# STUDY OF THE DEFORMATION STATE DURING THE PULLING OF THE WORKPIECE IN A SPECIAL DIE

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The article presents the results of a study of the deformation state when pulling a metal workpiece in a special die, preventing unidirectional metal flow in the longitudinal direction. The study of the deformation state and the construction of fields of moving during pulling of workpieces in a special die was carried out using variational methods, as well as mathematical modeling. The results of the studies on variational methods and mathematical modeling showed that the values of shear deformations in the central zone of deformation focus are respectively within 0,081 – 0,1226 and 0,085 – 0,175, which in turn proves the possibility of obtaining long-term metal workpieces with increased physical and mechanical properties in the die.

Keywords: pulling, deformation state, special die, workpiece, mathematical modeling

# INTRODUCTION

One of the main problems when pulling metal blanks or drawing a wire through the working channel of a drawing die is strictly unidirectional flow of the metal, which leads to a difference in physical and mechanical properties and the probability of metal destruction in the direction of pulling when the tensile stress reaches the yield stress of the material [1–3]. Therefore, to eliminate the above undesirable phenomena, a special die is proposed, where the unidirectional flow is prevented by creating an obstacle to the unidirectional metal flow from the side of the inclined portion of the die. Hence, it is necessary to assess the nature of the metal flow and the deformation state when the workpiece passes through the working channel of a special die. It is proposed to use a special die in a technological cycle with a closed die [4].

#### THEORETICAL BASIC AND METHOD [5, 6]

To study the deformation state and construct displacement fields when pulling workpiece in a special die, preventing unidirectional flow in the longitudinal direction, a variational method is used. It should be noted that the variational method for determining deformations, displacements and forces is based on the energy principle.

The work of external forces (double area integral F),

$$\mathbf{A}_{\text{ext}} = \iint_{F} (\mathbf{X}\mathbf{u} + \mathbf{Y}\mathbf{v} + \mathbf{Z}\mathbf{w}) d\mathbf{F},$$

 $A_{ext}$ :

where X, Y, Z – projection of external forces on the coordinate axis; u, v, w – corresponding to the coordinates of movement.

The work of internal forces (triple volume integral V),  $A_{i,...}$ :

$$A_{int} = \iiint_V \sigma_i \varepsilon_i dV$$

in which replacing the intensity of normal stresses  $\sigma_i$  with the intensity of shear stresses  $\tau_i$ , denoting it by the letter T, and the intensity of deformations  $\varepsilon_i$ , by the intensity of shear deformations  $\gamma_i$ , denoting the letter  $\Gamma$ , and also expressing the intensity of the shear stress T in terms of the yield stress  $\sigma_T$  of the deformed material, i.e.  $T = \sigma_T / \sqrt{3}$  and taking into account that k = T, and as a constant, taking k outside the integral sign can be represented as follows:

$$\mathbf{A}_{\mathrm{int}} = \mathbf{k} \iiint_{V} \Gamma d\mathbf{V},$$

where k – constant of plasticity or shear yield strength.

To construct displacement and deformation fields when pulling a workpiece with a diameter D in a special die, variational methods are used. A round cross section workpiece is pulled in a special die, consisting of three sections: I - exit section, II - lead-in section and III - inclined shear section (Figure 1).

In this case, the development of intense shear deformations in the shear section III leads to the elaboration of the metal structure, the elimination of internal defects and the prevention of excessive drawing of grains. The deformation can be considered axisymmetric and flat. Due to symmetry, only one half of the workpiece can be viewed by placing the origin at the midpoint of the broached workpiece height. The work of external forces

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**Figure 1** The scheme of roaching the workpiece in a special die and possible displacement fields: I – exit section (length I<sub>1</sub>); II – lead-in section (length I<sub>1</sub>); III – inclined shear section (length I<sub>1</sub>); D – diameter of the pulled workpiece;  $\alpha$  – angle of inclination of the die; P – pulling force; U<sub>y</sub> – vertical displacement;  $\tau_{\kappa}$  – friction stress; P<sub>a</sub> – total reduction force on the inclined section of the workpiece; x, y – abscissa and ordinate

is equal to the work of active forces and friction forces on an inclined section of the workpiece shear, which is small compared to the shear deformation, since the reduction of the workpiece in diameter is insignificant, i.e. the diameter of the workpiece changes slightly during pulling. Therefore, the main work will be spent on shear deformation of the workpiece in section III.

The work of active forces  $A_a$  is equal to the product of the total reduction force  $P_a$  on the inclined section of the workpiece by the vertical displacement  $U_y$ , with y = D or  $A_a = P_a \cdot U_{y=D} = P_a \cdot \Delta D$ , where  $\Delta D$  is the absolute reduction of the workpiece in terms of diameter.

