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Implementation of MPMM ballistic model for calculation of differential coefficients for TFTs according to NATO STANAG 4119

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Abstract

MPMM ballistic model (modified point mass model) is used to calculate all necessary ballistic trajectory elements (range, time of flight, angle of fall, maximum ordinate, differential coefficients, etc.) for classical spin-stabilized artillery projectiles. In this paper, differential and algebraic equations that fully define MPMM ballistic model and its program solution (MPMM_D30) are presented. The MPMM_D30 enables calculation of differential coefficients for H122mmD30 howitzer (and other ballistic trajectory elements) that are an integral part of TFTs according to NATO STANAG[2] 4119 (Adoption of a Standard Cannon Artillery Firing Table Format, Brussels, 2007), which should become HRVN STANAG[3] 4119.

Keywords

external ballistics, ballistic model, standard atmosphere, projectile trajectory, meteorological report, differential equations, H122mmD30

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2 STANAG (Official Gazette No. 88/2019, Article 6)

3 HRVN STANAG (Official Gazette No. 88/2019, Article 5)

Introduction

As a branch of the Croatian Army, artillery has adopted two NATO STANAGs as HRVN Croatian military standards that define the development of the TFTs according to NATO STANAG 4119. Those are HRVN STANAG 4355 (ballistic model MPMM) and HRVN STANAG 4044 (ICAO, International Civil Aviation Organization), respectively. The format of TFTs is established by STANAG 4119. It should soon be fully ratified and shall become HRVN STANAG 4119. The Croatian Armed Forces possess a variety of weapons from the so-called Eastern Country, where TFTs were developed according to "Russian" standards (M. Gajić and J. Viličić 1978, pp. 396-421). Those TFTs should be converted to NATO STANAG 4119 for compatibility with the artillery units of NATO member states. Differential coefficients or standard corrections according to which ballistic trajectory in the real atmosphere is calculated are the basic data in TFTs of any format. In this paper, a detailed description of the method for calculating differential coefficients for density and virtual temperature (humidity air feature included), which are the essential TFTs elements for the H122mm howitzer D30 according to NATO STANAG 4119, is given. The calculation is conducted based on the available TFTs according to the "Russian" standard. Other differential coefficients (muzzle velocity, projectile mass, wind, etc.) are calculated in the same manner. The MPMM ballistic model enables the realization of two main ballistic tasks, the direct and indirect one. The direct task of the external ballistics, based on the known initial elevation angle (elevation) in standard or real atmosphere calculates two main ballistic trajectory elements (as well as all other elements). Those are range and deflection from the firing plain. The direct task of the external ballistics is used for the development of TFTs by which the initial elements (elevation and deflection) for fire are calculated. Based on the known range in standard or real atmosphere, the indirect task of the external ballistics (INMPMM) calculates the optimal initial elevation angle and all other elements of the ballistic trajectory. It is used for the development of fire control systems for classical stabilized artillery projectiles. For the calculation of differential coefficients (and all other elements of the ballistic trajectory), a program called **MPMM_D30** was developed according to the block diagram (Figure 3), which can solve direct (DMPMM) and indirect tasks of the external ballistics (INMPMM). The **MPMM_D30** enables the calculation and use (TC 3-09.81, 2016) of all tables (F(i), F(ii), G(i), H, etc.) that are an integral part of the TFTs according to NATO STANAG 4119, based on the "Russian" standard (M. Gajić and J. Viličić 1978, p 396-421) and their use (TC 3-09.81,2016).

MPMM ballistic model

The MPMM is the basic ballistic model (HRVN STANAG 4355) that determines all necessary parameters of the ballistic trajectory for classical artillery spin-stabilized projectiles. The basic differential equations representing this model (Figure 1) are as follows:

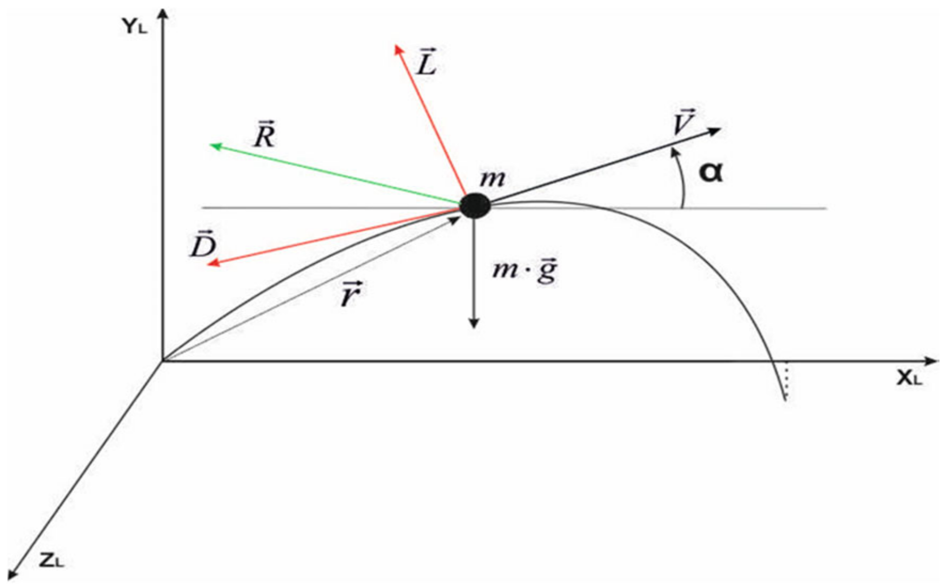


Figure 1. Graphic presentation of forces that influence the projectile mass *m* in the MPMM.

