drummer*. In this game the educator is the puppet Luta, who has received a new drum as a gift and hits it all day and night. He plays loudly during the day, and plays quietly at night, so as not to disturb other dolls while they sleep. Children (who also play dolls) should watch the drumming of the Luta doll and guess when it is daytime and when it is night. If the drummer is playing loudly, the puppets wake up, accompanied by the movements of the drum. The movements are solid, strong and energetic. When the drummer is playing quietly the dolls are still moving in accordance with the rhythm, but the movements are performed more softly, more gently and preparing for sleep. In this game, the educator can perform a rhythm that involves tones of different durations, but the dynamics needs to be altered to be either loud or quiet. The game is repeated several times. If older children are involved in the game, one of them may assume the role of the drummer's doll. Figure 1 shows examples of rhythms that can be performed on a drum.

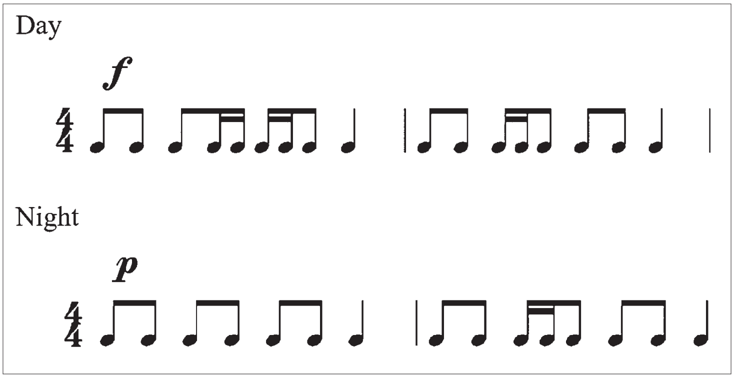

Figure 1 Examples of rhythm that can be played on a drum during the game *Little Drummer*

Through various music games children can experience and compare high and low tones. The *High and Low Tones* game is performed with children standing by their chairs. When they hear the low-octave (a) tone played by their educator on the instrument, everyone should squat. When they hear the tone in the first octave (a1) everyone should stand up. The game can also be played using black and white paper. When children hear a low tone, they raise the black paper and when they hear a high tone, they raise the white paper (Domonji, 1986).

We can also perform a certain rhythm on a drum or by clapping slowly. We then ask the children to move along that rhythm and to align their steps with the pace, at which the rhythm is performed. After that we will do the opposite – we will perform the same rhythm at a fast pace, and the children will adjust their movements to the rhythm of the new pace. This activity can also be related to a story.

The *Bear and the Bees* game can be used to compare long and short tones. The game begins by telling the story of a bear and a bee. The bear woke up from a winter sleep and, being very hungry, decided to look for food in the forest. The educator shows that the bear is walking slowly and asks the children to move in the same way. Each step goes to the first period in the musical tact (half notes) (Figure 2). The bear suddenly saw a beehive. He took a honeycomb from one beehive and started eating it. At that moment the bees escaped from the beehive and began to circle around the bear while stinging him. When describing them the educator moves fast like the bees, and on each period performs two steps (eight note value) (Figure 2). The educator asks the children to move in the same way, with their arms held on the sides, with elbows bent, as if they had wings. The bear roared, ran away and was no longer approaching the bees. The game can continue with the educator alternating the rhythm in longer and shorter durations, with the rhythm being the bear for the longer duration and then asking the children to move slowly. When the educator starts performing a rhythm in shorter durations, representing bees, children should move quickly. This can be changed several times (Stefanović, 1958).

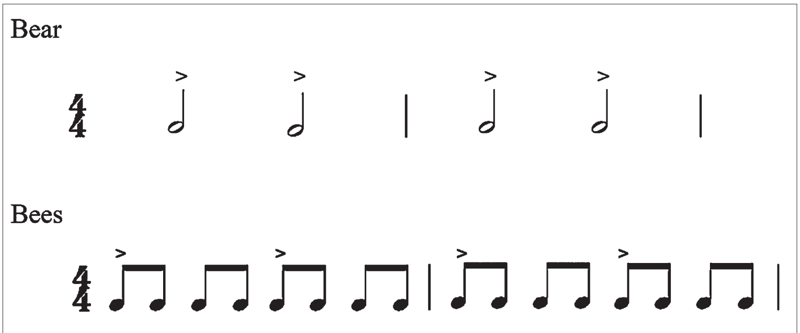

Figure 2 Rhythms representing the bear and bees in the game *The Bear and the Bees*

Children can be involved in comparison games by letting them listen and create a different sound or rhythm than what we performed for them. For example, we can perform a long and high pitch and ask the children to repeat it. Then we ask them to make the opposite sound, therefore, low and loud. We can also ask the children to perform the sound on percussion instruments. They will do this by producing a quiet sound, then a slow sound, then fast and finally slow rhythm. Throughout these exercises, the children improvise the rhythm.

