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The Mathematics-Natural Sciences Analogy and the Underlying Logic. The Road through Thought Experiments and Related Methods

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The aim of this paper is to point to the analogy between mathematical and physical thought experiments, and even more widely between the epistemic paths in both domains. Having accepted platonism as the underlying ontology as long as the platonistic path in asserting the possibility of gaining knowledge of abstract, mind-independent and causally inert objects, my widely taken goal is to show that there is no need to insist on the uniformity of picture and monopoly of certain epistemic paths in the epistemic descriptive context. And secondly, to show the analogy with the ways we come to know the truths of (natural) sciences.

Keywords: Thought experiment, epistemology, philosophy of mathematics, natural sciences, descriptive epistemic context.

1. *Introduction*

To endorse standard platonism in the philosophy of mathematics is not to be confined to platonic perception, as usually thought. In the same way, to defend other, non-standard, versions of platonism is not to be limited to some specific epistemic paths either. The aim of this paper is to show why this is the case and in which sense the plurality of epistemic paths in the domain of mathematics is analogous to the epistemic routes in the descriptive epistemic context given the domain of the natural sciences.

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Standardly, platonism in epistemology is "the view that mathematics is about objects of which we have a priori knowledge", where by "object" is meant a mind-independent entity. According to standard or Gödelean Platonism we gain mathematical knowledge via platonic perception.

Usually when the background philosophy, i.e. ontology is taken to be platonism, the epistemology is concentrated on the discussion about the existence of the platonic perception and those who endorse Platonism but not the Platonist perception, offer alternative epistemic routes, in order to avoid platonic intuition, taken to be a mysterious and unclear procedure of direct grasp of abstract objects. Given Benacerraf's argument against Platonism in epistemology, that is against the possibility of grasping truths concerning objective, abstract, non spatio-temporally located objects, other versions of platonism take other routes to have the epistemic monopoly (such as e.g. recarving in neo-Fregean platonism) or offer a variety of alternative routes to the platonistic intuition (e.g. ante rem structuralism).

In this paper the goal is to redirect the attention on two different points: the plurality of platonic epistemic paths and the analogy with the epistemic paths in the natural sciences.

I shall try firstly to show that there are more than one possible epistemic routes of gaining mathematical knowledge compatible with platonism (in the sense of grasping mathematical objects and their relations). Secondly, my aim is to show how it is possible to dig up other epistemic routes without giving up on platonism and without giving up the mathematics-natural sciences analogy.

Within the domain of standard platonism, the idea is hence to both dethrone platonistic intuition and situate it amongst other epistemic routes, and to show the analogy with the way we gain knowledge in the natural sciences. The focus will be on experiments, in particular on thought experiments.

2. *The historically oriented research*

The development of mathematical knowledge as well as the process of discovery in the (natural) sciences could be standardly analysed from different perspective: if could be analysed within the cognitive science oriented research, or within the historical oriented research, o a computationally oriented research, and so on.

In this paper I shall focus on the critical analysis of the mathematical descriptive epistemic paths as well as the epistemic mathematics-natural sciences analogy through the prism of the historically oriented research.

Even though it might come as a surprise, given that platonism is here taken to be the underlying ontology, I take the accepted methodology for epistemology of science and mathematics to be (Kitcher's) pragmatic naturalism, in particular his view that we ought to look at the history in order to determine the epistemology since "history is the teacher of epistemology" (italics mine). The underlying idea is hence that the epistemological route follows the historical one, and that "the epistemological order of mathematics broadly recapitulates the historical order." Even though one of the tenets of pragmatic naturalism is the denial of a priori knowledge (in the domain of mathematics as much as elsewhere), here the idea is rather merely to dethrone platonistic intuition and situate it amongst other epistemic routes. And the justification for doing so comes from history itself, which offers reasons for endorsing platonist intuition as much as other epistemic modes and that thus ironically ends up as a turn-the-table for Platonism. The importance of the historical analysis is threefold: it firstly justifies the endorsement of the platonic perception, secondly it justifies the plurality of epistemic modes in gaining mathematical knowledge, and finally it justifies the endorsement of the mathematics-natural sciences analogy.

