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# Stability analysis of network-controlled generator excitation system with interval time-varying delays

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**ABSTRACT**

This paper deals with the problem of stability analysis of generator excitation system (GES) with interval time-varying delays. Network-controlled GES involves transmitting measured data from the plant site to controller and control signals from the controller to the plant site. Open communication channels impart time-varying nature to the delays. These time-varying delays can vary between intervals which would affect the system stability. The model of a GES is developed with the proportional integral (PI) controller including delay effects. In this paper, a less conservative delay-dependent stability criterion is derived using Lyapunov-Krasvoskii functional (LKF) for GES with interval time-varying delays. The bounding technique for derivative of LKF is developed by using Wirtinger inequality and free-weighting matrices. The relationship between the delay margins of GES and gains of the PI controller are investigated. This delay margin is used to tune the PI controller. The adequacy of the proposed result is confirmed by using simulation studies.

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**KEYWORDS**
Delay-dependent stability; delay margin; generator excitation system; interval time-varying delay; Lyapunov-Krasovskii functional

## 1. Introduction

In a power system, the generator excitation control is employed to maintain the voltage between specific limits during changes in power system load [1–3]. The major challenge in power system under market environment is to integrate computing, communication, and control. The communication network is the backbone of smart grid technologies by integration of power system with information technologies. The usage of phasor measurements units (PMUs) for measuring current, voltage, frequency, angle and distributed communications network involves transmitting measured data between plant and the controller have introduced significant amount of time-delay in wide-area monitoring systems (WAMS). The instant between measurement of signal and signal available to controller are major problems in WAMS. These time-delays have impact on power system stability and further lead to loss synchronism [4].

Network control systems (NCSs) are meant for distributed control where plant and control centre is located far away from each other [4]. In closed-loop network control, the master controller is employed to control the system located at the remote place and the forward and feedback path of the system is completed through a communication channel [5]. Open communication channel and processing elements introduce time-delays in the feedback as well as the forward path. A delay in a network happens due to geographical distance, network congestion or processing time, etc. Stability is an important aspect of control system performance assessment [4–7]. The time-delay is introduced inevitably in the closed-loop control of dynamical systems under the open communication channel. These delays are generally time-varying in nature which affects the stability and the system performance of the closed-loop system. Hence, it is necessary to consider these delays in modelling and stability analysis [4,7,8]. The maximum amount of time-delay up to which system remains stable is called delay margin. There are several methods to compute the delay margin of time-delay system. Mainly they are grouped into two categories, frequency domain and time domain methods. Frequency domain methods determine the stability of time-delay system by the distribution of roots of the characteristic equation. It is suitable only for a system with constant delays. Time-varying delays or interval time-varying delays and system with uncertainties are difficult to handle in the frequency domain method [6].

In NCS, when the delay margin lower bound differs from zero, such a delay with non-zero lower bound is considered as interval time-varying delay [14–16,39]. There are two approaches to ascertaining the stability of the time-delay system. They are delay-independent stability approach and delay-dependent stability (DDS) approach. In delay-independent stability approach does not consider time-delay in the stability analysis. Since NCS involves time-delay in the control loop. Ignoring

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such a time-delay in analysis and designing controller for NCS may not be efficient under practical operating condition whereas in DDS approach time-delay are considered in stability analysis and controller are designed with concern to time-delay. Therefore, results obtained from DDS approach is less conservative as compared to delay-independent stability approach [17]. For any closed-loop system, DDS analysis computes the maximum value of the time-delay by which the system remains asymptotically stable [18].

Many results for DDS of power systems control methods such as generator excitation control, load frequency control (LFC), etc. have been appeared [4,7,8,22,26,27]. LFC systems with constant delay and time-varying delay have been discussed in [4,8,19,40]. Stability analysis of the generator excitation control system with constant delay is reported in [7]. Recently, a DDS criterion with interval time-varying delay by using the Lyapunov-Krasvoskii functional (LKF) approach has been reported [14]. This stability criterion has been derived by bounding time-derivative of the LKF using finite sum inequality and Wirtinger's inequality. However, finite sum inequality that are developed by using Jenson's inequality leads to more conservative criterion [14]. Hence, there is the further scope of constructing LKF and tighter bound of integral terms on derivative of LKF to improve the stability region. In this paper, by diligently combining Wirtinger's inequality with free-weighting matrices, a new DDS criterion is derived. It is also shown that the proposed result is less conservative than that of [14].

The objective of DDS analysis is to compute the delay margin for a time-delay system. The challenge in DDS analysis is the construction of LKF and choosing bounding technique for integral terms on the time-derivative of LKF [23–24âĂŞ25, 28, 32–41]. In this paper, a new DDS criterion is derived for computing the delay margin of the generator excitation system (GES) controlled by a communication network that introduces time-delays in the feedback path and forward path. These two delays are combined into a single delay component. This criterion is applied to determine the DDS analysis of a network-controlled GES with interval time-varying delays. The proposed result is in a linear matrix inequality (LMI) setting therefore, can be easily solved by Matlab/LMI toolbox [29]. Further, the relationship between the gain of the proportional integral (PI) controller and delay margin is investigated. Simulation studies are carried out by using Matlab/Simulink to validate the effectiveness of the proposed criterion. This paper is arranged as follows: Section 2 introduces a model of the NCS in the state- space framework with delay effects. DDS criterion for NCS with interval time-varying delay is established in Section 3. In Section 4, the relationship between the delay margin of GES for different sets of the PI controller gain is discussed. Finally, Section 5 concludes the paper.

**Table 1.** Notations.

| | |
|---|---|
| $\tau^{sc}(k)$ | Delay exists between the sensor of the plant site and controller |
| $\tau^{ca}(k)$ | Delay exists between controller and actuator of the plant site |
| $K_A$ | Amplifier gain |
| $T_A$ | Amplifier time constant |
| $K_G$ | Generator gain |
| $T_A$ | Generator time constant |
| $K_R$ | Rectifier gain |
| $T_R$ | Rectifier time constant |
| $K_E$ | Exciter gain |
| $T_E$ | Exciter time constant |
| $\tau_1$ | Lower bound of the delay |
| $\tau_2$ | Upper bound of the delay |
| $K_P$ | Proportional gain of PI controller |
| $K_I$ | Integral gain of PI controller |

The notations used in the paper are given in Table 1.

