The Comparison of the Amount of Backlash of a Harmonic Gear System

Maroš MAJCHRÁK*, Róbert KOHÁR, Slavomír HRČEK, František BRUMERČÍK

Abstract: This article presents a study of the amount of harmonic gear system backlash. The study shows a method for an effective measuring of the amount of backlash in harmonic gear systems. Given that this calculation tasks demand a lot of iterative operations, this article introduces a new method for creating 2D and 3D models of a harmonic gear system. At the same time, this study presents a new way of generating the geometry of a wave generator with a parametrically modified geometry. Introduction of the parametric model of a wave generator allows a rapid optimization of construction parameters, thus securing correct dimensioning and evaluation of the amount of backlash. When creating the computational model, operational characteristics were taken into consideration, therefore, the 2D and 3D computational models were created to correspond with real operation conditions to the greatest extent possible. The results of the analysis imply that the suggested method of examining the design parameters of the wave generator due to the harmonic gear system backlash is effective and correct. The benefit of this article is mainly to introduce a new dynamic computational model of a harmonic gear system and to optimize the construction parameters of the wave generator in line with the amount of backlash, which is manifested in the higher accuracy of the harmonic gear system. In the conclusion of this article we state our recommendations and acquired knowledge.

Keywords: backlash; computational model; harmonic drive gears; wave generator

1 INTRODUCTION TO THE TOPIC OF HARMONIC DRIVE GEARS

Harmonic gear systems have been used mainly where it is necessary to transmit high torque within a small and light design; therefore, these gear systems have been mainly used in the field of precise positioning, robotics, aerospace and precise industrial applications, in Fig. 1. Harmonic gear mechanisms owe their advantages predominantly to zero clearance and high solidity. On the other hand, they have an inherent positional error, known rather as a kinematic error. This error is primarily responsible for decreasing the performance and transmission of torque. Dong et al. [1] and Gravagno et al. [2] modeled and simulated kinematic error.
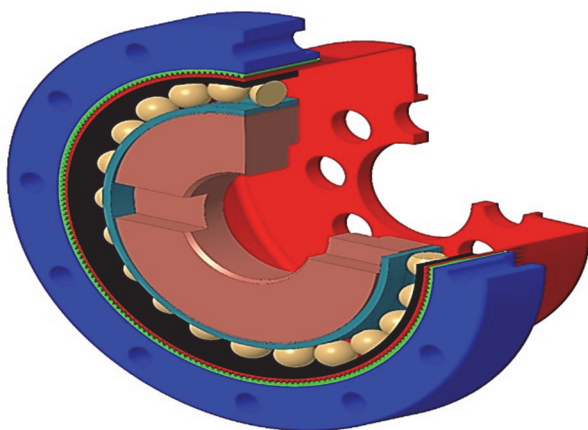

Figure 1 Harmonic Gear System in Creo

A harmonic gear system is composed of a wave generator, a flexible bearing, a flexible spline and a circular spline. In comparison to conventional gear systems, these gear systems use the principle of generating waves which are characterized by a specific harmonic movement of a flexible spline. Oh et al. [3] describe this problem in their research. It is the flexibility of this actuator that secures an ideal filling of the space in between the teeth, thus increasing the accuracy and rigidity of a harmonic gear system. Dong et al. [4] and Chen et al. [5] describe the issue

study on parameters tooth of the harmonic drive. Kondo et al. [6] describe the issue study on profiles tooth of the harmonic drive. The harmonic movement also secures an even distribution of the contact stress on gearing on both sides; this makes it possible to transmit rather high torques (1:50, 1:100 and 1:150). Popović et al. [7] and Ostapskiet al. [8] studied distribution of the contact stress on alloyed steel.

Backlash is definitely among the basic operational characteristics of harmonic gear systems. It is defined as clearance or loss in movement caused by gaps between elements. It may also be defined as a maximum distance or an angle through which any part of the mechanic system may move in one direction without exerting considerable force or movement on another part in the mechanical system. Jeon et al. [9] study stress and vibration analysis of a steel and hybrid flexspline for harmonic drive.
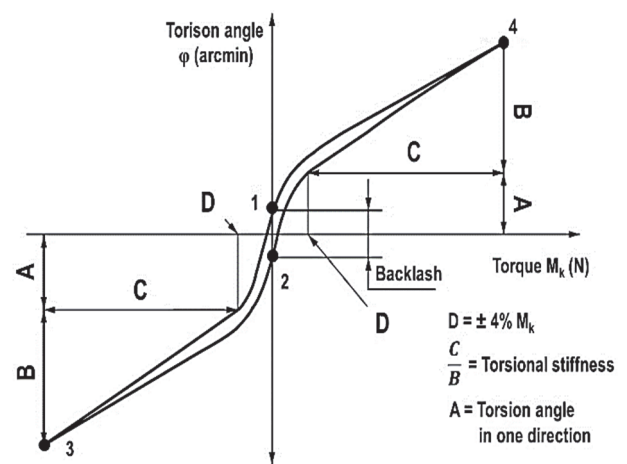

Figure 2 Backlash Diagram

Backlash is defined as a distance between Point 1 and Point 2 in Fig. 2. In this article, we will mainly focus on researching the amount of backlash of a harmonic gear system for 2D and 3D computational models. The total torsion angle is defined as a distance between Point 3 and 4, in Fig. 2. Backlash is normally up to 4 arcmin. Li et al. [10] and Chen et al. [11] describe this problem in their researches.

