Conjoining and the Weak/Strong Quantifier Distinction

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Pietroski’s model of semantic composition is introduced and compared to the standard type hierarchy. Particular focus is then given to Pietroski’s account of quantification. The question is raised of how the model might account for the weak/strong distinction in natural language quantification. A number of options are addressed and one proposal is tentatively recommended.

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1. Introduction

Pietroski (2018) offers a sparse theory of natural language semantic composition. According to the theory, the meanings of lexical items and syntactic structures are ‘instructions’ to fetch concepts, and such concepts are combinable by just two operations that allow for the construction of denumerably many complex concepts (‘sentence meanings’). Pietroski does not affect to have solved all problems, or even to have offered a complete framework. His goal, at least as I read him, is to show how semantics might be done in a new way unburdened by many of the assumptions of the truth-conditional tradition, both in terms of compositional technology and philosophical baggage. I applaud the endeavour. The open question is how well the framework serves to capture the gamut of phenomena traditional theories target, and whether it offers novel insight. As a methodological precept, I think it is invidious to hold new frameworks to higher standards than we ask traditional ones to meet. In other words, one should always ask whether the traditional accounts are really so successful according to whatever objective standards are appropriate, for standards of explanation are often fixed by the received theoretical framework at issue. Such considerations
are particularly germane to the present case, as we shall see. Traditional semantics marries a highly expressive meta-theory with mostly descriptive desiderata. Its success at descriptive coverage, therefore, flows from the expressive resources that stand in need of explanatory justification. It is to Pietroski’s great credit that he seeks to show how explanatory traction can be achieved by such minimal resources. These morals will be given some substance in the second section.\footnote{For further discussion of these broad issues, see Collins (2020).} Thereafter, my concern will be for how Pietroski’s framework might explain the weak/strong distinction in the domain of quantification. I have no settled answer here, but do examine a few options. At any rate, the distinction is an interesting topic for further inquiry into the framework on offer.

2. Sparseness

Pietroski’s model is sparse because it eschews the ‘full Frege’ of semantic types:

(FF) (i) $e$ and $t$ are types.
(ii) If $[<x>]$ and $[<y>]$ are types, then $[<x, y>]$ is a type.
(iii) These are all the types.

This gives us denumerably many types to map onto lexical items and their composition into phrases:

(T) (i) $<e, t>$: monadic predicates, such as predicative adjectives, relative clauses, and intransitive verbs ($\text{red}$, $\text{sleep}$, etc.)
(ii) $<e <e, t>>$: dyadic relations, such as transitive verbs ($\text{loves}$, $\text{kicks}$).
(iii) $<e, t>, <e, t>, t>$: 2-place determiners ($\text{every}$, $\text{most}$, etc.)
(iv) $<e, t>, t>$: determiner phrases ($\text{every man}$, etc.).
(v) $<e, t>, <e, t>$: attributive adjectives ($\text{red in red car}$, etc.)
(vi) $<t, <t, t>>$: dyadic sentence connectives ($\text{and}$, $\text{or}$, etc.)
(vii) $<t, t>$: sentence-level adverbs ($\text{necessarily}$, $\text{possibly}$, etc.)
(viii) $<<e>, <<e, t>, <e, t>> >$: prepositions, understood as VP modifiers ($\text{about}$, $\text{to}$, etc.)

In place of (FF), Pietroski presents just two compositional types, with primitive lexical content being virtually wholly monadic, and syntactic structure introducing restricted dyadic relations. Crucially, the model does not involve denumerably many non-applicable types as does the full Frege, i.e., types that do not correspond to any linguistic structure. The model does help itself to the ‘full Chomsky’ of syntactic structures, but that is independently required, unlike the Fregean hierarchy.
3. Two types

The first operation conjoins two predicates (simple or complex) and identifies a shared single argument position:

\[(M-Join) \Phi(_\ ) \land \Psi(_\ )\]

Don’t think of the gaps here as variables, but simply as a way of specifying the adicity of both the constituent and complex types, where the blanks are read as identical (having a co-application). Thus, it does no harm to render the result of M-Join as ‘\(\Phi \land \Psi(_\ )\)’. For example, (1a) has the expected interpretation:

(1) a BROWN(_\ ) \land COW(_\ )

b ‘BROWN(_\ ) \land COW(_\ )’ applies to \(e\) iff \(e\) is brown and a cow.

