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Different Perspectives on Success in Solving Stand-Alone Problems by 14 to 15-Year-Old Students

Nives Baranović[1] and Branka Antunović-Piton[2]
[1]*Faculty of Humanities and Social Sciences, University of Split*
[2]*Faculty of Educational Science, Juraj Dobrila University of Pula*

Abstract

The paper defines a special type of problem tasks and considers its didactic potential, as well as the success of students in solving the selected problem. The research instrument used is a geometrical task from the National Secondary School Leaving Exam in Croatia (State Matura). The geometrical task is presented in three versions: as a verbal problem, as a verbal problem with a corresponding image and as a problem in context. The material analysed in the present paper was collected from 182 students in 7th and 8th grade of Croatian urban elementary schools. The didactic potential is considered from the aspect of use of mathematical concepts and connections. The success of students in problem-solving is considered from the aspect of implementation of the problem-solving process and producing correct answers, depending on the manner in which the tasks are set up. The results show that the stand-alone problem, as a special type of problem task, has considerable didactic potential. However, the students' skills of discovering and connecting mathematical concepts and their properties are underdeveloped. In addition, the manner in which the tasks are set up considerably affects the process of solving the task and consequently the success of that process. Based on the results of the research, proposals are given for application of stand-alone problems in teaching mathematics.

Key words: *isolated problem; mathematical task; problem solving; problem evaluation.*

Introduction

Contemporary mathematics education is student-centred and focused on the development of mathematical knowledge, skills and competences students will use in their personal, social and professional life (KM, 2019). In order to achieve this, the requirements of the contemporary mathematics curriculum include teaching mathematical concepts, development of mathematical processes and creating suitable, creative and innovative environment for application of acquired knowledge, skills and procedures in various contexts. The problem-solving process is particularly highlighted as one of key mathematical processes (KM, 2019). A number of educational researches in the field of problem tasks and solving strategies during the last few decades confirm that the abovementioned curriculum requirements can be fulfilled using problem tasks (e.g. Kordaki, 2015; Schoenfeld 1992).

Taking into consideration all educational stages, it is evident that solving tasks is the most common activity of learners. For this reason, mathematical tasks present a very powerful tool in teaching mathematics. The goals of contemporary math education should be attained through proper application of various types of tasks. One of the aims of this paper is to reflect on the didactic role of geometry problem tasks.

Mathematical tasks can be used as motivational introduction to the topic or as basis for practicing certain procedures, algorithms and formulas. They can also be used for revision of previously acquired knowledge, application of knowledge in new situations and contexts and learning new content. Considering that the tasks can range from very simple standardised tasks to highly complex and non-standard tasks, through solving different types of tasks students can develop and improve various procedural skills, acquire corresponding conceptual knowledge and apply the knowledge and skills in more complex situations. Problem solving skills enable the students to acquire and connect mathematical concepts, acquire and apply mathematical processes and develop and demonstrate mathematical competences.

Since the teacher is the creator and implementer of teaching activities, he/she plays a crucial and responsible role in the teaching process, in spite of the wide accessibility of teaching material. By choosing various types of tasks, the teacher should ensure student-based learning environment, providing each student with an opportunity for acquiring and developing the abovementioned knowledge, skills and competences. If students usually solve only cognitively simple tasks and achieve success, they could refrain from further efforts because they think they have completed all necessary work. On the other hand, if students usually solve only high-level, cognitively complex tasks which are beyond their abilities, they do not achieve success and give up due to lack of motivation and confidence. If we take into account the fact that classes are heterogeneous, especially in elementary school, it is necessary to create balance and use tasks with different levels of complexity. In this way, all students are given the opportunity to develop their mathematical abilities and potential.

Various educational research confirm that the problem tasks create an appropriate environment for students to connect previously acquired and gain new knowledge, develop different skills and competences such as visualisation, choosing a correct and economical problem-solving path and appropriate method, coping with new and unfamiliar situations, etc. (e.g. Levav-Waynberg & Leikin, 2012; Natsheh & Karsenty, 2014; Schoenfeld, 1992).

Although the contemporary mathematics curriculum requires interrelationship between domains and processes and intra- and cross-curricular relations, teaching mathematics is often implemented based on a certain number of topics/subjects taught within one subject area (e.g. Triangles, Quadrilaterals, Linear Function, Vectors, etc.). Correspondingly, tasks are chosen for the purpose of learning, practising and applying certain concepts and processes. After all topics within a subject area are completed, assessment of students' skills and competences is carried out using similar tasks. In such a system, students usually have no difficulties in completing course contents. The system also seems very efficient, as most students move on to the next stage.

However, if students are assessed outside of the context of the subject area, for example in international benchmarking tests such as PISA, TIMSS, etc., and national level tests (State Matura in Croatia), they usually achieve poorer results (PISA 2012; Handbook SM, 2017; Handbook TIMSS, 2017). Those results show that students did not actually achieve the expected learning outcomes according to the requirements of the contemporary mathematics curriculum: they neither have the permanent knowledge nor knowledge applicable in different contexts.

When we select one mathematical task, e.g. a mid-level task which can be solved using different paths and strategies, and give this task to students outside of the subject area's context, this task becomes an isolated problem (called a stand-alone problem or SA problem). In order to solve the task, one has to activate an entire network of mathematical knowledge and skills. If this is not the case, i.e. if knowledge and skills are not developed enough or have remained at the level of disjunct subject areas, students will not be able to successfully solve the given problem. This paper investigates the potential of an isolated geometrical problem and the students' success in solving such problem outside of the standard context.

Theoretical background

Types of mathematical tasks

There are different classifications of tasks, depending on the classification criterion (Foster, 2013; Jones & Pepin, 2016). This paper gives an overview of classifications necessary for interpretation of results obtained in empirical research.

Based on how the tasks are conceived, i.e. according to the main components of the task, mathematical tasks can be grouped under two categories: "tasks to find" and "tasks to prove". **"Tasks to find"** consist of known and unknown elements and conditions which connect the unknown with the known; the unknown element needs to be determined. **"Tasks to prove"** consist of an assumption and a statement; it needs to be determined whether the statement is true. In order to successfully solve tasks to prove or tasks to find, it is very important to know how to recognize its main components and establish connections between them (Polya, 1966).

As the term "task solution" can have several meanings (Polya, 2003, p. 145), for the purpose of this paper, the following definition applies: task solution is a mathematical object which fulfils the conditions of the task. The process of determining the required object which fulfils the conditions of the task is "solving method" or a "solving process".

Tasks to find can have no solution (there are no objects fulfilling the conditions of the task), one solution (single-solution task) or two or more solutions (multiple-solution task). It follows that "to solve the task" means to determine all mathematical objects which fulfil the conditions of the task, i.e. determine a set of solutions (or determine that there are no such objects).

According to the level of demands required from the student in the process of determining the solution, tasks can have lower-level demands and higher-level demands (Smith & Stein, 1998).

Lower-level demands tasks involve reproducing previously learned facts, rules, formulas or definitions (**memorisation tasks**) and algorithmic tasks solved using specific procedures (**procedure without connections tasks**). These tasks are not ambiguous and do not require further explanations. They require limited cognitive demand and have no connection to the concepts or meaning that underlies the procedure being used.

The primary goal is to develop **procedural knowledge and skills**, i.e. how to reach the correct solution quickly. For example, determining the value of arithmetic expressions or simplifying algebraic expressions can have the purpose of acquiring certain operations and establishing routine.

Using this type of tasks in assessment of students' knowledge mostly evaluates only the correctness of the final answer. Indirectly, feedback given to students is that the solution is more important than the process. Considering the share of these tasks in teaching mathematics and course materials, students quickly fall into the habit of considering the final answer as reaching the goal. Students have no need to check the correctness or the meaning of the solution or process, let alone consider the concepts and meaning underlying the procedures.

Higher-level demands tasks require more cognitive effort in order to discover and establish relationships between fundamental mathematical ideas and concepts. By acquiring conceptual understanding, connections between given and unknown elements are found and the solution is reached. In the problem solving process, different solving methods and strategies can be used. In addition to applying and connecting existing knowledge, it is also possible to discover new knowledge and concepts which were not set as the initial goal of the task **(procedures with connections tasks, deductive reasoning tasks)**. These tasks are usually represented in multiple ways: visual, symbolical and contextual, and they require making connections among all three representations. The procedural skills and knowledge are used only as an auxiliary tool for the purpose of developing **conceptual knowledge** underlying the procedures, rules and algorithms. Focus is on the solving process, not the final answer, and the tasks are used to develop the solving competences, establishing functional relationships and developing understanding and mathematical reasoning (e.g. Boonen et al., 2013; Schoenfeld, 1992; Smith & Stein, 1998, p. 348). The share of these tasks in the teaching practice is much smaller, as they require thorough preparation and complex critical assessment.

Problem tasks

During the past few decades, a number of studies were conducted in the field of defining problems, various types of problems and problem-solving methods and implications of using problems in teaching mathematics (e.g. Jones et al., 2004; McIntosh & Jarret, 2000; Schoenfeld, 1992; Strong, 2009). In this paper, the term 'problem' or 'problem task' describes any task for which the solution or problem-solving method cannot be immediately found.

Research confirms multiple benefits of solving such problems: developing mathematical thinking in students (Foong, 2002; Leikin & Lev 2007), stimulating and developing creativity (Klavir & Gorodetsky, 2011), ensuring engagement of majority of students during classes (Klavir & Herskovitz, 2014) and, at levels suitable for students (Sullivan, 2009), enabling identification of mathematical giftedness (Leikin & Lev, 2007), developing communication and positive atmosphere in math classes (Schukajlow et al., 2012), etc. Research also indicates difficulties in evaluating different solutions to open-ended tasks (Bingolbali, 2011) and changes in the role of the teacher "from a transmitter of knowledge to a facilitator of learning" (Cavanagh, 2008, p.123).

Considering that problem tasks have higher-level demands, Polya proposes four phases in effective problem-solving process and finding the solution: understanding of the problem, devising a plan, carrying out the plan and looking back (Polya, 1966). In most cases, this process is not linear, but requires constant interchanging of the phases and going back to the understanding phase, repeated devising of the plan and attempts to find a clear path to the solution (Hodnik Čadež & Manfreda Kolar, 2018).

