





# Formation control of first-order multi-agents with region constraint

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## ABSTRACT

Formation problem of multi-agents systems has been investigated studied by more and more scientists. This paper studies the formation problem of the first-order multi-agent systems with region constraint. Not only formation control but also the constraint area is considered. The state of each agent is constrained by a common convex set. A formation control protocol is proposed based on the local information of neighbourhood and region constraint. If the communication topology is connected, and the target framework is rigid, all agents would enter the target area while reaching the desired formation. The convergence of the formation control to the final formation is guaranteed as proven by Lyapunov framework. Compared with the leader follower controller, the controller does not depend on the leader and has good robustness. In addition, the controller can quickly and accurately control all agents to all agents to enter the constrained area. Finally, the results are illustrated by numerical simulations.

## ARTICLE HISTORY

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## KEYWORDS

Formation; multi-agent systems; set constraint; asymptotic convergence

## 1. Introduction

Formation problem of multi-agents systems has been investigated by more and more scientists of the control community. The purpose of the formation is to control all agents to achieve and maintain pre-defined geometry. The formation problem is widely used in unmanned aerial vehicles formation, satellite formation flying, and load transportation [1].

In the past, formation problem of multi-agent has been considered from different aspects such as the leader–follower, behaviourism, virtual structure, and graph theory [2]. Lafferriere et al. [3] used the algebraic graph theory to study the distributed formation control of multi-agent systems, and gave the relationship between formation convergence rate and Laplacian eigenvalue. Xie and Wang [4] gave a sufficient condition for formation convergence with fixed and undirected topology. Xiao et al. [5] proposed a new control framework to solve the finite-time formation of multi-agents systems. Sun et al. [6] proposed an improved gradient controller, which realizes the formation of first-order multi-agent system in finite time. Lee et al. [7] proposed a formation control method combining global orientation estimation and formation control. Rezaee et al. [8] used the cyclic matrix method to solve the cyclic pursuit formation problem.

In recent years, some scholars have studied formation control with constraints. Egerstedt and Hu [9] proposed a strategy for multi-agent formation control defined by a formation constraint in combination with

a desired reference path. Mastellone et al. [10] devised a controller to ensure collision avoidance and tracking with bounded errors outside the collision area. Basiri et al. [11] proposed an angle-constrained formation control problem for triangular formation. Chen and Jia [12] studied the formation control of differential-drive mobile robots with diamond-shaped input constraints. Gao and Guo proposed [13] a leader–follower formation control method for AUVs with line-of-sight and angle constraints, and designed a finite-time velocity observer to estimate the velocity of leader. Ge et al. [14] proposed a control law based on artificial potential field method for formation tracking control of multi-agent in constrained space.

As far as we know, there are few studies on formation control with regional constraints. But in the actual control system, we need to control the agent to the required area. That is to say, we should not only consider the formation control, but also consider the constraint area. This paper studies the formation control of first-order multi-agent systems with regional constraints. We know that the leader–follower formation can also control all agents converges to the desired position on the plane. However, compared with the leader–follower controller, the controller in this paper does not depend on the leader and has good robustness. In addition, the controller can quickly and accurately control all agents to all agents to enter the constrained area. Moreover, compared with the previous research, the contribution of this paper is as follows: first, we

propose a new control law, which has the sum of a formation part and a projection part; second, the method to prove the asymptotic convergence is provided.

The outline of this paper is as follows. Some preliminaries and problem formulation are provided in Section 2. In Section 3 a new control law is presented, and convergence analysis of the controller is also given. In Section 4, numerical examples are given. Concluding remarks are then provided in Section 5.

## 2. Preliminaries and problem description

*Notations:*  $\|s\|$  is the Euclidean norm of the vector  $s$ .  $\|s\|_1$  represents the 1-norm of the vector  $s$ . The projection of the vector  $s$  onto the closed convex set  $\Omega$  is denoted by  $P_\Omega(s)$ .

