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Influence of design parameters on the behaviour of cross-spring pivots

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Abstract

Compliant mechanisms gain at least part of their mobility from the deflection of the flexible member. They are characterised by high precision, possibility of monolithic manufacturing, as well as the absence of backlash and wear. Numerical methods are used in this work to characterise the behaviour of compliant rotational mechanisms, known as cross-spring pivots, aimed at ultrahigh-precision positioning applications. The results obtained by using nonlinear finite element calculations are compared with experimental data reported in literature. The finite element model developed in this way makes it possible to conider the influence of lateral loads and of non-symmetrical pivot configurations where the angle or point of intersection of the leaf springs can be varied. This allows assessing the influence of the cited design parameters on the minimisation of the parasitic shifts of the geometric centre of the pivot, as well as on the minimisation of the variability of the rotational stiffness of the pivot while ensuring its stability. The obtained results allow determining design solutions applicable in ultrahigh-precision positioning applications, e.g. in the production or in handling and assembly of MEMS devices.

Keywords: *compliant devices, cross-spring pivots, ultrahigh-precision positioning, FEA modelling, parasitic displacements, rotational stiffness*

1. Introduction

Compliant mechanisms, a viable alternative to sliding and rolling mechanisms used to transfer motion, energy and power, are specific in the sense that they gain at least part of their mobility from the deflection of flexible members. Compliant devices are hence characterised by high precision, the possibility of monolithic manufacturing (thus enabling the adoption of the 'design for no-assembly' approach), reduced costs and the absence of backlash and wear. They are thus widely used in mechanical engineering design, precision engineering as well as the micro- and nanotechnologies [1].

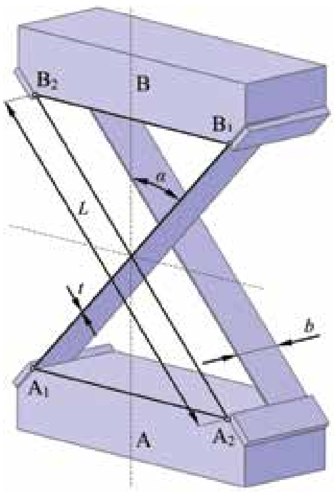


Fig. 1. Typical configuration of a cross-spring pivot

The design of compliant rotational mechanisms known as the cross-spring pivot (Figure 1) is characterised by high compliance along the 'in plane' rotational degree of freedom. Cross-spring pivots are then configured by using two rigid bodies A and B connected via leaf springs which intersect at their midpoint O. The employment of spring-strips enables the prediction of the behaviour of the pivots, which has, however, to be based on the analysis of the characteristic parameters of the strips themselves [1].