2 CONSTRUCTING A PARAMETRIC MODEL OF A WAVE GENERATOR

To construct the geometry of the wave generator, it was necessary to introduce a correct and simple mathematical definition of the shape forming the wave generating curve. As an auxiliary generating curve of the mathematical calculation model, we chose a circle describing the diameter of the inner ring of a rolling bearing, into which the wave generator shall be flexibly pressed. To make it possible to press the flexible bearing with the wave generator, we had to consider a flexible deformation of the bearing. The bearing was produced of flexible spring steel. The second condition we chose was that the outer circuit of the wave generator must be equal to the inner circuit of the ring of a rolling-element bearing. Cook et al. [12] and Tjahjowidodo et al. [13] describe this research in more detail.
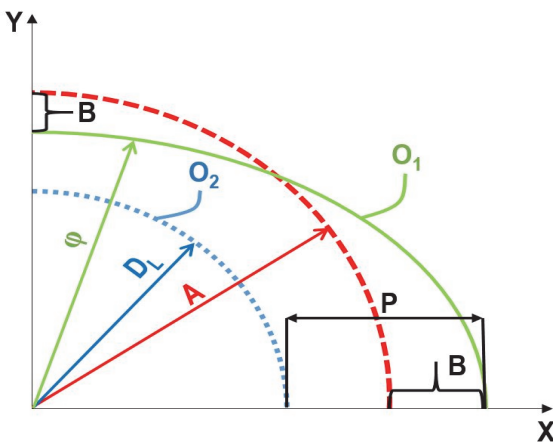

Figure 3 Construction Parameters of a Wave Generator

To describe the generating curve of the wave generator, we used a modified ellipse equation (5). The ellipse needed to be modified so that we would be able to slightly change the shape of the generating curve of the wave generator; this was important for dimensioning the amount of the harmonic gear system backlash. Coefficients A and B (5) were the unknown parameters of the system of equations (1) and (2) and it was necessary to find them using a numerical calculation method, in Fig. 3. Parameter n was used for the exponential change of the character of the generating curve of the wave generator. Parameter P was used for shifting and flattening the shape of the generating curve of the wave generator. Maiti et al. [14] describe this process in detail.

$$A + B = D_L + P \tag{1}$$

$$O_1 = O_2 \tag{2}$$

$$O_2 = D_L \cdot \frac{\pi}{2} \tag{3}$$

$$O_1 = \int_0^{\frac{\pi}{2}} \sqrt{1 + \left(\frac{d_x}{d_s}\right)^2 + \left(\frac{d_y}{d_s}\right)^2} \, d_s \tag{4}$$

$$d_s = A + B \cos\left\{2\left[\frac{\pi}{2}\left(\frac{2\varphi}{\pi}\right)^n\right]\right\} \tag{5}$$

We selected Matlab to analyze and numerically calculate the system of Eqs. (1) and (2). This programme is able to numerically solve this task in an effective and rapid way. Another advantage of this programme is the connection between PTC Creo Parametric. In practice, a curve with values entered as parameters n and P is created in the 2D sketchbook in Creo Parametric. Its parameters are sent directly to Matlab. Based on these parameters, A and B are numerically calculated in Matlab and sent back to Creo Parametric, where they modify the generating curve of the wave generator after regeneration. In this way, we can modify and dimension the geometry of the generating curve of the wave generator in real time. Bertollini et al. [15] describe this research in more detail.

3 CONSTRUCTING THE COMPUTATIONAL MODEL OF THE HARMONIC GEARBOX

To construct the calculation model of the harmonic gear system, it was necessary to create rapidly and effectively modifiable 2D and 3D computational models. We chose Creo Parametric as a tool. We had already put the generating curve of the wave generator geometry into the programme. We reflected this curve symmetrically to create a continuous total outline curve of the wave generator. A flexible bearing was later modelled in line with the dimensions defined in the catalogue of the producer. The flexible and circular spline was then generated in a non-deformed shape. The geometry of the circular and flexible spline gearing was generated similarly to the wave generator in Matlab like a parametric curve describing the nature of the shape of the gearing/teeth. We modelled the whole task in a non-deformed shape, like circles, in Fig. 4.
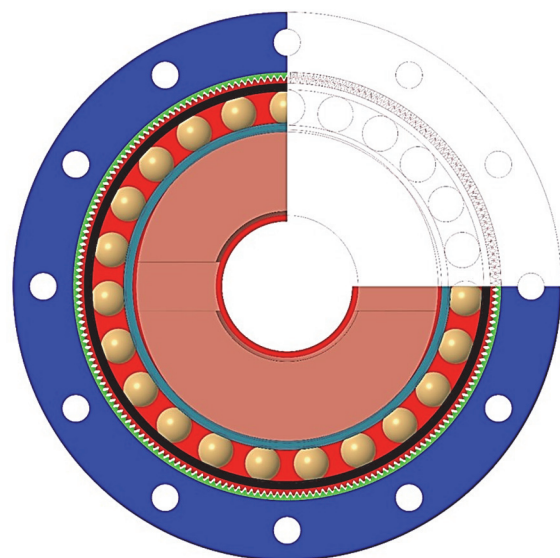