Problem solving in geometry encompasses all the above mentioned Polya phases. Moreover, only one geometric task can have considerable didactic potential, since it includes a number of mathematical processes and concepts, enables making connections among different representations and extending the levels and forms of students' thinking (Duval, 2006; Panaura & Gagatis, 2009; White et al., 2016). This paper contains the analysis of one stand-alone geometric problem from the aspects of its didactic potential, cognitive demand and success in solving.

The stand-alone problem

In spite of the fact that the term stand-alone problem is rarely used in mathematical research, description of certain problems shows that they fall under the category of stand-alone problems and correspond to the description from the introductory part of this paper. Here we present a definition of the stand-alone problem.

The stand-alone problem (**SA** problem) is a high-level, cognitively complex "problem to find", given a clearly defined purpose and removed from standard learning context.

The definition implies that the task has three main components: known and unknown elements and conditions connecting the unknown with the known. The tasks can be represented in various ways: visually, symbolically and contextually. In order to determine the unknown, it is required to make connections among all three representations. In this type of task, it is not immediately clear which procedure to use or which strategy to choose; it is necessary to conduct an analysis, devise a plan and establish functional relationships before starting computation.

Finally, since the task is isolated from its usual educational environment, it can be used for different didactical purposes, but in some other educational environments: introduction to subject area, knowledge, acquisition and revision, research, application, assessment, etc. For example, the educational environment of our selected SA problem (Figure 1) is the Circle topic, but it can be used for knowledge revision as part of the Triangle topic. Successful solving of an SA task requires activation of mathematical knowledge and processes and creating a strong network of mathematical concepts and their properties.

The above description suggests that many problem tasks have the potential to be used as SA problems, when isolated and used for a specific purpose.

Aims and research questions

The aim of the research is to investigate the potential of a selected geometrical task in terms of using mathematical concepts and connections, as well as the success of students in the final grades of elementary school in problem solving, depending on the manner in which the tasks are set up. The selected task is based on the concept of inscribed angle. The task is given to 7^{th} and 8^{th} grade elementary school students, since the subject is first taught in 7^{th} grade and in the 8^{th} grade the subject is revised as part of the geometry curriculum (MZOS, 2006).

The research questions are as follows:

1. What is the potential of the selected SA problem, i.e. which mathematical concepts, their properties and relationships do students recognise and use in the problem-solving process?
2. What is the students' success rate in solving SA problems in different versions of the task, i.e. in what manner and to what extent does the way in which the problem is posed affect the success in problem solving?

Method

Descriptive and quantitative analyses were used to obtain results from the tasks completed by students.

Participants

The research involved 182 randomly selected 14 to 15-year-old students from 7^{th} and 8^{th} grade from different Croatian urban elementary schools. Personal information about the participants were not requested. The participation was voluntary and anonymous – each student was assigned a unique ID code, in accordance with ethical research practice (Cohen et al., 2007, p. 61).

Instrument

The task used in this research was selected from the State Matura Exam in Mathematics (DM, 2012), given to students in three different versions.

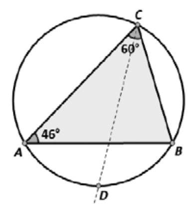
The first version (Task 1) is completely identical to the selected task and is used to make a comparison with the State Matura results. At the State Matura the task is given as a multiple-choice task with one correct answer out of four (Figure 1(a)). The students are allowed to draw and write down the steps in the problem-solving process; however, only the final answer is assessed (correct answer - 1 point, incorrect answer or no answer - 0 points). According to the exam criteria, this task is classified as an intermediate-level task (success rate 40-59 %), assessing the application of inscribed angle rule (Handbook SM, 2017, p. 20). After the conducted exams, the results showed that the task should be classified as advanced-level task (success rate 20-39 %) and that in most parts, the students used the guessing method.

The second version (Task 2) contains the text of the original task without multiple-choice answers, but with added drawing. The effect of the drawing on problem-solving will be analysed. The drawing is incomplete, but still allows the students to fill in the missing elements. In this case, successful problem-solving requires higher-level thinking: the drawing contains the given elements (scalene triangle, measurements of angles A and C, circumscribed circle, angle bisector and point D). The missing component is the angle CBD (line BD and corresponding angle arc CBD) (Figure 1(b)).

Consider triangle ABC. The measure of angle at vertex A is 46° and the measure of angle at vertex C is 60°. Angle bisector at vertex C intersects the circumscribed circle of the triangle at points C and D. What is the measure of angle ∠CBD?

Answers: (a) 104° (b) 120° (c) 134° (d) 150°

a)



b)

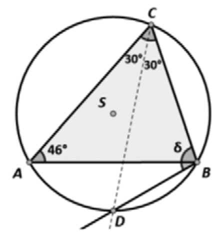
Figure 1. State Matura Exam task with added drawing

The third version (Task 3) contains the original task presented as an astronomical problem, without multiple-choice answers. The goal was to investigate whether the context motivates the students to engage in problem-solving, i.e. the effect of the context on problem-solving success. A complete drawing was given with the task (Figure 2). In addition, new terms are used: instead of "measure of the angle", the descriptive term "at what angle... is seen" is used. This version represents a "dressed up" word problem, since we are not interested in modelling, but the manner in which connections are established between the text, drawing and computations (Schukajlow et al., 2012).

Young astronomer and mountain climber Sanjin (marked as point S) is observing 4 mountain peaks (marked as A, B, C and D), located at the edge of the horizon, as shown in the drawing.

After climbing the mountain peak A, with the help of astronomical tools, he measured that from the point A, the imaginary line segment $\overline{BC}$ is seen at an angle of 46°.

He also climbed the mountain peak C. He measured that from the point C the imaginary line segment $\overline{AB}$ is seen at an angle of 60° and the imaginary line segment $\overline{BD}$ at an angle of 30°.



Sanjin did not make any more measurements but instead he drew a drawing (as shown) and he applied his knowledge of mathematics and determined at which angle the line segment $\overline{CD}$ is seen from point B.

Explain how Sanjin determined at what angle the line segment $\overline{CD}$ is seen from point B; i.e. what is the measure of the angle δ in the drawing?

Figure 2. Context-based task with drawing

All versions of the task included a note instructing the students to illustrate their work with drawings, write down computations and explain how they reached the result.

Data collection

The research was conducted at the end of the school year, in June 2019, after completing all the curriculum content. The participants were given the task at the beginning of the class session, with a 15-minute time period allowed to solve the task. The participants were not prepared for solving the task. They were informed of the purpose and aims of the research and their role in this research. Using random selection, students were divided into three groups. Each group was given one version of the task. Out of 182 randomly selected students, 66 (36.26 %) were given task 1, 63 (34.62 %) were given task 2 and 53 (29.12 %) were given task 3.

Data analysis

After collecting and reviewing students' work, three assessment criteria were defined: (1) accuracy of the presented problem-solving process and the final answer; (2) creating and use of a drawing; (3) utilisation of concepts, their properties and connections. Although it was initially expected that there would be a difference in the results of 7th and 8th graders, this was not the case. The results are almost uniform; therefore this parameter was excluded from the analysis. The only difference was observed in the level of use of mathematical language.

According to the criteria of the correct answer, students can be divided into three categories: students who answered correctly (T), students who gave the wrong answer (F) and students who did not give an answer (O). **According to the criteria of the presented problem-solving process**, five groups are identified: no explanation (1), completely incorrect procedure (2), partially correct procedure (3), fully correct procedure, but unfinished (4) and fully correct procedure (5). For example, if the answer is correct, but the procedure is unfinished, the identifier T4 is used.

The drawings are assessed according to consistency with the text of the task (emphasising given and unknown elements), emphasising additional elements and markings, and the correlation with the computations. In the first task, where students had to produce their own drawings (D1), five categories were identified: no drawing and no computation (1 - without anything); the drawing is not consistent with the text and computation, i.e. certain highlighted elements were mismatched with the task conditions, and the computation was incorrect accordingly (2 - misaligned); the drawing is partially consistent with the text and computation, e.g. not all the given elements were highlighted and there were no required elements (3 - partially aligned); the drawing is correct, but incompletely consistent with the text and computation, e.g. all the given elements were highlighted, but not all the required ones (4 – aligned, but unfinished); the drawing is fully consistent with the text and computation (5 – completely aligned). In the second and third version of the task, four similar categories were identified (there is no category 'aligned, but unfinished'). Surprisingly, the students also added their drawings to the drawings given in the task, so in some cases there were parallel categories (D2 and D2*; D3 and D3*).

Assessment of activated and used mathematical concepts, their properties and connections is carried out by selecting a corresponding code for each concept and assigning numerical value for application of the concept/property to different configurations. For example, the code SAT is used for the sum of interior angles of a triangle and a number is added for applying this property to a specific triangle (e.g. SAT5 means that the property was applied to triangle BCD). All the codes are listed in the results overview.

After the assessment criteria were defined, all completed tasks were assessed separately and collectively, assigning codes to each student. After descriptive analysis of code tables, data were interpreted using quantitative analysis. All the documents with students' work were scanned and archived.

Results and discussion

Analysis and discussion of the results is carried out based on the research questions, from the aspect of the potential of SA problems and the aspect of success in problem-solving.

Analysis of the results from the **aspect of potential** implies analysis of all mathematical concepts, properties and connections used and applied by the participants for the purpose of finding the solution. Analysis of the results from the **aspect of success in problem-solving** implies the rate of success in conducting the procedure and reaching the final answer.

The potential of the selected stand-alone problem

The original task was conceived to assess only one outcome (Handbook SM, 2017, p. 17), which is the application of properties of inscribed angles intercepting the same circular arc. However, in the solving process, the task also activated memory of other concepts, properties and connections. Some sub-configurations were recognisable 'at first sight', therefore the students used the properties and connections of these elements more often, establishing connections between them and consequently finding the measure of the unknown angle. Large number of properties was used less often (Table 1), while some properties were used incorrectly. Below is the description of properties, with corresponding codes listed in Table 1.