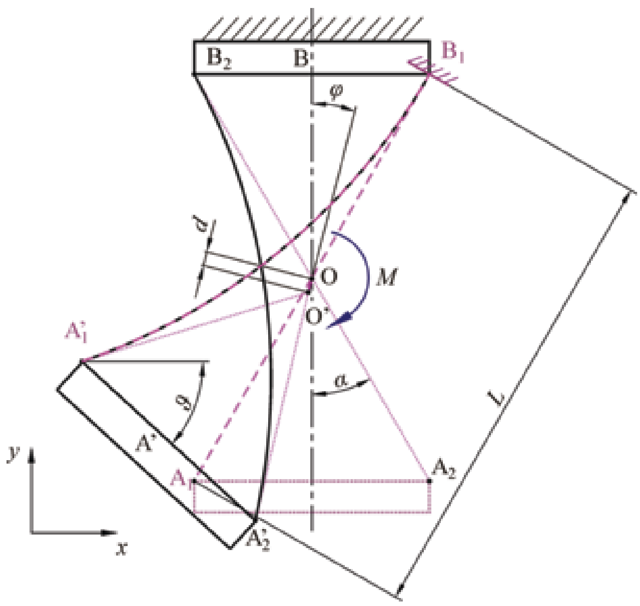


Fig. 2. Analytical model of the cross-spring pivot

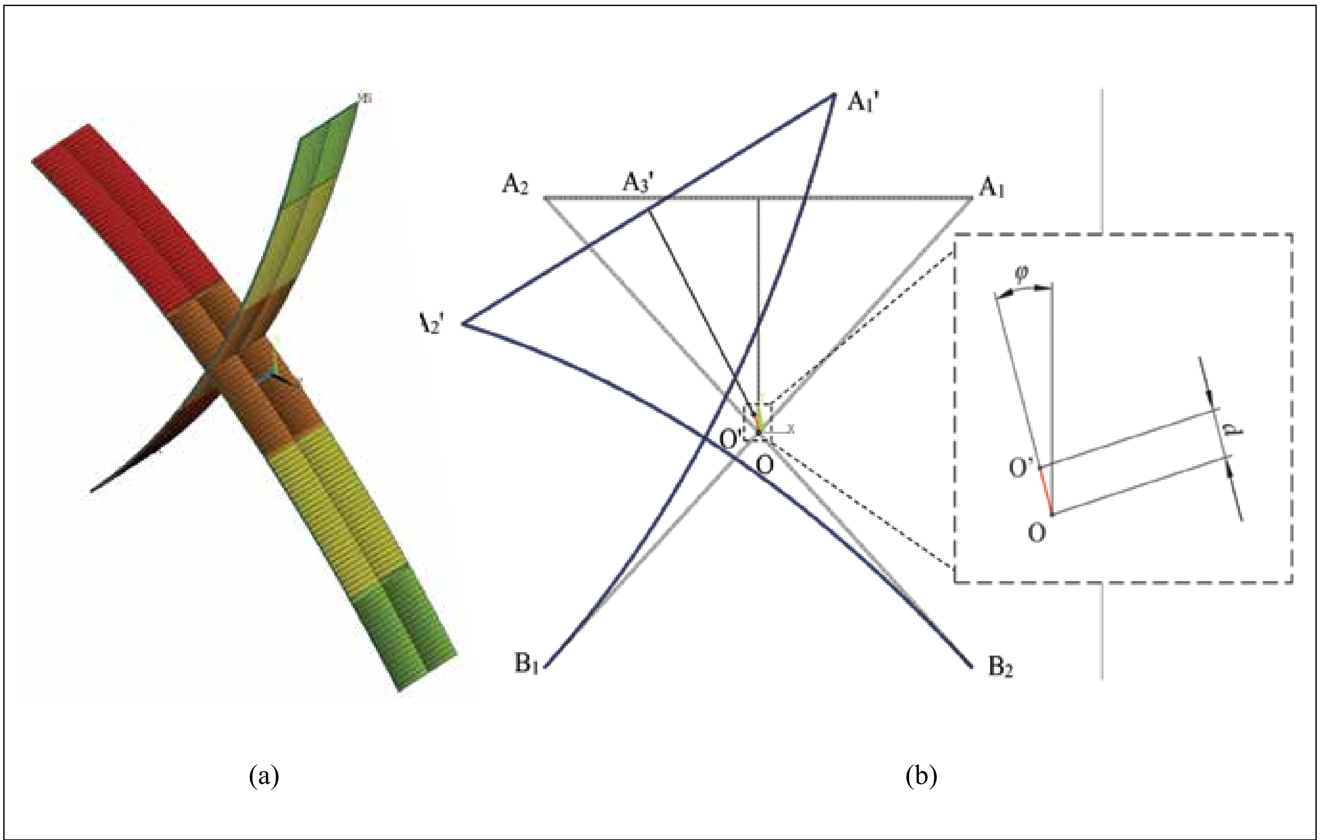


Fig. 3. Meshed FEA model (a) and parasitic motion of the pivot (b)

A typical pivot design configuration, where both leaf springs have the same length L, width b and thickness t, and they are made of the same material, is considered in this work. When loaded with a pure couple M, this configuration allows hence the movable block A to rotate, via the deflection of the leaf springs, with respect to the fixed block B. For larger rotation angles θ, the 'geometrical' centre of the pivot O moves, however, to O', giving rise to a parasitic shift of amplitude d and phase φ that is detrimental to the precision of the analysed mechanisms (Figure 2) [1-3].

By using the finite element analysis (FEA) approach, a numerical model is developed in the ANSYS software environment, enabling to perform nonlinear large deflection analyses of cross-spring pivots in the described loading condition. The considered geometry of the spring-strips in all the subsequent treatise is: L = 115 mm, b = 15 mm and t = 0.5 mm, 2α = 90°. Line elements (BEAM189), based on Timoshenko beam theory, are used to create the one-dimensional idealization of three-dimensional structure, since such an approach is computationally more efficient with respect to the one where solids and shells are used, while supporting also nonlinear analyses, including the effects of large (geometrically nonlinear) deformations [1]. The mashed model of cross-spring pivot is shown in Figure 3a.