## 2. Network control system

The block diagram of the NCS is presented in Figure 1. Time-delay in NCS causes the signal to shift rightwards in time domain axis and overall, affects system stability and leads to an unstable system.

The open communication channel is a medium for all signals. In the network control system, the plant output signals and control inputs are delayed under the open communication channel. Therefore, the delay phenomenon comes into the picture in both feedback path as well as in the forward path. This delay causes the instability of the system and affects the performance of NCS.

Figure 2 shows model of NCS in the state-space framework. The state variable $x(t)$ is observed, discretized and transmitted to an open communication channel. The signal from the plant to the controller is delayed by $\tau^{sc}(k)$, Control signal $u(t)$ is transmitted to plant with delay $\tau^{ca}(k)$.

The state feedback control input is given by $u(t) = Kx(kT)$. The state equation of the delayed system with time-varying delay is given as follows:

$$\dot{x}(t) = Ax(t) + A_d x(t - \tau(t)), \tag{1}$$

$$x(t) = \Phi(t),\ t \in [-\tau_2,\ 0], \tag{1a}$$

$$\tau_1 \le \tau(t) \le \tau_2, \tag{1b}$$

$$\dot{\tau}(t) \le \mu, \tag{1c}$$

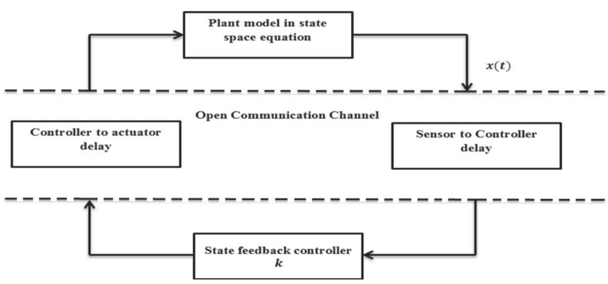


**Figure 1.** Block diagram of the NCS.

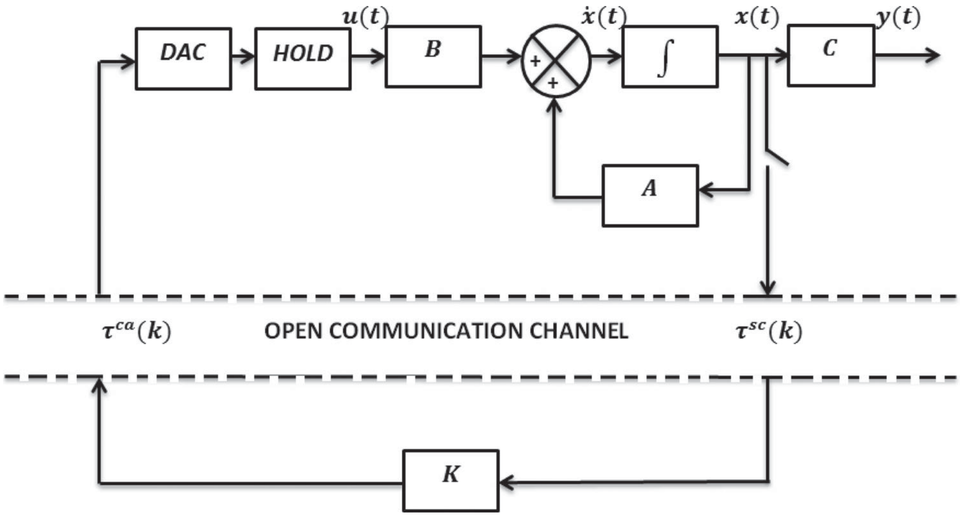


**Figure 2.** State-space model of NCS.

where $x(t) \in \mathbb{R}^n$ is the state vector, $A \in \mathbb{R}^{n\times n}$ and $A_d \in \mathbb{R}^{n\times n}$ represent system matrices, $\Phi(t)$ is the initial condition expressed in $t \in [-\tau_2, 0]$, $\tau_2$ is delay margin, $\tau(t)$ is a time-varying delay, and $\mu$ represent delay derivative. Delay derivative is assumed to be $\mu \leq 1$.

In the LKF analysis, a positive definite energy functional $E(t)$ is constructed and its derivative is expressed as follows:

$$\dot{E}(t) \leq \xi^T(t)\, \pi\, \xi(t), \tag{2}$$

where $\xi(t)$ is any augmented state vector and $\pi$ is a symmetric matrix.

Optimization algorithms for computing interval time-varying delay are given by

$$\max[\tau_1, \tau_2], \tag{3}$$

subject to $\pi < 0$.

Before presenting the main results let us recall important lemmas.

**Lemma 1:** [20]: *For given matrix $R = R^T > 0$ and for any differential signal $x$ in $[t-\tau_1, t]$, the following inequality holds (Wirtinger's Inequality)*

$$-\tau_1 \int_{t-\tau_1}^{t} \dot{x}^T(s)R\dot{x}(s)ds \leq \begin{bmatrix} x(t) \\ x(t-\tau_1) \\ \frac{1}{\tau_1}\int_{t-\tau_1}^{t} x(s)ds \end{bmatrix}^T \times \begin{bmatrix} -R - \frac{\pi^2}{4}R & R - \frac{\pi^2}{4}R & \frac{\pi^2}{2}R \\ * & -R - \frac{\pi^2}{4}R & \frac{\pi^2}{2}R \\ * & * & -\pi^2 R \end{bmatrix} \times \begin{bmatrix} x(t) \\ x(t-\tau_1) \\ \frac{1}{\tau_1}\int_{t-\tau_1}^{t} x(s)ds \end{bmatrix}.$$

**Lemma 2:** [30]: *when $\tau_1 \leq \tau(t) \leq \tau_2$, where $\tau(.)$: $W_+(\text{or } Z_+) \rightarrow W_+(\text{or } Z_+)$. Then, for any $W = W^T > 0$, following integral inequality holds*

$$-\int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s)W\dot{x}(s)ds \leq \delta^T(t)[(\tau_2\tau(t))UW^{-1}U^T + (\tau(t)-\tau_1) \times VW^{-1}V^T + \varphi + \varphi^T]\delta(t),$$

