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Capacity sharing, product differentiation and welfare

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ABSTRACT

This article constructs a duopoly market with product differentiation and analyses profits, consumer surplus and social welfare under three conditions: (a) two enterprises have sufficient capacity; (b) one enterprise has insufficient capacity, and another enterprise has excess capacity that is not shared; and (c) one enterprise has insufficient capacity, and another enterprise has excess capacity and engages in capacity sharing. Through comparison, the implementation conditions for and effects of capacity sharing and the role of product differentiation are revealed. The results show that capacity sharing helps increase producer surplus and social welfare. Capacity constraints reduce social welfare but can be solved by capacity sharing. Capacity sharing can only be realised when both enterprises are profitable, and the charge for capacity sharing should not be too high or too low. Product differentiation has impacts on output, profit, consumer surplus and social welfare, and these impacts are restricted by the existence of capacity constraints and capacity sharing.

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1. Introduction

Unbalanced capacity (overcapacity or insufficient capacity) is very popular in the market economy and can lead to the inefficient allocation of capacity resources. Traditional governance ideas are limited to individual corporate governance behaviour. Capacity sharing can provide an effective way for enterprises to optimize capacity decisions, resolve overcapacity and avoid recurring overcapacity. If capacity can be shared, enterprises with overcapacity can share with enterprises with insufficient capacity so the latter can achieve more output. Therefore, a feasible idea for solving unbalanced capacity is sharing capacity resources in time and space.[1]

Capacity sharing is characterised by the separation of ownership and use rights. Capacity sharing is an economic behaviour between the supply side and the demand side to realise the common use of capacity through some channels and technical

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means, aiming to save resources and improve the utilisation rate of capacity. Capacity sharing is a nonzero-sum game with win-win results. On the one hand, the sharing economy has evolved and spread to various sectors of the economy (Geissinger et al., 2019; Schivinski et al., 2016; Lee et al., 2016; Sutherland & Jarrahi, 2018). On the other hand, there are widespread capacity trading phenomena in retail, automobile manufacturing, shipping and power markets (Çömez et al., 2012; Hu et al., 2013; Wang et al., 2018; Yan et al., 2019).

A large number of scholars use game theory to explore the reasons for, effects of and means of capacity sharing. In terms of the effects, some scholars believe that the effects are positive, helping alleviate shortages of capacity, realize multiparty win-win situations and improve the efficiency of resource allocation in the whole economy. Renna and Argoneto (2011) propose a capacity sharing coordination mechanism in a production network composed of independent factories based on cooperative game theory and verify the effectiveness of this mechanism in improving plant benefits through simulation. Li and Zhang (2015) study capacity sharing behaviour between two freight forwarders and argue that capacity reservation between agents could achieve a win-win situation for both carriers and agents. Van Goeverden and Correia (2018) argue that the shortage of bicycle capacity in railway stations can be alleviated through point-to-point bike sharing. Some scholars also note that the realisation of positive effects of capacity sharing needs to be based on certain conditions; otherwise, capacity sharing may be unbeneficial or even negative. Yu et al. (2015) note the potential benefits of capacity sharing and the circumstances under which benefits can be realised. However, in an environment where enterprises have heterogeneous work content and service variability, capacity sharing may not be beneficial. Tinoco et al. (2017) show that the stability and long-term viability of a partnership strongly depends on the cost sharing agreement, in combination with the allocation mechanism used to share the costs (or gains) of the coordination. Tian and Jiang (2018) find that the sharing economy affects the earnings of manufacturers and retailers in the distribution channels, and the impact depends on the cost efficiency of manufacturers' capacity. When capacity costs are relatively high, both manufacturers and retailers benefit from sharing. When capacity costs are low, sharing makes manufacturers and retailers worse off. In terms of strategies, many scholars use game theory to explore the optimal capacity sharing strategies of enterprises. Yoon and Nof (2011) study the joint or separate decisions on demand and the capacity sharing protocol in an enterprise collaborative network. Aloui and Jebsi (2016) analyse the optimal capacity sharing management strategy of a bilateral monopoly intermediary platform with positive cross-externalities and bilateral congestion and argue that the capacity sharing management of any bilateral business platform should not only consider the expected participation level of both parties but also consider the size of the network benefits. Yang et al. (2017) study the cost allocation of capacity investment between manufacturers and retailers in two cases of partial and total cost sharing. Wang et al. (2018) study the optimal strategy of airline capacity allocation by using a dynamic programming model.

However, there are some issues that need to be addressed in the study of capacity sharing. The first is the effects of capacity sharing. Capacity sharing affects a series of market competition factors, such as market price, competition intensity, profit, and utility functions, which affect the equilibrium results and performance of the entire market. Nevertheless, existing research mainly reveals the effects of capacity sharing on the profits of the enterprise and emphasises the win-win results between enterprises. In addition, the current literature pays little attention to the effects on consumers, governments and other stakeholders and fails to fully consider the multiple effects of capacity sharing. This article compensates for these deficiencies by investigating the impacts of capacity sharing on consumer surplus and social welfare.

