AN INVESTIGATION INTO SHARE PRICES' CONDITIONAL HETEROSCEDASTICITY AND NON-SYMMETRICAL MODEL IN THE CONTEXT OF SOUTH AFRICA, NIGERIA, AND EGYPT

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Abstract

This paper investigates the leverage effect in African countries by applying normal and non-normal distribution densities. Furthermore, we investigate the possible opportunities for portfolio diversification in South Africa, Nigeria, and Egypt. We find that negative stock returns do not generate higher volatility in further returns than past positive returns. All three countries are subject to the ARCH effect, where past stock information (volatility) influence the current stock returns (volatility). We also find that Gaussian distribution produces a better estimate as compared to non-normal distribution. In terms of portfolio diversification, returns are also subject to the ARCH effect, however, the leverage effect does not determine that past negative returns influence the current stock returns asymmetrically.

Keywords: *volatility, GARCH models, ARCH effect, portfolio diversification, correlation, normal and non-normal distribution*

1. INTRODUCTION

In financial time series data, leptokurtosis and volatility clustering are the commonly observed phenomena that indicate the higher level of risk involved (Mandelbrot 1963). There is another measure, the leverage effect, that has acquired great attention since it inculcates that the fluctuation in security prices is inversely correlated to the fluctuation in security's volatility. Characteristically, a rising stock price is accompanied by a decline in volatility and vice versa. The term leverage effect is linked with the economic interpretation that is developed by Black (1976) and Christie (1982), showing how stock prices fall, when companies become more leveraged, due to an increase in value of their debt, relative to their value of equity. Consequently, it is a common belief that the stock of a higher leveraged firm becomes risker and more volatile. Although it is a hypothesis, however, it is generally believed in the literature that the leverage

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effect places a question mark on a statistical regularity. Moreover, it is also documented that the leverage effect is mostly asymmetric; ceteris paribus, decreases in stock prices is associated with large increases in stock volatility, as compared to declines in volatility, associated with increases in stock prices (see, e.g., Nelson, 1991; and Engle and Ng, 1993). Encountering these types of risks in financial time series data requires the use of a wide variety of models that encapsulate variances to estimate current and predict future volatility.

Engle (1982) has proposed the Autoregressive Conditional Heteroskedasticity (ARCH) model that is based on conditional time-varying variance, which considers the lagged disturbance. (It models the change in variance over time in financial time series data). The extension of the ARCH model is proposed by Bollerslev (1986), Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model, that includes the higher-order ARCH model to capture the dynamic behavior of conditional variance. Although ARCH and GARCH models encounter leptokurtosis and volatility clustering due to symmetric distribution, they fail to encapsulate leverage effect in financial time series data. To estimate the leverage effect some non-linear extensions of the GARCH model have been proposed, for instance, EGARCH (Nelson 1991), GJR (Glosten *et al.* 1993), and asymmetric power ARCH (APARCH) (Ding *et al.* 1993).

Another drawback of the GARH model is the lack of embracing the thick tail property of financial time series high frequency. Bollerslev (1987) and Baillie and Bollerslev (1989) provided the solution to this problem, by using Student's t-distribution. Furthermore, Liu and Brorsen (1995) used asymmetric stable density to capture skewness. To model kurtosis and skewness, Fernandez and Steel (1998) supported skewed Student's t-distribution that, later on, Lambert and Laurent (2000,2001) incorporated into a GARCH framework. To make GARH and EGARCH model fit for stock markets, Harris *et al.* (2004) use skewed generalized Student's t-distribution to estimate skewness and leverage effect for daily stock returns.

Despite an extensive amount of research on symmetric and asymmetric GARCH models, less attention has been paid to comparing alternative density forecast models, especially in the context of the African stock market, after Nor and Shamiri (2007) drew the comparison between different alternative density forecast models for Malaysia and Singapore. The financial time series data, especially stock returns, are mostly high-frequency data, which means that stock returns are subject to fat-tailed distribution. It is a well-known fact in finance literature that the stock returns mostly experience kurtosis higher than three (Simkowitz and Beedles 1980; Kon 1984). Kurtosis higher than three ascertains that stock returns are characterized by more extreme returns than the normal distribution. Mittnik and Paolella (2001) argued that fat-tailed distribution density is important to model the daily exchange rate of emerging countries' currencies against the dollar.