*where $\varphi = \begin{bmatrix} V & -V+U & -V \end{bmatrix}$, $\delta(t) = [x^T(t-\tau_1)\ \ x^T(t-\tau(t))\ \ x^T(t-\tau_2)]^T$ and $U = [U_1^T\ \ U_2^T\ \ U_3^T]^T$, $V = \begin{bmatrix} V_1^T & V_2^T & V_3^T \end{bmatrix}^T$ are free-weighting matrices of appropriate dimension.*

**Lemma 3:** [21]: *when $\tau_1 \leq \tau(t) \leq \tau_2$, where $\tau(.)$: $\mathbb{R}_+(\text{or } Z_+) \rightarrow \mathbb{R}_+(\text{or } Z_+)$. Therefore, any constant matrices $\Gamma_1$, $\Gamma_2$, and $\Gamma$ with proper dimensions, the following matrices inequality*

$$\Gamma + (\tau(t)-\tau_1)\Gamma_1 + (\tau_2 - \tau(t))\Gamma_2 < 0,$$

*holds, if and only if*

$$\Gamma + (\tau_2 - \tau_1)\Gamma_1 < 0,$$
$$\Gamma + (\tau_2 - \tau_1)\Gamma_2 < 0.$$

## 3. Main results

The DDS criterion for the system given by Equation (1) is stated below:

**Theorem 1:** *For given positive scalars $\tau_1, \tau_2$ the system represented by Equation (1) is stable if there exist real symmetric matrices $P_1 > 0$, $P_3 > 0$, $Q_i > 0 (i = 1, 2, 3)$,*

*$R_j > 0 (j = 1, 2)$ and $P_2$ of appropriate dimensions such that the following LMIs holds true:*

$$\begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix} > 0, \tag{4}$$

$$\pi(i) = \begin{bmatrix} A_{11} & A_{12} & A_{13} & 0 & A_{15} & 0 \\ * & A_{22} & A_{23} & A_{24} & A_{25} & A_{26}^i \\ * & * & A_{33} & A_{34} & A_{35} & A_{36}^i \\ * & * & * & A_{44} & 0 & A_{46}^i \\ * & * & * & * & A_{55} & 0 \\ * & * & * & * & * & A_{66} \end{bmatrix} < 0, \tag{5}$$

*For $i = 1, 2$ with*

$$A_{11} = P_1 A + A^T P_1 + P_2 + P_2^T + \sum_{i=1}^{3} Q_i - R_1 - \frac{\pi^2}{4} R_1 + A^T(\tau_1^2 R_1 + \tau_{12} R_2)A, \tag{6}$$

*where*

$$\tau_{12} = \tau_2 - \tau_1 \tag{6a}$$

$$A_{12} = -P_2 + R_1 - \frac{\pi^2}{4} R_1, \tag{6b}$$

$$A_{13} = P_1 A_d + A^T(\tau_1^2 R_1 + \tau_{12} R_2) A_d, \tag{6c}$$

$$A_{15} = \tau_1(A^T P_2 + P_3) + \frac{\pi^2}{2} R_1, \tag{6d}$$

$$A_{22} = -Q_1 - R_1 - \frac{\pi^2}{4} R_1 + V_1 + V_1^T, \tag{6e}$$

$$A_{23} = -V_1 + U_1 + V_2^T, \tag{6f}$$

$$A_{24} = -U_1 + V_3^T, \tag{6g}$$

$$A_{25} = -P_3 \tau_1 + \frac{\pi^2}{2} R_1, \tag{6h}$$

$$A_{26}^1 = \tau_{12} U_1, \tag{6i}$$

$$A_{26}^2 = \tau_{12} V_1, \tag{6j}$$

$$A_{33} = -(1-\mu) Q_3 + A_d^T(\tau_1^2 R_1 + \tau_{12} R_2) A_d - V_2 - V_2^T + U_2 + U_2^T, \tag{6k}$$

$$A_{34} = -U_2 - V_3^T + U_3^T, \tag{6l}$$

$$A_{35} = \tau_1(A_d^T P_2), \tag{6m}$$

$$A_{36}^1 = \tau_{12} U_2, \tag{6n}$$

$$A_{36}^2 = \tau_{12} V_2, \tag{6o}$$

$$A_{44} = -U_3 - U_3^T - Q_2, \tag{6p}$$

$$A_{46}^1 = \tau_{12} U_3, \tag{6q}$$

$$A_{46}^2 = \tau_{12} V_3, \tag{6r}$$

$$A_{55} = -\pi^2 R_1, \tag{6s}$$

$$A_{66} = -\tau_{12} R_2. \tag{6t}$$

***Proof:*** Construct the following LKF

$$E(x_t) = \sum_{i=1}^{3} E_i(x_t), \tag{7}$$

where

$$E_1(x_t) = \begin{bmatrix} x(t) \\ \int_{t-\tau_1}^{t} x(\alpha) d\alpha \end{bmatrix}^T \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix} \times \begin{bmatrix} x(t) \\ \int_{t-\tau_1}^{t} x(\alpha) d\alpha \end{bmatrix}, \tag{8}$$

$$E_2(x_t) = \int_{t-\tau_1}^{t} x^T(\alpha) Q_1 x(\alpha) d\alpha + \int_{t-\tau_2}^{t} x^T(\alpha) Q_2 x(\alpha) d\alpha + \int_{t-\tau(t)}^{t} x^T(\alpha) Q_3 x(\alpha) d\alpha, \tag{9}$$

$$E_3(x_t) = \tau_1 \int_{-\tau_1}^{0} \int_{t+\theta}^{t} \dot{x}^T(\alpha) R_1 \dot{x}(\alpha) d\alpha d\theta + \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^{t} \dot{x}^T(\alpha) R_2 \dot{x}(\alpha) d\alpha d\theta. \tag{10}$$