Second, there are not sufficient studies that combine capacity constraints with capacity sharing. Capacity constraints can lead to insufficient capacity, and many industries face such constraints due to funding, technology, policies, etc. (Gelhausen, Berster, & Wilken, 2013; Heo et al., 2013; Nie, 2018; Wang & Dargahi, 2013). Capacity constraints, as a common phenomenon, have multiple effects on output, competition strategies, regulation mechanism design, innovation, etc.; these effects have been widely studied (Chen et al., 2018; Mayo & Sappington, 2016; Nie, 2014; Nie & Wang, 2019). There is an important issue in the existing research on whether capacity constraints are unfavourable to enterprises (Chen, He, & Paudel, 2018; Chen, Nie, & Wang, 2015; Esó, Nocke, & White, 2010; Genc & Reynolds, 2011; Nie & Chen, 2012). Following this research, this article examines the effects of capacity constraints on the equilibrium results and judges whether capacity sharing can solve the problem of insufficient capacity.

Third, competition has an impact on capacity investment decisions (Goyal & Netessine, 2007), but there are few studies on capacity sharing among competitive enterprises. Most studies assume that capacity sharing is a cooperative relationship between two parties and that the game of capacity sharing between competing enterprises is less relevant. Wu et al. (2013) and Wu et al. (2014) study two prevailing types of contracts that address horizontal-capacity-coordination issues between two possible sources: an integrated device manufacturer and a foundry. Qi et al. (2015) argue that whether a firm invests in sharing capacity is affected by whether the capacity is used by its competitor. Guo and Wu (2018) study the optimal strategies and firm profitability related to capacity sharing between competing firms but do not consider product differentiation. This article analyses capacity sharing between two competitive enterprises with different products.

Fourth, product differentiation is an important factor affecting capacity choice. Chen et al. (2019) construct a duopoly model to investigate the influence of heterogeneity on capacity decisions under Cournot and Bertrand competition. However, research on capacity sharing focuses less on product differentiation (Chen, Xie, & Liu, 2020), and the product homogeneity assumption does not fit reality. There are two perspectives on the role of product differentiation. One perspective argues that differentiation can enhance the market competitiveness of products and increase corporate profits (Caves, 1971; Ju et al., 2013). Another view is that product differentiation improves financial performance but does not improve market performance (Chen et al., 2015). In this regard, this article incorporates product differentiation into the capacity sharing analysis framework and examines the impacts of product differentiation on enterprise output, profit, consumer surplus, social welfare and capacity sharing.

This article constructs a duopoly model to investigate three cases, including no capacity constraints, capacity constraints without sharing, and capacity constraints with sharing, and to examine the multiple effects of capacity sharing on profits, consumer surplus, and social welfare. The article also examines whether capacity sharing is feasible. In addition, product differentiation is a prerequisite, and its effects are examined.

The remainder of the article is organised as follows. The second part builds a duopoly model of capacity sharing. The third part is the model analysis. The last part presents the conclusions of the article.

2. The model

To construct a duopoly model of two enterprises under Cournot competition, the assumptions are as follows.

Assumption 1. Assume that there are two enterprises that engage in Cournot competition around production, namely, Enterprise 1 and Enterprise 2. The demand function is given by $p_i = a - q_i - rq_j$ (i, j =1,2 and $i \neq j$), where a is a positive constant and q_i is the output of firm i. Product homogeneity is defined as $r \in (0, 1]$, $1 - r$ is product differentiation, and a larger r means a lower level of product differentiation.

Assumption 2. The production technologies of the two enterprises are the same, so the marginal cost is assumed to be a constant c, and $a > c > 0$; thus, the profit function of Enterprise i is $\pi_i = (p_i - c)q_i$, and enterprises pursue profit maximisation. The capacity of enterprise i is denoted as k_i, which is a constant, and $i = 1$ or 2. Assume that the total social welfare (SW) is the sum of producer surplus (PS) and consumer surplus (PS), i.e. $SW = \pi_1 + \pi_2 + CS$, where $CS = \frac{q_1^2+q_2^2+2rq_1q_2}{2}$.[2]

Assumption 3. The outputs of enterprises depend on the profit maximisation objective and whether it has sufficient capacity. It is assumed that the capacity of enterprises is exogenous. If capacity is sufficient, the output of the enterprises can be produced according to the principle of profit maximisation. Otherwise, production can only be carried out under capacity constraints, and production with maximum profit cannot be achieved.[3] Although the products of the two enterprises may be different, it is assumed that their capacity can be used by the other party. For example, some overcapacity garment manufacturers often help other brands to produce while producing their own products in China.

Under the above assumptions, three cases are investigated and compared: (a) both enterprises have sufficient capacity; (b) one enterprise has insufficient capacity, while one enterprise has overcapacity but does not share; and (c) one enterprise has insufficient capacity, while one enterprise has overcapacity and does share. The impacts of product differentiation and capacity sharing on profit, consumer surplus and social welfare are revealed.

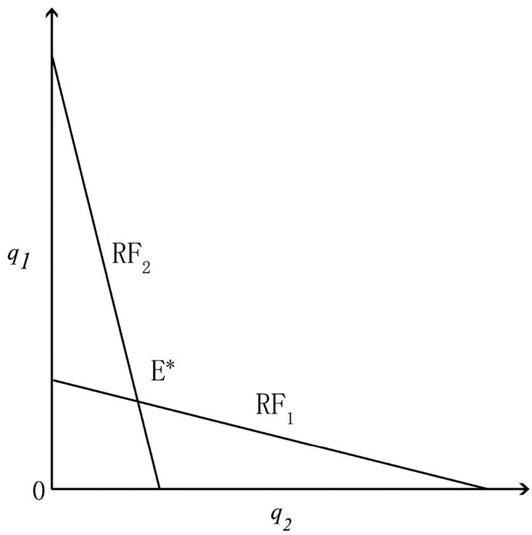

Figure 1. The reaction functions and the corresponding equilibrium when enterprises have sufficient capacity.