Figure 4 The Model of HGS in Creo

The next step was to create 2D and 3D models. The geometry of the contour curves of the bodies was pulled into a "solid" volumetric shape. The 3D model of a

harmonic gear system was fully parametric with more than 10 parameters that modified the shape of the wave generator and the shape of the harmonic gear system's gearing. Then, using the 3D task, a 2D model was created by first creating a section through the harmonic gear system in the median plane of the rolling-bearing elements. Using the function "intersect" in Creo Parametric, a new system generating a 2D curve of the harmonic gear system was created. For the 2D plane analysis, it was necessary to join the circumferential curves of the bodies of the harmonic gear system, using the "fill" function to achieve the "surface" plane geometry. In this way, we achieved a 2D and 3D modifiable parametric model for the 3D and 2D analysis. The 2D and 3D model was exported to *step* format, as MSC Marc Mentant is not able to parametrically work with PTC Creo Parametric. To create the 2D and 3D computational models, it was necessary to subsequently create a mesh.

4 CREATING 2D AND 3D MESH

It is necessary to realize that this is a difficult calculation task with many elements and nodes. This task is also demanding in terms of calculation time and hardware equipment. For this reason, we decided to first create and try out the 2D model. Based on a great number of measurements, our testing implied that it is better to simplify the 2D model as much as possible. Individual parts of the harmonic gear system needed to be divided into as many small units as possible, and the units needed to be separately meshed and subsequently reflected. Marc Mentant requires an external mesher. To create a plane and volumetric mesh, we used MSC Apex Iberian.
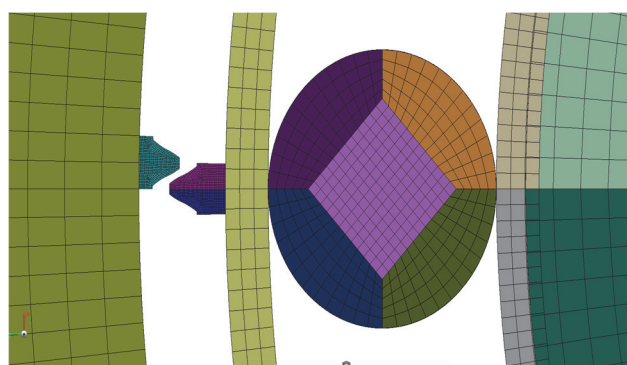

Figure 5 2D Mesh in Apex Iberian

The geometry of rolling bodies was distributed so that the linear quadrilateral surface mesh could be used. This meshing type secures high quality results with a low number of nodes. Subsequently, the bodies of the harmonic gear system were reflected in Apex Iberian. This gradual reflecting of the basic bodies resulted in a nearly perfect homogeneous linear mesh.

In Figs. 5, 6 and 7, the resulting 2D mesh of the harmonic gear system may be seen. The main advantage of keeping this procedure is the homogeneous nature of the mesh in the area of gearing. In Apex Iberian, it is possible to define the same number of nodes on the generating curve of the flexible spline gearing/teeth and circular spline gearing/teeth at the same time. General mesh density for the gearing was determined as 0.1 mm. For the rest of the elements of the harmonic gear system, the value was 0.5

mm. The total number of nodes was 413,520 and the number of elements was 392,706. The 2D plane mesh was exported in bdf.
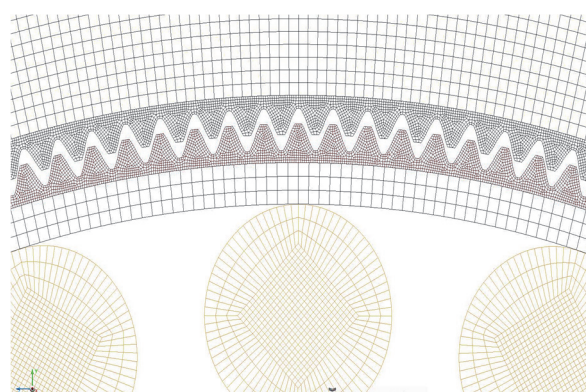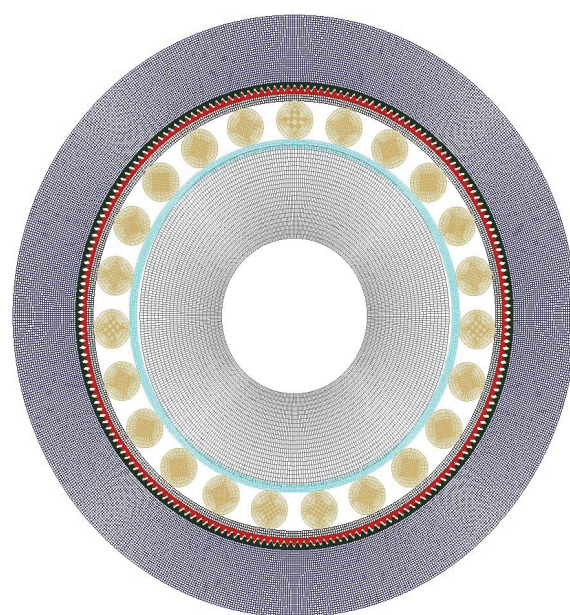