■

The time derivative of functional $E_i(x_t), i = 1$ *to* $3$ along the trajectory of the equation is given by

$$\begin{aligned} \dot{E}_1(x_t) &= x^T(t)(P_1 A + A^T P_1 + P_2 + P_2^T) x(t) \\ &+ 2x^T(t) P_1 A_d x(t-\tau(t)) \\ &+ 2(x^T(t) - x^T(t-\tau_1)) P_3 \int_{t-\tau_1}^{t} x(\alpha) d\alpha \\ &- 2x^T(t) P_2 x(t-\tau_1) \\ &+ 2[Ax(t) + A_d x(t-\tau(t))]^T P_2 \\ &\times \int_{t-\tau_1}^{t} x(\alpha) d\alpha, \end{aligned} \tag{11}$$

$$\begin{aligned} \dot{E}_2(x_t) &\le x^T(t) \left( \sum_{i=1}^{3} Q_i \right) x(t) - x^T(t-\tau_1) Q_1 \\ &\times x(t-\tau_1) - x^T(t-\tau_2) Q_2 x(t-\tau_2) \\ &- (1-\mu) x^T(t-\tau(t)) Q_3 x(t-\tau(t)), \end{aligned} \tag{12}$$

$$\begin{aligned} \dot{E}_3(x_t) &= \dot{x}^T(t)(\tau_1^2 R_1 + \tau_{12} R_2) \dot{x}(t) \\ &- \tau_1 \int_{t-\tau_1}^{t} \dot{x}^T(\alpha) R_1 \dot{x}(\alpha) d\alpha \end{aligned}$$

$$-\int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(\alpha)R_2\dot{x}(\alpha)d\alpha, \qquad (13)$$

now, applying Lemma 1, and Lemma 2 to Equation (13) to handle the integral terms $-\tau_1 \int_{t-\tau_1}^{t} \dot{x}^T(\alpha)R_1\dot{x}(\alpha)d\alpha$ and $-\int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(\alpha)R_2\dot{x}(\alpha)d\alpha$ and expressing the derivative of LKF as follows

$$\dot{E}(t) = \sum_{i=1}^{3} \dot{E}_i(x_t) \le \xi^T(t)\,\pi\,\xi(t), \qquad (14)$$

where

$$\xi(t) = \Big[x^T(t) \quad x^T(t-\tau_1) \quad x^T(t-\tau(t)) \quad x^T(t-\tau_2) \quad \frac{1}{\tau_1}\int_{t-\tau_1}^{t} x^T(\alpha)d\alpha\Big]^T,$$

by applying the Schur complement [31] and Lemma 3 to Equation (14), the LMIs given by Equation (5) stated in Theorem 1 is obtained. The maximum value of delay margin for which $\pi < 0$ holds will be the maximum allowable margin for the time-varying delays. If $\pi < 0$ then $\dot{E}(t) < 0$ the delayed system given by Equation (1) is absolutely stable for $\tau_1 \le \tau(t) \le \tau_2$. This completes the proof of Theorem 1.

### 3.1. Numerical Example

The nonlinear system given by Equations (1) and (1a) is reduced and denoted as linear time-delay system by the equation $\dot{x}(t) = Ax(t) + Bx(t-\tau(t))$ with the following parameters:

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix} \qquad (15)$$

To analyse the stability of the system given by Equation (15) with unknown delay derivative ($\mu$) has been reported in many literatures. Table 2 gives the upper bound for different values of lower bound of the time-varying delay. Delay margin obtained by solving Theorem 1 are compared with recently reported results [9–14]. Table 3 gives delay margin such that information on delay derivative is assumed to be known [9–14]. Table 4 gives the upper bound for different values of the delay derivative.

**Table 2.** Comparison of $\tau_2$ for different values of $\tau_1$.

| Method | $\tau_1 = 1$ | $\tau_1 = 2$ | $\tau_1 = 3$ | $\tau_1 = 4$ |
|---|---|---|---|---|
| [9] | 1.64 | 2.39 | 3.20 | 4.06 |
| [10] | 1.74 | 2.43 | 3.22 | 4.06 |
| [11] | 1.80 | 2.52 | 3.33 | 4.18 |
| [12] | 1.87 | 2.50 | 3.25 | 4.07 |
| [13] | 2.06 | 2.61 | 3.31 | 4.09 |
| [14] | 2.11 | 2.69 | 3.44 | 4.25 |
| Theorem 1 | 2.12 | 2.73 | 3.47 | 4.27 |

**Table 4.** Upper delay bound $\tau_2$ for $\tau_1 = 0.1$ with different values of $\mu$.

| Method | $\mu = 0$ | $\mu = 0.3$ | $\mu = 0.5$ | $\mu = 0.9$ |
|---|---|---|---|---|
| [14] | 0.243 | 0.231 | 0.226 | 0.225 |
| Theorem 1 | 4.548 | 2.860 | 2.372 | 1.877 |

## 4. Model of the GES

In GES, the generator output voltage is sensed, sampled and transmitted through a communication network. At the control centre, it has been compared with a reference voltage. The error signal is amplified and fed to the exciter field winding to change the field current for maintaining the terminal voltage and reactive power. Let us consider that the terminal voltage is reduced due to the switching of the large inductive load. Error changes concern with voltage changes. The controller processes this error and brings back the terminal voltage to the rated value. The components of GES are generator, exciter, sensor, controller and amplifier. The block diagram is shown in Figure 3.

The delay in forward path ($\tau_1(t)$) and delay in feedback path ($\tau_2(t)$) are summed into a single component ($\tau_1(t) + \tau_2(t) = \tau(t)$). The model of GES is represented in the state-space framework. The state vector