With the song *Seven Steps*[2] (Figure 3), children develop the ability to verbally count from a given number back and forth. The poem counts from one to seven, one to three, seven to one and seven to five. By singing this song, the children will first learn the words that represent the numbers, and later they will begin to understand that they are interconnected. Numbers one through seven can be cut from a collage of paper and hung against a wall so that children can indicate them while they sing. Performance of the song can flow in the way that one child shows numbers from one to seven and another child shows seven to one.

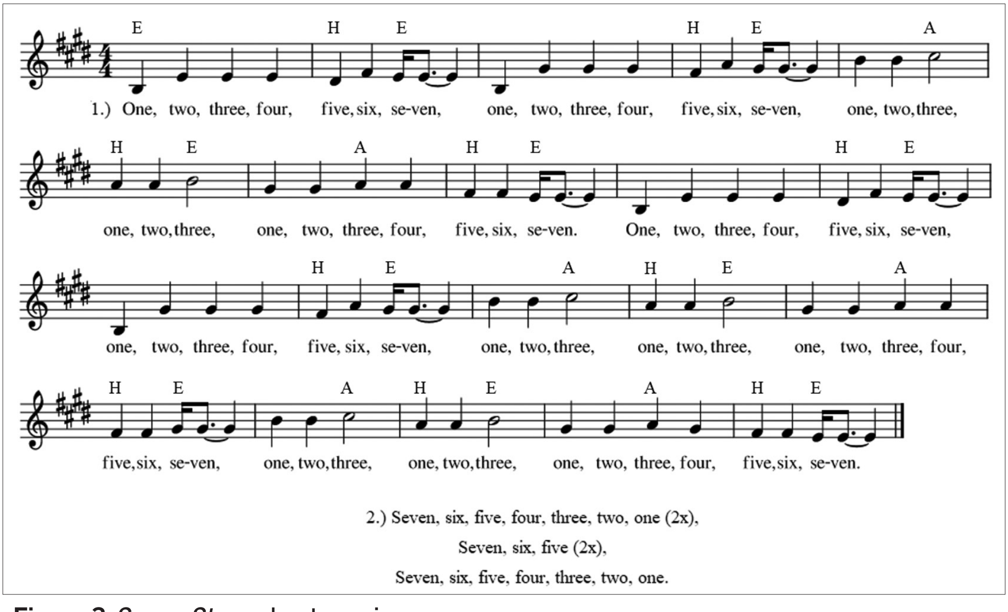

Figure 3 *Seven Steps* sheet music

Children can also practice subtraction through various music games and by singing different songs. One of these games is called *Little Train*. Children are divided

[2] Super Simple Songs – Kids Songs https://www.youtube.com/watch?v=pTLtcno5_cY

into four groups of five children, which then form a queue of “wagons” in each group, with the first child in the queue being the locomotive. Children then sing the song *Little Train* (Figure 4).

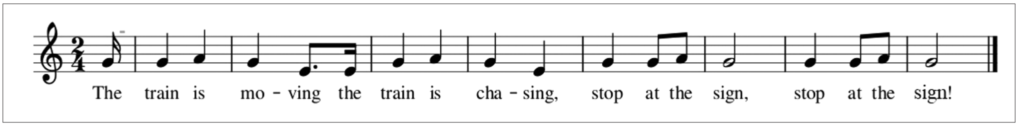

Figure 4 *Little Train* sheet music

Then the educator’s command follows: “The last wagons should go to their place”. The last child in the queue from each group goes to their place (Figure 5). The educator then asks the children how many wagons are left. Children respond aloud – three, two, one, zero. The game goes on until the wagons are gone. By gradually reducing the number of children in the game, we will get a subtraction exercise. In addition to the song, this game also features movement and space handling. The combination of these elements enables high-quality pattern recognition with the development of motor skills (Čupić, Sarajčev, and Podrug, 2017).

