Marijana Borić: “Marin Getaldić – A look on a new era. On occasion of the 450th anniversary of his birth”

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Abstract

Among the numerous Croatian humanists who left a mark in the development of Western European science and culture, Dubrovnik nobleman Marin Getaldić (1568-1666) is the greatest Croatian mathematician and physicist of the 16th and 17th centuries. Although he was educated in the hometown for performing of administrative jobs in the service of the Republic of Ragusa, in a very specific way, he became involved in scientific research and his ideas were accepted by the most prominent scientific circles and names of contemporary Europe, such as Galileo Galilei, François Viète etc. He was developing the most current area of Renaissance mathematics - the symbolic algebra. Getaldić also wrote the first complete manual of new algebraic analysis applied to diverse materials. Having achieved remarkable results, he came closer to founding of a new mathematical field - analytical geometry. Getaldić particularly focused on the development of mathematical methods and their application in the research of natural sciences. With that approach, different from the previous, traditional, and largely quantitative description of the world, he laid down the main tendencies of the modern approach to the research in natural sciences.

Key words: Marin Getaldić, the beginnings of modern science, experimental approach, mathematical methods, Galileo Galilei, François Viète, algebraic analysis, symbolic algebra
Introduction
Marin Getaldić lived in the period when the influence of Renaissance philosophy gradually shaped and based modern science. One of the main characteristics of modern science is a significant reliance on mathematics in interpreting and approaching the research of the natural world. Under the influence of those changes at the beginning of the 17th century, there is a transition from qualitative to quantitative description of the world. Consequently, an experimental method emerges from the established affirmation that full knowledge of a process in nature is not merely a mere observation, but it is necessary to repeat the events which are to be later investigated in particularly created conditions. The appearance of modern science is undeniably one of the most important events of modern civilization, and Marin Getaldić contributed and participated in the creation of the assumptions of modern science by his mathematical and physical work. From this aspect, the article analyses Getaldić’s opus and establishes links between his traditional starting point and the modern age novelties he achieved. The paper describes his life path, development and specific way of engaging in the scientific work, depicting the opus as a whole, as well as the influence of his ideas on the 17th and 18th century scholars and the further development of mathematical and natural sciences.

A nobleman from Ragusa
Marin Getaldić (Marino Ghetaldi, Marinus Ghetaldus) is the most prominent Croatian mathematician and physicist at the turn of the 16th to the 17th century. He was born on 2nd of October 1568 in the Republic of Ragusa (modern Dubrovnik) as the oldest child in a distinguished noble family, whose genealogy can apparently be traced back to the second half of the 12th century through Marin Getaldić’s father Mato and mother Anica, daughter of Andrija Restić. For eight centuries the Getaldić family held prominent place in the public, political, diplomatic, scientific and cultural life of Dubrovnik. The family has given a number of distinguished persons, and by crossing the boundaries of local heritage, certainly stands out the mathematician Marin Getaldić. About the reputation of this family in Dubrovnik best speaks the fact that they gave a number of Ragusan rectors (heads of executive power). Towards the
end of the 16\textsuperscript{th} century Marin Getaldić and all his brothers (Andrija, Šimun, Jakov and Martolica) were members of the Ragusan Grand Council. Interestingly, at the same time, next door to Marin Getaldić, a mathematician, lived another Marin Getaldić, rector of the Republic, who came from the different branch of the Getaldić family. According to tradition, which might well be legendary, the Getaldić family claimed their origins from Taranto in Apulia in southeastern Italy in the 10\textsuperscript{th} century. Over time, they were fully assimilated, and even Marin Getaldić himself, feeling patriotism and connection to the birth place, called himself ‘Apulian Illyrian’ in one of his works.

\textbf{Figure 1:} Drawing of Marin Getaldić by Antonio Nardello. Illustration from the book P. F. Martechinni, \textit{Galleria di Ragusei Illustri}, Ragusa 1841.

\textbf{Education in the Dubrovnik humanistic tradition}

Marin Getaldić was educated in his native Dubrovnik. He attended the Dubrovnik Gymnasium, located in the Divon Palace, today Sponza, which from the 16\textsuperscript{th} century was in the rank of lyceum, some kind of higher school, where grammar, rhetoric, literature, arithmetic, physics, philosophy, theology, law, music and astronomy were taught. There as
teachers worked many prominent foreign humanists, philosophers and writers from various cultural centres in Europe. They came to the city at the invitation of the Ragusan Senate and conveyed the influences of Humanism and Renaissance from neighbouring Italy and the school enjoyed a good reputation. To the spiritual climate of the city also contributed the cultural circles gathered around the prominent Dubrovnik residents, such as philosopher Nicolò Vito di Gozze (Nikola Gučetić) and poetess Flora Zuzzeri (Cvijeta Zuzorić). Today, it is not possible to fully reconstruct everything what Getaldić could have learned in the grammar school from the field of exact sciences, but according to the preserved documents it is known that the teachers of mathematics at the Dubrovnik school were engaged in teaching from the second half of the 16th century, much later than the teachers of other subjects of humanities.\(^1\) Getaldić mastered elementary calculus in mathematics because nothing more than that was taught in the program of the grammar school at that time.\(^2\) However, he has received excellent education in classical languages in Dubrovnik. Being educated in the humanistic atmosphere of Dubrovnik he already shaped his intellectual tendencies as a young man. The knowledge he had acquired was sufficient to stimulate his interest in mathematics and physics, areas in which he would later improve abroad when the circumstances allowed him to do so. In the hometown, as in other Dalmatian cities in the 16th century, there were no institutions and schools where he could continue his studies.\(^3\)

In 1588, having completed his education at the age of twenty, Getaldić was admitted as a nobleman into the Grand Council of the Ragusan Republic. According to the wishes of the family, who over time was sinking into financial problems, he was preparing for various jobs in the service of the Republic. Educated and trained for performing legal services, he takes over various administrative jobs. So since 1590, he had been performing administrative and judicial duties in a town of Janjina.