When talking about different epistemic modes in grasping mathematical objects, the mathematics-science analogy it's imposing itself to us and turns out to be particularly strong in such descriptive epistemic context. Let us have a look at such mathematics-science epistemic analogy in more details.

3. *Three modes of epistemic access and the mathematics-(natural) sciences link.*

I shall propose three main modes of initial epistemic access to both mathematical and scientific reality (objects and properties): (1) *Perception: Visual and Platonic*, (2) *Experiment* and (3) *Introduction (or hypothetical positing) and positing (or categorical positing) of objects*.

The epistemological science-mathematics analogy turns out to be overall, each epistemic path in science having its counterpart in mathematics. The plenitude of such paths is (to be) determined and classified by looking at the history of science, that is mathematics.

Let us hence have a look at the mentioned epistemic paths in the given order.

(1) *Perception: Visual and Platonic*

In scientific research, one epistemic way is sensory, primarily visual, direct perception of objects and phenomena. The analogy in mathematics would be the platonic/"pi in the sky", direct access to the mathematical objects and statements, often called platonic perception/intuition. When talking about it, J. R. Brown points out:

> The main idea is that we have a kind of access to the mathematical realm that is something like our perceptual access to the physical realm. This doesn't mean that we have direct access to everything: the mathematical realm may be like the physical where we see some things, such as white streaks in bubble chambers, but we don't see others, such as positrons. (Brown 1999: 13)

The platonic intuition—visual perception analogy is something standard platonists traditionally heavily insist on. Brown says:

> Just as the mathematical mind can grasp (some) abstract sets, so the scientific mind can grasp (some of) the abstract entities which are the laws of nature. (Brown 1991)

Gödel in particular famously insists on the analogy while saying that

> the assumption of such objects is quite as legitimate as the assumption pf physical bodies and there is quite as much reason to believe in their existence. They are in the same sense necessary to obtain a satisfactory system of mathematics as physical bodies are necessary for a satisfactory theory of our sense perceptions. (Gödel 1944: 456f)

and, in one of the most famous quotations in the philosophy of mathematics, that

> despite their remoteness form sense experience, we do have something like a perception also of the objects of set theory, as is seen from the fact that the axioms force themselves upon us as being true. I don't see any reason why we should have any less confidence in this kind of perception, i.e. mathematical intuition, than is sense perception. (Gödel 1947: 484)

Such an analogy has been criticised by many, and for several reasons. Apart from being unnatural and forced, the analogy seems to take the existence of the platonic perception for granted while in effect it is most contentious. It hence has been heavily criticised by both platonism's friend and foes. Kitcher remarks that

> …what some mathematicians call "intuition" or even (in the case of Ramanujan) the visitation of the goddess (Namakiri), *can* be explained as 'fine-tuned abilities […] rooted in extant mathematical practice. (Kitcher 2011)

While Shapiro, who is endorsing *ante rem* structuralism (a version of non-standard platonism) follows the same line as Kitcher's when underlying that

> …the axioms do not force themselves on a first (or second, or third) reading. For virtually any branch of mathematics, the psychological necessity of the axioms and inferences, and the feeling that the axioms are natural and inevitable, comes only at the end of a process of training in which the student acquires considerable practice working within the given system, under the guidance of teachers. (Shapiro 1997: 212)

How to respond to such criticisms?

The main point to be underlined is that mathematical intuitions are not just theoretical presuppositions of the philosophers of mathematics but are being asserted by working mathematicians themselves (Cantor, Gödel, Ramanujan, Hardy etc.). We hence have good reasons to transpose this fact from the history of mathematics, right into the core of our epistemology. Kitcher's main point being that what is explained as platonic perception could easily be explained by invoking the mathematicians' "fine-tuned abilities" that are "rooted in extant mathematical practice", instead of being some mysterious faculty of the mind. Wanting to have a closer look at Kitcher's remark, and to see what such “fine-tuned abilities” wold amount to, we are to face a dilemma. Namely, if what is meant by “fine-tuned” is just the perfect conformity to the extant practice in the profession, it is difficult to see how Gödel’s non-conformistic example, not to speak about Ramanujan, would fit in the picture. If, on the other hand, fine-tuning refers to an impressive ability to reach the truth beyond the extant research paradigm, then it certainly is compatible with the platonistic account on which such a fine-tuning (to the mathematical reality) might culminate in intuitive insights.