**Table 3.** Comparison of $\tau_2$ for different values of $\tau_1$ and $\mu$.

| $\mu$ | Method | $\tau_1 = 1$ | $\tau_1 = 2$ | $\tau_1 = 3$ | $\tau_1 = 4$ | $\tau_1 = 5$ |
|---|---|---|---|---|---|---|
| 0.3 | [10] | 2.6335 | 2.6615 | 3.2234 | 4.0643 | - |
| | [12] | 2.6490 | 2.6972 | 3.2591 | 4.0744 | - |
| | [14] | 3.0403 | 3.1292 | 3.4438 | 4.2596 | 5.1127 |
| | Theorem 1 | 3.0781 | 3.1452 | 3.4760 | 4.2780 | 5.1196 |
| 0.5 | [10] | 2.0764 | 2.4328 | 3.2234 | 4.0643 | - |
| | [12] | 2.1276 | 2.5043 | 3.2591 | 4.0744 | - |
| | [14] | 2.3672 | 2.6982 | 3.4438 | 4.2596 | 5.1127 |
| | Theorem 1 | 2.4623 | 2.7384 | 3.4760 | 4.2780 | 5.1196 |
| 0.9 | [10] | 1.7424 | 2.4328 | 3.2234 | 4.0643 | - |
| | [12] | 1.8737 | 2.5048 | 3.2591 | 4.0744 | - |
| | [14] | 2.1112 | 2.6982 | 3.4438 | 4.2596 | 5.1197 |
| | Theorem 1 | 2.1290 | 2.7384 | 3.4760 | 4.2780 | 5.1196 |
| $> 1$ | [10] | 1.7424 | 2.4328 | 3.2234 | 4.0643 | - |
| | [12] | 1.8737 | 2.5048 | 3.2591 | 4.0744 | - |
| | [14] | 2.1112 | 2.6982 | 3.4438 | 4.2596 | 5.1127 |
| | Theorem 1 | 2.1289 | 2.7384 | 3.4760 | 4.2780 | 5.1196 |

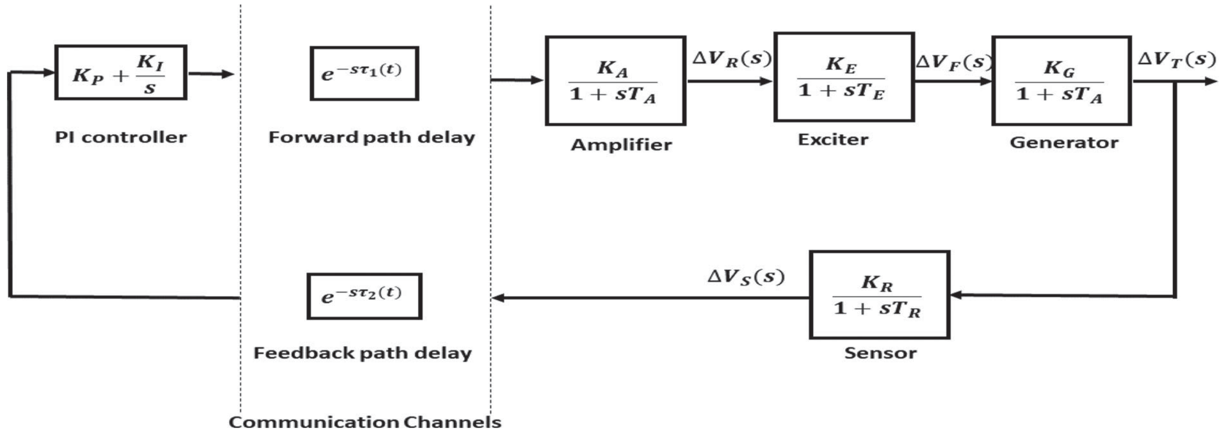


**Figure 3.** Block diagram of the GES

of GES is $x(t) = [\Delta v_R(t) \quad \Delta v_F(t) \quad \Delta v_T(t) \quad \Delta v_S(t) \quad \int \Delta v_S(t)dt]^T$.

The state-space equation of GES is given by

$$\dot{x}_1(t) = -\frac{1}{T_A}x_1(t) - \frac{K_P K_A}{T_A}x_4(t-\tau(t)) - \frac{K_I K_A}{T_A}x_5(t-\tau(t)), \quad (16)$$

$$\dot{x}_2(t) = \frac{K_E}{T_E}x_1(t) - \frac{1}{T_E}x_2(t), \quad (17)$$

$$\dot{x}_3(t) = \frac{K_G}{T_G}x_2(t) - \frac{1}{T_G}x_3(t), \quad (18)$$

$$\dot{x}_4(t) = \frac{K_R}{T_R}x_3(t) - \frac{1}{T_R}x_4(t), \quad (19)$$

$$\dot{x}_5(t) = x_4(t), \quad (20)$$

the above equations are represented in standard form given by Equation (1) as follows:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \\ \dot{x}_5(t) \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_A} & 0 & 0 & 0 & 0 \\ \frac{K_E}{T_E} & -\frac{1}{T_E} & 0 & 0 & 0 \\ 0 & \frac{K_G}{T_G} & -\frac{1}{T_G} & 0 & 0 \\ 0 & 0 & \frac{K_R}{T_R} & -\frac{1}{T_R} & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & -\frac{K_P K_A}{T_A} & -\frac{K_I K_A}{T_A} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} x_1(t-\tau(t)) \\ x_2(t-\tau(t)) \\ x_3(t-\tau(t)) \\ x_4(t-\tau(t)) \\ x_5(t-\tau(t)) \end{bmatrix}. \quad (21)$$

The parameters of GES are $K_A = 5$, $K_E = K_G = K_R = T_G = 1$, $T_A = 0.1$, $T_R = 0.05$, $T_E = 0.4$. [7].

### 4.1. Controller capability curve of GES

The controller capability curve of GES is shown in Figure 4. It shows for delay-free GES, $K_P$ ranges from