In this study, we fill the gap by using normal distribution density (Gaussian) and non-normal distribution densities (Student's t-distribution and Generalized Error Distribution - GED) to evaluate the leverage effect in African stock markets. With this aim, we will explore whether a change in distributional densities under ARCH, GARCH, and TGARCH models leads to any substantial change in volatility and leverage effect in African countries of South Africa, Nigeria, and Egypt. We will also explore the portfolio diversification opportunities in African countries and estimate the forecast. We employ the volatility model to test its ability to forecast and capture the volatility clustering, leptokurtosis, and impact of negative vs positive stock returns (leverage effect) in financial time series data. We investigate the forecasting ability of the ARCH, GARCH, and TGARCH models, with the use of normal and non-normal distributional densities.

The article is structured as follows: Section 2 reviews the previous studies, Section 3 highlights the data and economic modeling ARCH, GARCH, and TGARCH in African stock markets, Section 4 discusses empirical results whereas Section 5 concludes the paper.

2. LITERATURE REVIEW

Engle (1982) has recommended the ARCH model based on conditional time-varying variance, which considers the lagged disturbance. (It models the change in variance over time in financial time series data). The GARCH extension of the ARCH model is proposed by Bollerslev (1986), by including the higher-order ARCH model to estimate the dynamic performance of conditional variance. Although the ARCH and GARCH models encounter leptokurtosis and volatility clustering, due to symmetric distribution, they, however, fail to encapsulate the leverage effect in financial time series data. To estimate the leverage effect, some non-linear extensions of the GARCH model have been proposed, for instance, EGARCH (Nelson 1991), GJR (Glosten *et al.* 1993), and asymmetric power ARCH (APARCH) (Ding *et al.* 1993).

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The focus is mostly given to the application of heteroscedastic models to financial data. Several empirical studies have addressed the volatility of stock markets through the ARCH and GARCH models. It is important to analyze the volatility for investors to measure and manage the risk associated with stock returns, which is, in turn, beneficial for pricing the capital assets, financial instruments, and selecting stock for portfolios. For instance, Ahmed and Suliman (2011) and Naimy (2013) estimated the symmetric and the asymmetric model, from the GARCH family, to study stock return volatility. Ahmed and Suliman (2011) study the Sudan stock market, whereas Kalu (2010) analyses volatility in the Nigerian stock exchange. Naimy (2013) models the volatility of the returns, by using the GARCH (1,1) model and compares them with the results of the exponential weighted moving average (EWMA). Likewise, Shamiri and Isa (2009) show the comparison between the conventional GARCH models and the asymmetric non-linear NAGARCH models in the Malaysian stock market. Similarly, the Nairobi securities exchange is studied by Wagala et al. (2012) using the ARCH model[1].

Drachal (2017) carried out a cross-country study in Europe to estimate the effect of leverage on weekly stock returns, by using the ARCH-M, T-GARCH, EGARCH, GJR-GARCH, and APARCH models. Results favor the suitability of the normal GED (generalized error distribution). Certain countries, such as Latvia, Bulgaria, Montenegro, and Lithuania, are subject to the significant negative risk-returns association, whereas the positive association is found in Estonia, which is contrary to the normal expectations. On a practical ground, these findings implicate the consistency of the CAPM model in the Estonian market only. On the other hand, countries, with a negative trade-off between risk and returns, do not show higher returns for the riskier investments. Drachal (2017) associate the leverage effect with the investors' behavioral aspects because investors are more influenced by the bad news on the market, as compared to good ones.

Present literature fails to explore the leverage effect under normal and non-normal distribution densities in African stock markets. This study fills the gaps, by exploring how the TGARH model performs through normal and non-normal distribution densities.