(2) *(Thought) Experiment*

Experiments have been usually perceived as the lynchpin of empirical sciences, a method for discovering the facts of nature and hence as belonging to the sphere of practical research. Mathematics being an armchair activity—how can the mathematical domain be related to any experimental epistemic route?

I am using the entry from Stanford Encyclopedia to provide a mainstream characterisation of the role of experiment:

> Physics, and natural science in general, is a reasonable enterprise based on valid experimental evidence, criticism, and rational discussion. It provides us with knowledge of the physical world, and it is experiment that provides the evidence that grounds this knowledge. […] It can also call for a new theory, either by showing that an accepted theory is incorrect, or by exhibiting a new phenomenon that is in need of explanation. Experiment can provide […] evidence for the existence of the entities involved in our theories. Finally, it may also have a life of its own, independent of theory. Scientists may investigate a phenomenon just because it looks interesting. Such experiments may provide evidence for a future theory to explain. […] a single experiment may play several of these roles at once. (Franklin and Perovic 2016)

The standard taxonomy, when talking about experiments, includes the distinction between confirmatory (or demonstrative) on one hand and the exploratory experiments on the other. The former having the goal of testing theories, while the latter has as the primary goal the experimentation that is not guided by hypotheses but it rather a process or searching.

My aim, at this point, is to show that, no matter which of the two main sub-species of the experiment we prefer to concentrate on, either the confirmatory or the exploratory (non-demonstrative) one, the analogy with the mathematical case holds throughout.

If talking about the confirmatory experiments, there are examples from the mathematical practice that could be treated as examples of such experiments. Let us here mention the proof that number π is irrational. The number π has been studied for centuries (since ancient time) and so was the notion of irrational numbers. Aryabhata apparently hinted at number π being irrational in 500 CE. Such an outcome was accepted as a new mathematical result not prior to the 18th century when Lambert (in 1761) proved it to be irrational. And then again, in the same paper in which he proved π's being irrational, Lambert conjectured that number π is transcendental too, which was accepted in mathematics in 1882, when proved by Lindemann.

Other mathematical results and proofs are analogous to the exploratory, non-demonstrative experiments. In such experiments the experimentation is not guided by hypotheses. An example in mathematics could be the problem of trisection of an arbitrary angle. The attempts to solve the problem, can be seen as an exploratory experiment that had been going on for centuries.

Notwithstanding the mentioned mathematical examples, when comparing the experiments in science with those in mathematics, we might still find the proposed analogy implausible. And basically for two reasons: (a) experiments in science are practical procedures, done in laboratories, unlike in the mathematical domain, and (b) if anything, given the possibility to directly intervene on the objects in scientific experiments, which is not possible in the mathematical domain given the abstract nature of mathematical objects (and hence their being causally inert). Let us have a closed look at the possible replies at the two just-mentioned reasons.

(a) Experiments in science are practical procedures, done in laboratories, unlike those in the mathematical domain. Well, ought experiments to be practical in the first place? When thinking about, e.g. hight school experimentation, than we all have in mind the paradigmatic example of the laboratory and the practical procedures that we were performing there during the natural-sciences classes. The taxonomy however includes three types pf experiments: the real, the imaginary and the thought experiments. The real ones are those that have been performed, the imaginary are those that haven't been formed but could have been, while the thought experiments are those that those that could not be performed due to the lack of technology or because impossible in principle.