A communication topology among agents of the group is modelled by an undirected graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ , which is referred to as the sensing graph of the group.  $\mathcal{V} = \{1, 2, \dots, N\}$  and edges set  $\mathcal{E}$  are node set and edge set of  $(\mathcal{G}, \mathcal{E})$  respectively.  $j$  is called a neighbour of node  $i$  if  $(i, j) \in \mathcal{E}$ . The neighbours of vertex  $i$  are given by  $N_i = \{j : j \in \mathcal{V} : (i, j) \in \mathcal{E}\}$ . If  $j \in N_i$ , the agent  $i$  can get the information of the agent  $j$ . The adjacency matrix  $\mathcal{A} = [a_{ij}] \in R^{N \times N}$  of the graph  $\mathcal{G}$  is defined as  $a_{ij} = a_{ji} = 1$ ,  $a_{ii} = 0$ , if  $j \in N_i$ , otherwise  $a_{ij} = 0$ . The communication graph considered in this paper does not change with time and is fixed.

In this paper, a system of  $N$  agents governed by the following equation is considered

$$\dot{x}_i(t) = u_i(t) \quad i \in \mathcal{N}, \quad (1)$$

where  $x_i \in R^m$  is the position state, and  $u_i \in R^m$  is the input vector of the agent  $i$ .

A framework is defined as a pair  $(\mathcal{G}, X)$  where  $X = [x_1^T, x_2^T, \dots, x_N^T]^T \in R^{mN}$ . A rigidity function  $r_G(X) : R^{mN} \rightarrow R^{|\mathcal{E}|}$  of the framework  $(\mathcal{G}, X)$  is defined as:

$$r_G(X) = \frac{1}{2}[\dots, \|x_i - x_j\|^2, \dots]^T, \quad (i, j) \in \mathcal{E}. \quad (2)$$

**Definition 2.1** ([15]): A framework  $(\mathcal{G}, X)$  is rigid if there exists a neighbourhood  $\mathcal{U}$  of  $R^{mN}$  such that  $r_G^{-1}(r_G(x)) \cap \mathcal{U} = r_{\mathcal{K}}^{-1}(r_{\mathcal{K}}(x)) \cap \mathcal{U}$  where  $\mathcal{K}$  is the complete graph on  $N$ -vertices.

A rigidity matrix  $R(X)$  of  $(G, X)$  is given by the  $R(X) = \frac{1}{2} \frac{\partial r_G(X)}{\partial X}$ , where we have that  $\text{rank}[R(X)] \leq 2N - 3$ . Frameworks  $(G, X)$  and  $(G, \hat{X})$  are equivalent if  $r(X) = r(\hat{X})$ , and are congruent if  $\|x_i - x_j\| = \|\hat{x}_i - \hat{x}_j\|$ ,  $\forall i, j \in \mathcal{V}$  [15]. This paper studies multi-agent system that have a region constraint  $\Omega \in R^m$ , and the task is to control all agents to enter the constraint region when they reach the formation. To do that, let's first give the definition of formation. Give a state  $X^* = [x_1^{*T}, x_2^{*T}, \dots, x_N^{*T}]^T$  such that  $(\mathcal{G}, X^*)$  is rigid. The formation of multi-agent system is to control the position

of all agents to satisfy the following conditions.

$$E_X := \{X : \|x_i - x_j\| = \|x_i^* - x_j^*\|, j \in N_i, i \in \mathcal{V}\}, \quad (3)$$

where  $x_i^*$ ,  $i \in \mathcal{V}$  is the relative position of agent  $i$  of the desired formation, and  $x_j^*$ ,  $j \in N_i$  is the relative position of the neighbour of agent  $i$ . The problem described above can be transformed into the problem that  $E_X$  is the set of all formations congruent to  $X^*$ . It must be noted that  $x_i^*$  may not belong to the target region  $\Omega$ .

Thus, the goal of the formation control of this paper is to control the position state  $X$  to satisfy  $\|x_i - x_j\| = \|x_i^* - x_j^*\|$ ,  $j \in N_i$  and  $x_i \in \Omega$ ,  $i \in \mathcal{V}$ , as  $t \rightarrow \infty$ . That is, if we give a constrained area and a desired relative formation, the goal is to control all multi-agent to enter the constrained area while reaching the formation. Now let's give a formal description of the problem of this paper.