Table 1

Used concepts, properties and connections

	SAT	AA	EA	VA	IT	ET	SAA	SAQ	CQ	BA	BS	IA	CA	CCS	Σ
Tisak 1	84.85	12.12	12.12	9.09	21.21			1.52	3.03	51.52	25.76	30.30	4.55	22.73	18.18
Tisak 2	93.65	30.16	9.52	17.46	3.17		1.59	1.59		63.49		19.05	1.59		15.87
Tisak 3	73.58	22.64	5.66	15.09	9.43		1.89	5.66	3.77	11.32		32.08	1.89		24.53
N	154	39	17	25	21	0	2	5	4	80	17	49	5	15	35
%	84.62	21.43	9.34	13.74	11.54	0.00	1.10	2.75	2.20	43.96	9.34	26.92	2.75	8.24	19.23

As expected, students mostly recognised corresponding triangles (Table 2, Figure 3), applied the rule on the sum of interior angles of a triangle and determined the third angle based on given measurements of the other two angles (**SAT**). Although most students mastered this concept, some students failed to complete this part (Table 2). As much as 15.38 % (28 out of 182 students) did not calculate the measurements of angle B in given triangle ΔABC. Most students used only one triangle (94 out of 182 students, 51.65 %), while approximately one third of students combined two or more triangles (60 out of 182, 32.97 %). Most students concentrated on the angles in ΔABC and ΔBCD. The result was expected, since the ΔABC was dominant and the unknown angle is in ΔBCD.

Table 2

Frequency of SAT use

SAT	ABC	AEC	BCE	BED	BCD	ABS	ADB	ADC	O	1	2	3	4 ili 5
Tisak 1	68.18	3.03	7.58	25.76	21.21				10	40	14	2	0
Tisak 2	90.57	22.22	19.05	3.17	22.22		1.59	1.59	4	33	21	5	0
Tisak 3	41.27	17.46	12.70	6.35	20.63	1.59	3.17	1.59	14	21	12	4	2
N	119	27	25	11	44	1	2	1	28	94	47	11	2
%	65.38	14.84	13.74	6.04	24.18	0.55	1.10	0.55	15.38	51.65	25.82	6.04	1.10

Most participants recognised adjacent angles at vertices E and B and applied the rule of the supplement angle up to 180° to determine the measurement of the unknown angle (**AA**). In Task 2, slightly less than one third of the students (19 out of 63, 30.16 %) identified adjacent angles. In Task 1, some students failed to determine the sum up to 90°, resulting from incorrect drawing (Figure 3(b)). To a lesser extent, students identified exterior angles (**EA**) and vertical angles (**VA**) at vertices E and B (Table 1). Students applied the rule of equality with the sum of two internal non-adjacent angles to determine exterior angles ΔAEC, ΔBCE and ΔBED at vertex E. To determine the vertical angles, the property of equality of these angles was applied.

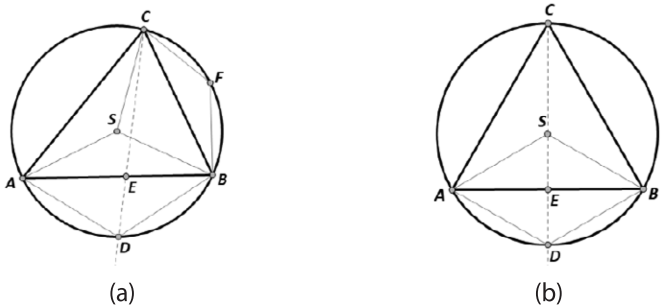


Figure 3. Completion of drawings and identifying sub-configurations

One part of the students recognized the isosceles triangle based on the equality of sides (ITS) and by applying the property of equality of opposite angels (ITA), they determined the third angle. Almost one third of participants (24 out of 66, 36.36 %) started drawing the isosceles triangle in Task 1 (Figure 3(b)) and continued work based on the initial mistake. One portion of students (30 out of 182, 16.48 %) used the new equilateral triangles by marking the centre of a circle and connected it with vertices of ΔABC. Some students made a wrong conclusion that the given triangle is an isosceles triangle, based on their perception of the drawing.

The task also provided the option to use equilateral triangles (**EA**) ΔADS and ΔDBS to determine the unknown angle, but not one student used this option.

In general, the participants mostly focused on triangle angles (interior, exterior, adjacent, vertical), not because this was part of some plan, but because they recognised these elements immediately. These properties were activated instantly because the knowledge was stored in accessible part of memory; possibly because the knowledge was used often and successfully during education (Gal & Linchevski, 2010, p. 179).

Furthermore, two participants noticed the similarities between ΔBED and ΔBCD and ΔAEC (**SAA**), based on the theorem on congruence of corresponding angles (KK), based on which the measurement of the unknown angle could be determined. Unfortunately, this idea was not fully implemented. Similarly, five participants came up with an idea to use the rule on the sum of interior angles in the quadrilateral (**SAQ**) and determine the fourth angle based on other three angles in quadrilateral ADBC, but without success. Two participants in each group (Task 1 and 3) recognised cyclic quadrilaterals (ADBC, ABFC, Figure 3) and applied the rule on the sum of opposite sides of a cyclic quadrilateral (**CQ**) to find the measurement of the unknown angle, but only one student succeeded in finding the answer.

Although the bisector of an angle (**BA**) is one of basic concepts in geometry, taught in 6th grade of elementary school in Croatia, the property described by the definition (angle bisector is a line that divides an angle into two congruent angles) was used by less than 50 % (80 out of 182) of participants. In Task 1, this property was used by 51.52 % (34 out of 66, with 2 incorrect applications) of participants, and in Task 2 by 63.49 % (40 out of 63 participants), i.e. total of 57.36 % (74 out of 129) of participants. In Task 3, this property is already given in the drawing, but some students (6 out of 53, 11.32 %) used the properties of angle bisector at vertices A and B incorrectly. The inscribed angle bisector divides the circular arc into two equal parts, therefore the corresponding chords are congruent, but this fact was used by only 2.2 % (4 out of 182) participants.

In Task 1, almost one third of the students drew isosceles triangle ABC, the angle bisector at vertex C became congruent to the bisector of line segment AB. Almost one fourth of participants (17 out of 66, 25.75 %) in this case used the properties of the line segment bisector (vertical to the line and/or passes through midpoint, i.e. cuts the line segment into two equal parts). One part of the students followed this false assumption and placed the centre of the circle (**CCS**) at the line segment bisector and/or identified point D as the centre of the circle (15 out of 66, 22.73 %).

The results indicate that students are not well-acquainted with the key concept of this task – the inscribed angle. In order to find an elegant solution for the unknown angle, one must remember the concept of inscribed angle and the correlation between a circumscribed circle and triangle angles, i.e. that triangle angles are inscribed angles of a circumscribed circle, and the property that inscribed angles (**IA**) of the circle are equal at the same circular arc (chord of a circle). Slightly more than one fourth of participants (49 out of 182, 26.92 %) used this property (20 participants or 30.30 % in Task 1, 12 participants or 19.05 % in Task 2 and 17 participants or 32.08 % in Task 3). This partly influenced the fact that one part of the students (5 out of 182, 2.75 %) used the correlation between the inscribed angle and central angle (**CA**). On the other hand, almost three quarters of the participants did not activate the concept of inscribed angle (133 out of 182, 73.08 %). It is possible that this concept was used either rarely or for a short period of time; therefore the participants could not activate it without external assistance (Gal & Linchevski, 2010, p. 179). This knowledge is irretrievable, which raises a question: does it make sense to teach the subject if the knowledge remains irretrievable for most students? The new mathematics curriculum in Croatia (KM, 2019) follows this proposal, placing the subject matter in the secondary school curriculum. There is another side to this issue: when and how to teach a certain topic to create proper relationships between concepts, to avoid disregarding or downplaying the importance of some concepts and their properties?

It can certainly be concluded that this SA problem has considerable didactic potential, since it contains a number of mathematical concepts, properties and connections, which the participants identified in all three versions of the task. However, only a small number of students use the observed concepts and their relations for the purpose of finding the solution. The share of students who fail to establish connections between observed concepts is much bigger, and there are students who fail to recognise even the basic properties, such as the sum of angles in a triangle (SAT). One possible reason might be the time period devoted to a certain subject matter. The obtained results indicate the necessity of introducing changes in the process of learning and teaching, with increased emphasis on teaching. Although the choice of a mathematical task is important, the manner in which such tasks are implemented in the teaching process is even more important (Hsu, 2013).

Success in solving the selected stand-alone problem

Considering that the task was taken from the State Matura (SM), first we looked into the relationship between the participants' results and the SM results (N = 10929), based on provided answers (Figure 4).

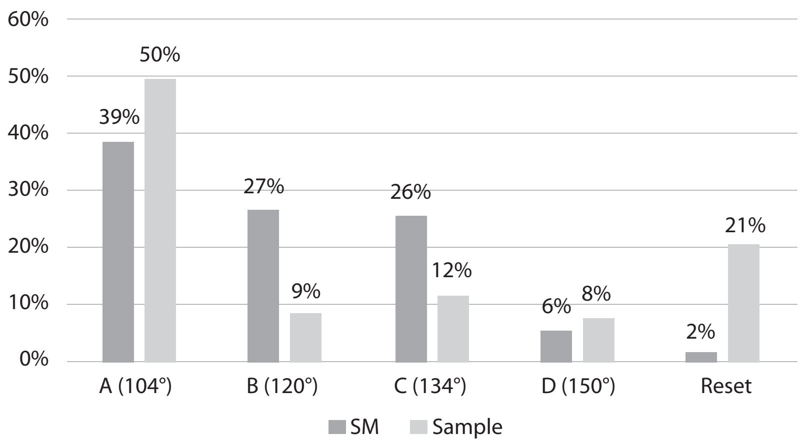


Figure 4. Comparison of SM and sample results

At first, it appears that larger portion of participants in our sample gave correct results (answer A, 50 % in the sample, 39 % at SM) and larger portion of participants did not give an answer (21 % in the sample, 2 % at SM). Based on these results, one possible conclusion is that fewer students in our sample used the guessing method than students at the SM. Since the SM is a high-risk examination, having a guess is better than leaving the question unanswered, since there are no negative points. However, a more detailed analysis of the sample shows there are more similarities than it first appeared. The task used in the sample is classified as intermediate-level task, according to the SM classification before the exam is taken. Further analysis is carried out from the perspective of answers and the problem solving process.

Global analysis of results according to the **correctness of answers** (T – correct, F – incorrect, O – no answer) shows that almost one third of participants answered correctly, one third answered incorrectly and one third gave no answer (Table 3). If we compare the results per task, we see differences in the results for three versions of the task. The students solving Task 1 were the most successful (33 out of 66, 50 %), followed by students solving Task 3 (18 out of 53, 33.96 %), and the students solving Task 2 were the least successful (10 out of 63, 15.87 %). Therefore, the design of the task influences the success in problem solving.