3. DATA AND METHODOLOGY

This section discusses the data and the methodological approach to estimate the leverage effect, using the TGARCH model, and compares the three different densities under GARCH and TGARCH models.

3.1. Data and models

We selected three stock market indices daily data from African stock markets, namely, South Africa (JTOPI 40 index), Nigeria (NSE), and Egypt (EXG 100) from January 2015 to May 2020[2]. We compute the returns of these three indices through the following formula:

$$r_{it} = \ln\left(\frac{p_{it}}{p_{it-1}}\right) * 100.$$

In this formula, is the compounded return of stock *i* at the time *t,* is current prices of stock *i* at the time *t*, and is the previous year's price of the stock *i* at the time *t*. Table 1 provides the descriptive statistics, test for normality, and the presence of the ARCH effect. It can be observed that Nigeria experiences negative mean returns, whereas South Africa and Egypt experience positive, but very small mean returns. South Africa's stock index is risker than Nigeria and Egypt, as indicated by the unconditional standard deviation. The distribution of Nigeria is negatively distributed, as opposed to South Africa and Egypt. The skewness null hypothesis, supporting normal distribution with a zero coefficient, is rejected for all three indices. These returns exhibit fat-tail, which is confirmed by the kurtosis coefficient value of 3. The higher value of kurtosis in a fat-tailed distribution density indicates that Student's t-distribution or GED is a more accurate distribution and produces better results than Gaussian distribution. Jarque-Bera's test of normality shows that all three indices are not normally distributed at the 1% level of significance. Finally, Engle's (1982) LM test confirms the presence of the ARCH effect in all three indices. Hence, this supports the use of GARCH and TGARCH models.

[1] See also: Sharma and Vipul (2016), Maqsood et al., (2017), and Coffie (2014, 2015).

[2] Data downloaded from the investing.com website.

Table 1. Descriptive statistics of daily indices returns

	South Africa	**Nigeria**	**Egypt**
Mean	0.0000	-0.00002	0.0000
Median	0.0003	-0.0005	0.0003
Standard Deviation	0.0123	0.0102	0.0110
Max	0.0790	0.0383	0.0626
Min	-0.1045	-0.0503	-0.0735
Skewness	-0.8262	0.0366	-0.6543
Kurtosis	12.1615	5.8883	7.9565
Jarque-Bera Test	4687.89***	429.90***	1427.82***
ARCH (5), Chi2 > Prob	**0.0000**	**0.0000**	**0.0000**
Observation	1332	1332	1332

3.1.1. Measuring the volatility through ARCH, GARCH, and TGARCH models

Stock market daily returns fluctuate in response to various firm-related news and other economic exogenous events (Philip and Fransens 1988). It is also observed that large positive (negative) observations mostly appear in stock returns clusters (Gujarati 2004). Thus, linear estimation techniques (OLS) are incapable of explaining the number of important features that are common to the stock daily returns:

- *Leptokurtosis* – daily stock returns experience fat-tailed distribution.
- *Volatility clustering* – the tendency towards stock returns volatility that appears in clusters in stock returns. E.g., the large returns of stocks, of either sign, are followed by large returns, and small returns of stock, of either sign, are expected to have small returns in the following period. One of the explanations for volatility clustering is the arrival of information that creates volatility clustering in the stock returns.
- *Leverage effect* – the likelihood of volatility to increase more following the large dip in the prices, compared to the price rise of the magnitude.

3.1.2. The Autoregressive Conditional Heteroscedasticity Model (ARCH)

Constant variance (homoscedasticity) is one of the most important assumptions of the classical regression model: $var(\mu_t) = \sigma^2(\mu_t),\ \mu_t \sim N(0, \sigma^2)$ A series with changing variance over time is characterized as heteroscedastic, which is very likely in financial time series data. Such financial series require an estimator that does not assume that the error term possesses constant variance and it should also ascertain how error term variance evolves.