Figure 6 Detail of 2D Gearing Mesh

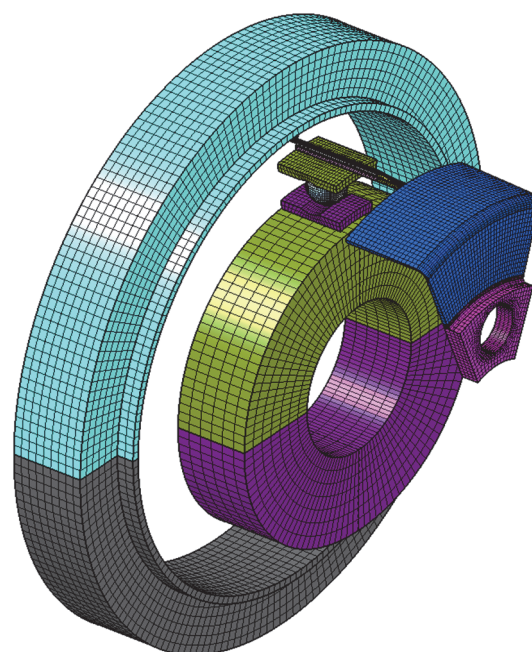**Figure 7** 2D Plane Mesh of the Harmonic Gear System

Figure 8 Preparation of a 3D Model for Meshing

To create the 3D mesh, we proceeded similarly to creating the 2D mesh. However, the 3D division of bodies has some specific properties. We needed to consider a higher number of elements and much higher computing time. For this reason, rather complicated bodies such as the flexible spline were further divided into smaller sections. When meshing such a large task, we recommend a procedure similar in Fig. 8 and in Fig. 9 to preserve the accuracy of the calculation.

Apex Iberian allows using linear "2.5D Meshing". This meshing secured a homogeneous 3D hex linear element mesh. The results of our research revealed that it is better to reflect individual teeth in Marc Mentant, the calculation programme. We wish to support our recommendation mainly with the fact that it allows using the "Sweep" function, merging duplicate points that are generated when reflecting symmetrical bodies.
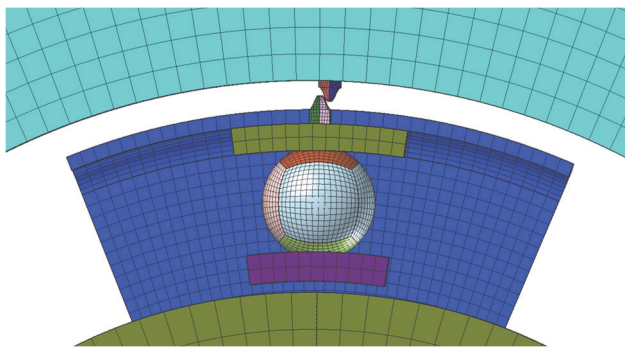

Figure 9 Detail of the 3D mesh

In Apex Iberian, a homogeneous 3D mesh was generated with a total number of elements amounting to 544,336 and nodes 693,975. The mesh density was defined as lower compared to the density for the 2D task, in particular, 1 mm, in the area of gearing/teeth, the density was 0.5 mm.