As pointed out, the parameter that determines the rotational accuracy of cross-spring pivot is the parasitic shift of its geometric centre. The x and y components of the parasitic shift can be easily determined from the calculated shift of the free end O of a thin stiff beam attached to the movable block of the pivot as shown in detail in Figure 3b.

The usage of BEAM189 elements implies that the plain strain state is assumed. To enable a high-precision assessment of the parasitic shifts, the anticlastic curvature effect has, however, to be also taken into account. Anticlastic curvature occurs in the leaf springs in the transversal direction (i.e., along their width) and induces stiffening that is nonlinearly proportional to the deflection induced by bending. To duly consider this effect, in the equation of beam curvature the nominal value of Young's modulus E has to be gradually modified towards $E/(1-\nu^2)$, where ν is the Poisson's ratio of the spring-strip material. The dependence of the spring-strip stiffness on its bending has, thus, to be expressed in terms of a modified modulus, defined as [4-5]:

$$E' = \Phi E \tag{1}$$

where Φ, for an instantaneous curvature r of the spring (i.e., its bending curvature), and the herein considered geometrical configuration with the respective ratio b/t = 30, is shown in Figure 4. Values of Young's modulus are thus modified in accordance with Figure 4 and imported in the parameters defining the material model of leaf-springs in the FEA calculations. The resulting analysis showed that the influence of anticlastic curvature is not too large and, for pivot's rotations $\theta \leq 30°$, does not exceed 1 % [1].

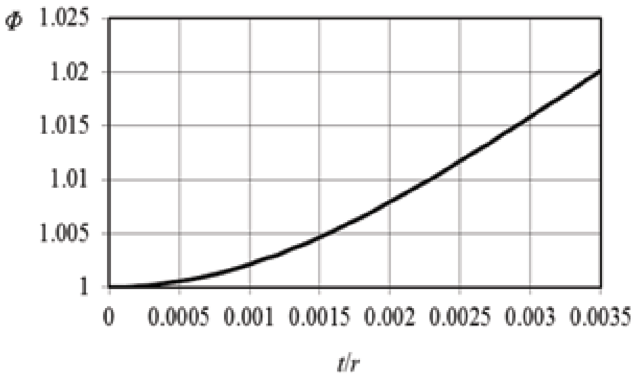


Fig. 3. Change of the transversal stiffness of the leaf springs vs. its instantaneous bending radius r

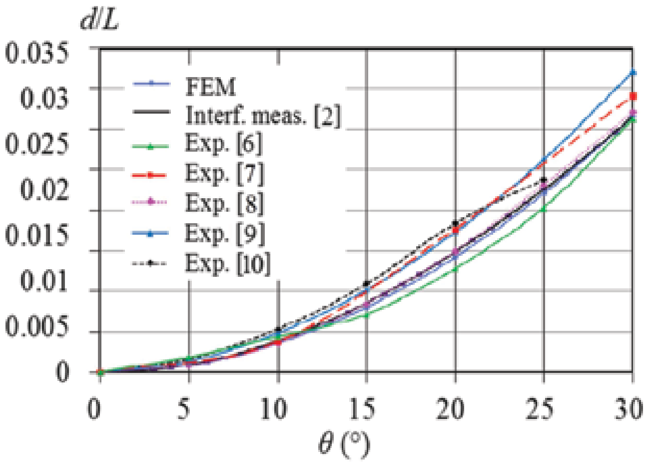


Fig. 5. Comparison of normalized parasitic shift values d/L obtained via experiments and by using the herein developed FEA model

To assess then the applicability of the developed numerical model in predicting the stress-strain behaviour of the considered class of mechanisms, results obtained via the developed FEA model, in terms of the calculated normalized parasitic shift amplitude d/L, are compared with the data of the experimental measurement reported in available literature [2, 6-10]. From the data depicted in Figure 5 it can thus be deduced that the results obtained via FEA are in excellent agreement to those attained via interferometric measurements [2], i.e., conducted by using a high-resolution measurement technology characterised by high accuracies and small intervals of uncertainty. In fact, the difference between these two sets of data is always smaller than 2 % throughout the considered range of rotations of the pivot ($0 < \theta \leq 30°$). The errors inherent in less accurate measurements result, in turn, generally in bigger differences with respect to both the interferometric measurement data and the results of the performed FEA calculations [1]. What is more, FEA results were shown also to be practically coincident to those obtained by a canonical Elastica-type approach of calculation of geometrically nonlinear deflections of spring-strips loaded with torques and forces of various orientations [11].