**Problem 2.1:** Given  $N$  single-integrator agents, an undirected graph  $(\mathcal{G}, E)$  and a state  $X^* \in R^{mN}$  of  $G$ , suppose that  $(\mathcal{G}, X^*)$  is rigid. Design a control  $u_i$  for each agent  $i$  in terms of  $x_i - x_j$ ,  $j \in N_i$  such that  $\|x_i - x_j\|$  converges to  $\|x_i^* - x_j^*\|$ , and  $x_i \in \Omega$  as  $t \rightarrow \infty$ .

The following lemma is needed in this paper.

**Lemma 2.1** ([16]): Given a closed convex set  $\Omega \subset R^m$ ,  $\forall y \in R^m$ ,  $\forall x \in \Omega$ , it has

$$(P_\Omega(y) - x)^T (y - P_\Omega(y)) \geq 0.$$

## 3. Main results

In this section, a new formation controller is proposed. The controller has the sum of the formation part and a projection part. The new distributed formation controller is as follows:

$$u_i(t) = - \sum_{j \in N_i} a_{ij} (x_i(t) - x_j(t) - \gamma_{ij}^*) - \beta (x_i(t) - P_\Omega(x_i(t))). \quad (4)$$

where  $\gamma_{ij}^* = x_i^* - x_j^*$ ,  $\beta$  is positive constant. In this algorithm, the first term is the reference relative distance between agents, so as to control the formation of agents, and the second term is designed to drive the agents to enter the constraint area.

Let  $x_i(t) - x_j(t) - \gamma_{ij}^* = x_i(t) - x_i^* - (x_j(t) - x_j^*) = \tilde{x}_i(t) - \tilde{x}_j(t)$ , where  $\tilde{x}_i(t) = x_i(t) - x_i^*$ ,  $\tilde{x}_j(t) = x_j(t) - x_j^*$ .

The following assumptions will be used throughout the paper:

**Assumption 3.1:** The undirected graph  $(\mathcal{G}, \mathcal{E})$  is connected, and the desired framework  $(\mathcal{G}, X^*)$  is rigid.

**Assumption 3.2:** In the framework of the formation problem, the tracking objective  $(\mathcal{G}, X^*)$  is given in advance.

Considering Assumptions 3.1 and 3.2 the main result of this paper is given as follows.

**Theorem 3.1:** Consider the multi-agents system (1) under the control algorithm (4). If  $\beta > 0$ , all agents can asymptotically track the desired formation framework, and  $x_i(t) \in \Omega$  as  $t \rightarrow \infty$ .

**Proof:** Let the Lyapunov candidate function for system (1) be defined as

$$V(t) = \frac{1}{2} \sum_{i=1}^N (x_i(t) - x_i^*)^T (x_i(t) - x_i^*). \quad (5)$$

where  $x_i^*$  is the position vector of the rigid framework, and may not belong to  $\Omega$ .

The time derivative of  $V(t)$  along the system (1) and the controller (4) is

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^N (x_i(t) - x_i^*)^T \dot{x}_i(t) \\ &= \sum_{i=1}^N (x_i(t) - x_i^*)^T \left[ - \sum_{j \in N_i} a_{ij} (x_i(t) - x_j(t) - \gamma_{ij}^*) \right. \\ &\quad \left. - \beta (x_i(t) - P_{\Omega}(x_i(t))) \right] \\ &= - \sum_{i=1}^N \sum_{j \in N_i} a_{ij} (x_i(t) - x_i^*)^T (x_i(t) - x_j(t) - \gamma_{ij}^*) \\ &\quad - \beta \sum_{i=1}^N a_{ij} (x_i(t) - x_i^*)^T (x_i(t) - P_{\Omega}(x_i(t))) \end{aligned} \quad (6)$$