The work of the forces of contact friction on an elementary surface  $\Delta F$  is equal to the product of displacement along the contact surface by the friction stress  $\tau_{\kappa}$ . The work of the friction forces is opposite to the displacement, then the work of the friction forces during axisymmetric plane deformation, expressing the value of contact friction in fractions n of the constant plasticity k, i.e.  $\tau_{\kappa} = n_{k}$ , can be expressed as follows:

$$A_{fr} = -\tau_{\kappa} \int_{0}^{1} U_{y=D} dx = -nk \int_{0}^{1} U_{y=D} dx.$$

Then the work of external forces, taking into account that  $A_{fr}$  is taken with a plus sign, is equal to:

$$A_{ext} = A_a - A_{fr} = P_a \cdot U_{y=D} - -nk \int_0^1 U_{y=D} dx.$$

Based on this formula, the work of internal forces under plane deformation can be represented as follows:

$$A_{int} = k2\pi \int_0^D \int_0^1 \Gamma y dy dx,$$

with  $\Gamma$  – intensity of shear deformations in an axisymmetric, plane strained state, D – diameter of the pulled workpiece. k is taken out of the integral sign as a constant number.

The work of active forces will be equal to:

$$A_{a\alpha} = P \Delta D = k2\pi \int_0^D \int_0^1 \Gamma y dy dx + nk \int_0^1 U_{y=D} dx.$$

Accordingly, the work of external and internal forces on possible displacements will be:

$$\mathrm{nk}\delta\int_{0}^{1}\mathrm{U}_{\mathrm{y}=\mathrm{D}}\mathrm{dx}$$
 and  $\mathrm{k}2\pi\delta\int_{0}^{D}\int_{0}^{1}\Gamma\mathrm{y}\mathrm{dy}\mathrm{dx}$ 

and according to the principle, the beginning of possible displacements can be represented as follows:

$$nk\delta\int_0^1 U_{y=D}dx + k\delta 2\pi\int_0^D \int_0^1 \Gamma y dy dx = 0.$$

If we express the displacement and the intensity of shear deformations using the direct Ritz method, then the last expression after reduction will take the following form:

$$\begin{split} &n\frac{\partial}{\partial a_{0}}\int_{0}^{1}U_{y=D}dx + \frac{\partial}{\partial a_{0}}2\pi\int_{0}^{D}\int_{0}^{1}\Gamma ydydx = 0, \\ &n\frac{\partial}{\partial a_{1}}\int_{0}^{1}U_{y=D}dx + \frac{\partial}{\partial a_{1}}2\pi\int_{0}^{D}\int_{0}^{1}\Gamma ydydx = 0, \end{split}$$

where  $\delta$  - possible displacement near the equilibrium state. As already noted when pulling the workpiece in a special die, and as the experimental data show, the displacement along the y axis is carried out due to pure shear. The movement along the x-axis is minimal, since the workpiece undergoes slight compression deformation along the height y and along the length x.

To select the displacement functions  $U_y$ , the direct Ritz method was applied, which consists in the fact that the sought function is represented as a series, for example:

$$U = a_0 \phi_0(x, y, z) \pm a_1 \phi_1(x, y, z) + ...,$$

where  $a_0, a_1, \ldots$  – undefined parameters;  $\phi_0(x,y,z)$ ,  $\phi_1(x,y,z), \ldots$  – functions of coordinates corresponding to boundary conditions. The functions  $\phi_1$  are taken arbitrarily, with the condition that they must only meet the given boundary conditions.

Hence, in the first approximation, the displacement function is taken in the following form:

$$U = a_0 x \tan \alpha + a_1 x \left(\frac{1}{2} - \frac{y}{D}\right) \left(1 - \frac{x}{D}\right)$$

which meets the accepted boundary conditions, i.e. at x = 0 the displacements are equal to zero, and at x = 1are equal to the value of U at the boundary between the plastic and rigid zones of the broached workpiece

Deformation components are determined as follows:

$$\varepsilon_{x} = \frac{\partial u}{\partial x} = a_{0} \tan \alpha + a_{1} \left(\frac{1}{2} - \frac{y}{D}\right) \left(1 - \frac{2x}{D}\right),$$

from the condition of volume constancy, i.e.  $\varepsilon_x + \varepsilon_y + \varepsilon_z = 0$ ; with axisymmetric-plane deformation  $\varepsilon_x = \varepsilon_y + \varepsilon_z$ , hence  $\varepsilon_y = \varepsilon_z = -0.5\varepsilon_x$  or

$$\varepsilon_{y} = -0, 5\varepsilon_{x} =$$

$$= -0, 5\left(a_{0} \tan \alpha + a_{1}\left(\frac{1}{2} - \frac{y}{D}\right)\left(1 - \frac{2x}{D}\right)\right),$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial u_{y}}{\partial x} =$$

$$= \frac{a_{1}x}{D}\left(\frac{x}{D} - 1\right) + \frac{a_{1}y}{2D}\left(\frac{y}{D} - 1\right) =$$

$$= a_{1}\left(\left(\frac{x^{2}}{D^{2}} - \frac{x}{D}\right) + \left(\frac{y^{2}}{2D^{2}} - \frac{y}{2D}\right)\right).$$