\(^1\) Korbler (1914): 136-68.
\(^2\) di Gozze (1589): 91-96. Among other things, di Gozze in his work talks about the Grammar School in Dubrovnik at that time and the level of teaching in sciences, and how school programs should be improved by introducing new content in the natural sciences and a better representation of arithmetic and geometry.
on the Pelješac peninsula. He was an Appellate Judge for a time, and also worked as one of two civil servants in the Ragusan State Office for Arms and the Salt Sales Office on the Neretva river.

**Study trip**

After returning to Dubrovnik in 1601, Getaldić continues with experimental work begun in Europe. In that period he completed and prepared his first works to be published, which he began writing already during the study tour. Getaldić was, particularly, interested in the construction of parabolic mirrors and optical experiments, which he conducted in a cave by the sea, on a family estate located outside the city walls at the foot of Srd hill in the area of Ploče (see Figure 2). Faced with a series of geometric optic problems encountered during construction, fabrication and experimentation with parabolic mirrors, he conducted mathematical research of parabolas and published results in the work *Nonula propositione de parabola (Some Items About the Parabola)* in Rome 1603. Although the work is inspired by physical interests and optical experiments, its main scientific result is mathematical. Until then it was considered that flammable mirrors can be obtained only by slicing rectangular and upright cones. Getaldić explored the properties of the parabola by mathematical methods and concluded that all the parabolas were obtained by cross-section of rectangular, sharp-angle, obtuse-angle, and scalene cone mutually congruent and are all suitable for the construction of flammable mirrors.

The parabolic mirrors which he constructed served to ignite objects in focus, and therefore to determine the melting point of various substances, as well as to determine the position and size of the image at different positions of the object relative to the parabolic mirror. Getaldić probably determined the focal distance first and then constructed a parabola for the desired characteristics. He constructed several parabolic mirrors, of which only one, over 2m in size, has been preserved till this day. Getaldić melted the lead, silver and steel with mirrors, as he writes to a respected naturalist, Clavius, in Rome in 1608, which means that he achieved a temperature of 1200 to 1500 degrees Celsius, depending on the type of steel he was using. The preserved mirror was made of very thin metal, fragile as glass, and had a glow from the front and the back.
After Getaldić’s death, his brother Jacob gave that mirror as a gift to Cardinal Francesco Barberini, urging him to take it upon himself to publish posthumously the most important work of Getaldić called *De resolutione et compositiome mathematica* (About Mathematical Analysis and Synthesis).\(^4\) The mirror was exhibited at the Barberini Museum in Rome for two centuries. Prince D. Francesco Barberini gave the mirror to be restored, and the restoration inspired the poet Santa Pieralisi to write the poem *Lo specchio concavo barberiano* (Barberini’s Concave Mirror). It is not known when and why the mirror had changed the owner later, and is now kept in the British Maritime Museum in Greenwich. Even during Getaldić’s lifetime, unusual stories have been inspired by his experimental work. They attributed him with his magical and astrological skills, what hundred years later the Croatian biographer and historian of the 18th century, Seraphinus Maria Cerva (Serafin Marija Crijević), recorded the following in his work *Bibliotheca Ragusina* (Dubrovnik Library):

> “On the coast of the Adriatic Sea between the old Epidaurus and modern Dubrovnik, overlooking the beautiful island of Lokrum, in odd contradiction to the lush greenery of the meadows and rich olive groves is a deep and spacious cave at the foot of Mount Srd. An oval hole leads into it that gives it a shape of the abyss. At the end of the sixteenth and early seventeenth century, this cave was ill-famed, and it is still the case today with fishermen and ignorant common people. There, they say, stayed all day the wizard named Bete or Betino. He read the future in the stars, ruled over things, and with special movements of his hellish machines he would burn all kinds of ships, so no fishing boat dared to approach that unlucky coast.”\(^5\)

Many did not understand Getaldić’s experiments, so it was believed that he burns the boats on the open sea by instruments in the cave. F. Appendini alleges that Getaldić repeated the Archimedes’ experiments in front of numerous spectators and burnt the small boats at the sea,

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\(^4\) Banfi (1938): 322-45.

causing great fear in the common people. Getaldić lived at the time of great turmoil in science and philosophy. There is a large number of diverse opinions and theories that are incompatible with a unique and coherent whole, as well as an intensified interest in various forms of mysticism and superstition, characteristic of the whole 16th century. Though today we are looking at them as pseudoscience, at this time in studies like astrology, cabal, various forms of prophecy, alchemy, and so on they were considered as serious scientific disciplines. So it would not be surprising that even Getaldić showed his interest in astrology in accordance with his time. However, although the legend says that he, like the wizard Bete, read the future from stars, there is no document or written record that would confirm his interest in astrological disciplines. Legend about Getaldić as a Dubrovnik wizard Bete and his powers has been preserved through the centuries. It probably has its roots also in the legend of the great Archimedes, who was so respected in the Renaissance so much that he was called ‘the prince of all mathematicians’. It was then that the story of Archimedes’ defence of native Siracusa was revived by burning ships at the open sea at city’s doorstep. Even today, the cave at the family Getaldić estate is named Betina cave after his nickname.

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6 Appendixi (1803): 46.
7 Getaldić’s Cave is located on the edge of the family estate, near his house in the Ploče area, located on the way to the monastery of St James. At the front door of the Getaldić estate in Ploče, the Latin inscription can still be read today, which reads: “Be far away, envy, strife, vanity, worry! Peace and tranquility adorn the caves, gardens, cliffs.”