Table 3
Rates per task according to the correctness of answers

	Task 1		Task 2		Task 3		All	
Answer	N	%	N	%	N	%	N	%
T	33	50,00	10	15,87	18	33,96	61	33,52
F	20	30,30	19	30,16	20	37,74	59	32,42
O	13	19,70	34	53,97	15	28,30	62	34,07
SUM	66	36,26	63	34,62	53	29,12	182	100,00

The highest rate of success is observed in the multiple-choice task, as the answers guided the participants in the problem-solving process, also giving them the opportunity to estimate and guess. It is surprising that the participants were more successful in Task 3 than in task 2, which can be attributed to motivating context and the complete drawing given in Task 3, guiding the students toward the unknown angle. In spite of the fact that a choice of possible answers was given in Task 1, it was expected that the success rate would be higher in Tasks 2 and 3, because the drawing was provided. However, this was not the case. If the students' operational understanding is not sufficiently developed, they are not able to identify relevant information in the drawing and are consequently not able to complete the task successfully (Duval, 2000).

Success in using problem-solving procedure is analysed in five categories (Table 4), with the correctness of provided answers considered in each category (T, F and O).

In the complete sample, slightly over one fifth of participants (37 out of 182, 20.33 %) uses the procedure correctly, reaching the correct answer, with some cases of very elegant reasoning. All correct procedures resulted in a correct answer, as expected. Slightly less than one fifth of students (31 out of 182, 17.03 %) started the procedure correctly, but failed to finish. Some incomplete procedures resulted in giving answers based on guesses and estimates, but most were just left unanswered. These are the students who initiated the procedure correctly, but failed to finish because they were either unfamiliar with the concept of inscribed angle or could not find another way to solve the task. Slightly less than one sixth of students (29 out of 182, 15.93 %) had incorrect procedures, with 6 of them providing correct answers (again based on guesses and estimates). Less than 10 % (15 out of 182) of the students gave no explanation, with 5 of them providing correct answers.

Table 4

Rates per tasks according to the correctness of the procedure

	Task 1		Task 2		Task 3		ALL	
problem solving process	N	%	N	%	N	%	N	%
completely correct	14	21,21	8	12,70	15	28,30	37	20,33
correct, but unfinished	7	10,61	19	30,16	5	9,43	31	17,03
partially correct	23	34,85	26	41,27	21	39,62	70	38,46
incorrect	14	21,21	8	12,70	7	13,21	29	15,93
without explanation	8	12,12	2	3,17	5	9,43	15	8,24
SUM	66	100,00	63	100,00	53	100,00	182	100,00

Finally, the largest share of students initiated the procedure correctly (70 out of 182, 38.46 %), but lost their way, making various mistakes or using wrong assumptions (e.g. drawing isosceles triangle) and continuing work based on that mistake. Most of these students gave wrong answers (44 participants, 24.17 %), while some gave correct answers (10 participants, 5.49 %) or no answer (16 participants, 8.79 %). These students were not able to control the problem-solving process, for various reasons (Polya, 1966).

If we combine the incomplete, partially correct and incorrect procedures and correct answers in Task 1, we see that 28.79 % (19 out of 66) of the participants based their answers on estimates and guesses, and only 21.21 % (14 out of 66) of the participants justified their answers correctly. When compared with NE results (Figure 1), these results are worse (Table 4) than it seemed at first sight (Table 3).

The observations indicate that the task can be classified as advanced-level task, with only 20.33 % of participants demonstrating problem-solving skills. Participants were most successful in Task 3 (28.3 %), i.e. in a "dressed up" problem. The conclusion is that the real-world situation problem contributed to higher motivation for completing the problem-solving process. In addition, it is possible that the complete drawing given with the task guided the students' attention towards the solution. Participants were least successful in Task 2 (12.7 %), which provided no multiple-choice answers or motivating context, while the unknown angle was not marked in the drawing. It can be observed that the representation of the task influences the success in problem solving. Although the main components of the "task to find" remained the same, different task design guided the students' solving process in different directions (Boonen et al., 2014).

Conclusion

This paper provides the definition of the stand-alone problem (SA problem), followed by the analysis of one isolated geometrical problem identified as an SA problem. Analysis of students' work was carried out for three versions of the task, from the aspect of potential of identifying mathematical concepts and connections and the aspect of success in problem-solving.

Based on the obtained results, it can be concluded that many students of the selected sample do not reach the satisfactory level of acquiring basic geometrical concepts (e.g. the concept of angle) by the final years of primary education. Furthermore, these students have underdeveloped skills of discovering and connecting geometrical concepts and their properties, which prevented them from finding a systematic way to reach the solution to the problem. Based on the results, a case can be made for using SA problems in teaching.

If we wish to gain insight into actual reasoning and knowledge of students, special attention must be devoted to the **evaluation process**. It is not advisable to use multiple-choice tasks (especially in tasks with higher-level demands), as students are prone to 'fishing' for correct answers, lacking the problem-solving culture. If we provide the answers and disregard the problem-solving process, we can never be sure whether the correct result is an outcome of actual knowledge, estimate or a lucky guess. Evaluating the problem-solving process in problem tasks, especially in SA problems, is a complex and demanding endeavour. Nevertheless, it provides valuable feedback on students' thinking processes and background reasoning processes. Based on these insights, the teacher can productively change the way of teaching and the mode of implementing problem tasks.

SA problems can **provide environment** for cooperative learning, giving the students the opportunity to investigate, discover and connect various mathematical concepts, consequently developing their mathematical literacy, mathematical communication skills and the problem-solving culture.

Although the process of selecting, preparing and implementing SA problems in teaching is highly demanding and complex, **it pays off in the long term** because solving SA problems makes learning more meaningful, it creates functional relationships within knowledge areas and produces their intersection and union, instead of current disjunction resulting from learning and teaching individual areas.

It would be better and more useful to concentrate efforts on one task, e.g. an SA problem, and use its potential to establish a strong network of conceptual knowledge and skills, than to keep solving one type of tasks and remain at the level of procedural knowledge and skills. Problem-based instruction helps students discover the horizontal and vertical connections among mathematical knowledge and improve their interest in learning mathematics (Yanhui, 2018).

It is certainly necessary to undertake further research into factors enabling or disabling students to successfully reach the final solution, the application of SA problems in specific didactic environment and the posing of SA problems with the purpose of guiding classroom activities and encouraging learning with understanding.

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Nives Baranović
Faculty of Humanities and Social Sciences
University of Split
Poljička cesta 35, 21 000 Split, Croatia
nives@ffst.hr

Branka Antunović-Piton
Faculty of Educational Science
Juraj Dobrila University of Pula
Ronjgova 1, 52100 Pula, Croatia
bpiton@unipu.hr

Različite prespektive o uspješnosti 14 i 15-godišnjaka u rješavanju izoliranoga problema

Sažetak

U radu se definira posebna vrsta problemskoga zadatka te se razmatra njegov didaktički potencijal kao i uspješnost učenika u rješavanju odabranoga problema. Instrument istraživanja je geometrijski zadatak s državne mature koji se postavlja u tri inačice: kao tekstualni problem, kao tekstualni problem uz odgovarajuću sliku te kao zadatak u kontekstu. U istraživanju je sudjelovalo 182 učenika 7. i 8. razreda hrvatskih gradskih osnovnih škola. Didaktički potencijal razmatra se s aspekta iskoristivosti matematičkih koncepata i veza, a uspješnost učenika u rješavanju problema razmatra se s aspekta provedbe procesa rješavanja i otkrivanja točnoga rješenja ovisno o načinu postavljanja zadatka. Rezultati pokazuju da promatrani problem kao posebna vrsta problemskoga zadatka ima veliki didaktički potencijal, ali da učenici imaju nedovoljno razvijene vještine otkrivanja i povezivanja matematičkih koncepata i njihovih svojstava. Osim toga, način postavljanja zadatka značajno utječe na proces rješavanja, a posljedično i na uspješnost određivanja rješenja. Na temelju rezultata daju se prijedlozi primjene opisane vrste problema u nastavi Matematike.

Ključne riječi: *izolirani problem; matematički zadatak; rješavanje problema; vrednovanje problema*

Uvod

Suvremena nastava Matematike u centar pozornosti stavlja učenika te razvoj njegovih matematičkih znanja, vještina i sposobnosti kojima će se koristiti u osobnom, društvenom i profesionalnom životu (KM, 2019). U tu svrhu, suvremeni matematički kurikul pred nastavnike stavlja zahtjeve poučavanja odgovarajućih matematičkih koncepata, razvoj matematičkih procesa te stvaranje prikladnoga, kreativnoga i inovativnoga okruženja za primjenu usvojenih znanja, vještina i postupaka u različitim kontekstima. Kao jedan od važnih matematičkih procesa posebno je istaknut proces rješavanja problema (KM, 2019). I mnoga obrazovna istraživanja, proučavajući uloge raznih vrsta problemskih zadataka te strategije rješavanja zadnjih nekoliko desetljeća, potvrđuju da se upravo kroz njih mogu realizirati prethodno opisani zahtjevi (npr. Kordaki, 2015; Schoenfeld 1992). Tako kroz problemske zadatke učenici mogu povezivati stara i izgrađivati nova znanja, razvijati razne vještine i umijeća poput vizualizacije, odabira ispravnoga i ekonomičnoga puta, najprikladnije metode, naučiti snalaziti se u raznim novim i nepoznatim situacijama, itd. (npr. Levav-Waynberg i Leikin, 2012; Natsheh i Karsenty, 2014; Schoenfeld, 1992).

S obzirom na to da zadatci mogu graduirati od vrlo jednostavnih i tipiziranih do vrlo složenih i neuobičajenih, rješavajući razne vrste zadataka učenici imaju priliku razvijati i usavršavati razne proceduralne vještine, usvajati odgovarajuća konceptualna znanja, kao i primjenu tih znanja i vještina u složenijim situacijama.

U nastavnom procesu, vrlo važnu i odgovornu ulogu ima učitelj jer je, bez obzira na dostupnost nastavnoga materijala, upravo **učitelj kreator i realizator nastavnih aktivnosti**. Stoga je pred učiteljem izazov da, primjerenim odabirom različitih vrsta zadataka, osigura učenicima prikladno okruženje učenja. Jer, ako su učenici izloženi rješavanju samo jednostavnijih, kognitivno nezahtjevnih zadataka i pri tome su uspješni, mogli bi olako odustati od daljnjega rada misleći da su svladali sve što je potrebno. S druge strane, ako su izloženi samo rješavanju složenih, kognitivno zahtjevnih zadataka za koje nisu spremni pa ne postižu ni očekivani uspjeh, mogli bi lako odustati od daljnjega rada misleći da to nema smisla jer ionako nisu sposobni za matematiku. S obzirom na to da su razredni odjeli heterogeni, posebno u osnovnoj školi, važno je uspostaviti ravnotežu između zadataka različite složenosti i kognitivne zahtjevnosti kako bi se omogućilo svim učenicima da razviju svoje matematičke sposobnosti i potencijale.