The second operation introduces a dash of dyadicity:

\[(D-Join) \exists[\Delta(..., _\ ) \land \Psi(_\ )]_2\]

The dyadic concepts correspond to, for example, prepositions (above, with, etc.), and also, thematic concepts. So,

(2) ‘\(\exists[\text{PATIENT}(..., _\ ) \land A-BROWN(_\ ) \land COW(_\ )]\)’ applies to \(e\) iff \(e\) involves a brown cow being affected.

The theory does not tell us that \(e\) ranges over events, as if we had an independent understanding of what events are; rather, semantically, we simply take such a complex concept as applying to ‘things’ that can have participants being affected, where such participants are the kind of things that can the applicands of monadic concepts that we can M-Join to the results of D-Join.

(3) CHASE(...) \land \exists[\text{PATIENT}(..., _\ ) \land A-BROWN(_\ ) \land COW(_\ )]

This gives us the content for the VP chase a brown cow (forget about the determiner, which complicates the presentation for my purposes).

(3) above gives way to

(4) CHASE(...) \land \exists[\text{INTERNAL}(..., _\ ) \land A-BROWN(_\ ) \land COW(_\ )]

That a brown cow is construed thematically as PATIENT is a property determined by the verb chase, not the very syntax or the mere labels <V, N>. A similar story can be readily told for external arguments provided by a functional head \(v\) that projects to \(vP\) by taking a VP as a complement and a DP/(NP) as its SPEC, which is the external argument position. Thus \([vP, v [vP V N]]\) fetches a dyadic concept that M-Joins with whatever concept the external argument fetches (A-DOG(_\ ), say). Thus:

(5) \(\exists[\text{EXTERNAL}(..., _\ ) \land A-DOG(_\ )] \land \text{CHASE}(...) \land \exists[\text{INTERNAL}(..., _\ ) \land A-BROWN(_\ ) \land COW(_\ )]\)

\(^2\) D-Join is not to be confused with Kratzer’s (1996) event identification rule. The latter introduces an external argument of a verb as an agent participant of an event already specified. Thus, Kratzer’s rule, like Pietroski’s, involves the co-identification of a position in two composed predicates/functions, but that is where the similarity ends. D-Join specifies the character of an internal argument and introduces a further position without any thematic specification of it.
Generalising, the tactic is to treat the syntax of a phrase as fetching what we might think of as a functional dyadic concept expressed by the meaning of the lexical item as syntactically projected that provides the applicand for the open argument of the predicate (Pietroski calls such concepts *adapter concepts*).

With so much in place, let’s now turn to quantification.

4. Quantification: the generalised view

The problem with first-order quantificational theory (as a model of NL) is that it (i) doesn’t generalise across all determiners (Dets) (*most, few*, etc.); (ii) is wedded to an invented syntax+composition; and (iii) fails to express generalisations across Dets and within the classes of Dets.

The basic fact about first-order quantification is that it depicts natural language Dets as *sortally reducible*:

\[
\text{(SR)} \quad \text{Where } R \text{ is a Boolean relation and } U \text{ is the universe, } Q[A, B] \leftrightarrow Q_U[R(A, B)]
\]

Here we take a determiner to express a quantifier as a relation over the Cartesian product of \( U \) that specifies a cardinality for the pairs. This general approach is nowadays referred to as generalised quantifier theory (see Peters and Westerståhl, 2006, for extensive overview). SR holds for *every, some, no*, but not for *most* and other comparative relations that cannot be rendered as relations over the whole of the universe.\(^3\) All Det relations, however, are specifiable as functions defined over the Cartesian products:

\[
\begin{align*}
\text{a) every: } f & : < A \times B> \Rightarrow |A \setminus B| = \emptyset \text{ (i.e., } A \subseteq B) \\
\text{b) some, a: } f & : < A \times B> \Rightarrow |A \cap B| \neq \emptyset \\
\text{c) no: } f & : < A \times B> \Rightarrow |A \cap B| = \emptyset \\
\text{d) most: } f & : < A \times B> \Rightarrow |A \cap B| > |A \setminus B|
\end{align*}
\]