Because  $a_{ij} = a_{ji} = 1$ ,  $\gamma_{ji}^* = -\gamma_{ij}^*$ , the first part of Equation (6) can be reduced to

$$\begin{aligned} &- \sum_{i=1}^N \sum_{j \in N_i} a_{ij} (x_i(t) - x_i^*)^T (x_i(t) - x_j(t) - \gamma_{ij}^*) \\ &= - \frac{1}{2} \sum_{i=1}^N \sum_{j \in N_i} a_{ij} [(x_i(t) - x_i^*)^T (x_i(t) - x_j(t) - \gamma_{ij}^*) \\ &\quad + (x_j(t) - x_j^*)^T (x_j(t) - x_i(t) - \gamma_{ji}^*)] \\ &= - \frac{1}{2} \sum_{i=1}^N \sum_{j \in N_i} a_{ij} [(x_i(t) - x_i^*)^T (x_i(t) - x_j(t) - \gamma_{ij}^*) \\ &\quad - (x_j(t) - x_j^*)^T (x_i(t) - x_j(t) - \gamma_{ij}^*)] \\ &= - \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N a_{ij} (x_i(t) - x_j(t) - \gamma_{ij}^*)^T \\ &\quad \times (x_i(t) - x_j(t) - \gamma_{ij}^*) \end{aligned}$$

Therefore, Equation (6) can be rewritten by

$$\dot{V}(t) = - \frac{1}{2} \sum_{i=1}^N \sum_{j \in N_i} a_{ij} (x_i(t) - x_j(t) - \gamma_{ij}^*)^T$$

$$\begin{aligned} &\times (x_i(t) - x_j(t) - \gamma_{ij}^*) \\ &- \beta \sum_{i=1}^N a_{ij} (x_i(t) - x_i^*)^T (x_i(t) - P_{\Omega}(x_i(t))) \end{aligned} \quad (7)$$

By Lemma 2.1, we obtain  $(P_{\Omega}(x_i(t)) - x_i^*)^T (x_i(t) - P_{\Omega}(x_i(t))) \geq 0$ . Therefore, it reduces to

$$\begin{aligned} &(x_i(t) - x_i^*)^T (x_i(t) - P_{\Omega}(x_i(t))) \\ &= (x_i(t) - P_{\Omega}(x_i(t)))^T (x_i(t) - P_{\Omega}(x_i(t))) \\ &\quad + (P_{\Omega}(x_i(t)) - x_i^*)^T (x_i(t) - P_{\Omega}(x_i(t))) \\ &\geq \|x_i(t) - P_{\Omega}(x_i(t))\|^2 \\ &\geq 0 \end{aligned}$$

By the above inequality and Equation (7), we get the following inequality

$$\begin{aligned} \dot{V}(t) &\leq - \frac{1}{2} \sum_{i=1}^N \sum_{j \in N_i} a_{ij} (x_i(t) - x_j(t) - \gamma_{ij}^*)^T \\ &\quad \times (x_i(t) - x_j(t) - \gamma_{ij}^*) \end{aligned} \quad (8)$$

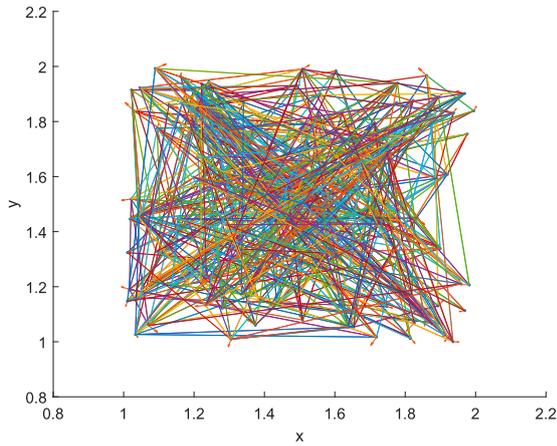
Therefore, based on Lasalle's invariance principle, we achieve that  $\|x_i(t) - x_j(t) - \gamma_{ij}^*\| \rightarrow 0$ , and  $\|x_i(t) - x_i^*\|$  is bounded, as  $t \rightarrow \infty$ . That is, the desired formation of multi-agents can be realized.