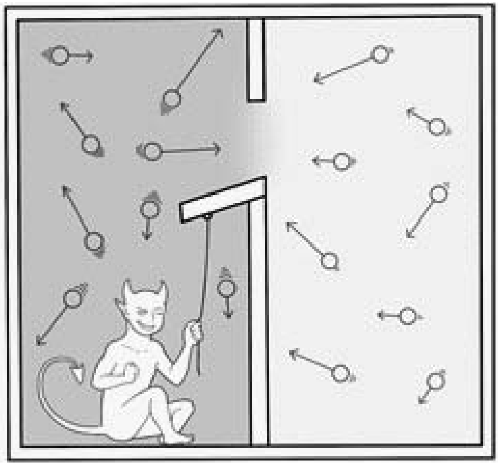
And when we look at the way experiments have been perceived by scientists through history, there is no uniformity of picture; not even a general agreement on experiments being real-world, practical methods for acquiring knowledge. Even in Galileo's writings the distinction between real and imaginary experiments is not a sharp one and it is, for some experiments, a contentious issue. Newton's bucket experiment is another example of an experiment that was originally an imaginary experiment but needn't be. What about thought experiments? Such experiments apparently played a major role in the development of scientific theories in the work of Galileo, Newton, Einstein, Heisenberg. Examples are legion: Galileo's experiment with the result that all bodies fall at the same speed, Schrödinger's cat, Maxwell's demon, Einstein chasing a light beam, Twin paradox and others.

Still—and here we shall focus on the above remark (b)—what happens in experimental science might seem at first sight to be remote from the standard mathematical practice.

If anything given the abstract nature of mathematical objects, i.e. the fact that they are not spatio-temporally located and are causally inert. Any intervention/manipulation on the objects of the domain fails to be possible in the case of mathematical experiments, and on abstract objects involved. So, while we can, to take the morally controversial example of tissue engineering—the Vacanti mouse, implant under the skin of a mouse a cartilage structure (and then the cartilage naturally grows by itself), is not clear what the counterpart of such a direct manipulation of objects in the case of mathematical experiment would amount to.

Could we, figuratively speaking, have a Vacanti number or a Vacanti geometric figure? Certainly not! Should we (again) infer from that that the analogy is, to put it blandly, farfetched and artificially imposed? Not so fast.

Namely, it is difficult to guess in which way we literally manipulate (concrete) objects in thought experiments. We actually do not. We can hence talk about experiments without presupposing any kind of direct manipulation of concrete objects. Let take the example of one of the most famous thought experiments: "Maxwell's demon". According to the Second law of thermodynamics, in any change of state entropy must remain the same or increase; it cannot decrease. In laymen's terms, heat cannot pass from a cold to a hot body. Maxwell's goal in the experiment is to show that the Law is to be read in probabilistic terms. which means that, in principle, it could be possible for the heat to pass from a hot body to a colder one. I order to show how this could be possible, Maxwell imagine to have two connected boxes, which the Devil at the door that connect those two boxes (see picture). The two boxes contain some gas, and in particular the gas in the left box is hot, while the gas in the right box is cold. What is to be expected, according to the Second law of thermodynamic, is for the heat to pass from the hot gas to the cold one. But, during the experiment, the Demon decides to let the fast molecules from the cold box into the hot box, and the slow molecules from the hot box into the cold one. By letting the fast molecules from the cold box into the hot box, and the slow molecules from the hot box into the cold one, there will be an increase in the average speed in the hot box and a decrease in the average speed of molecules in the cold. Since, on Maxwell's theory, heat is just an average speed of the molecules, there has been a flow of heat from the cold box to the hot one—contrary to what is expected according to the Second law of thermodynamic. Hence, the Law—and that is precisely Maxwell's point—has to be interpreted probabilistically.



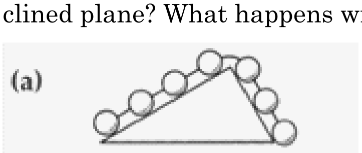
Maxwell's demon experiment.
Illustrated by Maja Grčki.

Let us further focus on some aspects of the analogy and the meeting points between the thought experiments in the natural sciences and some of the basic mathematical procedures. We shall analyse in more details the process of representations of abstract objects and the sameness of structure of some thought experiments in science with the *reduction ad* absurdum structure in mathematical proofs.

Let us start with some properties that the representations of abstract objects share both in the natural sciences and in mathematics.