3. Model analysis

This section first analyses the equilibrium outcomes in each scenario and then compares the equilibrium results in three cases.

3.1. Sufficient capacity

In this scenario, two enterprises have sufficient capacity. According to the profit maximisation objective, the first-order condition $\frac{\partial \pi_i}{\partial q_i} = 0$ needs to be satisfied. The quantity reaction functions and equilibrium outputs can be expressed as

$$q_1 = \frac{a - c - rq_2}{2} \tag{1}$$

$$q_2 = \frac{a - c - rq_1}{2} \tag{2}$$

$$q_1^* = q_2^* = \frac{a - c}{2 + r} \tag{3}$$

The reaction functions and the corresponding equilibrium (E^*) are shown in Figure 1.[4]

Lemma 1. *The equilibrium profits, consumer surplus and social welfare of the two firms are*
$\pi_1^* = \pi_2^* = \frac{(a-c)^2}{(2+r)^2}$, $CS^* = \frac{(a-c)^2(r+1)}{(2+r)^2}$, and $SW^* = \frac{(a-c)^2(r+3)}{(2+r)^2}$.

From the analysis of the effects of r on production, profit, consumer surplus, and social welfare by equation (3) and Lemma 1, Proposition 1 can be proposed.

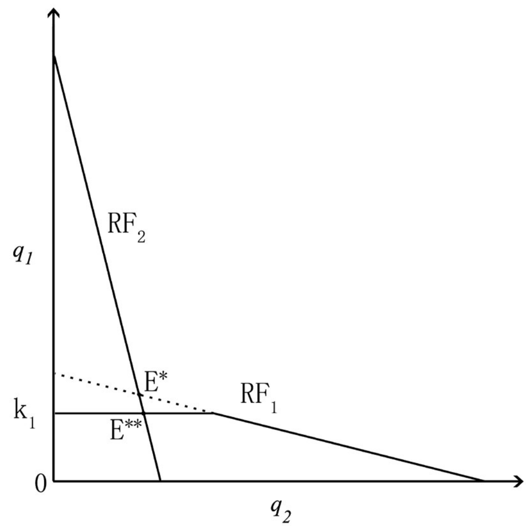

Figure 2. The reaction functions and the corresponding equilibrium when one enterprise has insufficient capacity and no capacity sharing.

Proposition 1. In the case of sufficient capacity, the effects of r on the equilibrium results are $\frac{\partial q_1^*}{\partial r} = \frac{\partial q_2^*}{\partial r} < 0$, $\frac{\partial \pi_1^*}{\partial r} = \frac{\partial \pi_2^*}{\partial r} < 0$, $\frac{\partial CS^*}{\partial r} < 0$, and $\frac{\partial SW^*}{\partial r} < 0$.

Proof. See Appendix A.

The economic implication of Proposition 1 is that in the absence of capacity constraints, increasing product differentiation increases profits and consumer surplus by increasing production, thereby increasing social welfare. In extreme cases, the products of the two enterprises are completely heterogeneous. Even if the monopoly causes prices to rise, consumer surplus also increases as consumers enjoy different commodities.

3.2. Insufficient capacity without sharing

Now change the original hypothesis and assume that Enterprise 1 has insufficient capacity and faces maximum capacity, which is defined as $k_1 < \frac{a-c}{2+r}$.[5] Enterprise 2 has sufficient capacity. If there is no capacity sharing, then the ceiling on production by the enterprise is k_1. According to the principle of profit maximisation, the reaction functions and equilibrium outputs are expressed as

$$q_1 = \begin{cases} k_1, & q_2 \in \left[0, \dfrac{a-c-2k_1}{r}\right] \\ \dfrac{a-c-rq_2}{2}, & q_2 \in \left(\dfrac{a-c-2k_1}{r}, \infty\right) \end{cases} \tag{4}$$

$$q_2 = \frac{a-c-rq_1}{2} \tag{5}$$

$$q_1^{**} = k_1, \quad q_2^{**} = \frac{a-c-rk_1}{2} \tag{6}$$

Compare with the previous case. Correcting the reflection function and the equilibrium point of Figure 1, the output of Enterprise 1 encountered a ceiling of k_1. The reaction functions and the corresponding equilibrium point (E^{**}) are shown in Figure 2. It can be found that RF_1 becomes a piecewise function and the equilibrium point changes from E^* to E^{**}, so the output of Enterprise 1 decreases and the output of Enterprise 2 increases.

Lemma 2. *Profits, consumer surplus and social welfare under equilibrium conditions are* $\pi_1^{**} = \frac{((r^2-2)k_1+(2-r)(a-c))k_1}{2}$, $\pi_2^{**} = \frac{(a-c-rk_1)^2}{4}$, $CS^{**} = \frac{(4-3r^2)k_1^2+2(a-c)rk_1+(a-c)^2}{8}$, and $SW^{**} = \frac{(3r^2-4)k_1^2-2(a-c)(3r-4)k_1+3(a-c)^2}{8}$.

From the above Lemma, it can be found that the equilibrium results are affected by capacity constraints (k_1) and product differentiation (r). From the investigation of the effects of k_1 and r on the equilibrium outputs, profits, consumer surplus and social welfare, respectively, Proposition 2 can be obtained.