A striking generalisation that issues from this approach is that all quantifier relations expressed by natural language Dets are *conservative* (Barwise and Cooper 1981):

\[
\text{(CONS) } Q[A, B] \Leftrightarrow Q[A, A \cap B]
\]

The truth of \( Q[A, B] \) ‘lives on’ the restriction \( A \) in the sense that how things are with the As alone determines truth value.

\[
\begin{align*}
\text{a) Some boy is a thief iff Some boy is a boy who is a thief} \\
\text{b) Every girl is a swimmer iff every girl is a girl who is a swimmer} \\
\text{c) Most women sing iff most women are women who sing}
\end{align*}
\]

\(^3\) In simple terms, a sentence such as *Most boys swim* cannot be rendered as a claim about the whole universe along the lines of *Most things are such that...* Although this expressive limitation of first-order quantification is widely recognised, its full philosophical consequences have yet to be properly registered. For example, deflationary approaches to truth often assume that the truth predicate is a device for generalisation over instances of a first-order scheme, but no such account can generalise to *Most things Bill says are true* (see Collins 2010).
Any theory of natural language Dets, therefore, should at least capture conservativity.

5. The Pietroski view of determiners

5.1. First pass

Take Dets to apply to ordered pairs, per the GQ approach, and to M-Join with internal arguments formed via D-Join:

(8) \text{SOME}(\ldots)^\lor \exists [\text{INTERNAL}(\ldots, _)^\lor \text{MAX:SPY}(_)]

where MAX is a concept expressing the maximisation of the concept to which it applies. A predicate is formed as expected:

(9) \exists [\text{EXTERNAL}(\ldots, _)^\lor \text{MAX:GERMAN}(_)]

M-Joining the two, we have

(10) \begin{align*}
\text{[SOME}(\ldots)^\lor \exists [\text{INTERNAL}(\ldots, _)^\lor \text{MAX:SPY}(_)]] & \lor \exists [\text{EXTERNAL}(\ldots, _)^\lor \text{MAX:GERMAN}(_)] \\
\end{align*}

This applies to all pairs that are such that the internal participant is an external participant, and the former is a spy and the latter is German. Polarising, a la Tarski, (10) applies to each pair so long as at least one pair satisfies the conditions (mutatis mutandis for other Dets).

So far so good, but the scoping behaviour of Dets is elided.

5.2. Second pass: QR-ed Dets

Assume that DPs undergo syntactic movement in order to acquire scope, creating structures akin to open sentences:

(11) \begin{align*}
\text{[DP Every girl]}_1 [\text{XP Every girl} & \text{likes Sam}]
\end{align*}

What concepts do the ‘open sentences’ map onto (fetch)?

Assume a concept TARSKI (Pietroski 2018: 321).

(12) \begin{align*}
\text{TARSKI}[i, P] &= ((\exists \alpha)[((\forall \alpha’)[\alpha’ \approx \alpha \land \text{ASSIGNED-BY-TO}[_, \alpha’, i]) \rightarrow \text{SAT}[\alpha’, P])] \rightarrow \text{SAT}[\alpha, P]]
\end{align*}

This is not how Pietroski presents it, but it is equivalent. The basic idea is to understand open sentences in terms of their satisfaction relative to an index i under the standard Tarski condition.

Thus, we can have:

(13) \begin{align*}
\text{[EVERY}(\ldots)^\lor \exists [\text{INTERNAL}(\ldots, _)^\lor \text{MAX:GIRL}(_)] & \lor \exists [\text{EXTERNAL}(\ldots, _)^\lor \\
\text{MAX:TARSKI}[i, _\text{likes Sam}] &]
\end{align*}

Again, so far, so good, but CONS is not reflected.

5.3. Third pass

On the standard treatment of natural language quantification offered by Heim and Kratzer (1998), which Pietroski uses as a foil, an open sentence is akin to a syncategorematic relative clause, but relative clauses
are not open sentences (Pietroski 2018: 337). Linguistically, raised DPs should merge with clause-like structures from which they serve as arguments of verbs. Rendering a Det as $<<e, t>, <<e, t>. t>>>$ makes it as if a second-order relation between monadic properties, which then creates the type-mismatch problems, for $<<e, t>, t>$ is not an $<e>$. Let’s take composition seriously, therefore: the DP both merges as an argument of a verb and merges with a clause (in its raised position). Thus, the internal and external arguments of a Det are fundamentally asymmetric, with the former restricting the range of the Det a la CONS. Pietroski implements this asymmetry via a modification on the TARSKI predicate.