The non-concrete objects which we (mentally) "manipulate" during imaginary or thought experiments are often related to their spatiotemporally counterparts. And the way these two kinds of objects are related might be analogous to the way in which representations of abstract objects—the subject of manipulations in mathematical experiments—are related to the abstract (mathematical) objects, i.e. their abstract counterparts. In the case of the trisection of an arbitrary angle, we do manipulate the representation of an abstract geometrical entity.

Thought experiments in the natural sciences can also share the same structure of standard proof methods in logic and mathematics. Let us take the example of Stevin's thought experiment. As well known, there are three possible planes: the horizontal, the vertical and the inclined plane. If we put a weight on each of these planes than we already know that on the horizontal plane the weight remains at rest, while on the vertical plane the wight freely falls. What about the inclined plane? What happens with a weight if put on an inclined plane?



Suppose we have a chain with weights and we put it on an inclined plane (picture (a)). How does the chain move? Well, there are obviously three possible answers:

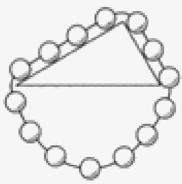
(1) it remains at rest (in so called static equilibrium)
(2) it moves to the left
(3) it moves to the right.

The right answer is (1), it remains at rest. The next step is to prove it!

Let us suppose not–(1) (notice the *reduction ad absurdum* structure of the proof!)

If not–(1), it means that the force of the left is not balanced by the force in the right. Let us now add the links at the bottom so to get a closed look (picture (b))

(b)



If not–(1) were the case, the look would rotate and hence, we would get a *perpetuum mobile*, which is impossible. Hence, the chain remains at rest.

The analogy is better presented in the following table:

Thought experiments in science as (quasi) RAA (*reductio ad absurdum*) proofs	
example: Stevin's thought experiment	
RAA proving method	structure of Stevin's thought experiment
A—the statement that we want to prove	**A**—the chain remains at rest
suppose not-A	**suppose** the chain does not remain at rest (**not-A**)
not-A leads to contradiction	if the chain moved (**if not-A**), we would **then** have a *perpetuum mobile*—**impossible**!
hence, A	**hence**, the chain remains at rest (**A**)

Table showing Stevin's thought experiment having the structure of the reductio ad absurdum deduction rule

Why the "quasi" in the "Thought experiments in science as (quasi) RAA (*reductio ad absurdum*) proofs" title? Well, the *reduction ad absurdum* method of deduction require the initial hypnoses to lead to *absurdum*, which means to (logical) contradiction, i.e. to a statement of the form A and not-A. Technically, a *perpetuum mobile* is not a logical impossibility but a nomic one. Hence it is not a (logical) *absurdum*. However, the idea is here that Stevin's proof and any *reductio ad absurdum one* (in logic or mathematics) do share the same structure (see the above table).

The natural science-mathematics analogy, rather surprisingly, holds even if we decide to concentrate on *real* experiments in science instead of imaginary and thought ones. Such experiments are, in fact, in many aspects similar to some examples of manipulative procedures/proofs in mathematics. Let us have a look at two examples.

	Mathematics	Natural sciences
Abstract object	✔ e.g. an angle	
Concrete object (spatio-temporally located)		✔ e.g. a molecule
Representation	 representation of an angle	 representation of a (serotonin) molecule

The mathematical one is Lakatos' historical proof case, while the one in the natural sciences is Hooke's observations made with microscope. We easily notice the same dynamic language used in both cases, that is in both domains. (See the table below.)

Mathematics domain Lakatos' historical proof case V–E+F=2, for any polyhedron	**Natural sciences domain** Hooke's observations made with microscope discovery of cells
Dynamic language	Dynamic language
– **take** an arbitrary polyhedron	– **take** a small piece of e.g. onion
– **remove** one of the faces	– **remove** (peel off) the membrane
– **stretch** the remaining figure out on flat surface	– **stretch** the part you want to analyse on a glass
– **remove** the lines one at the time, etc.	– **add** a few drops of water or solution, etc.