Today, the Renaissance mansion is no longer on the Getaldić estate. Only a few fragments have been preserved, but that is why Betina Cave is very attractive for visitors to Dubrovnik. It is accessible only from the sea because the steep stairway to it is part of a private estate that was later inherited by the Saraka family (Getaldić’s daughter Anica was married to Pavel Saraka) and is now owned by the Wagner family. On that part of the coast facing the peninsula of Lokrum, limestone cliffs rise from the sea. Among the numerous small caves there stands out the imposing Betina Cave, the largest and most beautiful among them, in fact a half-cave with a huge oval opening, covered with lush greenery from above, and open from the sea, and there is a beautiful natural beach hidden there. At the cave vault there is an opening through which light penetrates. The cave is spacious and, with its position, depth and opening, it is suitable for performing optical experiments.
Links with famous European Scientists
Getaldić spent most of his life in Dubrovnik, where he wrote and completed his works. Because of a stormy youthful event, probably a duel, he had to leave Rome in 1603 with a ban on return. In Dubrovnik, he lacked friends from whom he learned novelties in science. This is evidenced by a letter he sent to Galileo Galilei on 20th February 1608 with whom he exchanged published works. Getaldić writes in it: *Io sono qui come sepolto* (*Here I am like buried*). He corresponded with two of the most eminent mathematicians from the circle of the Roman Jesuits, Christopher Clavius and Christopher Grienberger. Mathematician Paul Guldin, who especially appreciated Getaldić’s work by declaring him as revived Apollonius (i.e. Apollonius of Perga, a great Greek mathematician from the 3rd century BC), persuaded him in a letter from 1617 that he be the editor of Viète’s collected works for the Munich printing company Zigler. It was only towards the end of Getaldić’s life that his faithful friends obtained permission from the Pope himself a permission for his return to Rome.

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8 *Manoscritti Galileani*, Parte 6., Biblioteca Nazionale di Firenze,
**Jobs in the Service of the Republic of Ragusa**

Although he proved to be an outstanding mathematician as a young man, Getaldić was never able to make a living from science. By the decision of the Council of Petitioners, he was hired in 1603 and 1604 as the leader of the reconstruction of the Podzvizd Fortress, the highest fortress in the fortified system of Ston (1604). In addition, he was appointed captain, military commander of Ston, the area where precious saltworks were in the possession of the Republic. As a confidential landlord, in 1606 the Ragusan Senate appointed him one of the two envoys of poll-tax. He travelled to Constantinople that year, taking to Sultan tribute of 12,500 ducats. Getaldić stayed there in the Dubrovnik colony for a year and represented the interests of the Republic. According to the Senate’s instructions, he asked for Ottoman support in connection with the Lastovo rebellion, one of the biggest crises in the history of the Republic. He also lobbied for the Ottoman support against the Venetian efforts to move the Balkan merchant routes, which have gone through Dubrovnik until then, to Venetian-ruled city of Split. According to the preserved diplomatic letters, it was demanded that, with the threat of Ottoman intervention, the Venetian ambassador was warned that the island of Lastovo must be returned to the Ragusan Republic. In addition to the diplomatic duties Getaldić successfully performed in Constantinople, measuring the geographical coordinates of the city more precisely than before. He was also unsuccessfully searching for an Arabic translation of Apollonius’ works on cones.

**Promotus Archimedes**

Among the first works, Getaldić published in Rome in 1603, in printery of Aloysius Zannetti, was the work entitled *Promotus Archimedes seu de variis corporum generibus gravitate et magnitudine comparatis* (*Improved Archimedes or About Comparing Body Weight and Volume of Different Types*). In addition to the six works of mathematical content written by Getaldić, *Promotus Archimedes* stands out as the only work from physics in his oeuvre. He completed it shortly after returning from a study trip in Europe. Incentives to deal with the problem of comparing different bodies in weight and volume were given to him by Michiel

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9 See the list of Getaldić’s works below.
Coignet and Federico Saminiati during his stay in Antwerp, which Getaldić himself mentions in the preface to the work.\textsuperscript{10} He worked on it for a few years. The occasions for its completion and final design were certainly the meetings with the renowned Jesuit Christopher Clavius in Rome, who persuaded Getaldić to publish this work, as well as the fruitful contacts he had with Galileo Galilei in Padua. The work begun on the study trip was supplemented by Getaldić’s measurements made during his stay in Dubrovnik in 1601 and 1602.

In \textit{Promotus Archimedes}, he discusses how to determine the relationship between the weights and volumes of various bodies, seven solid ones and five fluids: gold, silver, copper, iron, lead, tin, mercury, wine, water, wax, oil and honey. At the end of the work, he deals with the well-known problem of the Hieron’s Crown, which Getaldić cleverly and precisely solves.\textsuperscript{11} This problem, taken from Archimedes’ research, was addressed by many scientists after Archimedes, including Getaldić’s contemporary Galileo Galilei. He is also believed to have collaborated with Englishman Thomas Harriot, who also addressed these issues.\textsuperscript{12} Getaldić based the discussion on the application of Archimedes’ research and Archimedes’ Law, according to which every body immersed in a liquid becomes lighter by the weight of the amount of fluid that the immersed body displaces with its volume. In the preface, he notes that his goal is to refine the previous research because the problem of determining the relationship between weights and volumes of various bodies is nowhere more extensively interpreted.\textsuperscript{13} Nowhere in the work is the term ‘specific weight’ mentioned, although the term was introduced in the 13\textsuperscript{th} century in the pseudo-Archimedes’ work \textit{De insidentibus in humidum} (or \textit{De ponderibus}), but neither in this work nor in Getaldić’s time was it used in the meaning it has today.\textsuperscript{14} This specific weight was relative, as it was taken for one body in relation to another

\textsuperscript{10} Getaldić gives detailed incentives for the publication of this work in a passage entitled \textit{To the Benevolent Reader}, Getaldić (1972): 17.

\textsuperscript{11} Hieron’s problem is based on the legend that Syracusan king Hieron, in suspicion of goldsmith fraud, asked Archimedes to determine the proportion of gold and silver in his crown.

\textsuperscript{12} Tanner (1969): 161-70.

\textsuperscript{13} Getaldić (1972): 17.

\textsuperscript{14} The introduction and development of the notion of specific gravity is described in Clagett (1961): 91-97.
body of the same volume, which was not always the same.