Moving along the y-axis, i.e. by the diameter of the workpiece with a small reduction of the workpiece in the shear section:

$$U_{y} = \varepsilon_{y} \int dy =$$
$$= -0.5 \left( a_{0} \tan \alpha y + a_{1} \left( \frac{y}{2} - \frac{y^{2}}{2D} \right) \left( 1 - \frac{2x}{D} \right) \right).$$

From this equation, the value of the parameter  $a_0$  is determined for y = D and  $U_y = -0.5$  ( $a_0 \tan \alpha D$ ) i.e. on the contact surface, the movement or reduction of the workpiece in diameter is equal to, i.e.  $U_y = \Delta D/2$  then,  $\Delta D/2D = -0.5 a_0 \tan \alpha$  where  $\Delta D/2D = \varepsilon$  – relative reduction of the workpiece on the inclined section of the die, the value of which is significantly small, since on an inclined section, the diameter of the workpiece changes slightly. Hence,

$$a_0 = -\frac{2\varepsilon}{\tan \alpha}$$

i.e., the coefficient  $a_0$  shows the ratio of the relative compression  $\varepsilon$  to the intensity of shear deformation  $\Gamma$ , which are realized on the inclined portion of the die, which can be expressed through the tangent of the angle of inclination of the die  $\alpha$ , i.e.  $\Gamma = \tan \alpha$ . Coefficient  $a_0$ shows the ratio of two deformations, in particular, which of them prevails. In this case, the value of  $\Gamma$ should prevail, since shear deformation develops more on the inclined section.

The parameter  $a_1$  is determined from the second equation, obtained by the direct Ritz method, i.e. the first term is calculated. For this, taking into account  $a_0$  from the displacement function,  $U_{y=D}$  is determined:

$$U_{y=D} = -2\varepsilon x - a_1 x \frac{1}{2} \left( 1 - \frac{x}{D} \right).$$

Substituting this equation into the first term of the second equation, obtained by the direct Ritz method:

$$\frac{\partial A_{fr}}{k\partial a_1} = n \frac{\partial}{\partial a_1} \int_0^1 U_{y=D} dx =$$
$$= n \int_0^1 \left( -a_1 x \frac{1}{2} \left( 1 - \frac{x}{D} \right) \right) dx = -n l^2 \left( \frac{1}{4} - \frac{1}{6D} \right) \cdot$$

Next, the second term of the second equation is calculated, obtained by the direct Ritz method:

$$\frac{\partial A_{int}}{k\partial a_1} = \frac{\partial}{\partial a_1} 2\pi \int_0^D \int_0^1 \Gamma y dy dx.$$

Next, the value of the intensity of shear deformations  $\Gamma$  is determined, which, in case of axisymmetric-plane deformation, is determined by the following formula:

$$\Gamma = \sqrt{3\epsilon_x^2 + \gamma_{yx}^2},$$

where  $\varepsilon_x$  – the deformation of elongation of the workpiece along the x axis, which has a small value, since the cross-sectional area or reduction of the workpiece is insignificant,  $\gamma_{yx}$  is the angular deformation.

Substituting the corresponding values of  $\epsilon_{x}$  and  $\gamma_{yx}$  into this expression:

$$\Gamma = \sqrt{\frac{3\left(a_0 \tan \alpha + a_1\left(\frac{1}{2} - \frac{y}{D}\right)\left(1 - \frac{2x}{D}\right)\right) + \left(\frac{a_1 x}{D}\left(\frac{x}{D} - 1\right) + \frac{a_1 y}{2D}\left(\frac{y}{D} - 1\right)\right)^2}$$

However, the integration of the expression for the intensity of shear deformations, i.e. the second term leads to some difficulties, therefore, following the recommendations of [1], an approximate method was used. For this, the value of  $\Gamma$  is designated as follows:

$$\Gamma = \sqrt{f\left(\frac{x}{l}, \frac{y}{D}, a_{1}\right)},$$

which is first differentiated by a<sub>1</sub>:

$$\frac{\partial \Gamma}{\partial \mathbf{a}_1} = \frac{\frac{\partial}{\partial \mathbf{a}_1} f\left(\frac{\mathbf{x}}{\mathbf{1}}, \frac{\mathbf{y}}{\mathbf{D}}, \mathbf{a}_1\right)}{2\sqrt{f\left(\frac{\mathbf{x}}{\mathbf{1}}, \frac{\mathbf{y}}{\mathbf{D}}, \mathbf{a}_1\right)}} = \frac{\frac{\partial \Gamma^2}{\partial \mathbf{a}_1}}{2\Gamma}.$$