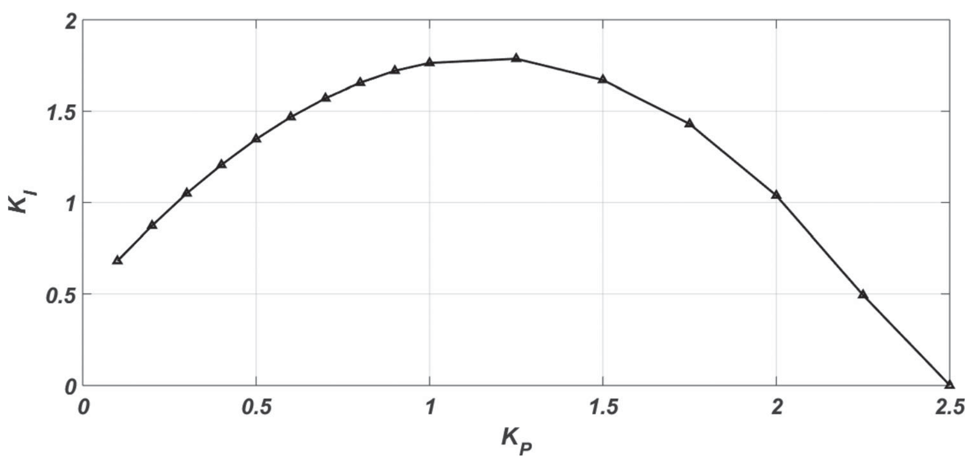


**Figure 4.** Controller capability curve for $K_P$ vs $K_I$.

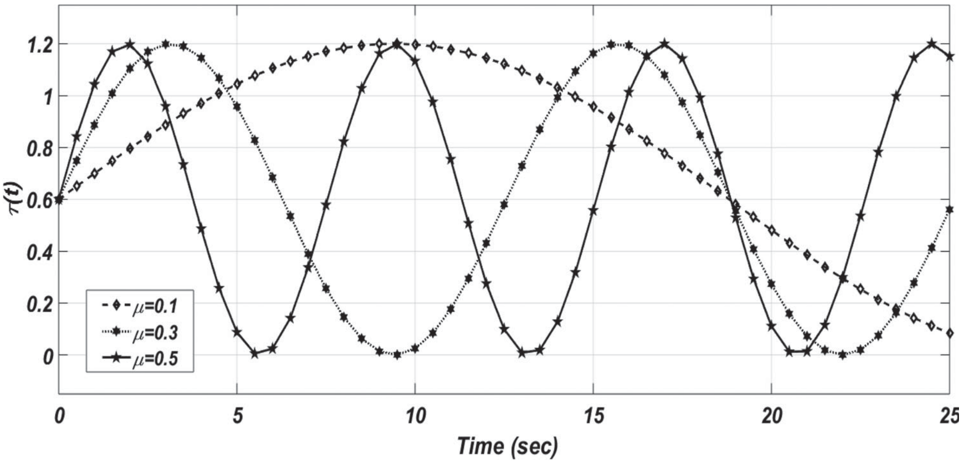


**Figure 5.** Time-varying delay.

0 to 2.5 and $K_I$ ranges from 0.471 to 1.786. To carry-out simulation studies, interval time-varying delay is assumed to be sinusoidal and varying between zero to upper bound of time-delay as shown in Figure 5.

### 4.2. DDS analysis of GES with interval time-varying delays

The proposed DDS criterion is used to determine the delay margin of GES. In a delay-dependent system when $\tau(t) < \tau_2$, the system remains stable. For $\tau(t) > \tau_2$, the system becomes unstable. Thus, the delay margin determines the stability of GES. Theorem 1 provides sufficient conditions for the stability of GES with time-varying delay or interval time-varying delay. The delay margin is obtained by solving Theorem 1 using feasp command provided in MATLAB. These results are presented in Table 5–7. The stable regions are given in Figures 6 and 7.

Table 5 gives a delay margin for different sets of the PI controller gain. The delay margin decreases for an increase in the value of $K_P$ and $K_I$.

Figure 6 shows that delay margin increases for an increase in $K_P$ value from 0 to 0.2 then, further increase in $K_P$ decreases the delay margin. Figure 7 shows that delay margin decreases as $K_I$ increases.

Tables 6 and 7 show that for an increase in delay derivative decreases the delay margin. These findings are used to tune the PI controller to achieve a stable performance for GES.

### 4.3. Voltage response of GES with time-varying delays

The simulation of GES is carried out using Matlab software. Using a block diagram of GES, simulation blocks are constructed in the simulink platform of the MATLAB. The voltage response of GES is plotted for the step input for different delay margins are shown in Figures 8–11.

From Table 5, controller parameters such as $K_P = 0.2$ and $K_I = 0.2$ are chosen. Time-varying delay is assumed to be sinusoidal and it varies between $0 \leq \tau(t) \leq 1.2$ with delay-derivative ($\mu = 0.1$). Figure

**Table 5.** Delay margin of GES with $\mu = 0.1$, $\tau_1 = 0$.

| $\tau_2$ | $K_P = 0.2$ | 0.5 | 0.8 | 1.0 | 1.25 | 1.4 | 1.6 | 1.8 |
|---|---|---|---|---|---|---|---|---|
| $K_I = 0.2$ | 1.0768 | 0.5081 | 0.2567 | 0.1753 | 0.1119 | 0.0852 | 0.0577 | 0.0370 |
| 0.4 | 0.4277 | 0.3725 | 0.2161 | 0.1515 | 0.0975 | 0.0740 | 0.0495 | 0.0306 |
| 0.6 | 0.1697 | 0.2551 | 0.1749 | 0.1217 | 0.0828 | 0.0626 | 0.0411 | 0.0242 |
| 0.8 | 0.0343 | 0.1619 | 0.1350 | 0.1027 | 0.0681 | 0.0512 | 0.0326 | 0.0179 |

**Table 6.** Delay margin of GES with $\mu = 0.3$, $\tau_1 = 0$.

| $\tau_2$ | $K_P = 0.2$ | 0.5 | 0.8 | 1.0 | 1.25 | 1.4 | 1.6 | 1.8 |
|---|---|---|---|---|---|---|---|---|
| $K_I = 0.2$ | 1.0359 | 0.4758 | 0.2411 | 0.1645 | 0.1056 | 0.0811 | 0.0559 | 0.0364 |
| 0.4 | 0.4172 | 0.3545 | 0.2040 | 0.1426 | 0.0928 | 0.0710 | 0.0482 | 0.0304 |
| 0.6 | 0.1669 | 0.2458 | 0.1660 | 0.1208 | 0.0795 | 0.0606 | 0.0403 | 0.0242 |
| 0.8 | 0.0343 | 0.1580 | 0.1298 | 0.0989 | 0.0660 | 0.0500 | 0.0323 | 0.0179 |

**Table 7.** Delay margin of GES with $\mu = 0.5$, $\tau_1 = 0$.

| $\tau_2$ | $K_P = 0.2$ | 0.5 | 0.8 | 1.0 | 1.25 | 1.4 | 1.6 | 1.8 |
|---|---|---|---|---|---|---|---|---|
| $K_I = 0.2$ | 1.0056 | 0.4556 | 0.2359 | 0.1631 | 0.1056 | 0.0811 | 0.0559 | 0.0364 |
| 0.4 | 0.4151 | 0.3470 | 0.2016 | 0.1424 | 0.0928 | 0.0710 | 0.0482 | 0.0304 |
| 0.6 | 0.1669 | 0.2446 | 0.1657 | 0.1208 | 0.0795 | 0.0606 | 0.0403 | 0.0242 |
| 0.8 | 0.0343 | 0.1580 | 0.1298 | 0.0988 | 0.0660 | 0.0500 | 0.0323 | 0.0179 |