Using the Archimedes’ principle and hydrostatic weighing, Getaldić determined the weight ratios of different bodies of the same volume or the volume ratios of different bodies of the same weights. His measurement results compared to the results of his contemporaries were very good, indicating that he was a good experimenter. However, he also did not emphasize the ratio of the weights of different bodies to one always the same reference body, which would result in a greater generality. Getaldić’s measurements are the result of carefully designed and conducted experiments. With a rich tabular data, he gives a systematic presentation of the results of his experimental work. The data are grouped into appropriate tables, which generally follow the presented theoretical and practical part. The entire tabular appendix contains several tables comparing weights and volumes for twelve different bodies, then tables for determining the weight of a sphere from a given radius and vice versa, and for determining the quality of gold. In addition, the theoretical considerations and the practical part are complemented by detailed descriptions of the experimentation procedures. Getaldić, in a separate chapter entitled *Quomodo ponderanda sint corpora solida in aqua* (How to Weigh Solids in Water), describes the process of weighing a body in water using a scale of equal arms with bowls. The text is a testimony to how carefully and precisely

15 The most famous researcher of these problems in the 16th century was considered to be the Italian scientist Nicolo Tartaglia. Three centuries after the publication of the afore-mentioned pseudo-Archimedes’ work, Tartaglia prints it in 1565 under the title *Jordani Opusculum de ponderositate*, adding to it his own experimental results for specific weights. However, Tartaglia’s experimental work produced significantly worse results of specific weights than those obtained and published by Marin Getaldić in *Promotus Archimedes*.

16 "The body to be weighed is hung on one bowl of scale by a horsehair. Weights are placed on the other bowl, and the hung body is lowered into the water so that it hangs freely in the water, so that the water does not touch the bowl on which the body hangs, or the other on which the weights are. And so the body is weighed as if hovering in the air. I said that the body to be weighed should be hung on horsehair, for it is almost as heavy as water, and therefore will not add or subtract anything to the weight of the body to be weighed. If the body we are weighing is so heavy that one hair cannot hold it, then hang it on several hairs tied together, and so that the hair tied in this way does not add any weight to the body being weighed, let the same amount of hair be placed on the other bowl equal to those hanging from the bowl on which the body is hung next to the hung body. With this addition of hair, both bowls will be equally heavy, and although those hairs on which the body hangs are longer than those on the other bowl,
Getalidić performed the experiments, trying to anticipate and avoid possible errors and deviations that occur during the experiments. To perform the experiment, he constructed one type of hydrostatic balance.

It is not clear whether Getalidić constructed the scale independently by himself before traveling around Europe, or it happened later when he could have obtained certain ideas from Galileo himself in Padua for making this instrument, or from some other source. At the beginning of his scientific work, Galilei wrote a treatise on the hydrostatic balance and its application, but during Getalidić’s lifetime this manuscript was not published. However, although there is no written confirmation, there is a possibility that he saw it, together with Galileo’s scale, during his stay in Padua. Getalidić’s scale differed from similar devices used by his contemporaries, and Galileo’s manuscript was published only in 1656, so Getalidić is considered to have priority in this regard.\(^{17}\)

The work *Promotus Archimedes* contains theoretical, practical and tabular parts, which are not completely separate, but are methodologically altered according to the content that is processed in individual units. It is of a specific methodological concept that significantly contributes to the value of this physical work with the already mentioned precise results of experimental work. Unlike the traditional, qualitative approach to describing the emergent world so far, Getalidić uses a new way of researching and approaching problems. He uses a quantitative approach and structures and presents the physical structure in the way that the Euclidean *Elements* are presented. In addition, Getalidić relies on mathematics as a major explanatory apparatus in accessing the natural world, which is one of the main features of modern science. He has structured the content of the work

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\(^{17}\) Kučera (1904): 371-72.
into ten theorems,\textsuperscript{18} nine problems,\textsuperscript{19} ten examples and seventeen tables with instructions for use. The theorems are fully formulated and proven in the spirit of ancient mathematics, modeled on Euclid’s \textit{Elements}, sometimes proven in two ways and accompanied by examples. Getaldić’s approach to material in the field of physics (natural philosophy) was a novelty at the time. Due to the methodology that Getaldić used in his work, deviating from the previous tradition, it is cited as an early example of a modern approach to science in which mathematics is increasingly used.\textsuperscript{20}

Although in \textit{Promotus Archimedes} there are no special units in which Getaldić explicitly explains the importance of introducing mathematics in physical research, nor conclusions what this step significantly brings to natural philosophy and the development of knowledge, he is very aware of the role that mathematics plays in understanding the world and in seeking the certainty of new knowledge. Understanding the true meaning of mathematics for the study of nature, Getaldić considers the geometric method to be the most suitable for presenting the investigated physical structure. He then proves the claims presented on the model of classical mathematical formulations by mathematical methods, relying precisely on the fact that mathematics was considered the ideal of the science of proof. It is evident that in this physical work Getaldić uses mathematical methodology in many ways, not only in research and proof of facts, but also in the way of presenting the obtained conclusions, so that he shapes the whole work on the model of the characteristic structure of ancient Greek mathematical works. That

\begin{itemize}
  \item \textsuperscript{18} E.g. The same heavy bodies of commensurable volumes have the same weight ratio as the volumes (Theorem 2); And incommensurable bodies of the same kind have the same weight ratio as the volumes. (Theorem 3); Bodies of the same type and weight, heavier than water, have equal weight in water even though they are of different shapes (Theorem 8); The weights of the same spheres relate among themselves as cubes of their diameters (Theorem 9).
  \item \textsuperscript{19} E.g.: Let us take two bodies of equal volume, one of which is solid, and the other is fluid. If the weight of the rigid body is given, let the weight of the fluid be found (Problem 1). If the given two bodies are equal in volume, one rigid and the other fluid, so if the weight of the fluid body is given, then find the weight of the rigid (Problem 2). If two given bodies are of the same weight, one rigid and the other fluid, and if the volume of the rigid body is given, the volume of the liquid body should be found (Problem 3).
  \item \textsuperscript{20} Napolitani (1988): 139-236; Høyrup (1990): 137-38.
\end{itemize}
rigorous mathematical approach, embodied by Euclidean *Elements*, is maintained by Getaldić in all segments of *Promotus Archimedes*. Aware of the need to prove all the claims made, he says at the end of the third theorem:

“What we have proved in the two previous theorems, some assume as something familiar in itself, and as if it were a very general axiom that they themselves supposedly saw quite well and wisely. However, Euclid could also assume that the 20th paragraph of his *Elements* is something quite familiar. Namely, it is better known to everyone that the sum of two sides of a triangle is greater than the third (every donkey knows that), than that heavy bodies of the same kind have the same ratio of weight as the volume, and yet Euclid proves this paragraph and does not assume it. Therefore, this paragraph as well, which is not so clear, had to be proven and not assumed.”