Replacing the value of  $2\Gamma$  in the denominator with the average value  $2\Gamma_a$  will make it possible to take it outside the integral sign as a constant number. The average value of  $2\Gamma_a$  in the shear section of the workpiece in a special die can be expressed through the shear angle  $\alpha$ . Then, from the expression  $\Gamma = \sqrt{3\epsilon_x^2 + \gamma_{yx}^2}$ , it is possible to determine the average value of the intensity of shear deformations, assuming that the elongation  $\varepsilon_x$  with small reductions of the workpiece in terms of diameter has an insignificant value ( $\varepsilon_x < 0,1$ ), therefore, it can be neglected. Then the value of the intensity of shear deformations will be equal to,  $\Gamma_a = \gamma_{yx} = \tan \alpha$ , hence,  $2\Gamma_m = 2\gamma_{yx} = 2\tan \alpha$ . Then

$$\frac{\partial A_{int}}{k\partial a_1} = \frac{\pi}{\tan \alpha} \int_0^D \int_0^1 \left( \frac{\partial \Gamma^2}{\partial a_1} \right) y dy dx$$

Differentiating  $\Gamma^2$  from the corresponding expression with  $a_1$  and integrating the resulting expression according to the above equation, taking the ratio l/D = 2,0 and n = 0,20,  $\varepsilon \approx 0,10$ , after the transformation, the following expression is obtained:

$$-nl^{2}\left(\frac{1}{4}\frac{\pi}{6D}\right) + \frac{1}{\tan\alpha}2\varepsilon l^{3} + \frac{6\pi}{\tan\alpha}l^{3}\frac{1}{18}a_{1} + \frac{4\pi}{\tan\alpha}a_{1}\cdot0, 139 = 0.$$

Hence the value of the parameter  $a_1$  will be equal to:

$$a_{1} = \frac{\frac{1}{6}nl^{2} - \frac{\pi}{\tan} 2\epsilon l^{3}}{\frac{\pi}{\tan\alpha}l^{3}\left(6\frac{1}{18} + 2\cdot0,139\right)} = -0,327$$

Similarly, you can determine the value of  $a_1$  at various preferred values of l/D, n and  $\varepsilon$ , for example, taking the ratio l/D = 1,5; n = 0,25 and  $\varepsilon \approx 0,05 - 0,10$ .

### **RESULTS AND DISCUSSION**

Thus, if the values of the length of the shift section of the die 1 are known, i.e. the ratio 1/D in section III, the coefficient of friction f and the relative reduction  $\varepsilon$  of the workpiece in the shear section, it is possible to determine the parameters  $a_0$  and  $a_1$ . Further, substituting these values into the displacement function, one can construct the displacement fields of points in the shear section or in the zone of plastic deformation of the workpiece, and give an estimate of the deformation state (Figure 2, a).



**Figure 2** Deformation state when pulling the workpiece in a special die: a) displacement and deformation fields in the deformation zone; b) deformation values obtained by computer simulation

It should be noted that the calculation results are in good agreement with the results obtained by mathematical computer modeling in the DEFORM software package (Figure 2, b).

Figure 2, b shows that the values of shear deformations in the central zone of the deformation zone are in the range of 0,085-0,175. Approximately the same results were obtained from calculations by variational methods, which are in the range of 0,081-0,1226. In addition, the calculation results show that the incompressibility condition is satisfied, i.e. meet the boundary conditions:  $\varepsilon_x = -2 \varepsilon = 0.5 \varepsilon_y$ , adopted in determining the components of deformation.

# CONCLUSIONS

Analysis of the obtained calculation results shows that the maximum value of shear deformations is observed in the zone between the coordinates x = 0 and x = 1, i.e. in the middle between the central point 0 and the extreme point corresponding to the boundary between the plastic and rigid zones l. At the central point, i.e. at x = 0 along the y axis, the displacement  $U_v$  and angular deformations  $\gamma_{xv}$ , (except for  $\varepsilon_{x}$ ) are equal to zero, and, as previously assumed, when choosing a suitable function U relative to this point, the values of displacement and deformations grow. In the deformation zone, shear deformations ( $\gamma_{yy}$  = -0,122) generally prevail (develop), which grow to a maximum, starting from x = 0 and up to x = 1. It should be noted that it is the development of shear deformations  $\gamma_{xy}$ that prevents the elongation of the microstructure in the deformation zone. In addition, due to shear deformations, not only the nature of the metal flow changes, but also internal defects and voids are closed, which leads to an increase in the quality of the drawn workpiece.

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- **Note:** The response for English language is D. Rahimbekova the translation professional of Karaganda industrial university