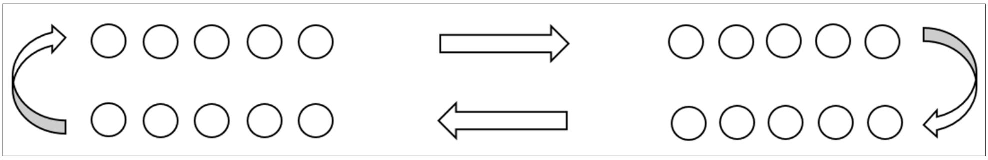

Figure 5 Arrangement of children in space during the performance of the game *Little Train*

With a musical game called *Ten Green Bottles* (and a song of the same name), children can practice subtraction up to the number ten (Figure 6). The poem consists of only one verse. The only thing that changes is the number of bottles. In the first verse there are ten bottles, in the second nine, in the third eight, etc. The sequence continues until the number of bottles is zero. The final verse ends with “There will be no green bottles hanging on the wall.” For this music game, it is necessary to prepare numbers from one to ten on separate cards that children will put around their necks before the game starts. Children will be in circle formation, holding hands, dancing and singing *Ten Green Bottles*[3].

[3] BBC – School Radio – Counting Songs https://www.bbc.co.uk/programmes/p038bdqh

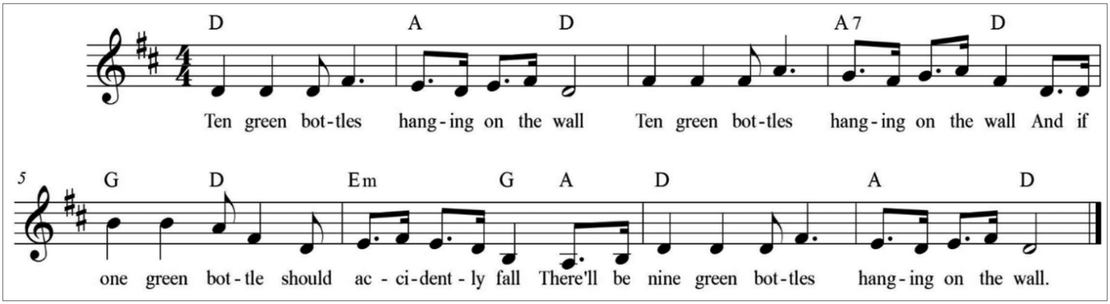

Figure 6 *Ten Green Bottles* sheet music

Each time the children say the verse, "If a green bottle should accidentally fall", the child who has the said number will fall to the floor. Therefore, a child who has the number 10 falls on the floor, child who has number 9 will fall after him, etc. When the children are better acquainted with the numbers, they can play the game without cards, so that each child remembers what their number is. Everything else is done the same way as when the game is played with cards.

Conclusion

In this paper, we wanted to explore the essential components of mathematics and music and determine the possibility of integrating them in preschool education. Although mathematics and music differ in their formal teaching methods in preschool, there are undeniable links that make them interesting partners for developing cognitive analytical skills.

The phenomenological potency of music and the rational harmony of mathematics are evident in numerous games, rhymes and songs. Through musical activities, children can acquire certain skills that precede the learning of mathematical operations. We can practice the skill of making comparisons with children by comparing the long and short tones, the treble and the deep tones, the loud and quiet sounds, and the skill of counting, adding and subtracting them by performing appropriate music games or rhymes and songs that mention numbers. Rhymes and songs help connect the beat with actions and numbers.

Music enhances the overall brain activity and is appropriate for different areas. However, it should be borne in mind that the musical components are explained precisely through mathematical terms and concepts. That is why it is important to integrate mathematics and music as often as possible in preschool education.

References

Andreis, J. (1967). *Vječni Orfej* [*The eternal Orpheus*]. Zagreb: Školska knjiga.

Berke, M. K. (2000). *The ability of preschool children to recognize chord changes and audiate implied harmony*. Unpublished doctoral dissertation, University of Arizona, Tucson.

Chailley, J. (2006). *Povijest glazbe srednjega vijeka* [*History of middle ages music*]. Zagreb: Hrvatsko muzikološko društvo.

Cheek, J. M., and Smith, L. R. (1999). Music training and mathematics achievement. *Adolescence,* 34, 136, 759-761. [Google Scholar]

Church, E. B. (2001). The math in music and movement. *Early Childhood Today,* 15, 4, 38-43. [Google Scholar]

Clements, D. (2001). Mathematics in the preschool. *Teaching Children Mathematics,* 7, 270-275. Doi: https://doi.org/10.5951/TCM.7.5.0270

Costa-Giomi, E. (1994). Recognition of chord changes by 4 and 5 year old American and Argentine children. *Journal of Research in Music Education,* 42, 1, 68-85. Doi: https://doi.org/10.2307/3345338