2. Influence of pivot's design parameters

The above analyses allowed verifying that FEA is suitable for modelling the behaviour of cross-spring pivots. With the goal of minimising the parasitic shifts and the variability of rotational stiffness, while taking into account also the stresses occurring in the pivots, the hence developed FEA model allows considering next alternative design configurations, i.e., varying the design parameters of the pivots. The considered variable design parameters are [1]:

- the angle α and the position of the intersection of the leaf springs (Figure 6),
- a monolithic configuration with the spring-strips joined in point O, and
- the effects of additional external loads of various orientations applied to the pivot.

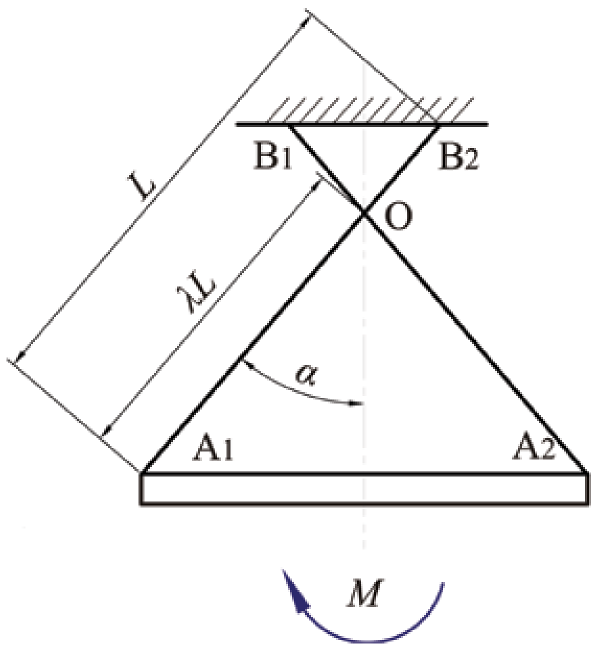


Fig. 6. Non-symmetrical cross-spring pivot configurations

In Figure 7 is hence depicted the influence of a change of the inclination of the leaf springs (i.e., the variation of the angle α) on the normalised parasitic shift amplitudes d/L (Figure 7a), on rotational stiffness $K = M/\theta$ (Figure 7b) and on the maximal stresses σ induced in the fixtures of the leaf springs (Figure 7c). It can be observed that an increase in the value of α and of pivot's rotation θ causes a rise of the values of the normalized parasitic shift amplitudes, an exponential increase of the rotational stiffness and an almost linear increase of the stresses in the clamped ends [1].