(14) \text{RESTRICTIVE-TARSKI}[i, P] = (\exists \alpha)\left[ (((\forall \alpha')[\alpha' \equiv \alpha \land \text{ASSIGNED-BY-TO}[\_, \alpha', i] \rightarrow \text{SAT}[\alpha', P] \land \text{SAT}[\alpha', \text{EXTERNAL}[P']]) \rightarrow \text{SAT}[\alpha, P] \right]

Again, this is my formulation, but the content is equivalent to Pietroski’s definition.

We arrive, therefore, at a model of quantification that is compatible with the generalised quantifier framework, respects the movement of DPs to take scope, and can be conditioned to respect CONS. It bears noting that the definitions here are part of the meta-theory, not the compositional principles themselves, i.e., we define the relevant concepts in terms of satisfaction, but satisfaction is not part of the compositional analysis.

Hereafter, we shall look at the weak/strong distinction between natural language Dets and consider what resources Pietroski might have to capture the distinction.

6. The distinction between weak and strong determiners

Prior to Milsark (1977), a general distinction prevailed between definites and indefinites, but a deeper distinction was hand that has been the focus of much attention:

\textbf{Weak: some, a, no, one, two, few, many, several,...}

\textbf{Strong: every, all, the, most, Sam, both, neither,...}

\textit{Pro tem}, think of the weak Dets as being existential, in some sense, whereas the strong Dets are universal, in some sense (clarity will be offered soon). Note that this distinction cross-classifies the definite/indefinite distinction; for example, numerical Dets are definite, but pattern with indefinite \textit{a}, and indefinite most patterns with definite \textit{every}. Also worth noting is some weak Dets can have strong construals, although not the reverse. \textsuperscript{4} For our purposes, imagine that the distinction cleanly divides determiners into two classes.

\textsuperscript{4} Strong DPs do not produce an ambiguity with individual- or stage-level predicates:

(i) a Every girl is clever/is in the garden

b Most boys wear shorts/have boarded the plane
7. Three conditions

The w/d distinction is grounded in three central semantic phenomena.

7.1. Existentials

There can be weak, but not strong, DP associates in existentials:

(16) a There is \([_{dp} a\ bee] \_{c}\ in\ the\ room\]
b There are \([_{dp} some\ cowboys] \_{c}\ here\]
c There are \([_{dp} few\ girls] \_{c}\ still\ to\ see\]
d */# There is \([_{dp} the\ man]\_{c}\ in\ the\ room\]
e */# There is \([_{dp} every\ cowboy]\_{c}\ here\]
f */# There are \([_{dp} most\ girls]\_{c}\ still\ to\ see\]

Here I take the predicates (labelled ‘c’ for coda) to be non-constituents of the DPs in order to preclude a so-called ‘list’ reading, which is a specifically focused use that can rescue the unacceptable case. There are also presentational readings, where there is locative, but, again, we are just interested in the existential readings.

7.2. Weak Dets have symmetrical arguments (Keenan 1987, 2003)

The condition here is self-explanatory, but to be precise:

\((SYM) \ Q(A, B) \leftrightarrow Q(B, A)\)

(17) a Some men are nurses
b Some nurses are men
c Few women are engineers
d Few engineers are women

If the first of the pairs is true, the second is true, too, so long as the determiners are weak. Strong determiners do not license the entailment:

If we bracket domain restriction, these sentences are unambiguously universal claims about the set of girls/boys. Similarly, weak DPs are typically uniform in construal across the two sorts of predicate. Obviously, the construal differs in being existential, in the sense in which some things are said to satisfy the restriction (i.e., exist), in a way (i) does not. Some determiners are atypical, such as few. Consider:

(ii)a Few girls are clever
b Few girls are in the garden

(jia) can only be construed as true where most girls are not clever. It has no partitive reading where some small number of girls are clever, but the rest might be smart. In contrast, (iib) precisely has such a duality of construal. It might be a claim that the garden contains a small number of girls (three, say), or the ‘strong’ claim that few of the girls (i.e., a small percentage) are in the garden, which might be a huge number, depending on the number of girls. This is said to be the ‘strong’ reading because it is about the set of the restriction as a whole, rather than some definite number of girls.