5 CONSTRUCTING 2D AND 3D COMPUTATIONAL MODELS

MSC Marc Mentant was selected as a calculation tool, mainly for the broad possibilities of defining "loadcase" steps and individual computational operations. We need to realize that a computational model of six bodies was created, of which a total number of three bodies is flexible and thus, it is necessary to secure a flexible deformation as well as mutual interaction without penetrations. The calculation task is also specific due to the fact that the harmonic gear system was left to roll with friction over a certain time frame. Therefore, this is not just a classic static task without any simulated rolling motion like in real practice. The principle of defining the flexibility in Marc Mentant consists in the right definition of boundary conditions, selecting the right material and right type of elements with required properties. Also, it is necessary to consider the mutual fixation of the rolling-bearing elements without a cage. Without the cage, upon the start of calculation, the rolling elements would roll down to the bottom of the bearing due to the force of gravity, and the bearing would be incorrectly pressed. In this chapter, we will marginally describe the procedure of generating the 2D and 3D computational models.
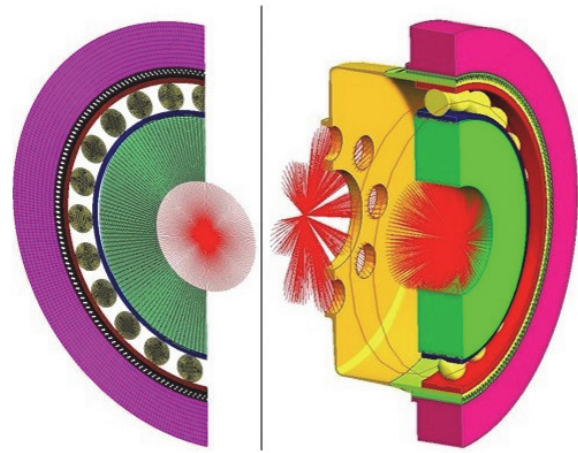

Figure 10 Defining Boundary Conditions

To define the boundary conditions, we introduced rigid point RBE2. Within these points, the torque applied to the harmonic gear system through the wave generator was entered, in Fig. 10. The amount of backlash was read using the second point RBE2 on the back side of the flexible spline. In this way, we tried to get as close as possible to the real stress. The cage was replaced by springs of high rigidity. Boundary points of the fixation of springs were defined at opposite points of the rolling elements. In the next step, the task of several contact bodies was introduced using contact tables.
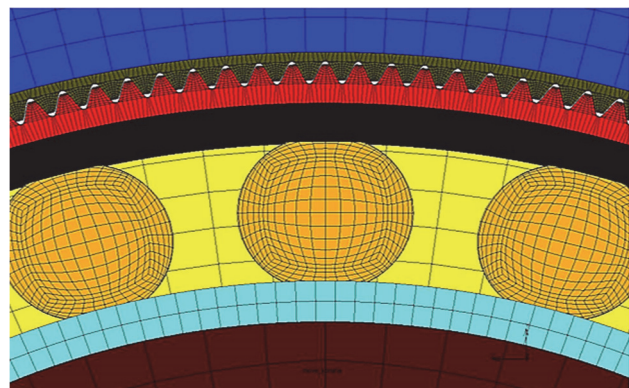

Figure 11 Contact Tables

Figure 12 Assembling the Harmonic Gear System

The contact tables secured mutual possible combinations of the contacts of individual bodies with a defined friction. Combinations of three contact tables were used for this task. The first contact table locked the wave generator and allowed for the pressing of the flexible bearing within the allowed deformation against the wave generator. In the second step of assembling the harmonic gear system, the possible penetration of the

gearing/teething of the flexible and circular spline was limited. In this way, we secured a correct layout of the teeth and a correct elliptical deformation of the flexible spline. In the last contact table, the contact switch between the gearing/teething was defined and thus rolling the harmonic gear system was enabled. The whole task was split into five loadcases-calculation steps. Within these steps, the duration of each operation was defined, as well as turning the boundary conditions on and off. In the last step, the "Job" was defined, within which recording desired results was activated. To check the calculation, an "mdf" file was created after assembling the harmonic gear system, in Fig. 12.

6 ANALYSIS AND PROCESSING THE RESULTS

The results were processed in Marc Mentant. The first and main difference between the 2D and 3D computational models was, no doubt, the shape of the deformed flexible spline.
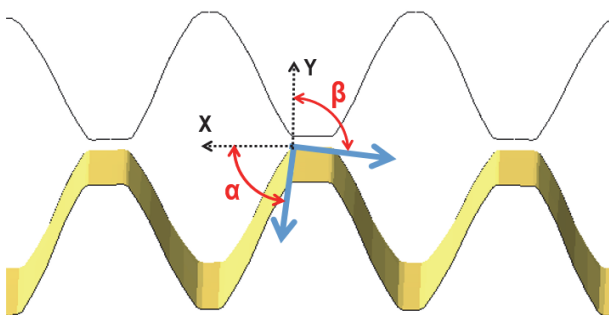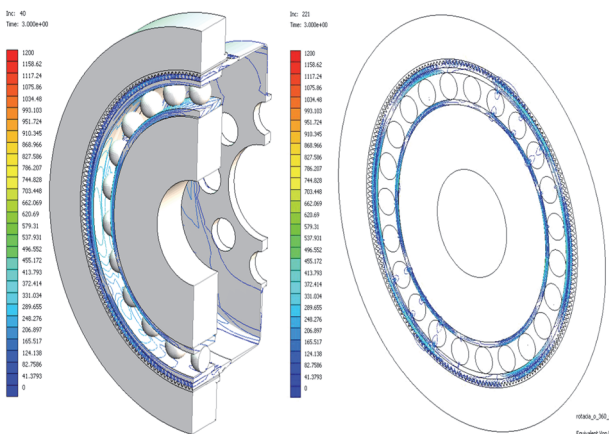

Figure 13 Deformed Flexible Spline

Figure 14 Stress Distribution

When pressing the harmonic gear system, the gearing/teeth of the flexible spline was deformed in the direction of axis y by angle β. When rotating the harmonic gear system, the flexible spline gearing/teeth was deformed also in the direction of axis x by angle α, in Fig. 13. This deformation arose due to attaching a flexible spline in the circular spline gearing/teething. We found out that this deformation of flexible spline significantly impacts the amount of the harmonic gear system backlash.