Next, we would prove that all agents can enter into the region constraint. First we point out that because the region constraint  $\Omega$  is closed and convex, there exists a constant  $M$  such that  $\forall x \in \Omega, \|x\| \leq M$ . The second, because  $\|x_i(t) - x_j(t) - \gamma_{ij}^*\| \rightarrow 0$ , and  $\|x_i(t) - x_i^*\|$  is bounded, as  $t \rightarrow \infty$ , there exist  $\varepsilon > 0, M_1 > 0$ , and  $t_1 > t_0$  such that  $\|x_i(t) - P_{\Omega}(x_i(t))\| \leq M_1$ , and  $\|x_i(t) - x_j(t) - \gamma_{ij}^*\| < \frac{\varepsilon}{2NM_1}$ , for all  $t > t_1$ . The third, we give a new Lyapunov candidate function of system (1) as

$$V_i(t) = \frac{1}{2} \|x_i(t) - P_{\Omega}(x_i(t))\|^2. \quad (9)$$

Its derivative along the system (1) is

$$\begin{aligned} \dot{V}_i(t) &= (x_i(t) - P_{\Omega}(x_i(t)))^T \dot{x}_i(t) \\ &= (x_i(t) - P_{\Omega}(x_i(t)))^T \left[ - \sum_{j \in N_i} a_{ij} (x_i(t) - x_j(t) \right. \\ &\quad \left. - \gamma_{ij}^*) - \beta (x_i(t) - P_{\Omega}(x_i(t))) \right] \\ &\leq \sum_{j \in N_i} a_{ij} \|x_i(t) - P_{\Omega}(x_i(t))\| \\ &\quad \times \|x_i(t) - x_j(t) - \gamma_{ij}^*\| - \beta \|x_i(t) - P_{\Omega}(x_i(t))\|^2 \\ &\leq 2NM_1 \times \frac{\varepsilon}{2NM_1} - \beta \|x_i(t) - P_{\Omega}(x_i(t))\|^2 \\ &\leq \varepsilon - \beta V_i(t) \end{aligned} \quad (10)$$



**Figure 1.** The initial positions and the communication links of 155-agent.

By the arbitrariness of  $\varepsilon$ , we get

$$\frac{\dot{V}_i(t)}{V_i(t)} \leq -\beta$$

Thus,  $V_i(t)$  converges to zero as  $t \rightarrow \infty$ . Namely, as  $t \rightarrow \infty$ ,  $\|x_i(t) - P_\Omega x_i(t)\| = 0$ . That is, under the control (4),  $x_i(t) \in \Omega$ .

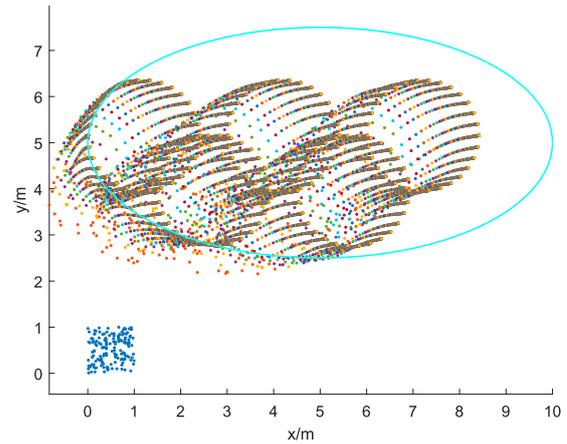
From the above two proof parts, we get that  $\|x_i - x_j\| = \|x_i^* - x_j^*\|, j \in N_i$  and  $x_i \in \Omega, i \in \mathcal{V}$ , as  $t \rightarrow \infty$ . That is, all agents would enter the desired area while reaching formation finally. ■

#### 4. Numerical examples

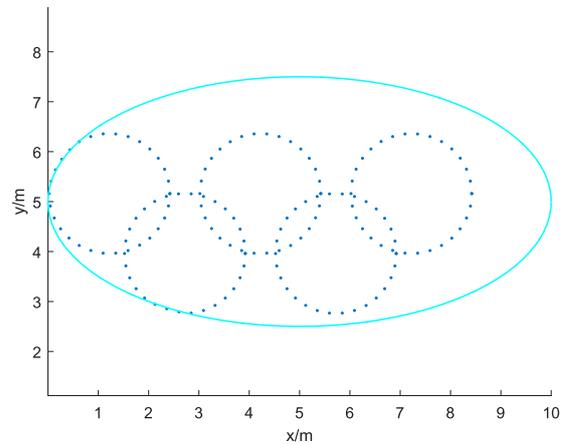
In this section, two examples are presented to illustrate the effectiveness of our results. If we give a constrained area and a desired relative formation, the controller (4) can control all agents to enter the constrained area while reaching the formation.