Another problem with the time-series financial data is volatility clustering, meaning that the period of high volatility is followed by a higher volatility period and the period of low volatility is characterized by a period of lower volatility. Using the ARCH model, the time series financial data with non-constant variance in error terms can be parameterized. It is also necessary to define a conditional variance of the error term to understand how the ARCH model works. The conditional variance of is represented by as follows:

$$\sigma_t^2 = var(\mu_t|\mu_{t-1}, \mu_{t-2} \dots) = E[(\mu_t - E(\mu_t)^2|\mu_{t-1}, \mu_{t-2}, \dots] \quad (1)$$

If $E(e_t) = 0$, then equation (1) can be expressed as:

$$\sigma_t^2 = var(\mu_t|\mu_{t-1}, \mu_{t-2} \dots) = E[(E(\mu|\mu_{t-1}, \mu_{t-2} \dots) \quad (2)$$

According to equation (2), a random variable, with zero-mean, conditional variance μ_t that is normally distributed is equal to the conditional expected value of the square of μ_t. In such a situation, the ARCH model is as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 + \mu_t^2 \quad (3)$$

Equation (5) is a part of the ARCH (1) model, which shows that conditional variance of the σ_t^2 error term is influenced by its immediate previous square root value. However, it should be noted that equation (5) only ascertains a part of the complete model, because it does not have anything to say about the conditional mean. The mean conditional equation that shows Y_t, as the dependent variable, can change over time and can take any form under the ARCH model. The full ARCH model is as follows:

$$Y_t = \beta_1 + \beta_2 x_{2t} + \beta_3 x_{3t} + \beta_4 x_{4t} + \mu_t \quad (4)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \mu_{t-1}^2, \quad (5)$$

where $\mu_t \sim N(0, \sigma^2)$

Equations (4) and (5) can also be expressed in a general form, where the variance of an error term is influenced by k lags of square errors. This type of model is called ARCH(k):

$$Y_t = \beta_1 + \beta_2 x_{2t} + \beta_3 x_{3t} + \beta_4 x_{4t} + \mu_t \quad (6)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \mu_{t-1}^2 + \alpha_2 \mu_{t-2}^2 + \dots + \alpha_k \mu_{t-k}^2, \quad (7)$$

where $\mu_t \sim N(0, \sigma^2)$

σ_t^2 is a conditional variance, with a positive value[3], which means that the variance regression must produce positive coefficients, e.g. $\alpha_i \geq 0, (\forall) i = 0,1,2 \dots . k$ GARCH is the extension of the ARCH(k) model.

3.1.3. *The Generalized Introgressive Conditional Heteroscedastic Model (GARCH)*

The GARCH model, created by Bollerslev (1986) and Taylor (1986) estimates conditional variance that is influenced only by its previous lagged values. The following equation is an example of conditional variance:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \mu_{t-1}^2 + \beta \sigma_{t-1}^2. \quad (8)$$

Equation (8) is a GARCH(1,1) model, where $\alpha_1 \mu_{t-1}^2$ expresses the information of volatility about the previous period, while the variance during the period is expressed by $\beta \sigma_{t-1}^2$. GARCH (1/1) model can also be written in a GARCH (k, p) form, where a conditional variance is influenced by k lags of squared errors and p lags of conditional variances:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \mu_{t-1}^2 + \alpha_2 \mu_{t-2}^2 + \dots + \alpha_k \mu_{t-k}^2 + \beta_1 \sigma_{t-1}^2 +$$
$$+ \beta_2 \sigma_{t-2}^2 + \dots + \beta_p \sigma_{t-p}^2. \quad (9)$$

Equation (9) can be rearranged as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{k} \alpha_i \mu_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \quad (10)$$

GARH (1,1) model is, in most cases, sufficient to estimate the evolution of volatility, with the GARH (1,1) model is as acceptable as the ARCH (2), while the GARCH (k, p) is as acceptable as the ARCH ($k + p$) model (Gujarati 2004).