Iako suvremeni kurikul matematike zahtijeva isprepletenost domena i procesa, unutarpredmetnu i međupredmetnu povezanost, nastava Matematike najčešće je organizirana tako da se određeni broj tema/sadržaja poučava unutar odgovarajuće **tematske cjeline** (npr. Trokuti, Četverokuti, Linearna funkcija, Vektori i sl.) pa se u skladu s tim odabiru i zadatci kroz koje se usvajaju, uvježbavaju i primjenjuju odgovarajući koncepti i procesi. Nakon **obrađene cjeline** sličnim zadatcima vrednuju se usvojena znanja i vještina učenika vezanih za tu cjelinu. U takvom sustavu rada učenici obično nemaju većih teškoća u svladavanju **sadržaja kroz cjeline**, a sustav djeluje vrlo učinkovito budući gotovo svi učenici više-manje uspješno prelaze u sljedeće **obrazovno razdoblje**.

Međutim, kada se učenici ispituju izvan konteksta tematske cjeline, kao što su međunarodna testiranja PISA, TIMSS i dr. te razna testiranja na nacionalnoj razini (kao što je državna matura u RH), njihovi rezultati obično su slabiji, a često i razočaravajuće loši (PISA 2012; Priručnik DM, 2017; Priručnik TIMSS, 2017). To znači da učenici ipak nisu ostvarili očekivane obrazovne ishode opisane zahtjevima suvremenoga matematičkog kurikula: njihova znanja nisu trajna niti su primjenjiva u različitim kontekstima.

Naime, kada izdvojimo jedan matematički zadatak, npr. srednje složenosti, do čijeg se konačnoga rješenja može doći različitim putevima i strategijama te ga ponudimo izvan konteksta tematske cjeline, on postaje izolirani problem (nazivamo ga *stand-alone* problem ili SA problem). U tom okruženju, za njegovo uspješno rješavanje potrebno je aktivirati cijelu mrežu matematičkih znanja i vještina. U protivnom, ako učenička znanja i vještine nisu dovoljno razvijene ili su ostale na razini disjunktnih tematskih cjelina, oni neće biti u mogućnosti uspješno riješiti ponuđeni problem. U ovom radu ispituje se potencijal jednoga takvog izoliranog geometrijskog problema i uspješnost učenika u njegovom rješavanju izvan uobičajenoga tematskog konteksta.

Teorijska pozadina

Vrste matematičkih zadataka

Postoje različite klasifikacije zadataka ovisno o kriteriju razvrstavanja (Foster, 2013; Jones i Pepin, 2016). Ovdje prikazujemo one koje su nam potrebne za razumijevanje rezultata dobivenih empirijskim istraživanjem.

Prema načinu kako su koncipirani, tj. prema glavnim dijelovima od kojih su sastavljeni, matematički zadatci mogu se razvrstati u dvije klase: odredbeni i dokazni. **Odredbeni zadatci** sastoje se od poznatih i nepoznatih elemenata i uvjeta koji povezuju nepoznato s poznatim te je potrebno odrediti nepoznati element. **Dokazni zadatci** sastoje se od pretpostavke i tvrdnje te je potrebno utvrditi istinitost tvrdnje uz danu pretpostavku. Za uspješno rješavanje odredbenoga odnosno dokaznoga zadatka veoma je važno znati prepoznati njegove glavne dijelove te uspostaviti vezu među njima (Polya, 1966).

Kako izraz „rješenje zadatka" može imati više značenja (Polya, 2003, str. 145), ovdje će se pod tim podrazumijevati matematički objekt koji tražimo, a koji udovoljava uvjetima zadatka. Za proces traženja odgovarajućega objekta koji udovoljava uvjetima zadatka u radu se koristi naziv „način rješavanja" ili „proces rješavanja".

S aspekta broja rješenja odredbenoga zadatka, zadatak može biti bez rješenja (kada nijedan objekt ne udovoljava uvjetima zadatka), može imati jedno rješenje (zadatak s jedinstvenim rješenjem), ali može imati dva ili više rješenja (zadatak s više rješenja) pa u skladu s tim „riješiti zadatak" znači odrediti sve matematičke objekte koji udovoljavaju uvjetima zadatka (ili reći da takvih objekata nema), tj. odrediti skup rješenja.

Prema razini zahtjeva koji matematički zadatak stavlja pred učenika kako bi odredio traženo rješenje, mogu se razlikovati zadatci sa zahtjevima niže razine i zadatci sa zahtjevima više razine (Smit i Stein, 1998).

Među **zadatcima sa zahtjevima niže razine** nalaze se oni koji traže samo puko reproduciranje prethodno naučenih činjenica, pravila, formula ili definicija (**zadatci memoriranja**), ali i oni koji se rješavaju algoritamski primjenom točno određenih postupaka (**zadatci s procedurama bez veza**). U njima je jasno što treba napraviti, ne traže se nikakva objašnjenja te nije potrebno uložiti veći kognitivni napor niti razumjeti pojmove i značenje koja se nalaze u pozadini provedenih postupaka. Primarni cilj im je razvijanje odgovarajućih **proceduralnih znanja i vještina**, odnosno kako brzo i točno doći do konačnoga rješenja. Na primjer, određivanje vrijednosti aritmetičkih izraza ili pojednostavljivanje algebarskih izraza u svrhu svladavanja i automatiziranja odgovarajućih operacija.

Pri vrednovanju znanja učenika korištenjem ovih zadataka, najčešće se vrednuje samo točnost konačnoga rješenja. Time se učenicima indirektno poručuje kako je važnije otkriti rješenje nego sam procesa rješavanja. S obzirom na veliku zastupljenost ovih vrsta zadataka u nastavi Matematike i nastavnom materijalu, učenici vrlo brzo razviju naviku da je otkrivanjem rješenja njihov cilj završen te nemaju potrebu provjeravati točnost i smislenost dobivenoga rješenja kao ni provedenoga postupka, a kamoli promišljati o pojmovima i značenjima koja se nalaze u pozadini provedenih postupaka.

Među **zadatcima sa zahtjevima više razine** nalaze se oni u kojima je potrebno uložiti veći kognitivni napor kako bi se otkrile i uspostavile veze među temeljnim matematičkim idejama i konceptima i steklo određeno konceptualno razumijevanje, odnosno kako bi se otkrile veze između zadanih i nepoznatih elemenata i time pronašao put do rješenja. Najčešće se tijekom toga puta mogu koristiti različite metode i strategije rješavanja pa su oni korisni u primjeni znanja i njihovom međusobnom povezivanju, a ponekad je moguće otkriti i neka nova znanja i koncepte koji nisu ni bili u polaznom cilju zadatka (**zadatci s procedurama i vezama**, **zadatci s čistim deduktivnim zaključivanjem**). Obično su predstavljeni na različite načine: vizualno, simbolički, kontekstualno te je potrebno uspostaviti i veze među različitim prikazima. U ovim zadatcima opisane proceduralne vještine i znanja su potrebna, ali samo kao sporedno sredstvo u svrhu razumijevanja i izgradnje **konceptualnih znanja** koja se nalaze u pozadini procedura, pravila i algoritama. Sada je puno važniji sam proces rješavanja od konačnoga rješenja stoga služe u svrhu razvijanja umijeća rješavanja, uspostavljanje funkcionalnih veza te stjecanje potrebnoga razumijevanja i matematičkoga promišljanja (npr. Boonen, van der Schoot, van Wesel, de Vries i Jolles, 2013; Schoenfeld 1992; Smit i Stein, 1998, str. 348). Njihova zastupljenost u nastavnoj praksi puno je manja, zahtijevaju dobru pripremu učitelja, a vrednovanja rada učenika na tim zadatcima je složeno.

O problemskim zadatcima

Zadnjih nekoliko desetljeća provode se brojna obrazovna istraživanja i rasprave o tome što je problem, o različitim vrstama i načinima rješavanja problema te koju korist impliciraju kada se koriste u nastavi Matematike (npr. Jones, Fujita i Ding, 2004; McIntosh i Jarret, 2000; Schoenfeld, 1992; Strong 2009). Pod izrazom „problem" u ovom radu podrazumijeva se svaki zadatak u kojem se ne može odmah odrediti rješenje ili metoda kojom bi došli do rješenja, a koristi se i izraz „problemski zadatak".

Istraživanjima je potvrđeno da je rješavanje takvih problema od višestruke koristi: razvija se matematičko mišljenje učenika (Foong, 2002; Leikin i Lev 2007), potiče se i razvija kreativnost (Klavir i Gorodetsky, 2011), osigurava se mogućnost angažmana većine učenika (Klavir i Herskovitz, 2014) tijekom nastave te na njima primjerenim razinama (Sullivan, 2009), omogućava se identificiranje darovitosti učenika za matematiku (Leikin i Lev, 2007), razvija se komunikacija i pozitivno matematičko/razredno ozračje (Schukajlow, Leiss, Pekrun, Blum, Müller i Messner, 2012) itd. Istraživanja ukazuju i na teškoće pri vrednovanju rješavanja zadataka na različite načine (Bingolbali, 2011), a uočeno je i da se uloga nastavnika korištenjem problemskoga pristupa mijenja „od prenositelja znanja prema vođenju učenja" (Cavanagh, 2008, p.123).

S obzirom na to da problemski zadatci pred učenika stavljaju odgovarajući viši kognitivni napor, Polya predlaže četiri faze koje mogu osigurati učinkovit proces rješavanja te uspješno otkrivanje rješenja: razumijevanje problema, stvaranje plana rješavanja, realizacija plana i osvrt na provedeno (Polya, 1966). Najčešće to nije linearan proces već je potrebna stalna izmjena faza, vraćanje na razumijevanje, ponovno planiranje i pokušavanje sve dok se ne ostvari jasan put do traženoga rješenja (Hodnik Čadež i Manfreda Kolar, 2018).