Viewed from a methodological point of view, it is important that Getaldić was among the first at the turn of the 16th and 17th century to choose to write a work in a geometric style, because this trend will be affirmed not only in physics (natural philosophy) but later also in philosophy of 17th century and be called the ‘geometric mode’, as well as become the ideal of presenting philosophical material.

**Restoration of Lost Ancient Works**

Like many Renaissance mathematicians, Getaldić sought to reconstruct and restore lost mathematical treatises, relying on claims in the surviving works of other ancient mathematicians. This work was prompted by Viète’s restoration of the lost writings *On Touches* by the Greek mathematician, Apollonius of Perga. Viète restored ten problems, while Getaldić noticed and reconstructed six more. The restoration was published in 1607 in the work *Revived Apollonius*. Satisfied with the results and with a considerable dose of patriotism, Getaldić wrote at the end of his work: “And so Apollonius of Gaul will not, without Apollonius...”

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22 Getaldić’s restorations are presented in more detail in Borić (2013/14a): 78-82. and Borić (2013/14b): 184-89.
of Illyria, revive Apollonius of Perga, who lay extinguished by the injustice of time or buried by barbarians.” Studying the Mathematical Proceedings of the great ancient mathematician Pope and his intricate interpretations, Getaldić made the first formulations of Apollonius’ problems from the lost work About Inclines, which were then used by future restorers. Although he intended to publish the restoration in one part, burdened with various jobs and due to the imminent departure to Constantinople, he published the first part of the restoration in 1607, entitled The Revived Apollonius. It contained the first four problems solved and the fifth one only formulated. Getaldić completed and published the fifth problem in 1613 in the work The Revived Apollonius, Book two. Interestingly, it was precisely the fifth problem that prompted a friendly competition between Getaldić and Scottish mathematician Alexander Anderson. Each of them solved the fifth problem by different methods twice. When one compares their published solutions, it can be said that in support of Getaldić’s work go his solutions in the books The Revived Apollonius (I, II), which are a complete restoration because they are methodically uniform and homogeneous whole in accordance with the original, since he used exclusively geometric synthesis. Getaldić had been involved in mathematical restoration for many years. With this work he joins a corps of Renaissance scholars who sought to establish a new science on the foundations of ancient heritage. Getaldić’s works had a diverse and rich echo in the natural sciences literature of the time. He has been cited and his results were used in various ways by numerous scholars, such as Kaspar Schott, William Oughtred, Pierre Herigon, John Lawson, Samuel Horsley, Ruben Burrow, Michelangelo Ricci, Alexander Anderson, Marin Mersenne, Jacob Christmann, Cyriaq de Mangin and others.23

At the threshold of analytical geometry
Getaldić began writing his two most important published works, Variorum problematum collectio (Collection of Various Problems) and De resolutione et compositione mathematica (On Mathematical Analysis and Synthesis) at the same time with the intention of using exclusively the methods of ancient mathematics in the first, and in to affirm Viète’s

symbolic algebra in the second part by applying it to a variety of materials. He completed and published his *Variorum problematum collectio* well before in 1607, while steadily completing his main work *De resolutione et compositione mathematica*, published posthumously in 1630. Getaldić died in 1626, just as his great work had taken the final form. Realizing the meaning and importance that Viète’s method brings in its generality, Getaldić had in fact designed his major work as its first complete and comprehensive manual. It is a methodical collection of problems and theorems, solved by applying a new algebraic method on a variety of materials. The main contribution of *De resolutione et compositione mathematica* is in the very development of the algebraic method, although the work contains numerous new original mathematical results. This is especially evident in the problems he repeats from older works of ancient tradition, where he addresses his own geometric problems from earlier works, and the problems and theorems of Euclid, Apollonius of Perga, Viète, Regiomontanus, and others.

Calculating with general quantities enabled Getaldić to re-interpret previous mathematical results. He modifies the results of geometrically solved problems and performs algebraic analysis within general algebra. It can be noticed that Getaldić, affirming a new algebraic method on a variety of materials, at the same time proves to be a faithful transmitter and interpreter of the traditional approach. The key difference in relation to the previous mathematical approach, based on admiration for everything ancient and attempts to transform certain concepts and procedures of existing mathematics into ancient attire, appears in the new understanding of the mathematical object. In other words, in the conception of general number, the introduction of which leads to a radical reform not only of algebra but also of mathematics as a whole.

Characteristical of Renaissance mathematicians, Getaldić’s *oeuvre* too is largely based on the works of the Greek mathematicians, among whom Euclid, Pappus, and Diophantus are prominent. He is also influenced by Eudoxus’ theory of proportions and Archimedes’ application of logistical methodology, that is, the arithmetic interpretation of geometry. Drawing on the ancient mathematical tradition and inspired by Viète’s algebraic method, Getaldić applies the integration of different tendencies of ancient Greek mathematics, strict
geometric methods and logistics, which implied the routine of ordinary mathematical calculus and allowed an approximate approach. In addition to the application of rigorous geometric methods, there was another pronounced tendency in mathematics at the beginning of a new era in Ancient Greek mathematics, which was the increasingly intensive introduction of logistics into theoretical mathematics. In dealing with geometric problems that can be interpreted algebraically, Heron added, for example, surfaces and lengths, which was unacceptable in ancient Greek tradition. In this way, contrary to the mathematical tradition of the past, he favoured the numerical aspect of the problem in relation to its geometric origin.24