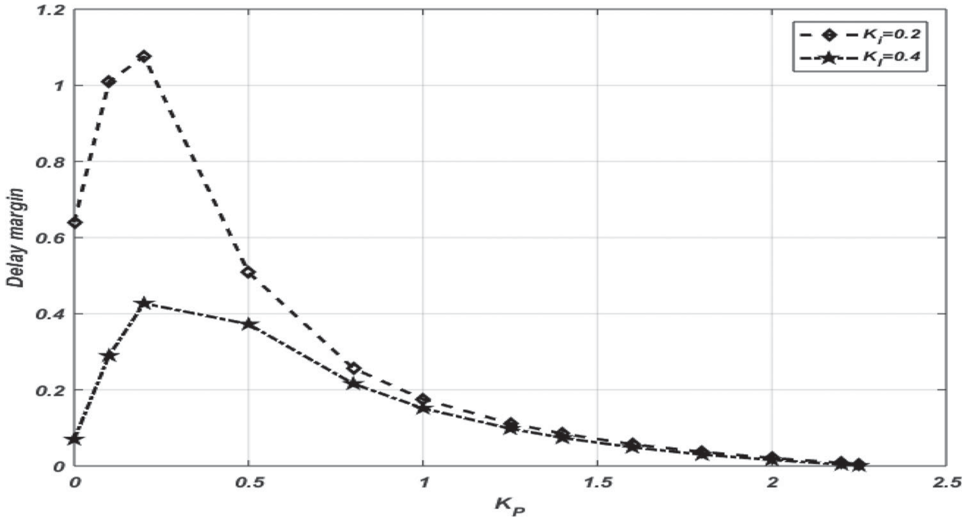


**Figure 6.** Stability region of GES ($K_P$ *vs* $\tau_2, \mu = 0.1, \tau_1 = 0$) with $K_I$ constant.

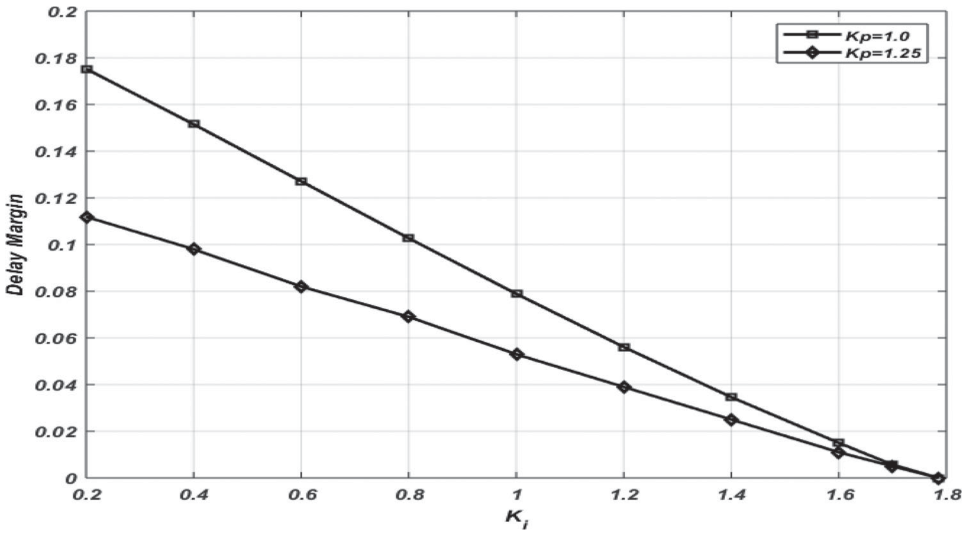


**Figure 7.** Stability region of GES ($K_I$ *vs* $\tau_2, \mu = 0.1, \tau_1 = 0$) with $K_P$ constant.

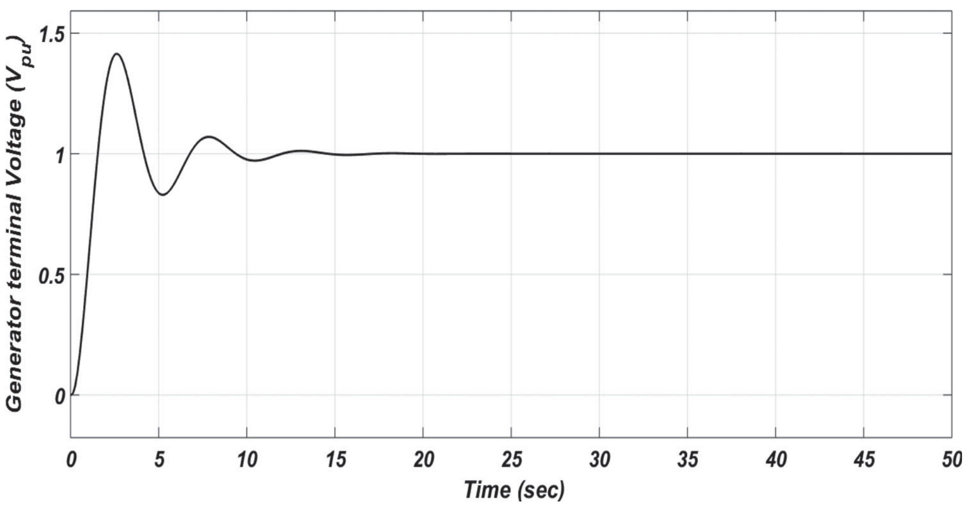


**Figure 8.** The voltage response of GES with $K_P = 0.2, K_I = 0.2, \tau_2 = 0.5s$.

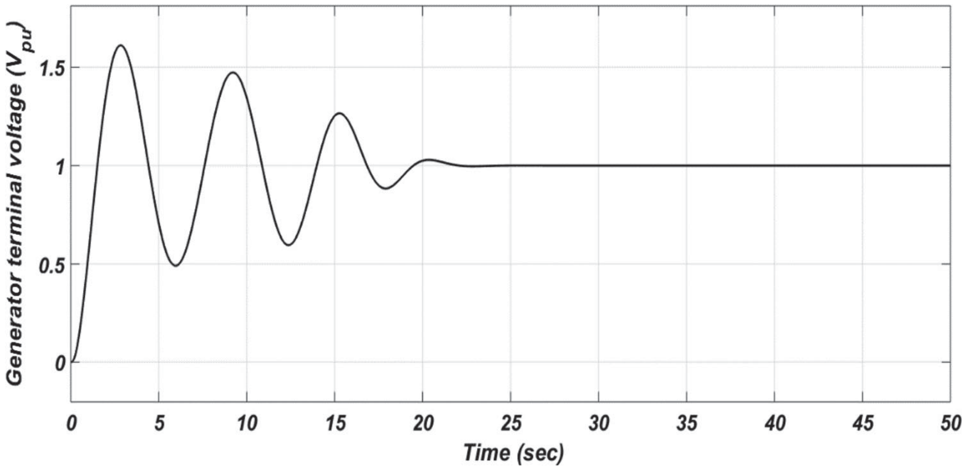


**Figure 9.** The voltage response of GES with $K_P = 0.2, K_I = 0.2, \tau_2 = 1.0768s$.

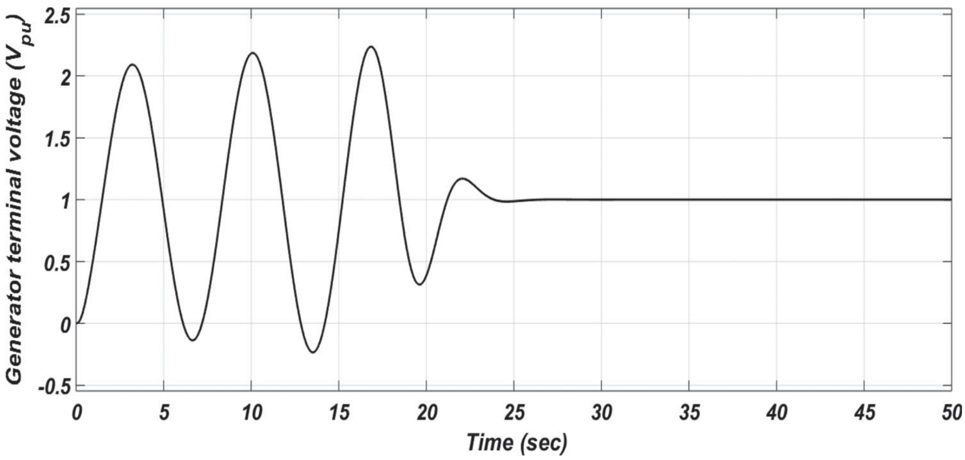


**Figure 10.** The voltage response of GES with $K_P = 0.2, K_I = 0.2, \tau_2 = 1.2s$.

8 shows the voltage response of GES for $K_P = 0.2$ and $K_I = 0.2$ with delay margin $\tau_2 = 0.5s$. The system reaches a steady state. Delay margin ($\tau_2 = 0.5$) is located far away from the delay vs gain pair in Figure 6. Hence the system is stable. Figure 9 shows that the system reaches steady state value with oscillations for an increase in delay margin ($\tau_2 = 1.0768s$). At $\tau_2 = 1.2s$ the system undergoes sustained oscillations as shown in Figure 10. Further, an increase in delay margin ($\tau_2 = 1.5s$) for the same value of the PI controller gain, the system undergoes an increase in oscillations as shown in Figure 11. After 20 s, time-varying decreases and trends