According to Von Misses theory, the total maximum stress was approximately the same for the 2D and 3D models 1200 MPa. The greatest stress was at the point contact pressure of the rolling-bearing elements, see the

model in Fig. 14. However, the stress distribution was totally different for the 2D and 3D models, in Fig. 15 and in Fig. 16.
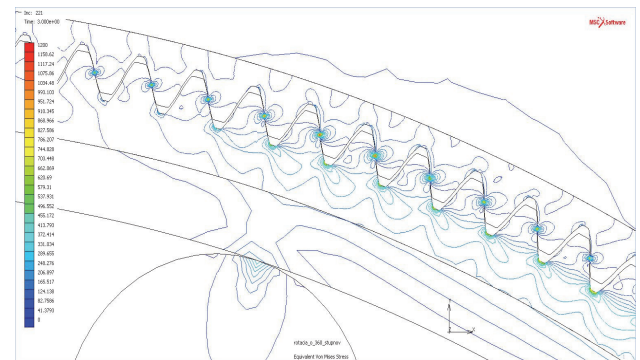

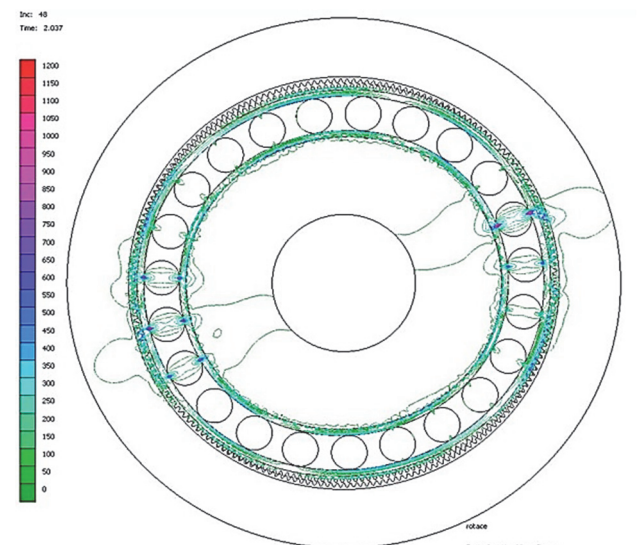**Figure 15** Detail Stress Distribution - 2D Task

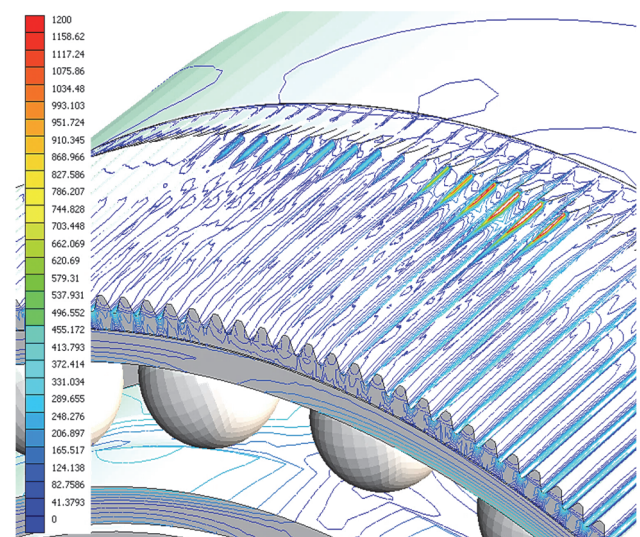**Figure 16** Stress Distribution - 2D Task

Figure 17 Stress Distribution - 3D Task

With the 3D task, the stress was distributed asymmetrically throughout the deformed sides of the gearing/teeth of the flexible spline, in Fig. 17.

The amount of the backlash of the harmonic gear system is also defined by the shape of the rear part of the flexible spline, mainly by the thickness of the transition wall between the front and rear part of the flexible spline.

The knowledge from this research enables us to better understand the significant difference between the 2D and 3D computational tasks.