[3] A negative variance at any time is meaningless.

3.1.4. The Threshold Generalized Introgressive Conditional Heteroscedastic Model (TGARCH)

The Threshold GARCH (TGARCH) model was introduced by Glosten *et al.* (1993), being also called GJR-GARCH, after its proponents. This model is similar to the simple GARCH model but adds an extra ARCH term which is restricted upon the direction of the past information. It is specified as follows.

$$\sigma_t^2 = \alpha_0 + \beta_1\sigma_{t-1}^2 + \alpha_1\mu_{t-1}^2 + \lambda_1\mu_{t-1}^2 d_{t-1}, \tag{11}$$

$$d_t = \begin{cases} 1 & \mu_t < 0 \text{ (bad news)} \\ 0 & \mu_t \leq 0 \text{ (good news)} \end{cases}.$$

In equation 11, estimates the leverage effect and is the dummy variable with a value of 1, if is negative. If stock returns are characterized by the leverage effect, the value of should be negatively significant.

3.2. Density distributions

The GARCH model uses Maximum Likelihood Estimation (MLE), which assumes that error distribution is Gaussian (normally distributed)., Nevertheless, in financial literature, there is evidence that error is subject to non-normal distribution densities (Nelson 1991). It is of utmost importance to select the most appropriate distribution for error terms during volatility modeling, as it reduces the problem, caused by the skewness and kurtosis, due to the conditional heteroscedasticity in the residuals. Hence, our study employs all three forms of distributions, Gaussian, Students' t-distribution, and Generalised Error Distribution, and compares the results.

4. EMPIRICAL RESULTS AND DISCUSSION

This section interprets the empirical results and provides insight into the estimations, using the GARCH and the TGARCH models under three different distribution densities.

Table 2. Estimated statistics-comparative distribution density Threshold-GARCH model

	South Africa			Nigeria			Egypt		
	Gaussian	**Student's t-distribution**	**GED**	**Gaussian**	**Student's t-distribution**	**GED**	**Gaussian**	**Student's t-distribution**	**GED**
α_0	0.000*** (0.007)	0.000*** (0.011)	0.000*** (0.014)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)
β_1	0.878*** (0.000)	0.874*** (0.000)	0.874*** (0.000)	0.652*** (0.000)	0.556*** (0.000)	0.597*** (0.000)	0.671*** (0.000)	0.658*** (0.000)	0.661*** (0.000)
α_1	0.247** (0.068)	0.021 (0.184)	0.021 (0.175)	0.223*** (0.000)	0.322*** (0.000)	0.261*** (0.000)	0.126*** (0.000)	0.123*** (0.000)	0.121*** (0.000)
λ_1	0.178*** (0.000)	0.194*** (0.000)	0.189*** (0.000)	0.036 (0.406)	0.157 (0.149)	0.083 (0.343)	0.272*** (0.000)	0.258*** (0.000)	0.262*** (0.000)
AIC	-8420.81	-8429.31	-8434.01	-8108.9	-8246.0	-8256.10	-8412.92	-8480.84	-8489.64
BIC	-8394.83	-8398.15	-8399.84	-8083.30	-8215.27	-8225.35	-8387.04	-8449.79	-8458.60

Table 2 reports the statistics, related to the use of the asymmetric TGARCH model, by using normal Gaussian, with the normal distribution density, the Student's t-distribution, and the generalized error distribution (GED) for three indices South Africa, Nigeria, and Egypt, respectively. Table 2 also provides a diagnostic test, by estimating the values of AIC and BIC (Akaike's information criterion and Bayesian information criterion), to compare the results of the TGARCH model, under normal and non-normal distribution densities. The evidence shows that the ARCH effect is significant at the 1% level across Nigeria and Egypt under normal and non-normal distribution density. However, it is insignificant in South Africa for non-normal distribution only. This implies that past stock returns information can influence the current stock returns across all three countries. Similarly, the coefficient of the GARCH effect is also significant at the 1% level for all three countries, under normal and non-normal distribution densities. Positive significant states that the past volatility of stock returns influences the present-day volatility of stock returns.