Upravo rješavanje geometrijskih problema objedinjuje sve gore navedene Polyinove faze. Štoviše, samo jedan geometrijski zadatak može predstavljati pravi didaktički potencijal jer uključuje mnoge matematičke procese i koncepte, omogućuje povezivanje različitih prikaza te otkrivanje i produbljivanje razina i oblika mišljenja kod učenika (Duval, 2006; Panaura i Gagatis, 2009; White, Warren i Quinlan, 2016). U ovom radu analizira se jedan izdvojeni geometrijski problem (eng. *stand-alone* problem, kraće SA problem) s aspekta njegovoga potencijala, kognitivne zahtjevnosti i uspješnosti rješavanja.

O izoliranom problemu

Unatoč tome što se u matematičkoj literaturi pojam izolirani problem rijetko koristi, iz opisa određenih problema može se prepoznati da se radi upravo o izoliranom problemu, u skladu s našim uvodnim opisom. Sada uvodimo definiciju izoliranoga problema.

Izolirani problem (tj. SA problem) je svaki odredbeni zadatak s višim kognitivnim zahtjevima koji se daje s jasno određenom svrhom izvan prirodnoga obrazovnog okruženja.

Definicija implicira da se radi o zadatku koji ima tri glavna dijela: poznate i nepoznate elemente i uvjete koji povezuju nepoznato s poznatim. Mogu biti predstavljeni na različite načine: vizualno, simbolički, kontekstualno te je za određivanje nepoznatoga potrebno uspostaviti veze među različitim prikazima. To nadalje implicira da u ovom zadatku nije odmah jasno koji postupak treba primijeniti ni koju strategiju odabrati, već je potrebno provesti analizu, napraviti plan, uspostaviti funkcionalne veze i tek onda započeti odgovarajući račun.

Konačno, s obzirom na to da se radi o zadatku koji je izdvojen iz prirodnoga obrazovnog okruženja može se ponuditi u različite didaktičke svrhe, ali u nekom drugom edukacijskom okruženju: uvođenje u temu, utvrđivanje, ponavljanje, primjena, istraživanje, primjena IKT-a, vrednovanje znanja i dr. Na primjer, edukacijsko okruženje za naš odabrani SA problem (Slika 1) je cjelina Kružnica, ali može se koristiti za ponavljanje u cjelini Trokut. Za njegovo uspješno rješavanje potrebno je aktivirati različita matematička znanja i procese te stvoriti kvalitetnu mrežu matematičkih koncepata i njihovih svojstava.

Prema opisanome može se zaključiti da su mnogi problemski zadatci potencijalno SA problemi, a to postaju kad se izdvoje iz prirodnoga okruženja s određenom svrhom.

Ciljevi i istraživačka pitanja

Cilj istraživanja bio je ispitati potencijal odabranoga geometrijskog zadatka u svrhu iskoristivosti matematičkih koncepata i veza kao i uspješnost učenika završnih razreda osnovne škole u rješavanja problema ovisno o načinu prikaza zadatka. U tu svrhu, odabrani zadatak koji se temelji na konceptu obodnoga kuta, ponuđen je učenicima 7. i 8. razreda osnovne škole jer se pojam obodnoga kuta obrađuje u 7. razredu, a u 8. razredu ponavlja se kroz geometrijske sadržaje (MZOS, 2006).

Postavili smo sljedeća istraživačka pitanja:

Koji je potencijal odabranoga SA problema, tj. koje matematičke koncepte, njihova svojstva i veze učenici prepoznaju i koriste u procesu rješavanja?

U kojoj mjeri učenici uspješno rješavaju SA problem kroz različite varijante zadatka, tj. na koji način i u kojoj mjeri postavljanje problema utječe na uspješnost rješavanja?

Metoda

U svrhu dobivanja odgovora na postavljenja pitanja, učenički su radovi obrađeni deskriptivno u kombinaciji s kvalitativnom analizom.

Sudionici

U istraživanju je sudjelovalo 182 slučajno odabranih 14 i 15-godišnjaka različitih sposobnosti, učenika sedmih i osmih razreda iz različitih hrvatskih gradskih osnovnih škola. Svi sudionici su dobrovoljno sudjelovali u istraživanju. Osobne informacije o sudionicima nisu tražene već je svakom učeniku pridružen jedinstveni kôd kako bi se zadovoljili etički aspekti anonimnosti edukacijskoga istraživanja (Cohen, Manion i Morrison, 2007, str. 61).

Instrument

U svrhu ovoga istraživanja odabran je jedan geometrijski zadatak s ispita Državne mature iz matematike (DM ljeto 2012) koji je učenicima ponuđen u tri varijante.

Prva varijanta (zadatak 1) podudara se u potpunosti s preuzetim zadatkom kako bi rezultate mogli usporediti s rezultatima s ispita. Na ispitu DM zadatak je ponuđen kao zadatak višestrukoga izbora s jednim točnim odgovorom od ponuđena četiri (Slika 1(a)). Učeniku je dopušteno da crta i zapisuje proces rješavanja, ali se vrednuje samo konačan odgovor (točan odgovor - 1 bod, pogrešan odgovor ili bez odgovora - 0 bodova). Prema kriterijima sastavljanja ispita, zadatak je klasificiran kao srednje težak zadatak (riješenost 40 – 59 %) kojim se mjeri primjena pravila o obodnim kutovima (Priručnik DM, 2017, str. 20). Nakon provedenoga ispita, rezultati su pokazali da se radi o teškom zadatku (riješenost 20 – 39 %), koji se u velikom postotku rješavao metodom pogađanja.

U **drugoj varijanti** (zadatak 2) zadržan je tekst zadatka bez ponuđenih odgovora, ali je dodana skica kako bi stekli uvid u doprinos skice pri rješavanju problema, s obzirom da su njihove skice često neuredne i pogrešne. Skica je nepotpuna (ali dovoljno velika da se može nadopunjavati) kako bi učenici ipak morali uložiti potreban kognitivni napor za uspješno pronalaženje puta do rješenja. Naime, na slici su istaknuti svi zadani elementi (raznostranični trokut, veličine kutova pri vrhovima A i C, opisana kružnica, simetrala kuta i točka D), ali nije istaknut traženi kut (krak BD i odgovarajući luk kuta CBD) (Slika 1(b)).

Slika 1.

U **trećoj varijanti** (zadatak 3) tekst odabranoga zadatka stavljen je u kontekst astronomskoga problema bez ponuđenih odgovora kako bi ispitali je li i u kojoj mjeri kontekst motivira učenike na ustrajno rješavanje problema, tj. hoće li kontekst utjecati na uspješnost rješavanja. Uz kontekst zadatka dali smo potpunu skicu (Slika 2), a koristi se i nova terminologija (umjesto ‚veličina kuta' nalazi se ‚kut pod kojim se vidi'). Ova inačica je *dressed up* problem jer nas u ovoj situaciji nije zanimalo modeliranje već način uspostavljanja veze između teksta, skice i računa (Schukajlow i sur., 2012).

Slika 2.

Uz svaki zadatak učenici su imali napomenu da na papiru prikažu sav svoj rad (skice, račun) te da objasne kako su došli do rezultata.

Prikupljanje podataka

Istraživanje je provedeno u lipnju 2019., na kraju nastavne godine, nakon što su obrađeni svi nastavni sadržaji kako bi zadatak ispunio sve uvjete SA problema. Sudionici su rješavali zadatak na početku nastavnoga sata, a na raspolaganju su imali 15 minuta. Prije pisanja sudionici nisu imali nikakve pripreme. Nadalje, informirani su o ciljevima i svrsi istraživanja te njihovoj ulozi u istraživanju. Učenici su slučajnim izborom bili podijeljeni u tri skupine. Svaka skupina rješavala je jedan od navedenih zadataka. Od 182 slučajno odabranih učenika, 66 (36,26 %) sudionika je pisalo zadatak 1, 63 (34,62 %) sudionika zadatak 2 i 53 (29,12 %) sudionika zadatak 3.

Analiza podataka

Nakon što su prikupljeni i individualno pregledani svi radovi, usuglašena su tri kriterija vrednovanja: (1) točnost prikazanoga procesa rješavanja i konačnoga odgovora, (2) izrada i korištenje skice i (3) korištenje koncepata, njihovih svojstava i veza. Nadalje, prije provedbe testa očekivalo se da će postojati razlika u rezultatima između sudionika 7. i 8. razreda, no ipak se pokazalo da to nije slučaj. Njihovi rezultati gotovo su ujednačeni pa smo taj parametar isključili iz analize. Jedina uočena razlika je u kvalitetnijem matematičkom izražavanju.

Prema točnosti odgovora svi se učenici mogu razvrstati u tri kategorije: oni koji imaju točan odgovor (T), oni koji imaju pogrešan odgovor (F) i oni koji nisu dali odgovor (O), a **prema točnosti prikazanoga procesa rješavanja** svi se mogu razvrstati u pet kategorija: bez ikakvoga objašnjenja (1), postupak u potpunosti nekorektan (2), postupak djelomično korektan (3), postupak u potpunosti korektan, ali nedovršen (4) i postupak u potpunosti korektan (5). Tako npr., ako netko ima točan odgovor i korektan, ali nedovršen postupak dobiva oznaku T4.

Skice su vrednovane prema usklađenosti s tekstom zadatka (isticanje zadanih i traženih elemenata), prema isticanju dodatnih elemenata i oznaka te načinu povezivanju s računom. S obzirom na to da su u prvom zadatku učenici sami trebali nacrtati skicu

(D1), postavljeno je pet kategorija: nema skice ni računa (1 – bez ičega); skica nije usklađena s tekstom i računom, tj. određeni zadani elementi su pogrešno istaknuti i prema tome se pogrešno proveo račun (2 - neusklađeno); skica je djelomično korektno usklađena s tekstom i računom, npr. nisu istaknuti svi zadani elementi i nisu istaknuti traženi elementi (3 – djelomično usklađeno); skica je korektno, ali nepotpuno usklađena s tekstom i računom, npr. istaknuti su svi zadani, ali ne i svi traženi elementi (4 – usklađeno, ali nedovršeno) i skica je u potpunosti usklađena s tekstom i računom (5 – potpuno usklađeno). U preostala dva zadatka postavili smo četiri kategorije na sličan način (ne postoji kategorija - usklađeno, ali nedovršeno). No, na naše iznenađenje, učenici su pored dane skice crtali i svoje skice pa smo u tom slučaju imali paralelne kategorije (D2 i D2* te D3 i D3*).