Following consistently the Viète’s method, Getaldić introduces general quantities into already known methods of analysis and synthesis. So, in De resolutione et compositione mathematica, through the application of the method, he also achieves a change in the conception of a mathematical object. Getaldić structured the work into five books. Introduction of the first book De resolutione outlines the general principles of the method and clarifies the concept of analysis, which first appeared in Ancient Greek philosophy. Getaldić interprets the basic principles of the analytical, i.e., synthetic procedure, comparing them with each other and explaining their essence in the broadest mathematical sense. Thus, in the introduction itself, he says that synthesis is a process in which we take what is given and proceed with a conclusion towards the ultimate goal, that is, towards what is required. He defines analysis as taking (assuming) what is required as if it were given, and then going by inference over what follows towards what is truly given. Getaldić says:

“Such proofs are twofold. Namely, they either confirm the given or deny it; those that confirm the given are called analytical proofs (analyses). In doing so, we reduce the sought conclusion to precisely those reasons by which it is proved.”25

24 More on Heron’s work has been written by Dadić (1992): 55-56.
25 Cited from the manuscript of unpublished translation by Jakov Stipišić, which was kindly given to me by colleague Žarko Dadić.
After defining analytical proofs, Getaldić methodologically opposes them to synthetic proofs:

“Namely, it is possible to turn from what is given to the same paths of analysis to what is required. Those actions that nullify the given are called reduction to the impossible. Namely, reduction to the impossible is taking what is opposed to what is truly given, because in reducing to the impossible we take as an assumption what is opposed to what is required. With such an assumption, we progress until we come across some absurdity, which, by annulling the assumption, confirms what was initially sought.”

After explaining the basic principles of analytical and synthetic procedure, Getaldić compares analytical and synthetic proofs, stating that it is evident from the above that the analysis of the reduction to the impossible differs only in the way of inference. They both go from the unknown to the known in the same order of progression, but the analysis, ending with the truth, concludes that what is assumed is true, while the reduction to the impossible, ending in the error, confirms that what is assumed is false and therefore that what is sought is true. Getaldić further points out that we distinguish between two types of analysis, theoretical and problematic.

Theoretical analysis has the ultimate goal of discovering the truth it formulates in the theorems, while problem analysis teaches how to find a way of constructing in problems and a way of proving construction. Here Getaldić emphasizes the strength and comprehensiveness of algebraic analysis and the role of Viète in its development. He emphasizes that almost all problems and theorems that fall under algebra are very easily analyzed and synthesized using symbolic algebra and algebraic analysis:

“Namely, the analysis carried out by means of invariant labels, and not by means of numbers, subject to change in whatever operation they are used, leaves clear traces through which it is not difficult to return to synthesis; synthesis in problems solved either algebraically or by

26 See previous footnote.
the old method, returns from the end of the analysis, returns from the end of the analysis, by traces of the analysis, to the beginning. In theorems, on the other hand, the truth of which is investigated algebraically, the proof proceeds in the same order in which the truth is found in the theorems.”

Then Getaldić instructs how to deal with theorems and problems that do not fall under algebra. He cites as an example those in which the comparison of angles is proved. Getaldić says that such theorems and problems are analyzed and synthesized by a method inherited from the ancient Greeks, and they exist in the books of Archimedes, Apollonius of Perga, Pappus of Alexandria and others, older and younger. And although all theorems and problems can be analyzed and synthesized by this method, nevertheless those that fall under algebra are mostly analyzed easier and faster algebraically, and then synthesized by traces of analysis. Having outlined the basic principles of algebraic analysis and synthesis, Getaldić presents the first theorems that will often be used in analyses and syntheses, together with their evidence, and thus introduces the reader to the principle of finding theorems by an algebraic approach. The comparison shows that Getaldić’s presentation at the beginning of the first book of *De resolutione* relies heavily on the introductory part of Viète’s first work on symbolic algebra *In artem analyticen isagoge* published in 1591. Here Viète also states that in mathematics there is a certain way of exploring truth, which was, it is claimed, first discovered by Plato. Theon of Alexandria gave this procedure the name of *analysis*, and precisely defined it as the process that begins by “*assuming what is requested as if it were given, and through consequences proceeds to the truth which has in fact been already given,*” as he in turn also defined *synthesis* as a process that begins with “*assuming what is given and through consequences proceeds towards the conclusion and understanding of what is required.*” It is important to emphasize that methods of analysis and synthesis were developed in ancient Greece on geometric problems, so this first analysis in mathematics (from which all other analyses later evolved) was precisely geometric analysis, while

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27 See n.25 above.
geometric construction was understood as a synthesis. Definitions similar to Theon’s also appear in Pappus of Alexandria (3rd/4th c. AD), but in a slightly changed and clarified form, at the beginning of the seventh book of his work *Treasury of Analysis*. Together with Diophantine Arithmetic, these are the two main ancient Greek sources on which Viète’s work relies.29

The key change taking place in mathematics at the turn of the 16th to 17th centuries is partly based on the transformations initiated in the way mathematical texts are written, influenced by Latin translations of Arabic original mathematical works made in the 12th and 13th centuries. Gradually, Arabic numerals came into use, which enabled a simpler presentation of mathematical expressions and operations. During the 14th, 15th and 16th centuries, mathematical knowledge was gradually improved, and some new mathematical symbols were created, which all preceded the emergence of symbolic algebra and was the basis for major changes in mathematical understandings at the end of the 16th century.