Moreover, the impact of news is significant in South Africa and Egypt. However, the positive sign indicates that previous day negative stock returns do not generate greater volatility in stock returns than previous day positive stock returns. On the other hand, for Nigeria for normal and abnormal distribution densities establishes the insignificance of the leverage effect in Nigeria. The leverage effect is more profound in the case of South Africa under abnormal distribution densities, such as Student's t-distribution and the generalized error distribution. However, this magnitude further falls, when applying normal (Gaussian) distribution. Thus, results indicate that South Africa and Egypt experience the leverage effect, but bad news does not generate higher volatility than good news. The significance of the leverage effect is the same, regardless of distributional densities across South Africa and Egypt. However, Nigeria is not subject to any leveraging effect.

4.1. Portfolio diversification: Threshold GARCH model

In addition, we investigate the portfolio diversification opportunities in South Africa, Nigeria, and Egypt, by constructing a portfolio with weights of 50% in the South African stock market, 25% in the Nigerian stock market, and 25% in the Egypt stock market. Table 3 shows the Pearson correlation among the three indices. The correlation coefficient is small and does not indicate the perfect correlation among the three indices, which is suitable for portfolio diversification.

Table 3. Pearson correlations

	South Africa	Nigeria	Egypt
South Africa	1.0000		
Nigeria	0.0776	1.0000	
Egypt	0.0178	0.0213	1.0000

Table 4 provides descriptive statistics for the portfolio, constructed based on three indices. It can be observed that the mean returns of the portfolio are lower than the indices of South Africa and Egypt, but greater than those of Nigeria, whereas the portfolio standard deviation is smaller than the individual standard deviation of each index. The skewness and kurtosis show that the portfolio returns distribution is negatively skewed and leptokurtic (fat-tailed). The probability value of the LM test is lower than 0.05 for the ARCH effect, with a lag of 5. Hence, Table 4 concludes the presence of the ARCH effect in the portfolio series.

Table 4. Summary statistics

Descriptive	Portfolio
Mean	-0.0001
Median	0.0003
Standard Deviation	0.0073
Skewness	-0.9286
Kurtosis	10.919
ARCH (5), Prob > Chi^2	0.0000

Table 5 shows ARCH (1,1), GARCH (1,1) and TGARCH (1,1) results. These models show the internal effect of the portfolio return time series. The squared error coefficients and conditional variance are statistically significant at the 1% level of significance, respectively. As comparable to Table 2, the coefficients of ARCH and GARCH in Table 5 are significant at the 1% significance level, which infers that the past portfolio returns have predictive power to explain the portfolio returns for the forthcoming day, while the past portfolio volatility can also explain the present-day portfolio volatility, respectively. Hence, Table 5 shows that the period of high volatility is followed by a period of higher volatility and the period of small volatilities is also followed by a period of smaller volatility.

However, the coefficient of the TGARCH model is significant, but not negative. This, however, suggests that although a diversified portfolio is subject to the leverage effect, the past negative portfolio returns do not generate higher volatility, as compared to the past positive returns.

Table 5. Mean and variance ARCH, GARCH, and TGARCH models

	(1)	(2)
Variables	**Portfolio**	**ARCH**
Portfolio (Mean Model)		
Constant	0.0003	
	(0.060)	
Variance Model		
Constant		-0.0000***
		(0.002)
L. ARCH		0.056***
		(0.001)
L.TARCH		0.087
		(0.000)
L. GARCH		0.879***
		(0.000)
Observations	1332	1332

Note: p value in parentheses
*** p<0.01, ** p<0.05, * p<0.1

It is important to check the autocorrelation in the residual of the TGARCH model to use the coefficients for future forecasting. Table 6 presents the Autocorrelation Function (ACF), Partial Autocorrelation Function (PACF), Q statistics, and associated probability value. The results indicate that the residuals are not subject to autocorrelation as the p-value is greater than 0.05 and fails to reject the null hypothesis. Based on this evidence, the estimated coefficients in Table 6 can be utilized to forecast the future volatility of diversified portfolios in South Africa, Nigeria, and Egypt. Thus, there is no ARCH effect on the residuals.