My conclusion at this point would be that there is a strong analogy between the experiment in science and some of the central procedures in mathematics. However, experiments in science and mathematics represent just one possible epistemic path in gaining knowledge. Another one is the positing of objects, which can be either hypothetical or categorical.

(3) *Introduction (or hypothetical positing) and positing (or categorical positing) of objects*

In science and mathematics there are objects that we introduce in our theories in order to make the overall theory complete and/or to explain the appearance of discrepancies, And there again, the mathematics-science analogy enters the picture.

An example in science might be the discovery of the existence of the planet Neptune. After the discovery of Uranus, it was noticed that its orbit was not as it should be in accordance with Newton's laws, which led the astronomers to the assumptions that a (still to be discovered) planet might be the cause of the discrepancy. Astronomers hence predicted the position of an unobserved planet perturbing the orbit of Uranus.

In mathematics a nice example of introducing objects is the one concerning the methods of solution of the quadratic and cubic equations. Negative square roots appeared in Cardano's *Ars Magna* (1545) that contains the first occurrence of complex numbers. Cardano introduced negative square roots as solutions of the quadratic equation $x^2-10x+40=0$. Since it was evident that solving the equation was impossible (in $\mathbb{R}$), Cardano decided to formally introduce the negative square roots. As the existence of the planet Neptune was first predicted by mathematical calculations and then empirically detected, i.e. seen by a telescope which gave the prediction the ontological force, analogously the complex numbers were first introduced formally and then, 300 years later, a more specifying meaning was attached to them. The strategy of introducing planets in the astronomy case corresponds to the formal introduction of complex numbers in the mathematical case. Similarly, in the following step, the astronomers pass from the hypothetical assumption to the categorical claim concerning the existence of the planet, while in the mathematical case, the analogous step goes from the formal introduction of negative square roots to the full-blown positing of new, complex numbers.

4. *The mathematics-natural sciences analogy. More aspects concerning the underlying logic*

One possible reaction at this point might be: when talking about the mathematics—natural sciences analogy, is there an insurmountable difference in methodology? Possible complaint: the reasons for asserting the analogy between mathematics and the natural sciences in the descriptive epistemic context are marginal. Namely, there is a (much more) essential parameter that should be taken into consideration and that might make the difference between the epistemic paths in maths and the natural sciences come to surface: the underlying methodology.

When referring to the underling methodology, i.e. logic, the standard view is that in maths, unlike the natural sciences, the basic methodology is the axiomatic-deductive method (of the geometric tradition). Contrary to that, in the natural sciences, the logic underlying the research is primarily inductive/abductive. It implies that the two domains are profoundly methodologically different given the difference at the core, that is at their underlying logic. To that remarks, I find the most plausible reply to be the following one.

Proofs/theorem/theories at the final stage (textbook) do not coincide with the heuristics (in the sense of the epistemic paths within the context of discovery). The structure of the polished theory and the underlying deductive system do not however correspond to the research process in the epistemic descriptive context.

Lakatos nicely underlines the difference between the historical development of mathematical results and the procedures we find in mathematics textbooks. Such a difference amounts to the difference between the *preformal* development (correlates to the context of discovery, i.e. the epistemic descriptive context we've been focused) on and the *formal* articulation (corresponding to the context of justification) of a branch of mathematics by offering reasons for asserting that preformal proofs are not simply drafts of the formal ones but rather heuristic explanatory and exploratory tools having a development on their own.

A very simple yet illuminating example is the one Pólya presents in his *How to Solve It* (Pólya 1945: 114–117). Let us have a closer look at it. And let us start by supposing that a mathematician is helping their child to write the homework in mathematics, and at some point the child is supposed to calculate 1+8+27+64 and solve it rightly by writing the result: 100. While waiting for the child to solve the exercise, the parent/mathematician notices that all four of the numbers/addends are cubs while the result (100) is a square. So that it is possible to write the mentioned equation in the form: $1^3+2^3+3^3+4^3=10^2$. He also notices that the mentioned sum is the sum of the cubes of the first four natural numbers. And then ask himself if it is a coincidence or it is not an isolated case to have the sum of the cubes of the first n natural numbers to be equal to a square. Pólya comments such a situation by comparing the parent/mathematician with the naturalist:

> In asking this,[1] we are like the naturalist who, impressed by a curious plant or a curious geological formation, conceives a general question. Our general question concerns with the sum of successive cubes $1^3+2^3+3^3+4^3+\ldots+n^3$. We were led to it by the "particular instance" n=4. (Pólya 1945: 115)

How would the mathematician procede at this point? What would he do? Pólya's answer is that the mathematician would do what the naturalist would do—investigate other special cases! And realise that:

$1^3=1^2$

$1^3+2^3=9=3^2$

$1^3+2^3+3^3=36=6^2$

$1^3+2^3+3^3+4^3=100=10^2$

$1^3+2^3+3^3+4^3+5^3=225=15^2$

…

[1] Pólya here refers to the question as to whether it is a coincidence or a general rule that the sum of the cubes of the first n natural numbers is equal to a square.

The mathematician might subsequently notice that the results on the right side of the equations, i.e. the squares, follow a regularity, a certain patter too. Namely:

$1^2=1^2$
$3^2=(1+2)^2$
$6^2=(1+2+3)^2$
$10^2=(1+2+3+4)^2$
$15^{2=}(1+2+3+4+5)^2$

…

Interestingly enough, the sum of the cubes of the first *n* natural numbers is equal to the square of the sum of the first *n* natural numbers. Given that this regularity seems to be general too, the assertion finally obtains the form:

$1^3+2^3+3+\ldots+n^3 = (1+2+3+\ldots+n)^2$

This initial procedure, as pointed out by Pólya, is based on observation and induction and as such corresponds to the procedures of investigation in the natural sciences where the naturalist "may also reexamine the facts whose observation has led him to his conjecture; he compares them carefully, he tries to disentangle some deeper regularity, some further analogy" (Pólya 1945: 116).

> In mathematics as in the physical sciences we may use observation and induction to discover general laws. But there is a difference. In the physical sciences, there is no higher authority than observation and induction but in mathematics there is such an authority: rigorous proof. (Pólya 1945: 117)

The difference Pólya is referring to is however beyond the scope of this article. This idea of the mathematics-natural sciences analogy is meant to be confined to the epistemic descriptive context, while the disanalogy enters the picture in the context of justification, which I am not addressing in this paper.

To summarise, in this paper I have taken the underlying ontology to be a version of standard platonism. I choose not to refrain from endorsing the platonic perception as one of the possible epistemic paths and hence from endorsing a version of standard platonism since I have hopefully showed that platonic perception is not to he banned from the epistemology of mathematics domain given that we do have good reasons for endorsing it.

I have then argued that, in the domain of mathematical entities and within the descriptive epistemic context, there is however a plurality of platonic epistemic paths and that such paths in the mathematical domain are analogous to the epistemic paths in the natural sciences.

Last but not least, I analysed the mathematics vs. natural sciences analogy from the perspective of the underlying logic. I have claimed the importance of keeping in mind the distinction between the context for discovery and the one of justification. The former being correlated with the so called *preformal* development of statements or theories in mathematics, while the latter being connected with the *formal* articulation of branches of mathematics. When concentrating on the mathematics-natural sciences analogy and the underlying logic, it is important to take into consideration that the analogy holds in the context of discovery, in which the *preformal* development plays the major role. Such a development is based, both in mathematics and in the natural sciences, mostly on induction. It is certainly true that in mathematics the basic logical apparatus is deduction and mathematical induction (and that differs from the logical apparatus used in the natural sciences). It is however crucial to take into account that the logical apparatus based on deduction enters the picture not before we take into account the context of justification. The context of justification, however, being outside the scope of this paper.

Hence—to conclude—if the claims about analogy hold ground and I hope that they do, they vindicate both a pluralist view on the epistemology of mathematics and a thorough analogy between the epistemic paths in mathematics and in the natural sciences (given the descriptive epistemic context). Given that the underlying ontology is taken to be (a version of standard) Platonism, the presented mathematics-science epistemic analogy will hopefully offer a new perspective in the platonistic epistemology debate.

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