What needs to be emphasized for 16th century mathematics is the fact that before the emergence of symbolic algebra, despite the rapid and powerful creation of many new algebraic concepts (new rules and examples of how to work, new abbreviations that facilitated mathematical expression), it still remains concrete, as mathematicians of the time think within this framework of a particular problem and concrete object. The acronyms of syncopated algebra are being refined and established, but algebraic operations are still not abstracted and separated from their concrete objects to which they applied. As Dadić points out:

29 Diophantus (the first half of the 3rd century AD) continued to develop mathematics in Heron’s approach and carried out a methodological transformation of numerical use. Separately from geometric problems, he developed theoretical logistics and equation theory and dealt with quadratic equations, linear equations, and systems of equations. He considered arithmetic in addition to arithmetic and algebraic problems in his major work, so in his work algebra went from geometric to arithmetic. Diophantus’ understanding of the number was influenced by Plato and Aristotle. Therefore, to reconcile such an attitude with the use of fractions, he conceives them as smaller units of an integer. He did not acknowledge irrational and negative numbers, so he rejected such solutions, setting certain conditions for the solution to exist. Diophantus also used the unknown in the process of seeking a solution. In accordance with ancient Greek rhetorical algebra, a rhetorical form of mathematical presentation is also present in his texts, but he transforms it by introducing abbreviations for mathematical terms and summarizing the sentences into shorter form. Such a mode of expression is called syncopated, and according to it the algebra thus written becomes syncopated algebra.
“It was considered that the operations and the object form an indivisible whole, it was thought within the framework of a particular (concrete) problem, and therefore in that period the concept of the formula has not yet come to light.”

Symbolic algebra in this sense brings about a crucial turnabout. Ancient Greek geometric analysis and synthesis are used, but in such a way that by introducing general quantities called *species*, they are modified and carried out algebraically as part of general algebra. The introduced general quantities can then be applied equally and on equal terms to both numbers and geometric objects. Therefore, this new algebra, which works with general quantities instead of just numbers or with geometric objects, is pure and general algebra, different from the previous ones.

Within the work *De resolutione et compositione mathematica*, his schematic presentation of algebraic analysis and synthesis of problems is considered a great methodological contribution. He brings them up in the first book, which contains a group of problems that boil down to first-order equations with one unknown. At the end of each processed problem and the performed procedure of algebraic analysis and synthesis, which are previously mentioned in the rhetorical record, Getaldić adds a *conspectus resolutionis et compositionis*, a specific, concise and symbolic record of the performed procedures. By adding *conspectus*, Getaldić, after conducting and rhetorically writing algebraic analysis and synthesis of problems, gives a new and specific presentation which presents the role of Viète’s algebra in solving geometric problems in the methodologically best way. The *Conspectus* precisely shows and determines the mutual relationship between analysis and synthesis and reflects Getaldić’s effort to formalize procedures with the symbolism of

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31 An account of the emergence of symbolic algebra is given in Busard (1981): 18-25, and Viète’s complete oeuvre is available in a modern reprint Viète (1970). In addition, from the Croatian language literature, the basic principles of Viète’s symbolic algebra are explained by Dadić (1992): 90-92.
32 Although the first general number was introduced into mathematical practice by Jordanus de Nemore in the 13th century, its general number referred only to numbers and not to geometric objects.
33 In Ancient Greece, algebra developed within the area of geometric problems, while in the Arabic and Diophantus algebra had a numerical character. The Greeks were concerned with geometry and arithmetic, and algebra only indirectly through geometric problems that were of such a character that they could be interpreted algebraically.
the mathematics of his time. He schematically depicts an algebraic procedure as two mathematically logical processes flowing in reverse directions. Getaldić’s *conspectus* presents a double chain of inference, so that on the left side of the tabular display, in the order characteristic of the analysis as a mathematical logical method, individual mathematical steps are stated, while on the other, on the right side, the synthetic procedure is presented, in the order characteristic of the synthesis as a mathematical-logical method. Getaldić supported the value of various mathematical methods that were practiced in the early 17th century. He used the geometric and algebraic method, and promoted the value and power of Viète’s symbolic algebra and algebraic analysis and used it to solve various geometric problems. However, Getaldić realized that even the geometric method did not cease to have its meaning. The results later obtained as part of the geometric method were abundantly used in the creation of a new area, the infinitesimal calculus.

Figure 3: Page from Getaldić’s main work *On Mathematical Analysis and Synthesis*. An example of a *conspectus* made from algebraic analysis and problem synthesis. Ghetaldi (1968): 405.
The fifth book of *De resolutione et compositione mathematica* is considered the most valuable because Getaldić here provides a classification of various problems. He determines it according to the results of algebraic analysis and classifies the problems into four parts. Thus, the first chapter deals with problems that do not require construction, but are solved by numbers. The second chapter is dedicated to impossible problems that can be seen in the analysis of porism, the third chapter deals with problems that are indeterminate, and the fourth with problems that do not fall under algebra. The division of problems created by Getaldić is extremely important, especially the third unit with indefinite problems (he calls them *futile* or *void*), which can be solved in infinitely many ways. In such cases, an identity emerges, in which the given and required magnitudes can take on any value. Getaldić distinguished between two types of such problems: those that can be solved in an infinite number of ways without any restrictions, and those that can be solved in an infinite number of ways, but still not in every way. In solving problems, he notes that the ambiguity of the problem consists in the fact that the required magnitude depends on the choice of the arbitrary magnitude used in the construction of the solution.

That Getaldić’s conclusion about the existence of a connection between two magnitudes, that is, two lengths or two points, was something new and therefore can in some way be related to the beginnings of a new field of mathematics, analytical geometry, although the focus of the work itself is not on that. Due to the fact that Getaldić came very close to the realization that all points that satisfy an indeterminate problem are on some curve, his work *De resolutione et compositione mathematica* has often evaluated in the literature from the point of view of its share in the origin of analytical geometry.\(^{34}\) It was argued that Getaldić, through his major work, indirectly participated in the preparation and creation of the synthesis of the arithmetic continuum

\(^{34}\) Several scholars and historians of science, such as Oton Kučera (1893), Eugen Gelcich (1882), Antonino Favaro (1910), and others, have researched Getaldić’s *oeuvre* and cited his merits in the field of mathematics development, and pointed out the importance of the work *On Mathematical Analysis and Synthesis*, with emphasis on its role in the foundation of analytical geometry. However, Getaldić’s most valuable contribution is to work on the development of mathematical methods, so he gets his true interpretation precisely as part of the reflection on the interaction of philosophy and mathematics, which fruitfully opened the door to modern mathematics.
of numbers and the geometric continuum of points, realized in Descartes’ analytical geometry, on the basis of which an infinitesimal analysis later developed. Namely, seven years after Getaldić, using another unspecified problem, this conclusion was drawn by René Descartes.