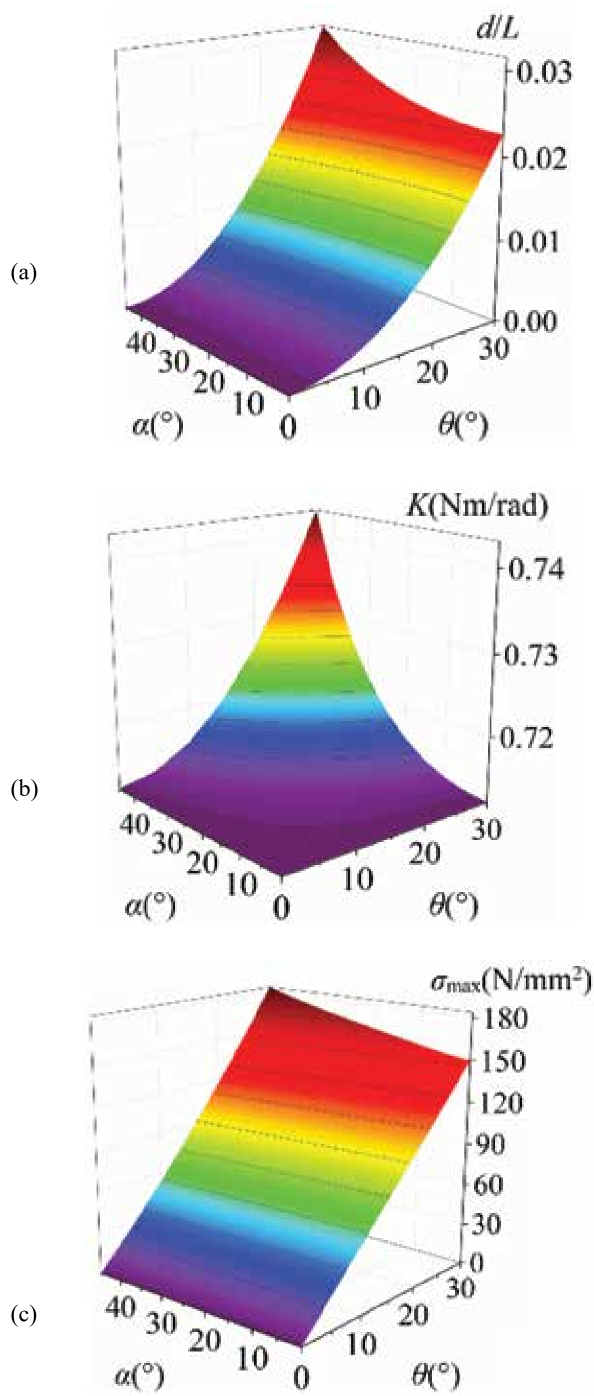


Fig. 7. Change of d/L (a), K (b) and σ (c) depending on the change of the inclination of the leaf springs α and pivot's rotation θ

When, in turn, the position of the geometrical centre of the pivot O, defined with the parameter λ (cf. Figure 6) is analysed, while $\alpha = 45°$, results depicted in Figure 8 are obtained. The thus induced variations of the normalised parasitic shift amplitudes d/L (Figure 8a), of the rotational stiffness K (Figure 8b) and of the stresses σ in the fixtures (Figure 8c), are again considered.

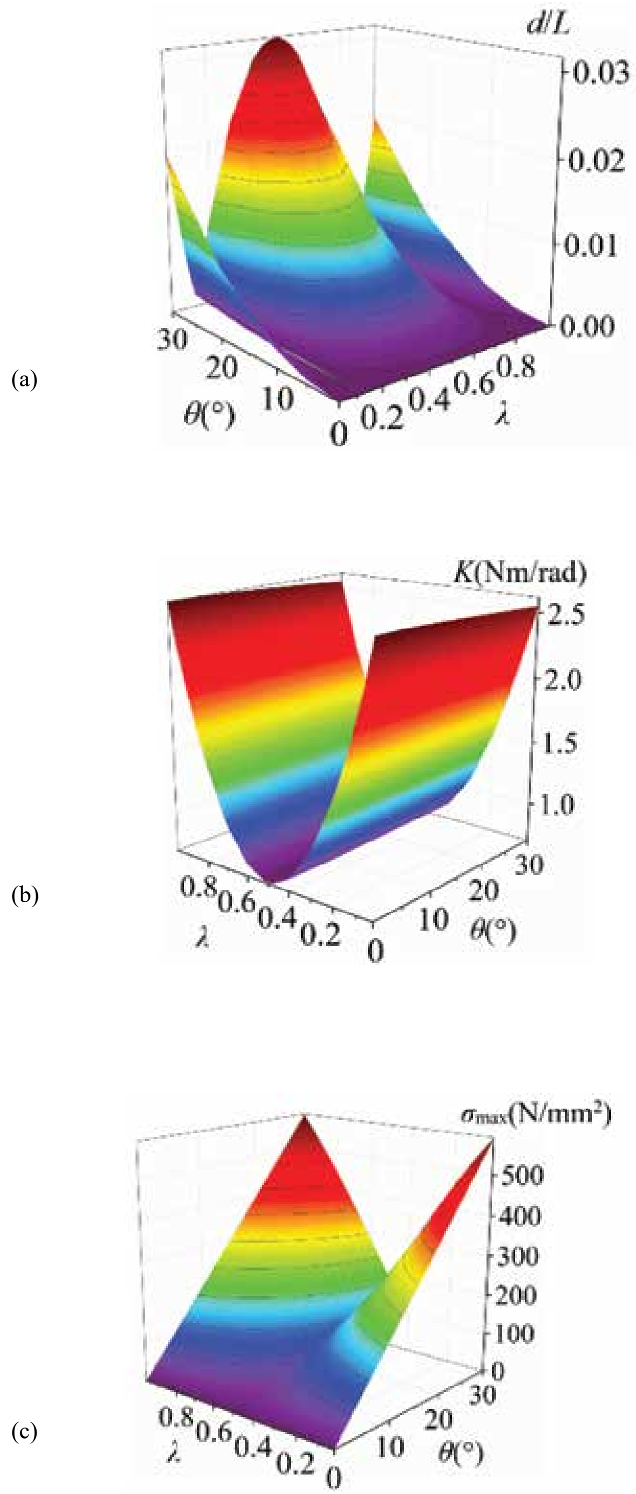


Fig. 8. Change of d/L (a), K (b) and σ (c) depending on the change of the position of the intersection of the springs λ and pivot's rotation θ