U svrhu **vrednovanja aktiviranih i korištenih matematičkih koncepata**, njihovih svojstava i veza, za svaki koncept odabran je odgovarajući kôd, a primjena istoga koncepta/svojstva na različite konfiguracije se numerirala. Tako primjerice, za korištenje svojstva zbroja unutarnjih kutova trokuta koristi se kod SAT, a za primjenu tog svojstva u odgovarajućem trokutu ističe se broj (npr. SAT5 znači primjenu tog svojstva na trokut BCD). Svi kodovi vidljivi su u prikazu rezultata.

Nakon usuglašavanja kriterija vrednovanja, svi radovi još su jednom vrednovani individualno, a zatim zajedno kako bi se svakom učeniku pridijelili jedinstveni kodovi. Tablice s kodovima obrađene su deskriptivno, a zatim su podatci tumačeni u skladu s kvalitativnom analizom. Svi radovi su skenirani i arhivirani.

Rezultati i rasprava

U ovom radu analizu i diskusiju vršimo po zadatcima s aspekta potencijala odabranoga SA problema te s aspekta uspješnosti u rješavanju, u skladu s istraživačkim pitanjima.

Pod analizom radova **s aspekta potencijala** podrazumijevamo analizu svih matematičkih koncepata, njihova svojstva i veze koje su sudionici uočili i primijenili u svrhu otkrivanja rješenja. Pod analizom radova **s aspekta uspješnosti rješavanja** podrazumijevamo mjeru uspješnosti provođenja postupka i dobivanja traženoga rješenja.

Potencijal odabranoga SA problema

Iako je originalni zadatak osmišljen u svrhu provjere usvojenosti samo jednog ishoda (Priručnik DM, 2017, str. 17), u ovom slučaju, primjena svojstva obodnih kutova nad istim kružnim lukom (tetivom), u procesu rješavanja zadatak je aktivirao prisjećanje raznih drugih koncepata, svojstava i veza jer je ponuđen kao SA problem. Neke podkonfiguracije bile su uočljive „na prvi pogled" pa su njihova svojstva i veze učenici očekivano koristili više, stvarajući veze među njima kako bi izveli zaključak o traženom kutu. No, veliki broj uočenih svojstava koristio se ipak u manjem postotku (Tablica 1), a neka i pogrešno. U nastavku se opisuju uočena svojstva, kodovi kojih su istaknuti u Tablici 1.

Tablica 1.

Očekivano, sudionici su najviše uočavali odgovarajuće trokute (Tablica 2, Slika 3), primjenjivali pravilo za zbroj unutarnjih kutova u trokutu te određivali treći kut kada su bila poznata preostala dva (**SAT**). Iako bi mogli reći da su taj koncept sudionici dobro savladali, ipak, postoje učenici koji ni to nisu napravili (Tablica 2). Čak 15,38 % (28 od 182 učenika) nije izračunalo ni veličinu kuta pri vrhu B u zadanom ΔABC. Najveći broj učenika koristio je samo jedan trokut (94 od 182 učenika, 51,65 %), dok je bilo i onih koji su kombinirali 2 i više trokuta (60 od 182, 32,97 %). Najviše su se bavili kutovima u ΔABC i ΔBCD što je očekivano budući da je ΔABC bio dominantna figura, a u ΔBCD se nalazio traženi kut.

Tablica 2.

Uz uočene trokute, uočavali su i sukute s vrhovima u E i B te primjenjivali pravilo nadopune do 180° u svrhu određivanja nepoznatoga kuta (**AA**). U zadatku 2 sukutima bavilo se nešto manje od trećine učenika (19 od 63, 30,16 %), a u zadatku 1 bilo je i pogrešnih nadopuna do 90°, što je proizišlo iz pogrešno nacrtane skice (Slika 3(b)). U nešto manjoj mjeri, uz sukute uočavali su i vanjske kutove trokuta (**EA**) kao i vršne kutove (**VA**) s vrhovima u E i B (Tablica 1). Za vanjske kutove ΔAEC, ΔBCE i ΔBED pri vrhu E primjenjivali su pravilo jednakosti sa zbrojem dvaju unutarnjih nesusjednih kutova tih trokuta kako bi odredili veličinu tog kuta, a za vršne kutove svojstvo jednakosti tih kutova.

Slika 3.

Dio sudionika u svom radu prepoznavao je jednakokračne trokute na temelju jednakosti stranica (**ITS**) te su primjenom svojstva jednakosti kutova nasuprot tih stranica (**ITA**) određivali treći kut uz poznata preostala dva. Tako je skoro trećina sudionika (24 od 66, 36,36 %) skicu u zadatku 1 započela crtanjem jednakokračnoga trokuta (Slika 3(b)), a zatim rad nastavili slijedeći tu grešku. No, bilo je i onih koji su isticanjem središta kružnice i povezivanjem s vrhovima ΔABC (30 od 182, 16,48 %) istaknuli i koristili nove jednakokračne trokute kao i onih koji su na temelju percepcije skice zaključili pogrešno da se radi o jednakokračnom trokutu.

U konfiguraciji ovoga zadatka sudionici su imali priliku baviti se i jednakostraničnim trokutima (**EA**) ΔADS i ΔDBS u svrhu određivanja traženoga kuta, ali nitko od učenika to nije iskoristio.

Generalno gledano, sudionici su se dosta bavili kutovima trokuta (unutarnjim, vanjskim, sukutima, vršnim kutovima), ali ne zato što je to dio nekog plana, već prije svega što su to bile prve figure koje su odmah uočili. Ova svojstva odmah su aktivirali jer je to znanje bilo spremljeno u dostupnom dijelu memorije, moguće zato jer su to tijekom dosadašnjega obrazovanja najdulje, najčešće i uspješno koristili (Gal i Linchevski, 2010, str. 179).

Nadalje, dva sudionika bila su na tragu uočavanja sličnosti ΔBED i ΔBCD s ΔAEC (**SAA**), prema poučku o podudarnosti dvaju kutova unutar promatranih trokuta (KK), na temelju čega su mogli odrediti veličinu nepoznatoga kuta, ali nažalost tu ideju nisu realizirali do kraja. Slično tome, pet sudionika imalo je ideju koristiti pravilo za zbroj unutarnjih kutova u četverokutu (**SAQ**) te odrediti četvrti kut uz poznata preostala tri u četverokutu ADBC, ali nažalost bezuspješno. Isto tako, po dvoje sudionika koji su rješavali zadatak 1 i 3, uočili su tetivne četverokute (ADBC, ABFC, Slika 3) te primijenili svojstvo zbroja nasuprotnih kutova tetivnog četverokuta (**CQ**) kako bi došli do veličine traženoga kuta, ali samo jedan od njih to je uspješno iskoristio u svrhu određivanja traženoga rješenja.

Iako je simetrala kuta (**BA**) jedan od temeljnih koncepata u geometriji koji se uz koncept kuta uči prema hrvatskom kurikulu u 6. razredu osnovne škole, svojstvo opisano definicijom (simetrala dijeli kut na dva jednaka dijela) koristilo je manje od 50 % (80 od 182) sudionika. U zadatku 1 to svojstvo koristilo je 51,52 % (34 od 66, ali 2 pogrešno) sudionika, a u zadatku 2 63,49 % (40 od 63 sudionika). Dakle, ukupno 57,36 % (74 od 129) sudionika. U zadatku 3 to svojstvo je već dano na skici, ali su neki učenici (6 od 53, 11,32 %) koristili simetralu kutova pri vrhu A i B, no nažalost pogrešno. Kako se radi o simetrali obodnoga kuta, ta ista simetrala dijeli pripadni kružni luk na dva jednaka dijela, pa su i pripadne tetive sukladne, no tu činjenicu koristilo je tek 2,2 % (4 od 182) sudionika.

Kako je skoro trećina učenika u zadatku 1 za ΔABC nacrtalo jednakokračni trokut, njihova simetrala kuta pri vrhu C podudarila se sa simetralom stranice AB te je skoro četvrtina sudionika (17 od 66, 25,75 %) u tom slučaju koristilo svojstva simetrale dužine (da je okomita na dužinu i/ili prolazi polovištem, tj. dijeli dužinu na dva jednaka dijela). Određeni broj sudionika je, slijedeći tu grešku i središte kružnice (**CCS**) smjestilo na simetralu dužine i/ili poistovjetilo točku D sa središtem kružnice (15 od 66, 22,73 %).

Ključni koncept ovoga zadatka - obodni kut - pokazalo se da učenicima nije baš blizak. Naime, za elegantno određivanje traženoga kuta trebalo se prisjetiti pojma obodnoga kuta te veze između opisane kružnice trokutu i kutova trokuta, odnosno da su kutovi trokuta obodni kutovi opisanoj mu kružnici, a zatim svojstva da su obodni kutovi (**IA**) iste kružnice nad istim lukom (tetivom) jednaki. Tek malo više od četvrtine sudionika (49 od 182 sudionika, 26,92 %) koristilo je to svojstvo (20 sudionika ili 30,30 % u zadatku 1, 12 sudionika ili 19,05 % u zadatku 2 i 17 sudionika ili 32,08 % u zadatku 3). Svakako to je jednim dijelom utjecalo na dio sudionika (5 od 182, 2,75 %) da koristi odnos obodnoga i središnjega kuta (**CA**). S druge strane, pojam obodnoga kuta gotovo tri četvrtine sudionika uopće nisu aktivirali (133 od 182 sudionika, 73,08 %). Moguće je da se radi o pojmu koji su vrlo kratko ili rijetko koristili te ga i nisu u stanju aktivirati bez pomoći sa strane (Gal i Linchevski, 2010, str. 179). To znanje za njih je neupotrebljivo pa se neočekivano nameće pitanje: ima li smisla poučavati teme ako za veliki broj sudionika to ostaje neupotrebljivo znanje? Novi kurikul matematike u Hrvatskoj (KM, 2019) upravo je to i napravio, obodni kutovi prepušteni su srednjoškolskoj matematici. No pitanje se može postaviti i na drugi način: kada i kako poučavati određene teme kako bi se ostvarile kvalitetne veze među konceptima, odnosno kako se ne bi zanemario ili umanjio značaj nekih koncepata i njihovih svojstava?