7 EVALUATION OF BACKLASH

In total, 40 measurements for the 2D and 40 measurements for the 3D model were carried out. The parameters n and P with a predefined step of 0.05 mm were modified for both tasks, in Fig. 18.
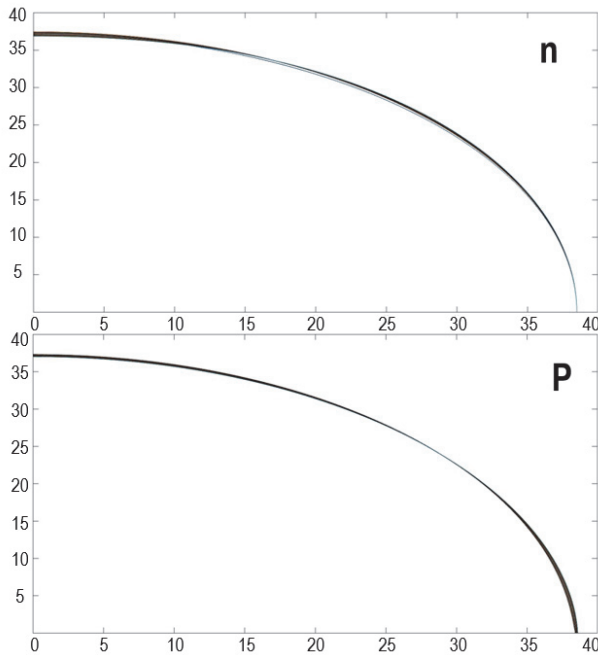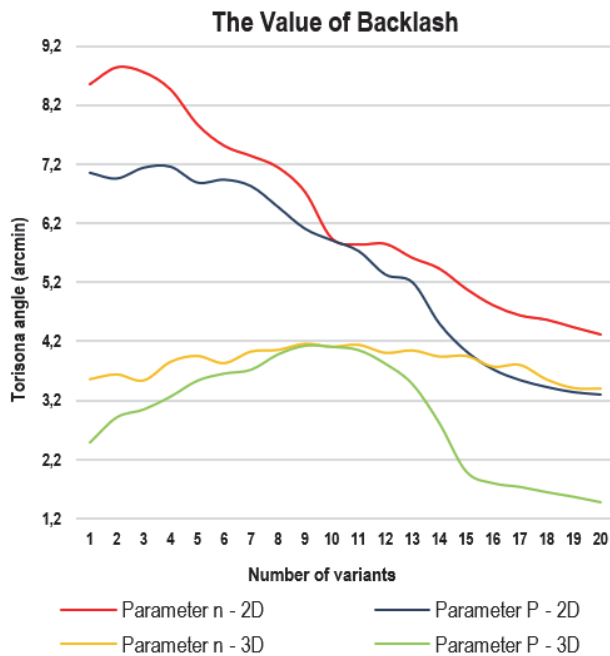

Figure 18 Geometry of the Wave Generator

Figure 19 Value of Backlash

The results of the measurements implied that parameters n and P significantly influence the amount of the backlash of the harmonic gear system, in Fig. 19.

The observed average difference in terms of the backlash for parameter n was 36.6% between the 2D and
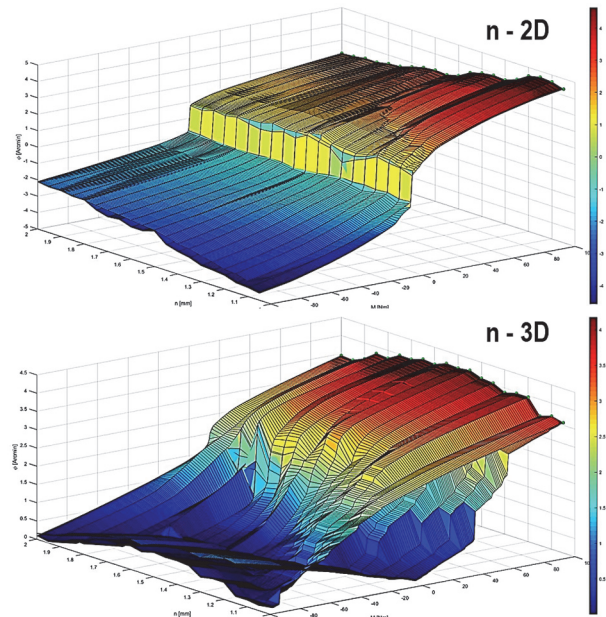
3D tasks. This parameter amounted to 46% for parameter P.

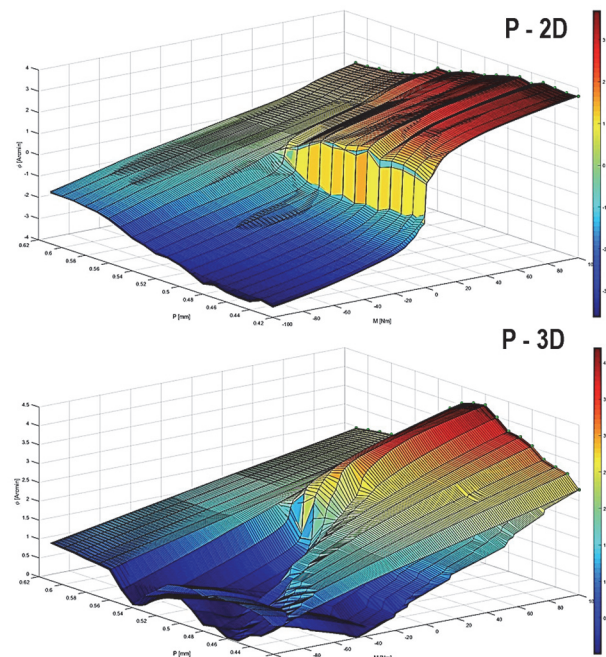**Figure 20** 3D Graph of the Backlash - Parameter n