It can thus be concluded that a change of λ causes a substantial nonlinear variation of all the resulting parameters defining the behaviour of the cross-spring pivots. It is very important to note here, however, that, even for large pivot rotations θ, a design configuration for which $\lambda \approx 0.13$, results in negligible parasitic shift amplitudes, although at the expense of a large increase of rotational stiffness as well as of the stresses. Contrary to what is generally reported in literature, the performed nonlinear FEA calculations also allow the conclusion that the λ value for which d/L is minimised is not constant, but changes depending on the value of α in the range $0.127 \le \lambda \le 0.175$ [1].

When the monolithic configuration of the cross-spring pivot, with the spring-strips joined in their midpoints, is considered (Figure 9), results shown in Figure 10 are obtained. It is clear that in this case values of the normalised parasitic shift amplitudes d/L of up to about 10 times lower with respect to the previous cases are attained. This is achieved, however, at the expense of a 5-fold increases of the stiffness and of a 4-fold increase of the stresses with respect to the conventional cross-spring pivot configuration of Figure 1.

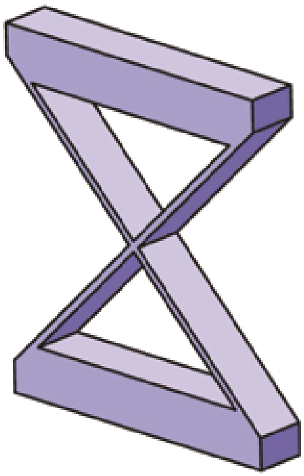
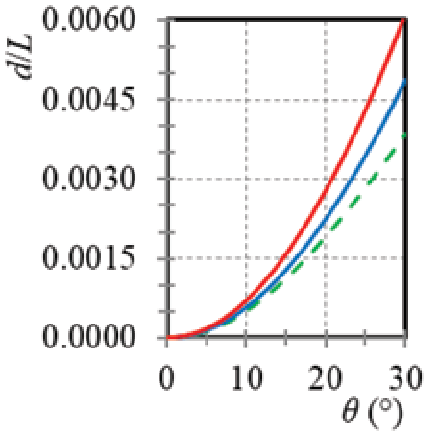
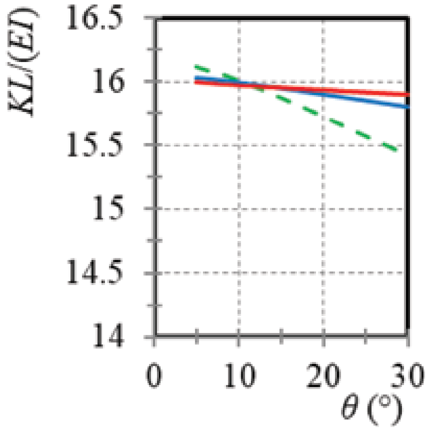


Fig. 9. Monolithic cross-spring configuration

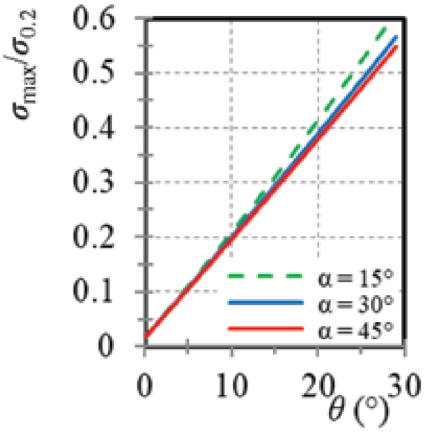


(a)

KL/(EI)



(b)



(c)

Fig. 10. Change of d/L (a), K (b) and σ (c) for the monolithic cross-spring pivot configuration depending on pivot's rotation θ for various α values

In [1] a composite pivot design configuration of Figure 11 is also considered. Although it allows diminishing even further the parasitic shifts with lower stress levels than in the monolithic pivot configuration, this configuration is certainly very complex from the technological point of view, so that its applicability is questionable.