5 A ‘list’ reading goes with a focused unit in response to a question, say. For example: A: Who will save us now the cavalry have left? B: Well, there is every cowboy still here. The weak cases can be expressed with the coda as a conjunct (There is a bee and it is in the garden) or relative clause (There is a bee, which is in the garden). List readings aren’t so supported. The DP+codas in list readings might thus be small clauses.
(18)  
   a. Every man is a nurse
   b. Every nurse is a man
   c. Most women are engineers
   d. Most engineers are women

### 7.3. Weak Dets are intersective

\[ Q(A, B) \leftrightarrow Q(A \cap B). \]

(INS) offers a different kind of test: if 'Det As are Bs' is weak, then it is equivalent to 'Det As, who are Bs, are Cs' where \( B \subseteq C \). The truth of a statement involving an intersective DP as subject wholly depends upon the intersection of the class of things that are both A and B—one may ignore the A things that are non-B. Thus:

(19)  
   a. Some man is a nurse
   b. Some man, who is a nurse, is a care worker
   c. Two men are nurses
   d. Two men, who are nurses, are care workers

(20)  
   a. Every man is a nurse
   b. Every man, who is a nurse, is a care worker
   c. Most men are nurses
   d. Most men, who are nurses, are care workers

The pairs in (19) cannot differ in truth value. Note, in particular, that both depend upon the men existing. Thus, (19b, d) are not tautological. If there are no men, say, then (19b) can't be true, and (19d) fails to be true if there is just one man. In (20), a difference in truth value between the pairs not only can obtain, but clearly does obtain in fact. (20a) is false, whereas (20b) is a tautology; ditto for (20d).

### 7.4. A significant fact

Given the conservativity of natural language determiners, (SYM) and (INS) are equivalent (see Peters and Westerståhl 2006: 210-11). That is: (CONS)+(INS) entails (SYM), and (CONS)+(SYM) entails (INS)

### 8. Pietroski’s options

#### 8.1. Pragmatics

A ready option is to seek to capture the w/d distinction in terms of pragmatics rather than compositional semantics. It is unclear how this tactic might be realised, notwithstanding the common thought that strong utterances, as it were, presuppose existence rather assert it. We should still want to know why such a difference is tethered to the Dets and their differential behaviour with existentials. For example, whereas a presuppositional account might explain the felt unaccept-
ability of an empty restrictor for strong Dets, the behaviour of the weak Dets remains opaque.\(^6\)

More generally, since CONS is not a pragmatic principle, and interacts with SYM and INS, it would be nice if all these algebraic properties fell together under the one semantic explanation; indeed, the W/S phenomena look (almost) as robust as the CONS phenomena.

8.2. Encode SYM or INS for the W cases, and let CONS do the rest

Recall our significant fact. CONS+SYM entails INS, and CONS+INS entails SYM. If CONS is encoded, therefore, we really only need to encode one of the other properties. Perhaps the easiest implementation of this idea is to let the internal predicate be RESTRICTED-TARSKI modified by the satisfaction of the external predicate for the weak Dets alone. Such would satisfy SYM, and so entail INS.

I can see two main problems with this thought. Firstly, it is not obvious how to make it work compositionally, i.e., why should there be a restriction going up the syntactic tree? The restriction going down tracks the syntactic movement of the DP. In short, it appears to be a stipulation. Secondly, the there-existential restriction to weak Dets would remain unexplained. I shall come back to this shortly below.

8.3. An alternative

Suppose that only the weak Dets encode or fetch an empty internal monadic concept that we may render intuitively as ‘x is in the domain’, but when combined with the Det effectively encodes the idea that a cardinality of things that satisfy the predicates exists. This captures the existential content of the weak Dets and simultaneously explains why the strong Dets admit empty restrictors. For example:

\[
(21) \quad \text{[SOME(...), DOMAIN(...), } (\exists[\text{INTERNAL(..., _, MAX:GIRL(_))}], \\
\quad \quad \exists[\text{EXTERNAL(..., _), MAX-RESTRICTED-TARSKI[i, _likes Sam]})]
\]

Being more speculative, we may think, on this story, that weak Dets ‘originate’ from a means of talking about some ‘relevant’ domain, whereas the strong Dets don’t. Thus, the latter don’t introduce a domain but have a global or universal meaning (more anon).