Figure 21 3D Graph of the Backlash - Parameter P

For a better depiction of the change in the amount of backlash in relation to parameter n (in Fig. 20) and parameter P (in Fig. 21), space graphs were also created, characterizing the process of measuring the amount of backlash at a maximum defined torque of $M_k = 100$ Nm. The measurement process consisted in pressing the harmonic gear system, subsequent rolling and stressing with the prescribed torque in the direction and in the opposite direction of the wave generator rotation. The circular spline and the wave generator were firmly fixed during the stressing. The torque was only applied to the flexible spline via a/the RBE rear point. The values of the rotation of the RBE point were written down in the result file. These spatial graphs imply a significant change between 2D and 3D computational models.

8 CONCLUSION

In the conclusion of this research, we wish to emphasize some facts we discovered. The construction of the 2D computational model was important for trying out/testing the computational model. This plane task gave us an opportunity to solve the problems related to creating the computational model in a relatively short computational time. However, we do not recommend considering this type of plane analysis as a type trustworthy for a final reading of the amount of backlash. The plane task does not allow carrying out a real deformation of the flexible spline just like with the 3D task. During this research, it was found out that it is the spatial deformation of flexible spline which plays a crucial role when reading the amount of backlash. To compare the amount of the backlash with the 2D and 3D tasks, the difference was in average about 41.3%. We may prove this statement by the fact that during the rolling of the harmonic gear system, spiral deformation of the flexible spline occurs. It is this deformation that is responsible for the irregular gear of the teeth (in Fig. 17), which became evident in a significant decrease in the amount of backlash (in Fig. 19). It was also found out that the geometry of the wave generator has a significant impact on the amount of backlash of harmonic gear systems.
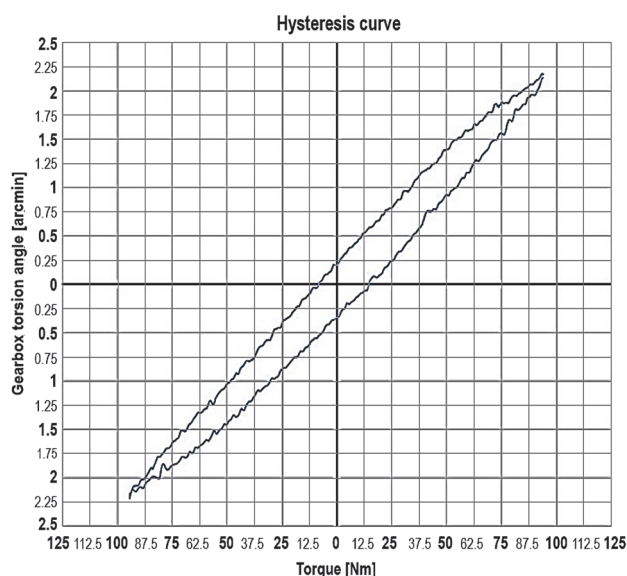

Figure 22 Hysteresis measurement

Fig. 22 shows the actual backlash measurement. The difference between 3D analysis and actual measurement was less than 2%. This measurement proves the accuracy of our research. In following research, we would like to concentrate on researching other construction parameters of the gearing/teething which impact the amount of backlash in these not mass-produced harmonic gear systems.

Acknowledgements

transmissions with a high gear ratio with respect to kinematic properties.

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Contact information:

Maros MAJCHRAK, Ing.
(Corresponding author)
University of Žilina, Faculty of Mechanical Engineering,
Univerzitná 8215/1, 01026 Žilina, Slovakia
E-mail: Maros.Majchrak@fstroj.uniza.sk

Robert KOHAR, doc., Ing., PhD
University of Žilina, Faculty of Mechanical Engineering,
Univerzitná 8215/1, 01026 Žilina, Slovakia
E-mail: Robert.Kohar@fstroj.uniza.sk

Slavomir HRČEK, Prof., Doc., Ing., PhD
University of Žilina, Faculty of Mechanical Engineering,
Univerzitná 8215/1, 01026 Žilina, Slovakia
E-mail: Slavomir.Hrcek@fstroj.uniza.sk

František BRUMERČÍK, doc., Ing., PhD
University of Žilina, Faculty of Mechanical Engineering,
Univerzitná 8215/1, 01026 Žilina, Slovakia
E-mail: Frantisek.Brumercik@fstroj.uniza.sk