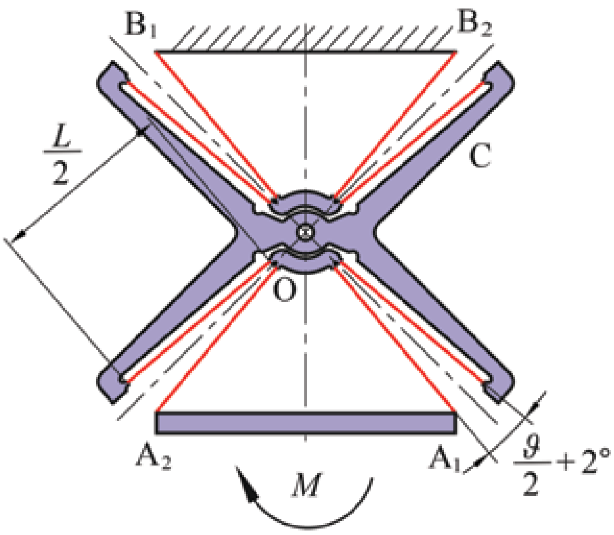


Fig. 11. Composite cross-spring pivot design configuration

The analysis of the influence of external loads on the variability of rotational stiffness and on the value of the parasitic shifts is finally performed [1]. The pivot is hence loaded as shown on Figure 12.

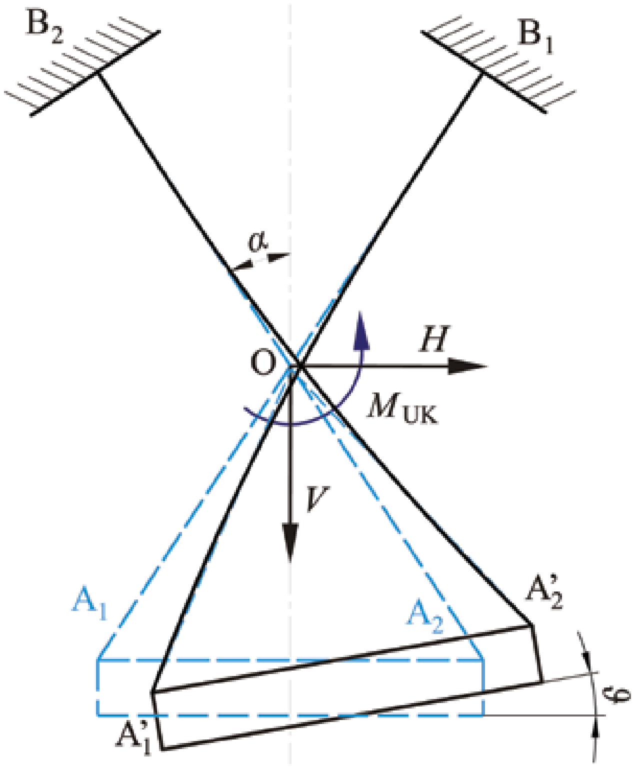


Fig. 12. Pivot loaded with external loads

Physically, the horizontal force H and the torque M superimpose, so the influence of H is taken into account as a contribution to M. The results of numerical analyses of the influence of the vertical force V loading the pivot, along with M, on rotational stiffness allow then evidencing that a compressive vertical force V_C narrows the stability range of the pivot (that where its rotational stiffness is positive), induces an increase of rotational stiffness but also a decrease of the parasitic shifts. On the other hand, the action of a tensile vertical load V_T results in a decrease of the rotational stiffness and an increase of parasitic shifts. When a variation λ is considered as well, V_T results, in turn, in widening the stability range (Figure 13a). A design configuration with $\lambda = 0.1$ allows thus attaining a very stable value of the rotational stiffness and, concurrently, small parasitic shifts, in the whole range where $V_T L^2/(EI)$ is smaller than 30 (Figure 13b), whereas the configuration with $\lambda \approx 0.13$ results in a stable rotational stiffness value in the range where $|VL^2/(EI)| \leq 10$ regardless of the vertical load orientation (Figure 13c) [1].