Let’s see how this basic idea might fair in accommodating the basic properties of the w/s distinction.

Firstly, the proposal accommodates SYM, for with weak Dets, a domain is populated with a kind of thing (or things) that has the internal property, which is also said to have the external property, and so

\(^6\) It is common to think of strong Dets as presupposing a non-empty restriction, hence the supposed infelicity of Every French king is bald. It is better, I think, to account for the infelicity as due to implicature.
whatever condition the Det places on the thing(s) having the internal property will hold of the external property too. Not so for strong Dets, where no domain is populated by things.

Secondly, the proposal accommodates INS, for, again, whatever kinds of things are in the domain share the properties of the internal and external predicates. With strong Dets, since there is no domain, we can have an empty-restrictor reading in line with the non-intersective relation of the strong Det.

Thirdly, the proposal also accommodates the fact that some weak Dets can be read strongly. For example, the weak construal of *few* is as expected, with the domain populated by few things that are both girls and in the park, as might be. On the strong reading, the domain is still populated (no empty-restrictor reading is available), but some broader group of girls than just those in the park must be understood to exist. In effect, the content is partitive.

Fourthly, the *there*-existential restriction to weak associates is nicely accommodated. In the weak cases the domain is populated, which is actually just what the bare existential says. The strong Dets encode no domain, and so they have no existential reading, unless a domain is explicitly introduced via presentation or ‘list’, which are, of course, supported by strong Dets.

In the following section I shall dwell somewhat on existentials, for they add some interesting support to my general proposal.

9: Some syntactic considerations (after Kayne 2019)\(^7\)

According to Kayne (2019) there are four possible construals of *there* exhibited in (22):

(22) **There**... two files *there*, on the desk, which *therefore*, need filing in them *there*, cabinets.

So, there can be existential, locative, rationale, and presentational readings. Instead of positing a 4-way ambiguity, let *there* have a basic presentational construal fixed in a low small clause with its associate; the other construals are configurationally fixed. In particular, for the existential, *there* obligatorily moves to SPEC-TP (subject). Of course, this makes sense of the general syntactic differences, between existential *there* and locative *there*.\(^8\)

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\(^7\) Herburger (2000) offers other syntactic reasons why strong Dets must move to SPEC-TP

\(^8\) Neither of the them receive a theta-role, but locative *there* is not an argument, while existential *there* is, albeit an expletive. Syntactically, this shows up in various ways. Firstly, locative *there* cannot occur in a tag question:

(i) *There is your dog, isn’t there?

Secondly, it does not admit raising:

(ii)a There seems to be a dog in the garden

b *There seems to be your dog

Thirdly, locative *there* cannot be negated:

(iii) *There isn’t your dog
None of this by itself essentially bears on the w/s distinction. Kayne merely notes that there must be ‘some constraint’. Note, however, that the existential *there* is still presentational content-wise; it is only existential configurationally. Suppose, then, that *there* is always first merged in a small clause with its associate and interpretation happens throughout the derivation, not just at LF or some other completed structure. If we now also posit a domain predicate with weak Dets, then the small clause will be interpretable with presentational *there*. If the Det is strong, and so lacks a domain, the presentational *there* will be uninterpretable. Thus, we get to explain the existential restriction on the assumption of Kayne’s model and the weak Dets introducing a domain predicate.

10. Conclusion

I hardly think what I have said here is the end of the matter, or even the beginning of the end of the matter. I suspect it might not even be the end of the beginning of the matter. I commend the topic for further inquiry within Pietroski’s basic framework.⁹

References


Kratzer A. 1996. “Severing the External Argument from its Verb.” In J.

Fourthly, locative *there* cannot occur in a subordinate clause:

(iv) *Sam wondered whether there was your dog.*

⁹ My thanks go to Paul Pietroski for conversations on the topic of this essay and many related issues, and to Michael Glanzberg for the same. I was hoping to find time to develop a more worked out discussion, but children, pandemic, and semantics are not a happy mix.