Proposition 2. When an enterprise has capacity constraints without sharing, the effects of k_1 on the equilibrium results are $\frac{\partial q_1^{**}}{\partial k_1} > 0$, $\frac{\partial q_2^{**}}{\partial k_1} < 0$, $\frac{\partial \pi_1^{**}}{\partial k_1} > 0$, $\frac{\partial \pi_2^{**}}{\partial k_1} < 0$, $\frac{\partial CS^{**}}{\partial k_1} > 0$, and $\frac{\partial SW^{**}}{\partial k_1} > 0$; and the effects of r on the equilibrium results are $\frac{\partial q_1^{**}}{\partial r} = 0$, $\frac{\partial q_2^{**}}{\partial r} < 0$, $\frac{\partial \pi_1^{**}}{\partial r} < 0$, $\frac{\partial \pi_2^{**}}{\partial r} < 0$, $\frac{\partial CS^{**}}{\partial r} > 0$, *and* $\frac{\partial SW^{**}}{\partial r} < 0$.

Proof. See Appendix B.

Compared with the first case, the relationship between consumer surplus and product differentiation has changed, which indicates that the impacts of product differentiation on consumption surplus are affected by capacity constraints. According to the function of CS, the higher the product differentiation, even when the total outputs are higher, the greater is the decline of $2rq_1q_2$, and the lower is the total consumer surplus.

3.3. Insufficient capacity with sharing

In introducing capacity sharing, it is assumed that Enterprise 2 has sufficient capacity to meet the needs of Enterprise 1 and is willing to share, but for each unit of capacity, Enterprise 1 is charged at $b > 0$. Therefore, Enterprise 1 does not worry about capacity but considers the cost of payment. The profit functions of the two enterprises are given by[6]

$$\pi_1 = (-rq_2 + a - c - q_1)q_1 - (q_1 - k_1)b \tag{7}$$

$$\pi_2 = (-rq_1 + a - c - q_2)q_2 + (q_1 - k_1)b \tag{8}$$

Solving the reaction functions and the equilibrium outputs of the two enterprises (E^{***}), the following results can be obtained.

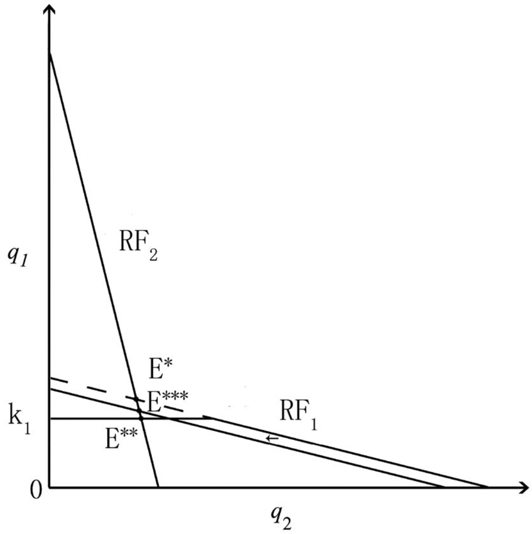

Figure 3. The reaction functions and the corresponding equilibrium when there is capacity sharing.

$$q_1 = \frac{a-c-b-rq_2}{2} \tag{9}$$

$$q_2 = \frac{a-c-rq_1}{2} \tag{10}$$

$$q_1^{***} = \frac{r(a-c)-2(a-b-c)}{r^2-4} \tag{11}$$

$$q_2^{***} = \frac{-2(a-c)+r(a-b-c)}{r^2-4} \tag{12}$$

The reaction functions and the corresponding equilibrium point (E^{***}) are shown in Figure 3. It can be found that RF_1 moves to the left compared with the first case and that the equilibrium point changes from E^{**} to E^{***}, where the output of Enterprise 1 is lower than the first case but higher than the second case and the output of Enterprise 2 is larger than the first case but lower than the second case.

Lemma 3. *The profits of the two enterprises, consumer surplus, and social welfare in this case are*

$$\pi_1^{***} = \frac{bk_1r^4 + (a^2-2ac-8bk_1+c^2)r^2-4(a-c)(a-b-c)r+4(a^2+b^2+c^2)-8a(b+c)+8b(c+2k_1)}{(r^2-4)^2}$$

$$\pi_2^{***} = \frac{\begin{array}{c}-bk_1r^4+br^3(a-c)+r^2(3b^2-4b(a-c-2k_1)+(a-c)^2)\\ -4r(a-c)^2-8b^2+8b(a-c-2k_1)+4(a-c)^2\end{array}}{(r^2-4)^2}$$

$$CS^{***} = \frac{2(a-c)(a-b-c)r^3 + \left(-6(a-c)^2 + 6(a-c)b - 3b^2\right)r^2 + 8(a-c)^2 - 8(a-c)b + 4b^2}{2(r^2-4)^2}$$

$$SW^{***} = \frac{\begin{array}{c} 2(a-c)^2 r^3 + \left(-2a^2 + (4c-2b)a + 3b^2 + 2bc - 2c^2\right)r^2 - 16(a-c)\left(a - \dfrac{b}{2} - c\right)r \\ +24a^2 - 8(b+6c)a - 4b^2 + 8bc + 24c^2 \end{array}}{2(r^2-4)^2}$$

If capacity sharing is to be achieved, it is necessary to satisfy Enterprise 1's willingness to purchase the capacity of Enterprise 2 ($\pi_1^{***} \geq \pi_1^{**}$) and Enterprise 2's willingness to sell its own capacity ($\pi_2^{***} \geq \pi_2^{**}$).