**Example 4.1:** In simulation, the multi-agent systems consist of 155 agents in  $R^2$ , and the parameter in control law (4) is  $\beta = 1$ . The communication topology is also a Random Network [17]. The generation algorithm of the Random Network is as follows: start with a set of  $N$  isolated vertices and add contiguous edges between them based on the probability of  $p$ . Here, we choose  $N = 155, p = 0.05$ . The symbols  $\cdot$  are the initial positions of agents which are chosen randomly in the unit square. The communication links between the agents are indicated by solid lines. The initial positions and the communication graph are shown in Figure 1. Clearly, the graph is Figure 1. Clearly, the graph is undirected and connected. The communication graph does not change with time and is fixed. Agent  $i$  can get the positions information of its required to preserve the Olympic Rings formation. That is to say, the Olympic Rings are chosen as the framework  $(\mathcal{G}, X^*)$ .  $(\mathcal{G}, X^*)$ .

The constraint area is a ellipse region, which is  $\Omega = \{(x, y) : (x - 5)^2 + 4(y - 5)^2 \leq 25\}$ . Figure 2 depicts



**Figure 2.** The moving process of 155-agent.

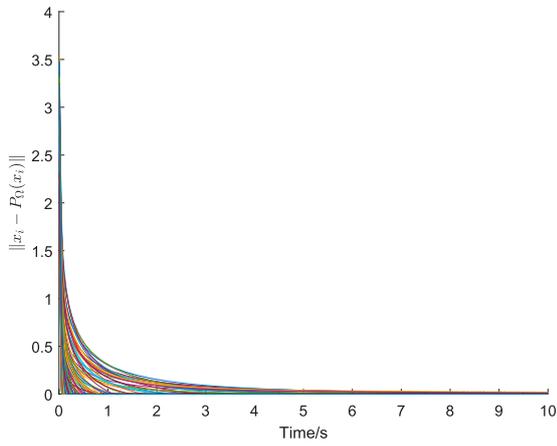


**Figure 3.** The final positions of 155-agent.

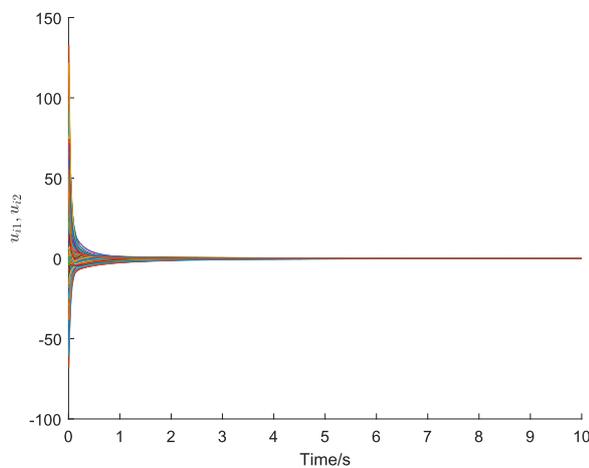
the motion trajectories of all agents within 10 s. Figure 3 depicts the final configurations of the group at 10 s. So we can see that the 155-agent group moves into the constraint set and then achieves the desired formation shape.

The convergence of the errors of the position and the region constraint are plotted in Figure 4. We can see that the errors converge to zero. That is into the constraint set. Also it would be nice to see where the energy of the group (Equation (5)) lowers during the formation transition. The control inputs of the simulation are plotted in Figure 5 which also converge to zero.