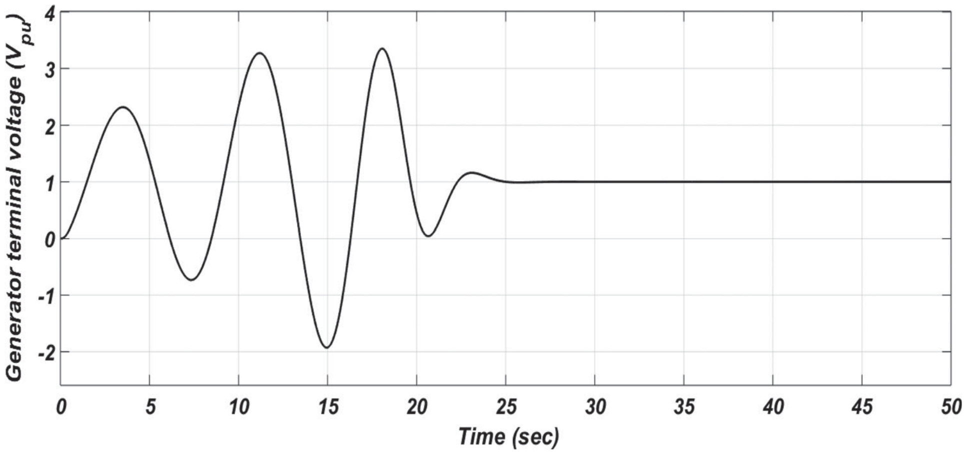


**Figure 11.** The voltage response of GES with $K_P = 0.2, K_I = 0.2, \tau_2 = 1.5s$.

towards zero as shown in Figure 5, therefore system reaches stable after 20 s. Hence it is proven that time-delays has an impact on the stability of a system. To maintain an absolute stable system, the PI controller is tuned with concern to the time-delays.

## 5. Conclusion

In this paper, the DDS analysis of network-controlled GES with the interval time-varying delay has been discussed. The chosen LKF and bounding technique for integral terms such as free-weighting matrices and Wirtinger's inequality tightly bound the integral terms on derivative of LKF. The results obtained from the proposed method for numerical example is less conservative when compared with other literature on interval time-varying delays. The proposed DDS criterion has been applied to the GES. GES stability is analysed for interval time-varying delays. The stability of a GES has been maintained asymptotically by choosing the particular value of $K_P, K_I$ with concern to time-varying delays. Thus, the proposed method provides a fine set of tuning to a network controller to maintain the system asymptotically stable with more delay margin.

## Disclosure statement

No potential conflict of interest was reported by the author(s).

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