Lemma 4. When $\frac{-k_1 r^4 + (a-c)r^3 + 2(a-c+2k_1)r^2 - 8(a-c)r}{6r^2-16} \leq b \leq \frac{k_1 r^2 - (a-c)r + 2a - 2c - 4k_1}{2}$, both parties will achieve capacity sharing, and k_1, r is negatively related to the upper and lower limits of b and positively related to $a-c$.

Proof. See Appendix C.

According to Lemma 3 and Proposition 3, product differentiation, the cost of capacity and capacity constraints affect the equilibrium results.

Proposition 3. When there are capacity constraints but capacity sharing, the effects of r on the equilibrium results are $\frac{\partial q_1^{***}}{\partial r} < 0$, $\frac{\partial q_2^{***}}{\partial r} > 0$, $\frac{\partial \pi_1^{***}}{\partial r} < 0$ $\frac{\partial \pi_2^{***}}{\partial r} < 0$, $\frac{\partial CS^{***}}{\partial r} < 0$, and $\frac{\partial SW^{***}}{\partial r} < 0$; the effects of k_1 on the equilibrium results are $\frac{\partial q_1^{***}}{\partial k_1} = 0$, $\frac{\partial q_2^{***}}{\partial k_1} = 0$, $\frac{\partial \pi_1^{***}}{\partial k_1} > 0$, $\frac{\partial \pi_2^{***}}{\partial k_1} < 0$, $\frac{\partial CS^{***}}{\partial k_1} = 0$, and $\frac{\partial SW^{***}}{\partial k_1} = 0$; the effects of b on the equilibrium results are $\frac{\partial q_1^{***}}{\partial b} < 0$, $\frac{\partial q_2^{***}}{\partial b} > 0$, $\frac{\partial \pi_1^{***}}{\partial b} < 0$, $\frac{\partial \pi_2^{***}}{\partial b} < 0$, $\frac{\partial CS^{***}}{\partial b} < 0$, and $\frac{\partial SW^{***}}{\partial b} < 0$; and the optimal capacity sharing charge $b^* = \frac{-k_1 r^4 + (a-c)r^3 + 2(a-c+2k_1)r^2 - 8(a-c)r}{6r^2-16}$.

Proof. See Appendix D.

Compared with the first two cases, there are some differences in the impacts of product differentiation in the third case. Increasing product differentiation is not conducive to increasing the output of Enterprise 2 but will increase the output of Enterprise 1, profits and consumer surplus, generally improving social welfare. Additionally, capacity constraints have no effect because of capacity sharing, and reducing capacity sharing costs helps increase social welfare.

3.4. Comparisons

Here, the difference between the second case and the first case in the equilibrium results are compared, and the impacts of capacity constraints are analysed.

Proposition 4. If the equilibrium results of the cases of insufficient capacity without sharing and sufficient capacity are compared, the results of the comparison are $q_1^{**} < q_1^*$, $q_2^{**} > q_2^*$, $\pi_1^{**} < \pi_1^*$, $\pi_2^{**} > \pi_2^*$; $CS^{**} < CS^*$ and $SW^{**} < SW^*$.

Proof. See Appendix E.

Proposition 4 means that insufficient capacity limits the enterprise's output and profit while benefiting its competitor. In addition, consumer surplus and social welfare may truly decline. Therefore, insufficient capacity is inefficient for social welfare.

When the equilibrium outputs, profits, CS and SW in the third case are compared with those in the second case, Proposition 5 can be obtained.

Proposition 5. If the equilibrium results of the cases of capacity sharing and insufficient capacity without sharing are compared, the results of the comparison are $q_1^{***} > q_1^{**}$, $q_2^{***} < q_2^{**}$, $\pi_1^{***} > \pi_1^{**}$, $\pi_2^{***} > \pi_2^{**}$, $CS^{***} > CS^{**}$, and $SW^{***} > SW^{**}$.

Proof. See Appendix F.

The meaning of Proposition 5 is that if capacity sharing can be implemented, then it helps to increase consumer surplus and social welfare, so capacity sharing should be actively encouraged. Although Enterprise 2 increases the production of competitors because of sharing, it compensates by charging (b). To achieve capacity sharing, the capacity charge must be within a reasonable range: if it is too high (Enterprise 1 will choose not to share) or too low (Enterprise 2 will choose not to share), no capacity sharing will occur.

Comparing the third case with the first case in the equilibrium results, the results are as follows.

Proposition 6. The comparison results of the cases of sufficient capacity and capacity sharing are $q_1^{***} < q_1^{*}$, $q_2^{***} > q_2^{*}$, $\pi_1^{***} < \pi_1^{*}$, $\pi_2^{***} > \pi_2^{*}$, $CS^{***} < CS^{*}$, and $SW^{***} < SW^{*}$.

Proof. See Appendix G.

The meaning of Proposition 6 is that if there is insufficient capacity, capacity sharing can improve the profit of Enterprise 2 but achieve lower consumer surplus and social welfare. Therefore, capacity sharing is an effective way to optimize the allocation of capacity resources when capacity is insufficient but still inefficient compared to the case of sufficient capacity.

4. Conclusion

This article sets up a capacity sharing model based on duopoly and analyses the outputs, profits, consumer surplus and social welfare in three market environments. In contrast to other studies, this article analyses capacity sharing between two competitive enterprises with product differentiation, reveals the multiple effects of capacity sharing and product differentiation on the equilibrium results, and has some significance for promoting the development of capacity sharing and the optimisation of capacity sharing strategies. The conclusions are as follows.