**Conclusion**
The concluding interpretation of Getaldić’s contribution covers various aspects of his work in the context of the development and transformation of modern science. In this sense, his role in the development of mathematics and its methods should be particularly emphasized. Getaldić’s work *On Mathematical Analysis and Synthesis* is the first complete manual of algebraic analysis, and in its methods is completely innovative. The symbolic algebra and algebraic analysis that he developed and affirmed in this work enabled many aspects of new knowledge and the emergence of simpler and more accurate interpretations of previous results and knowledge. In these, the general magnitudes and the letter account are fruitfully combined with the ancient tradition. In this way, the methods of analysis and synthesis, which until the 17th century developed in the geometric realm, pass into the algebraic realm. Until then, mathematical objects and operations formed an indivisible whole. From that time on, the concept of the formula was gradually developed. Getaldić’s rich and diverse mathematical opus was focused on the development of mathematical methods. He used both traditional and new methods (geometric and algebraic). Although the new algebraic analysis he affirmed yielded many new solutions and opened up new vistas and areas, he also practiced the traditional geometric method. The geometric method proved valuable even later when the results obtained within it were used in founding a new area - the infinitesimal calculus. Getaldić applied mathematics to a variety of materials, outside the mathematical field as well, at a time when such an approach was new and completely deviated from the previous tradition. In his experimentally based work, *Promotus Archimedes* he used mathematical methodology repeatedly in analyzing physical material, and not only in researching and proving facts, but also in the method of presenting the obtained conclusions, so that he modeled his entire work modeled on the characteristic structure of ancient Greek
mathematical works. Therefore, this work of his has a special value in the history of physics and stands out as an early example of the modern approach to the study of natural sciences. *Promotus Archimedes* also fits into the general philosophical tendencies of its time, where a new knowledge of nature is imposed as the first task of philosophy. Mathematics was indisputably a key link in the founding and shaping of modern science, and Getaldić’s work on the development of mathematical methods is an important contribution and part of the overall changes that have contributed to the development of new knowledge.

**The List of Getaldić’s works in chronological order:**

*Nonnullae propositiones de parabola* (Rome: Aloysius Zannetti, 1603).

*Promotus Archimedes seu de variis corporum generibus gravitate et magnitudine comparatis* (Rome: Aloysius Zannetti, 1603).

*Apollonius redivivus seu restituta Apollonii Pergaei Inclinationum geometria* (Venice: Bernardo Giunti, 1607).

*Supplementum Apollonii Galli seu exsuscitata Apollonii Pergaei Tactionum geometriae pars reliqua* (Venice: Vincenzo Fiorina, 1607).

*Variorum problematum collectio* (Venice: Vincenzo Fiorina, 1607).

*Apollonius redivivus seu restitutae Apollonii Pergaei De Inclinationibus geometriae, Liber secundus* (Venice: Baretti, 1613).

*De resolutione et compositione mathematica* (Rome: Reverenda Camera Apostolica, 1630).

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Matematičko–fizički list 64(3) (255): 184-189.


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Sažetak
Marin Getaldić (Marino Ghetaldi, Marinus Ghetaldus), najistaknutiji je hrvatski matematičar i fizičar na prijelazu iz 16. u 17. stoljeće. Potječe iz plemićke obitelji koja je osam stoljeća zauzimala istaknuto mjesto u javnom, političkom, diplomatsком, znanstvenom i kulturnom životu Dubrovnika. Školovao se u rodom gradu, a čitavog života bavio se poslovinama u službi Dubrovačke Republike. Presudne poticaje za bavljenje znanosti Getaldić je dobio za vrijeme studijskoga boravka u europskim znanstvenim središtima i u susretima sa znanstvenicima (Michel Coignet u Antwerpenu, François Viète i Alexander Anderson u Parizu, Galileo Galilei u Padovi, Christopher Clavius i Christopher Grienberger u Rimu). Bavio se područjima teorijske matematike i njene primjene. U prirodnoj filozofiji (fizici) zanimalo se za optiku, konstruirao je i pisao o paraboličnim zrcalima, a bavio se i eksperimentalnim radom s hidrostatskom vagom, što je vodilo do rešavanju gustoće tvari.
Njegovo je najznačajnije djelo De resolutione et compositione, metodička zbirka problema i teorema, rješavanih primjenom nove algebarske metode na raznorodnoj gradi. To je prvi cjelokupni priručnik algebarske analize, u kojem se postignutim rezultatima približio utemeljenju novog matematičkog područja – analitičkoj geometriji.
Getaldićev pristup eksperimentalnom dijelu istovjetan je principima koji su kasnije izloženi u Galilejevu djelu Il Saggiatore. Getaldićev eksperiment metodički je planirani zahvat svjestan svog cilja. Razmatranu pojavu svodi na jednostavne elemente, da bi se potom podvrgnuli mjerenju. Primjenom matematike ta mjerenja dobivaju određeno značenje, upotrebljavajući matematiku u daljnjem istraživanju. Djelom Promotus Archimedes Getaldić je potpuno na tragu novog vremena, svjestan uloge i značenja matematike u novoj spoznaji prirode i pripremi za početak novovjekovne prirodne znanosti. U istraživanjima se koristi čistom matematikom, njenim znanstvenim aparatom i metodama, bez simboličkih špekulacija i metafizičkog ruha mistike brojeva, nastojeći raspoloživim metodama dosegnuti istraživani segment prirode te ga pomoću matematike razumjeti, objasniti i dokazati iz raspoloživih činjenica. U skladu s time, može se zaključiti da je Getaldićev cjelokupan rad na razvoju i prakticanj matematičkih metoda upravo potaknut sviješću da se tek nastankom novih metoda za obradu teorijskih i praktičnih problema dolazi do novih činjeničnih uvida, na kojima se zatim mogu stvarati nova gledišta i teorije.