Clements, D. H., Sarama, J., and DiBiase, A. (2004). *Engaging young children in mathematics: standards for early childhood mathematics education.* Mahwah: Erlbaum. [Google Scholar]

Cranmore, J., and Tunks, J. (2015). Brain research on the study of music and mathematics: A Meta-Synthesis. *Journal of Mathematics Education,* 8, 2, 139-157. [Google Scholar]

Currie, R. N. (1974). A Neglected guide to Bach's use of number symbolism - part I. Bach, 5, 1, 23-32. [Google Scholar]

Čupić, A., Sarajčev, E., and Podrug, S. (2017). Edukativne igre u nastavi matematike [Educational games in mathematics teaching]. *Poučak:* časopis *za metodiku i nastavu matematike,* 18, 70, 66-80. [Google Scholar]

Domonji, I. (1986). *Metodika muzičkog vaspitanja u predškolskim ustanovama* [*Methods of music education in preschool institutions*]. Sarajevo: Svjetlost. [Google Scholar]

Drexler, E. N. (1938). A study of the development of the ability to carry a melody at the preschool level. *Child Development,* 9, 3, 319-332. Doi: 10.2307/1125444

Fauvel, J., Flood, R., and Wilson, R. J. (2006). *Music and mathematics: from pythagoras to fractals*. New York: Oxford University Press. [Google Scholar]

Foster, B. (2005). Einstein and his love of music. *Physics World,* 18, 1, 1. [Google Scholar]

Geist, K., and Geist, E. A. (2008). Do re mi, 1-2-3: That's how easy math can be: using music to support emergent mathematics. *Young Children,* 63, 2, 20-25. [Google Scholar]

Geist, K., Geist, E. A., and Kuznik, K. (2012). The patterns of music: children learning mathematics through beat, rhythm, and melody. *Young Children,* 67, 1, 74-79. [Google Scholar]

Hodges, D. A. (2000). Implications of music and brain research. *Music Educators Journal,* 87, 2, 17-22. Doi: https://doi.org/10.2307/3399643

Ho, S. Y. (2003). Young children's concept of shape*:* van Hiele Visualization Level of geometrix thinking. *The Mathematics Educator,* 7, 2, 71-85.

https://repository.nie.edu.sg/bitstream/10497/65/1/TME-7-2-71.pdf

Kieran, C. (Ed.) (2018). *Teaching and learning algebraic thinking with 5 to 12 year olds: the global evolution of an emerging field of research and practice*. New York, NY: Springer. [Google Scholar]

Mason, J., Graham, A., and Johnston-Wilder, S. (2007). *Developing thinking in algebra*. London: Sage. [Google Scholar]

McDonel, J. S. (2015). Exploring learning connections between music and mathematics in early childhood. *Bulletin of the Council for Research in Music Education,* 203, 45-62. Doi: https://www.jstor.org/stable/10.5406/bulcouresmusedu.203.0045

Moog, H. (1976). *The Musical Experience of the Pre-school Child.* (trans. C. Clarke) London: Schott. [ISBN-10: 0901938068]; [ISBN-13: 978-0901938060]

Partch, H. (1974). *Genesis of a Music*. New York: Da Capo Press. [ISBN-13: 9780306801068]

Plavša, D. (1981). *Muzika: prošlost, sadašnjost, ličnosti, oblici* [*Music: past, present, personalities, forms*]. Knjaževac: Nota Knjaževac. [Google Scholar]

Plomp, R., and Levelt, W. J. M. (1965). Tonal consonance and critical bandwidth. *Journal of the Acoustical Society of America,* 38, 548-560. Doi: https://doi.org/10.1121/1.1909741

Rocha, F., Rocha, A., Massad, E., and Menezes, R. (2005). Brain mappings of the arithmetic processing in children and adults. *Cognitive Brain Research,* 22, 359-72. Doi: https://doi.org/10.1016/j.cogbrainres.2004.09.008

Rumsey, D. (1997). Bach and numerology: 'Dry mathematical stuff'? *Literature & Aesthetics,* 7, 143-165. [Google Scholar]

Sam, R. (1998). *Glazbeni doživljaj u odgoju djeteta* [*Music experience in child education*]. Rijeka: Glosa. [Google Scholar]

Schmithorst, V. J., and Holland, S. K. (2004). The effect of musical training on the neural correlates of math processing: a functional magnetic resonance imaging study in humans. *Neuroscience Letters,* 354, 3, 193-196. Doi: https://doi.org/10.1016/j.neulet.2003.10.037

Scott, C. R. (1979). Pitch concept formation in preschool children. *Bulletin of the Council for Research in Music Education,* 59, 87-93. https://www.jstor.org/stable/40317550