First, capacity sharing has effects on the equilibrium results. When capacity sharing is implemented, consumer surplus and social welfare decrease compared with their levels in the case of sufficient capacity. Compared with their levels in case of the existence of constraints without sharing, consumer surplus, producer surplus and social welfare in the case of capacity sharing increase. Therefore, capacity sharing can enhance social welfare under certain conditions. Compared with their levels in the case of sufficient capacity, consumer surplus and social welfare in the case of insufficient capacity without sharing decline. Thus, capacity constraints reduce consumer surplus and social welfare.

Second, capacity sharing needs to be realised only when both enterprises are profitable. Capacity sharing is beneficial to enterprises with sufficient capacity and those with insufficient capacity if the capacity sharing cost (b) is not too high or too low, and the optimal capacity sharing charge for social welfare is $b^* = \frac{-k_1r^4+(a-c)r^3+2(a-c+2k_1)r^2-8(a-c)r}{6r^2-16}$. Therefore, capacity sharing is an effective way to solve capacity constraints, and the price of capacity sharing transactions is closely related to the existence of capacity sharing and social welfare. To optimize the transaction price, an alternative suggestion is that the government can provide price guidance, subsidies, etc., without destroying market competition.

Third, product differentiation has impacts on firm production and profits, consumer surplus, and social welfare, and the impacts are affected by the existence of capacity constraints and the existence of capacity sharing. In the cases of sufficient capacity and capacity sharing, increasing product differentiation can help increase profits, consumer surplus and social welfare, while in the presence of capacity constraints without sharing, increasing product differentiation reduces consumer surplus but increases profits and social welfare. Therefore, it is necessary to promote means for enterprises to develop in a differentiated way and avoid homogenous development to increase corporate profits and social welfare.

This article has some limitations that can be addressed in future research. First, this article only considers two enterprises and ignores capacity sharing among multiple enterprises. Second, this article does not consider the effects of third-party trading platforms and government intervention on capacity sharing. Third, other oligopolistic market structures (e.g. Stackelberg and Bertrand competition) are not considered. Fourth, one implicit assumption of this research is that the enterprise can hold overcapacity at no cost, while the impacts of overcapacity on enterprise cost are significant in reality.

Notes

1. In the study, the negative impacts of overcapacity on enterprises are not taken into account, and this paper focuses on the roles of capacity sharing in insufficient capacity.
2. In accordance with Singh and Vives (1984), Jain and Pal (2012), and Fanti (2016), let the utility function of the representative consumer be $U(q_1, q_2) = Aq_1 + Aq_2 - \frac{q_1^2+q_2^2+2rq_1q_2}{2}$, from which the demand function is deduced, and the consumer surplus $CS = \frac{{q_1}^2+{q_2}^2+2rq_1q_2}{2}$.
3. Insufficient capacity caused by capacity constraints implies that actual production is lower than the production that maximizes profit. Therefore, as long as the cost of capacity sharing is lower than the benefit, capacity sharing is an effective way to solve the problem of insufficient capacity (capacity constraints). In addition, it is assumed that the enterprise with sufficient capacity does not face capacity constraints (or there exist capacity constraints that do not affect the profit maximization).
4. The quantity reaction functions of Enterprises 1 and 2 are denoted as RF_1 and RF_2 respectively.

5. In the absence of capacity constraints, the optimal production of Enterprise 1 is $\frac{a-c}{2+r}$. Thus only when $k_1 < \frac{a-c}{2+r}$ is there insufficient capacity.
6. It is assumed that there is no difference in capacity and that enterprises can produce differentiated products based on the same capacity resources.

Disclosure Statement

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Appendix A

Proof of Proposition 1

In the case of sufficient capacity, the results can be obtained that $\frac{\partial q_1^*}{\partial r} = \frac{\partial q_2^*}{\partial r} = -\frac{a-c}{(2+r)^2} < 0$, $\frac{\partial \pi_1^*}{\partial r} = \frac{\partial \pi_2^*}{\partial r} = -\frac{2(a-c)^2}{(2+r)^3} < 0$, $\frac{\partial CS^*}{\partial r} = -\frac{r(a-c)^2}{(2+r)^3} < 0$, and $\frac{\partial SW^*}{\partial r} = -\frac{(r+4)(a-c)^2}{(2+r)^3} < 0$.

Appendix B

Proof of Proposition 2

Given $k_1 < \frac{a-c}{2+r}$, the results can be obtained that $\frac{\partial q_1^{**}}{\partial k_1} = 1 > 0$, $\frac{\partial q_2^{**}}{\partial k_1} = \frac{-r}{2} < 0$, $\frac{\partial \pi_1^{**}}{\partial k_1} = \frac{2(r^2-2)k_1+(2-r)(a-c)}{2} > 0$, $\frac{\partial \pi_2^{**}}{\partial k_1} = \frac{2(r^2-2)k_1+(2-r)(a-c)}{2} < 0$, $\frac{\partial CS^{**}}{\partial k_1} = \frac{2(-3r^2+4)k_1+2(a-c)r}{8} > 0$, and $\frac{\partial SW^{**}}{\partial k_1} = \frac{(3r^2-4)k_1-(a-c)(3r-4)}{4} > 0$; $\frac{\partial q_1^{**}}{\partial r} = 0$, $\frac{\partial q_2^{**}}{\partial r} = -\frac{k_1}{2} < 0$, $\frac{\partial \pi_1^{**}}{\partial r} = -\frac{(a-c-2rk_1)k_1}{2} < 0$, $\frac{\partial \pi_2^{**}}{\partial r} = -\frac{(a-c-rk_1)k_1}{2} < 0$, $\frac{\partial CS^{**}}{\partial r} = \frac{(a-c-3rk_1)k_1}{4} > 0$, and $\frac{\partial SW^{**}}{\partial r} = -\frac{3(a-c-rk_1)k_1}{4} < 0$.