Svakako, može se zaključiti da ovaj SA problem predstavlja veliki didaktički potencijal jer objedinjuje niz matematičkih koncepata, njihovih svojstava i veza, koja su i sudionici uočili kroz sve tri inačice zadatka. Ali unatoč tome, samo manji broj učenika uočene koncepte i njihove veze koristi u svrhu dobivanja rješenja, dok je puno više onih koji nisu u mogućnosti uspostaviti veze među uočenim konceptima, a ima i onih koji nisu prepoznali ni elementarna svojstva, poput zbroja kutova u trokutu (SAT). Mogući razlog može biti vremenski period koji se posveti određenim sadržajima. Dobiveni rezultati ukazuju na potrebne promjene u procesu učenja i poučavanja, s velikim naglaskom na poučavanje. Jer, iako je izbor matematičkoga zadataka važan još je važnije na koji će se način takvi zadatci implementirati u nastavnom procesu (Hsu, 2013).

Uspješnost rješavanja odabranoga izoliranog problema

S obzirom da je zadatak preuzet s ispita Državne mature (DM), zanimalo nas je prije svega kakvi su rezultati sudionika iz uzorka u odnosu na rezultate s ispita DM (N = 10929), prema danim odgovorima (Slika 4).

Slika 4.

Na prvi pogled može se uočiti da je veći postotak sudionika u našem uzorku ostvario točan rezultat (odgovor A, 50 % u uzorku, 39 % na DM) i da veći postotak sudionika nije uopće ponudio odgovor kada ga nije znao (21 % u uzorku, 2 % na DM). Na temelju ovih rezultata, moglo bi se zaključiti da je manji postotak onih koji su pogađali u uzorku nego na ispitu. Mogući razlog može biti to što je DM ispit visokoga rizika pa je bolje pogađati nego ostaviti bez odgovora, s obzirom da nema negativnih bodova. No, detaljnija analiza uzorka ukazuje na više sličnosti nego što se na prvi pogled čini. Prema skali težine, zadatak u uzorku bio bi u kategoriji srednje teškog zadatka što potvrđuje prvu kategorizaciju zadatka neposredno prije pisanja ispita DM. Daljnja analiza vrši se po zadatcima s aspekta odgovora i s aspekta postupka rješavanja.

Analizirajući rezultate globalno prema **točnosti danih odgovora** (T – točan odgovor, F – pogrešan odgovor, O – bez odgovora) može se uočiti da je skoro trećina svih sudionika imalo točan rezultat, trećina pogrešan i trećina sudionika nije dala odgovor (Tablica 3). Ako usporedimo rezultate prema zadatcima, mogu se uočiti razlike između različitih varijanti zadataka. Najuspješniji su bili učenici koji su rješavali zadatak 1 (33 od 66, 50 %), zatim učenici koji su rješavali zadatak 3 (18 od 53, 33,96 %) i najslabiji učenici koji su rješavali zadatak 2 (10 od 63, 15,87 %). Dakle, način postavljanja zadatka utječe na uspješnost rješavanja.

Tablica 3.

Najveću uspješnost postigli su u zadatku s ponuđenim odgovorima jer su ih odgovori mogli navoditi u procesu rješavanja, a imali su i mogućnost procjene i pogađanja. Iznenađuje da su sudionici bili uspješniji u zadatku 3 nego u zadatku 2, što se može tumačiti motivirajućim kontekstom, ali i potpunom slikom u zadatku 3 koja je usmjerila njihov rad na traženi kut. No, bez obzira na ponuđene odgovore u zadatku 1, očekivalo se da će uspješnost rješavanja biti veća u zadatcima 2 i 3 zbog ponuđene skice. Međutim to ipak nije bio slučaj jer ako učenik nema razvijeno operativno razumijevanje, onda ni na danoj slici nije u mogućnosti izdvojiti relevantne podatke, pa posljedično nije u mogućnosti ni uspješno završiti zadatak (Duval, 2000).

Uspješnost provođenja postupka rješavanja analizira se kroz pet kategorija (Tablica 4), a u svakoj od kategorija razmatra se točnost danih odgovora u tri kategorije (T, F i O).

Na razini cijeloga uzorka, nešto više od petine sudionika (37 od 182, 20,33 %) u potpunosti korektno provodi postupak i dolazi do točnoga traženog rješenja, među kojima ima i elegantnih zaključivanja. Odnosno, svi korektni postupci doveli su do točnoga rješenja, što je i očekivano. Nešto manje od petine učenika (31 od 182, 17,03 %) korektno je započelo postupak, ali ga nije dovršilo. Neki nedovršeni postupci doveli su do odgovora temeljem procjene i pogađanja, ali većina ih nije dala nikakav odgovor. To su oni učenici koji su proces rješavanja započeli korektno, ali ga nisu dovršili jer pojam obodnoga kuta nisu usvojili, a neki drugi put nisu pronašli. Nešto manje od šestine učenika (29 od 182, 15,93 %) ima u potpunosti nekorektan postupak, ali ipak 6 ih daje točan odgovor (opet temeljem procjene i pogađanja). Manje od 10 % (15 od 182) učenika nije dalo nikakvo objašnjenje, no i među njima ima 5 učenika s točnim odgovorom.

Tablica 4.

Konačno, najveći postotak je onih koji su započeli dobro (70 od 182, 38,46 %), ali su se putem negdje pogubili radeći razne vrste grešaka ili su započeli s pogrešnom pretpostavkom (npr. crtanjem jednakokračnog trokuta) pa su rad nastavili slijedeći tu grešku. Među njima je razumljivo najviše onih s pogrešnim odgovorima (44 sudionika, 24,17 %), ali ima ih i s točnim odgovorima (10 sudionika, 5,49 %) kao i bez odgovora (16 sudionika, 8,79 %). Za ove učenike možemo reći da nisu u mogućnosti kontrolirati svoj proces rješavanja, a razlozi mogu biti razni (Polya, 1966).

Objedinjujući u zadatku 1 one s nedovršenim, djelomično korektnim ili nekorektnim postupkom, a točnim odgovorom, vidimo da je 28,79 % (19 od 66) sudionika svoj odgovor temeljila na procjeni i pogađanja, a samo 21,21 % (14 od 66) sudionika ima korektno opravdanje svoga točnog odgovora. U usporedbi s rezultatima na ispitu DM (Slika 1), ovi rezultati ipak su slabiji (Tablica 4) nego što to na prvi pogled izgleda (Tablica 3).

Prema ovim zapažanjima, može se reći da zadatak spada u kategoriju vrlo teškog zadatka u kojem je samo 20,33 % sudionika pokazalo vještinu rješavanja. Najuspješniji su bili sudionici u zadatku 3 (28,3 %), tj. u *dressed up* problemu. Može se zaključiti da je realni kontekst ipak pridonio većoj motivaciji da se proces rješavanja izvede do kraja, a moguće je i slika doprinijela tome budući da je bila potpuna i usmjerila njihovu pozornost u pravom smjeru. Najneuspješniji su bili u zadatku 2 (tek 12,7 %), u kojem nije bilo ni ponuđenih odgovora kao putokaza, ni zanimljivoga konteksta koji bi ih motivirao, a ni slika nije pomogla jer na njoj nije bio istaknut traženi kut. U konačnici uočava se da prikaz zadatka utječe na uspješnost rješavanja. Naime, iako se nisu promijenili glavni elementi ovoga odredbenog zadatka, različito postavljanje zadatka različito je usmjerilo njihov proces rješavanja (Boonen, 2014).

Zaključak

U ovom radu daje se definicija izoliranoga problema (*stand-alone* problem, kraće SA problem), a zatim se analizira jedan geometrijski problem prepoznat kao SA problem. Analiza učeničkih uradaka napravljena je po zadatcima s aspekta potencijala otkrivanja matematičkih koncepata i veza odabranoga SA problema te s aspekta uspješnosti rješavanja, a obzirom na tri varijante istog zadatka.

Na temelju dobivenih rezultata može se zaključiti da pri kraju osnovnoškolskoga obrazovnog ciklusa mnogi učenici iz odabranoga uzorka nisu usvojili elementarne geometrijske koncepte (npr. koncept kuta) na očekivanoj razini. Nadalje, ti učenici imaju nedovoljno razvijene vještine otkrivanja, povezivanja koncepata i njihovih svojstava, a što je mnoge onemogućilo u sustavnom pronalaženju puta do traženoga rješenja. Temeljem rezultata iznose se razlozi korištenja SA problema u nastavnom procesu.

Ako se želi steći uvid u stvarno promišljanje i znanje učenika, **procesu vrednovanja** trebalo bi posvetiti značajnu pažnju. Nije uputno koristiti zadatke s ponuđenim rješenjima (pogotovo u zadatcima više kognitivne razine) jer su učenici, u nedostatku kulture rješavanja problema, skloni „navlačiti" točno rješenje. Ako ponudimo odgovore, a ne gledamo postupak rješavanja, nikada nismo sigurni je li točan odgovor rezultat stvarnoga znanja, procjene ili pukog pogađanja. Vrednovanje procesa rješavanja problemskoga zadatka, posebno SA problema, vrlo je složen i zahtijevan posao, ali zauzvrat nudi brojne korisne informacije o toku misli i pozadinskom procesu razumijevanja učenika. Temeljem tih spoznaja učitelj ima mogućnost produktivno mijenjati svoje poučavanje, ali i način implementacije problemskoga zadatka.

SA problemi mogu **osigurati okruženje** za kooperativno učenje unutar kojeg učenici imaju priliku istraživati, otkrivati i povezivati razne matematičke pojmove i na taj način razvijati svoju matematičku pismenost, vještine matematičke komunikacije te kulturu rješavanja problema.

Iako je proces odabira, pripreme i provedbe SA problema u nastavnom procesu vrlo zahtjevan i kompleksan, **dugoročno je isplativ** jer rješavanjem SA problema učenje postaje smislenije, znanje se funkcionalno povezuje te se postiže presjek i unija, a ne disjunktnost koja postoji u učenju i poučavanju tematskih cjelina.

Bolje je i korisnije zadržati se samo na jednom zadatku, npr. SA problemu te kroz njegov potencijal uspostaviti bogate mreže konceptualnih znanja i vještina, nego rješavati hrpu sličnih zadataka i zadržati se samo na proceduralnim znanjima i vještinama. Poučavati matematiku na taj način pomoći će učenicima da otkriju horizontalnu i vertikalnu povezanost matematičkoga znanja, kao i osnažiti njihovih interes za matematikom (Yanhui, 2018).

Svakako je potrebno provesti daljnja istraživanja o faktorima koji su o(ne)mogućili učenicima uspješan hod do rješenja, način primjene SA problem u nekoj odabranoj didaktičkoj situaciji, kao i način dizajniranja SA problema s ciljem usmjeravanja učioničkih aktivnosti te poticanje učenja s razumijevanjem.