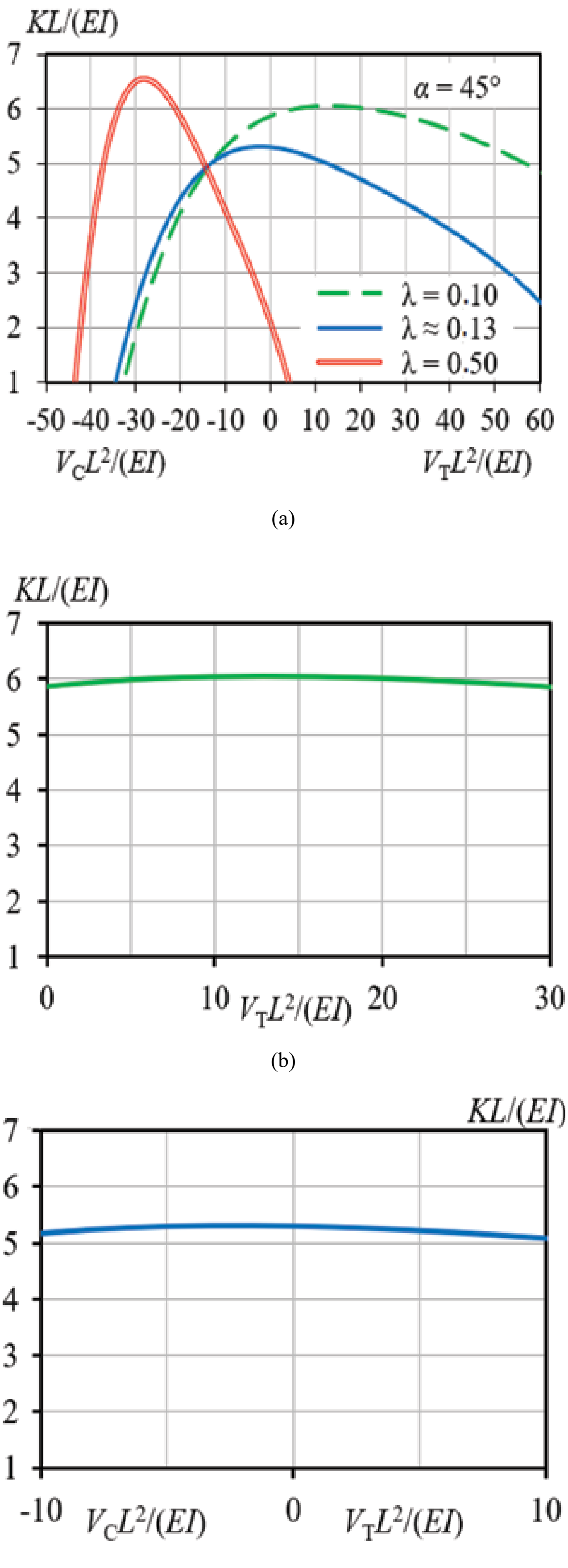


Fig. 13. Rotational stiffness dependence on vertical loads (a) minimized variation of rotational stiffness for $\lambda = 0.1$ (b) and $\lambda \approx 0.13$ (c)

3. Conclusions

The comparison of the results on the behaviour of cross-spring pivots aimed at ultrahigh-precision positioning applications obtained via the developed nonlinear FEA models to experimental data, performed in this work, allows confirming the validity of the FEA models even when large pivot rotations, inducing geometrical nonlinearities, are to be considered. Nonlinear FEA is hence used to attain quick, accurate and reliable results in thoroughly studying the influence of various pivots' geometric and loading conditions on the possibility to minimize the parasitic shifts and the variability of the stiffness of the pivots. Thus it could be established that an optimal design configuration will always be based on a compromise between configurations that allow improving some characteristic parameters of the pivots, while deteriorating, at least partially, some of the others.

A technologically easily achievable cross-spring pivot design with the value of the geometric parameter $\lambda \approx 0.13$ allows, then, attaining ultrahigh-precisions, as it is characterized by negligible parasitic shifts even for large pivot rotations, while concurrently guaranteeing the stability of the mechanism and the maintenance of the stress levels well within the allowable limits. The values of λ that allow minimising parasitic shifts depend, however, on spring-strips' inclination α, on the range of rotations θ, as well as on the external forces loading the pivot. A pivot design with $\lambda \approx 0.13$ and $\alpha = 45°$ is characterized by small parasitic shifts, but also by very limited variations of rotational stiffness. On the other hand, pivot configurations with $\lambda = 0.1$ allow achieving small rotational stiffness variations and small parasitic shifts for a rather large span of tensile vertical loads.

Simple and reliable cross-spring pivot design configurations with the values of the geometric parameter in the $0.1 \leq \lambda \leq 0.13$ range could, therefore, be applied in a broad range of ultrahigh-precision micropositioning applications such as, for instance, in the field of the production or of handling and assembly of micro-electromechanical systems (MEMS).

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