Appendix C

Proof of Lemma 4

The results can be deduced that when $0 < b < \frac{k_1 r^2-(a-c)r+2a-2c-4k_1}{2}$ or $b > \frac{-k_1 r^4+6k_1 r^2-2(a-c)r+2a-2c-4k_1}{4}$, $\pi_1^{***} > \pi_1^{**}$; when $\frac{-k_1 r^4+ar^3-cr^3+2(a-c+2k_1)r^2-8(a-c)r}{6r^2-16} < b < \frac{k_1 r^2-(a-c)r+2a-2c-4k_1}{2}$, $\pi_2^{***} > \pi_2^{**}$.

Let $b_2(k_1, r, a-c) = \frac{-k_1 r^4+(a-c)r^3+2(a-c+2k_1)r^2-8(a-c)r}{6r^2-16}$ and $b_1(k_1, r, a-c) = \frac{k_1 r^2-(a-c)r+2a-2c-4k_1}{2}$; then, this article can obtain that $\frac{\partial b_1}{\partial k_1} = \frac{r^2(4-r^2)}{6r^2-16} < 0$, $\frac{\partial b_1}{\partial r} = \frac{-3r^5+16r^3-32r}{(3r^2-8)^2} < 0$, $\frac{\partial b_1}{\partial (a-c)} = \frac{r^3+2r^2-8r}{6r^2-16} > 0$, $\frac{\partial b_2}{\partial k_1} = \frac{r^2-4}{2} < 0$, $\frac{\partial b_2}{\partial r} = \frac{2k_1 r-a-c}{2} < 0$, and $\frac{\partial b_2}{\partial (a-c)} = \frac{2-r}{2} > 0$.

Appendix D

Proof of Proposition 3

Given $\frac{-k_1 r^4+(a-c)r^3+2(a-c+2k_1)r^2-8(a-c)r}{6r^2-16} \le b \le \frac{k_1 r^2-(a-c)r+2a-2c-4k_1}{2}$ and $k_1 < \frac{a-c}{2+r}$, the following results can be deduced.

The effects of r on the equilibrium results are as follows.

$$\frac{\partial q_1^{***}}{\partial r} = \frac{(c-a)(4+r^2)+4(a-b-c)r}{(r^2-4)^2} < 0$$

$$\frac{\partial q_2^{***}}{\partial r} = \frac{(c-a+b)(4+r^2)+4(a-c)r}{(r^2-4)^2} > 0$$

$$\frac{\partial \pi_1^{***}}{\partial r} = -\frac{2((a-c)r+2(c-a+b))((a-c)r^2+4(c-a+b)r+4(a-c))}{(r^2-4)^3} < 0$$

$$\frac{\partial \pi_2^{***}}{\partial r} = \frac{\begin{array}{c}-b(a-c)r^4-2(a-b-c)(a-3b-c)r^3+12(a-c)(a-b-c)r^2\\ -8(3(a-c)^2-b^2)r+16(a-c)^2\end{array}}{(r^2-4)^3} < 0$$

$$\frac{\partial CS^{***}}{\partial r} = \frac{\begin{array}{c}r((a-c)(a-b-c)r^3+(6(a-b-c)(c-a)-3b^2)r^2\\ +4(a-c)(a-b-c)(3r-2)-4b^2)\end{array}}{(4-r^2)^3} < 0$$

$$\frac{\partial SW^{***}}{\partial r} = \frac{\begin{array}{c}-(a-c)^2 r^4+(2(a-c)(a-c+b)-3b^2)r^3+12(a-c)(a-b-c)r^2\\ +(8(a-c)(5c-5a+3b)-4b^2)r+16(2a-b-2c)(a-c)\end{array}}{(r^2-4)^3} < 0$$

The effects of k_1 on the equilibrium results are $\frac{\partial q_1^{***}}{\partial k_1} = 0$, $\frac{\partial q_2^{***}}{\partial k_1} = 0$, $\frac{\partial \pi_1^{***}}{\partial k_1} = \frac{br^4-8br^2+16b}{(r^2-4)^2} > 0$, $\frac{\partial \pi_2^{***}}{\partial k_1} = \frac{-br^4+8br^2-16b}{(r^2-4)^2} < 0$, $\frac{\partial CS^{***}}{\partial k_1} = 0$, and $\frac{\partial SW^{***}}{\partial k_1} = 0$.

The effects of b on the equilibrium results are as follows.

$$\frac{\partial q_1^{***}}{\partial b} = \frac{2}{r^2-4} < 0, \quad \frac{\partial q_2^{***}}{\partial b} = \frac{-r}{r^2-4} > 0$$

$$\frac{\partial \pi_1^{***}}{\partial b} = \frac{k_1 r^4 - 8k_1 r^2 + 4(a-c)r + 8(b-a+c+2k_1)}{(r^2-4)^2} < 0$$

$$\frac{\partial \pi_2^{***}}{\partial b} = \frac{-k_1 r^4 + r^3(a-c) + 2r^2(3b-2a+2c+4k_1) - 8(2b-a+c+2k_1)}{(r^2-4)^2} < 0$$

$$\frac{\partial CS^{***}}{\partial b} = \frac{-2(a-c)r^3 + 6(a-c-b)r^2 - 8(a-c-b)}{2(r^2-4)^2} < 0$$

$$\frac{\partial SW^{***}}{\partial b} = \frac{(-a+3b+c)r^2 + 4(a-c)r - 4(a-c+b)}{(r^2-4)^2} < 0$$