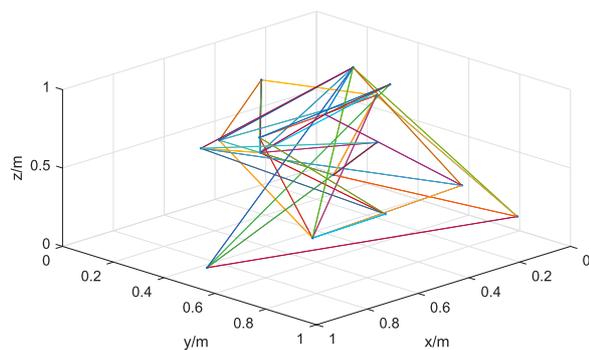
**Example 4.2:** In simulation, the multi-agent systems consist of 16 agents in  $R^3$ , and the parameter in control law (4) is also taken  $\beta = 1$ . The communication topology is also a Random Network. Here, we choose  $N = 16, p = 0.2$ . The symbols  $\cdot$  are the initial positions of agents which are chosen randomly in the unit square. The communication links between the agents are indicated by solid lines. The initial positions and the communication graph are shown in Figure 6. Clearly, the graph is 16 agents are required to preserve a tetrahedron with a top angle of 90 degrees. The side length corresponding to the top angle of tetrahedron is 1 m.



**Figure 4.** The errors of the position and area constraint of 155 agents.

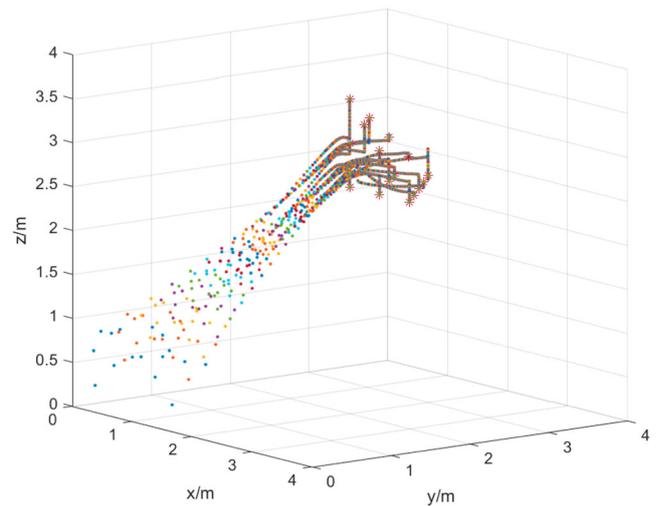


**Figure 5.** The control inputs  $u_i$  of 155-agent.

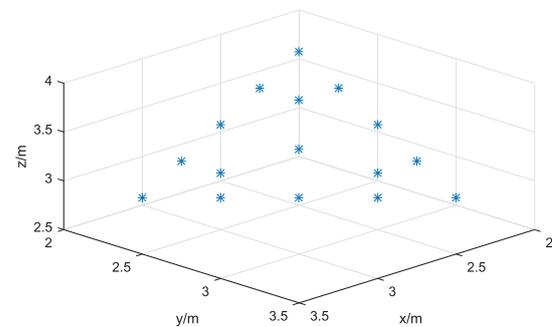


**Figure 6.** The initial positions and the communication links of 16-agent formation.

That is to say, a tetrahedron is chosen as the framework  $(\mathcal{G}, X^*)$ . The constraint area is a cuboid region, which is  $\Omega = \{(x, y, z) : 2 \leq x, y, z \leq 4\}$ . Figure 7 depicts the motion trajectories of all agents within 10 s. Figure 8 depicts the final positions of the group at 10 s. We can observe that the 16-agent group moves into the constraint set and then achieves the desired formation shape.



**Figure 7.** The simulation process of 16-agent formation.



**Figure 8.** The final positions of 16-agent formation.

## 5. Conclusion

This paper studied the formation problem for the first-order multi-agent systems with region constraint. A formation control protocol is proposed based on the local information of neighbourhood and local region constraint. The convergence of the formation control to the final formation is guaranteed as proven by Lyapunov framework. If the topology is connected, and the target framework is rigid, all agents would enter the global target area while reaching the formation. Finally, the results were illustrated by numerical simulations. In the future, we will consider each agent has local region constraint which is known only to an individual agent. We will also consider how to use the control algorithm to some other possible real applications.

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