Selimović, H., and Karić, E. (2011). Učenje djece predškolske dobi [The learning process of preschool children]. *Metodički obzori,* 11, 6, 145-160. [Google Scholar]

Shilling, W. A. (2002). Mathematics, music, and movement: exploring concepts and connections. *Early Childhood Education Journal,* 29*,* 3, 179-184. Doi: https://doi.org/10.1023/A:1014536625850

Spelke, E. (2008). Effects of music instruction on developing cognitive systems at the foundations of mathematics and science. In Asbury, C., and Rich, B. (Eds.), *Learning, Arts, and the Brain* (pp. 17-49). New York/Washington, D.C.: The Dana Foundation. [ISBN: 978-1-932594-36-2]

Stefanović, B. D. (1958). *Metodika muzičkog vaspitanja predškolske dece* [*Methods of music education of preschool children*]. Beograd: Nolit.

Tatlow, R. (2015). *Bach's Numbers: Compositional Proportion and Significance*. Cambridge: University Cambridge Press. [Online ISBN: 9781316105061]; Doi: https://doi.org/10.1017/CBO9781316105061

Vaughn, K. (2000). Music and mathematics: modest support for the oft-claimed relationship. *Journal of Aesthetic Education,* 34, 3-4, 149-166. DOI: 10.2307/3333641

Vidulin, S. (2016). Glazbeni odgoj djece u predškolskim ustanovama: mogućnosti i ograničenja [Music education of children in preschool institutions: opportunities and limitations]. Život *i* škola [*Life and School*], 62, 1, 221-233. [Google Scholar]

Vlahović Štetić, V., and Vizek Vidović, V. (1998). *Kladim se da možeš – psihološki aspekti početnog poučavanja matematike* [*I bet you can - psychological aspects of initial teaching mathematics*]. Zagreb: Korak po korak.

Voglar, M. (1980). *Kako muziku približiti deci* [*How to bring music closer to children*]. Beograd: Zavod za udžbenike i nastavna sredstva Beograd. [ISBN: 86-17-05425-5]

Welch, G.F. (2002). Early childhood musical development. In Bresler, L., and Thompson, C. (Eds.), *The Arts in Children's Lives: Context, Culture and Curriculum* (pp. 113-128). Dordrecht, NL: Kluwer. [ISBN 1-4020-0471-0]

White, P. (2005). Albert Einstein: the violinist. *The Physics Teacher,* 43, 286-288. Doi: 10.1119/1.1903814

Zentner, M., and Eerola, T. (2010). Rhythmic engagement with music in infancy. *Proceedings of the National Academy of Science,* 107, 13, 5768-5773. Doi: 10.1073/pnas.1000121107

Zimmerman, M. P. (1993). An overview of developmental research in music. Bulletin of the Council for Research in Music Education, 116, 1-21.
https://www.jstor.org/stable/40318552

Povezivanje matematike i glazbe u predškolskom odgoju i obrazovanju

Sažetak

U ovom smo radu željeli istražiti bitne sastavnice matematike i glazbe te utvrditi mogućnost njihove integracije u predškolski odgoj i obrazovanje. Osnovni matematički pojmovi, o kojima ovisi razvoj intelektualnih sposobnosti svakog djeteta, formiraju se u predškolskoj dobi. Kombinacijom komponenata elemenata iz matematike i glazbe možemo vidjeti njihovu povezanost u smislu simetrije, vrijednosti i mjerenja te prepoznavanja uzoraka.
Raznim glazbenim aktivnostima, djeca mogu steći određene vještine koje prethode učenju matematičkih operacija. Tako možemo vježbati matematičku vještinu uspoređivanja, uspoređujući s djecom duge i kratke tonove, visoke i duboke tonove, glasne i tihe zvukove i matematičku vještinu brojanja izvodeći prikladne glazbene igre, rime i pjesme u kojima se spominju brojevi.
Brojanje u rimama i pjesmama pomaže djetetu u učenju pojma broja i matematičkih operacija kao što su zbrajanje i oduzimanje. Igre koje kombiniraju glazbu i matematiku obično koriste glazbu kao pokretačku snagu produktivnog i dinamičnog obrazovnog okruženja. Ritam i melodija pomažu u procesu matematičkog razmišljanja jer djeca primaju informacije izravno i u cjelini. Zato je važno što češće povezivati glazbu i matematiku u predškolskom odgoju i obrazovanju.

Ključne riječi: glazba, igre, matematika, pjesme, predškolska djeca, rani odgoj i obrazovanje