Table 6. Correlogram

LAG	AC	PAC	Q	Prob > Q	[Autocorrelation]	[Partial Autocorrelation]
1	0.0587	0.0591	4.6057	0.0619		

5. CONCLUSION

The aim of the study is twofold: to evaluate the forecast and performance of asymmetric volatility (leverage effect) through the TGARCH model under normal and non-normal distribution densities, and to explore the possible opportunities for portfolio diversification in African countries, South Africa, Nigeria, and Egypt. Our study contributes to the existing literature as follows. First, we select an emerging stock market from Africa, where the asymmetric model has not been applied yet. Second, we model asymmetric volatility of the individual stock market, as well as a diversified portfolio, and capture the time-series feature of kurtosis, skewness, and volatility clustering. Third, we introduce the comparison among the normal and the non-normal distribution density of the asymmetric model, across all three countries. Our results show all three countries have the leptokurtic (fat tail) and the negatively skewed distribution.

TGARCH models for individual countries show that the past stock returns (volatility) influence the present stock returns (volatility), across all countries, under all distribution densities, whereas a leverage effect is only present in South Africa and Egypt. However, negative returns do not generate higher volatility, as compared to positive returns. On the other hand, the leverage effect is insignificant in Nigeria. The diagnostic tests show AIC and BIC produce the lowest estimates under Gaussian distribution for all three countries. This infers that normal distribution produces better and reliable estimates for all countries.

The existence of the ARCH effect indicates that investors in these markets should seek more information about volatility, before allocating their funds for portfolio investment. Investors and fund managers should not only be limited to the mean-variance analysis before the investment, but also consider asymmetric information, volatility, skewness, kurtosis, and correlation (see for example Bekaert *et al*, 1996). The presence of the ARCH effect should be compensated with higher returns, as it increases the cost of equity capital in South Africa, Nigeria, and Egypt stock markets.

Future research should be directed towards estimating and forecasting the realized volatility by utilizing intra-day returns. Moreover, future studies may apply models for conditional variances, such as APARCH and long memory models, e.g. FIEGARCH, FIAPARCH, and CGARCH. These models provide greater information on the dynamics of the returns series. Lastly, the same methodology can be applied to other African countries to explore the effect of economic and structural variables that influence returns volatility.

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ANALIZA UVJETNE HETEROSKEDASTIČNOSTI I NESIMETRIČNOG MODELA CIJENA DIONICE U JUŽNOJ AFRICI, NIGERIJI I EGIPTU

Sažetak

U ovom se radu istražuje efekt poluge u afričkim zemljama korištenjem gustoće normalne i ne-normalnih distribucija. Nadalje, analiziramo moguće prilike za diverzifikaciju portfelja u Južnoj Africi, Nigeriji i Egiptu. Rezultati pokazuju da negativni povrati na dionice ne dovode de veće volatilnosti budućih povrata, negoli je to slučaj kod pozitivnih povrata u prošlosti. U sve tri države, na snazi je ARCH efekt, prema kojem informacije o prošloj cijeni dionica i njenoj volatilnosti djeluju na tekuće povrate na dionice i njihovu volatilnost. Također dolazimo do zaključka da Gaussova distribucija dovodi do bolje procjene, u usporedbi s ne-normalnom distribucijom. S obzirom na diverzifikaciju portfelja, i povrati se ravnaju prema ARCH efektu, pri čemu, ipak, efekt poluge ne određuje asimetričan utjecaj prošlih negativnih povrata na tekuće cijene dionica.

Ključne riječi: *volatilnost, GARCH modeli, ARCH efekt, diverzifikacija portfelja, korelacija, normalna i ne-normalne distribucije*