Given $\frac{-k_1 r^4 + (a-c)r^3 + 2(a-c+2k_1)r^2 - 8(a-c)r}{6r^2-16} \le b \le \frac{k_1 r^2 - (a-c)r + 2a - 2c - 4k_1}{2}$ and $\frac{\partial SW^{***}}{\partial b} = \frac{(-a+3b+c)r^2 + 4(a-c)r - 4(a-c+b)}{(r^2-4)^2} < 0$, it can be deduced that $b^* = \frac{-k_1 r^4 + (a-c)r^3 + 2(a-c+2k_1)r^2 - 8(a-c)r}{6r^2-16}$.

Appendix E

Proof of Proposition 4

$$q_1^{**} - q_1^{*} = k - \frac{a-c}{2+r} < 0$$

$$q_2^{**} - q_2^{*} = \frac{r(-rk + a - c - 2k)}{4+2r} > 0$$

$$\pi_1^{**} - \pi_1^{*} = \frac{-((r^3 + 2r^2 - 2r - 4)k_1 + 2a - 2c)((-r-2)k_1 + a - c)}{2(2+r)^2} < 0$$

$$\pi_2^{**} - \pi_2^{*} = \frac{(-rk_1 + a - c - 2k_1)(-k_1 r^2 + (a-c-2k_1)r + 4a - 4c)r}{4(2+r)^2} > 0$$

$$CS^{**} - CS^{*} = \frac{(a-c-(2+r)k_1)\left(3k_1 r^3 + (a-c+6k_1)r^2 - 4(a-c+k_1)r - 4(a-c+2k_1)\right)}{8(2+r)^2} < 0$$

$$SW^{**} - SW^{*} = \frac{(a-c-(2+r)k_1)\left(-3k_1 r^3 + 3(a-c-2k_1)r^2 + 4(a-c+k_1)r - 4(3a-3c-2k_1)\right)}{8(2+r)^2} < 0$$

Appendix F

Proof of Proposition 5

$$q_1^{***} - q_1^{**} = \frac{r(a-c) - 2(a-b-c-2k_1)}{r^2-4} > 0$$

$$q_2^{***} - q_2^{**} = \frac{-k_1r^3 + r^2(a-c) - 2(a-b-c-2k_1)r}{8-2r^2} < 0$$

$$\pi_1^{***} - \pi_1^{**} = \frac{\begin{array}{c} -k_1^2r^6 + k_1(a-c)r^5 - 2k_1(a-b-c-5k_1)r^4 - 8k_1(a-c)r^3 \\ +(-32k_1^2 + 16(a-b-c)k_1 + 2(a-c)^2)r^2 - 8(a-c)(a-b-c-2k_1)r \\ +32k_1^2 - 32(a-b-c)k_1 + 8a^2 - 16(b-c)a + 8(b+c)^2 - 32ac \end{array}}{2(r^2-4)^2} > 0$$

$$\pi_2^{***} - \pi_2^{**} = \frac{\begin{array}{c} (-k_1r^4 + (a-c)r^3 + 2(a-3b-c+2k_1)r^2 - 8(a-c)r \\ +16b)(-k_1r^2 + (a-c)r - 2(a-b-c-2k_1)) \end{array}}{-4(r^2-4)^2} > 0$$

$$CS^{***} - CS^{**} = \frac{\begin{array}{c} (-k_1r^2 + (a-c)r - 2(a-c-b-2k_1)(3k_1r^4 + (a-c)r^3 \\ -2(3a-3b-3c+8k_1)r^2 + 4(a-c)r + 8(a-b-c+2k_1) \end{array}}{-8(r^2-4)^2} > 0$$

$$SW^{***} - SW^{**} = \frac{\begin{array}{c} -\left(-3r^4k_1 + 3(a-c)r^3 + (-2a-6b+2c+16k_1)r^2 + (-20a+20c)r + 24a + 8b - 24c - 16k_1\right) \\ (-k_1r^2 + r(a-c) - 2a + 2b + 2c + 4k_1) \end{array}}{8(r^2-4)^2} > 0$$

Appendix G

Proof of Proposition 6

$$q_1^{***} - q_1^{*} = \frac{2b}{r^2-4} < 0$$

$$q_2^{***} - q_2^{*} = \frac{rb}{4-r^2} > 0$$

$$\pi_1^{***} - \pi_1^{*} = \frac{\left(k_1r^4 - 8k_1r^2 + 4(a-c)r - 8(a-c) + 4b + 16k_1\right)b}{(2+r)^2(2-r)^2} < 0$$

$$\pi_2^{***} - \pi_2^{*} = \frac{\left(-k_1r^4 + (a-c)r^3 + (3b + 8k_1 - 4(a-c))r^2 + 8(a-c-b-2k_1)\right)b}{(2+r)^2(2-r)^2} > 0$$

$$CS^{***} - CS^{*} = -\frac{\left((a-c)r^3 - (3a-3c-1.5b)r^2 + 4(a-c) - 2b\right)b}{(r-2)^2(2+r)^2} < 0$$

$$SW^{***} - SW^{*} = -\frac{b\left((a-c-1.5b)r^2 + 4(c-a)r + 4(a-c) + 2b\right)}{(r-2)^2(2+r)^2} < 0$$