$$m \cdot \frac{d\vec{V}_k}{dt} = \vec{F}_G + \vec{D} + \vec{L} + \vec{F}_k \;, \tag{1}$$

$$\frac{d\vec{r}}{dt} = \vec{V}_k, \tag{2}$$

$$\vec{V}_k = \vec{V}_W + \vec{V} \;. \tag{3}$$

Equation (1) (S. Janković 1998, pp. 234-235) shows which forces (Figure 1) affect the projectile of mass *m*, caliber *d*, projectile cross-sectional area *S*, and aerodynamic velocity that represents the projectile velocity with wind velocity.

- $\vec{F}_G = m \cdot \vec{g}$ (force of gravity) ,
- $\vec{D} = \frac{\rho \cdot V^2}{2} \cdot S \cdot C_D(M) \cdot \frac{\vec{v}}{V}$ (aerodynamic drag force)
- $\vec{L} = \frac{\rho \cdot V^2}{2} \cdot S \cdot C_{L\sigma} \cdot \sigma \cdot \vec{i}_L$ (lift force)
- $\vec{F}_k = -2 \cdot m \cdot \vec{\Omega}_E \times \vec{V}_k$ (Coriolis force)

The flight velocity $\vec{V}_k$ shown by Equation (2) in the form of the vector and by Equation (3) in the windy atmosphere represents the projectile velocity (absolute) relative to the Earth. The force $\vec{F}_G$ is the intensity the gravity and is directed vertically downwards. The force $\vec{D}$ is a resistance drag force that lies on the velocity line $\vec{V}$ but is in the opposite direction. The force $\vec{L}$ is the lift force that appears due to the angle of attack σ between the projectile axis and aerodynamic velocity. The lift force is proportional to the magnitude of the angle and lies in the plane formed by the projectile axis and aerodynamic velocity $\vec{V}$. It is vertical to the aerodynamic velocity and has a direction from the aerodynamic velocity toward the projectile axis. The total aerodynamic force applied to the projectile of mass *m* is $\vec{R} = \vec{D} + \vec{L}$. The force $\vec{F}_k$ is Coriolis force due to the rotation of the Earth, where $\Omega_E = 7{,}27 \cdot 10^{-5}\, rad/s$ is angular velocity of the Earth. Dividing Equation (1) by the mass m gives the equation that represents a total change in acceleration action on all the following forces:

$$\frac{d\vec{V}_k}{dt} = -\frac{\rho \cdot V^2}{2} \cdot \frac{S}{m} \cdot C_D \cdot \frac{\vec{V}_k - \vec{V}_W}{V} + \vec{g} + \frac{\rho \cdot V^2}{2} \cdot \frac{S}{m} \cdot C_{L\sigma} \cdot \sigma \cdot \vec{i}_L - 2 \cdot \vec{\Omega}_E \times \vec{V}_k. \quad (4)$$

By projecting Equation (4) on the axis of the local coordinate system (S. Janković 1998, pp. 29-30) LKS (X_L, Y_L, Z_L), the final system of differential equations of the MPMM ballistic model is obtained (S. Janković 1998, pp. 241-242) with the range and deflection fitting $C_1(\alpha_0)$ and the projectile direction $C_2(\alpha_0)$:

$$\frac{du_k}{dt} = -E \cdot \left(C_D \cdot \frac{u_k - u_W}{V} + C_L \cdot \beta \cdot \frac{w_k - w_W}{\sqrt{V^2 - {v_k}^2}} \right) - 2 \cdot (-\Omega_{EZ} \cdot v_k + \Omega_{EY} \cdot w_k), \quad (5)$$

$$\frac{dv_k}{dt} = -E \cdot \left(C_D \cdot \frac{v_k}{V} \right) - g - 2 \cdot (-\Omega_{EZ} \cdot u_k - \Omega_{EX} \cdot w_k), \quad (6)$$

$$\frac{dw_k}{dt} = -E \cdot (C_D \cdot \frac{w_k - w_W}{V} - C_L \cdot \beta \cdot \frac{u_k - u_W}{\sqrt{V^2 - {v_k}^2}}) - 2 \cdot (-\Omega_{EY} \cdot u_k + \Omega_{EX} \cdot v_k), \quad (7)$$

$$\frac{dp}{dt} = \frac{S \cdot d^2}{4 \cdot I_x} \cdot \rho \cdot V \cdot C_{lp} \cdot p, \quad (8)$$

$$\frac{dx}{dt} = u_k, \quad (9)$$

$$\frac{dy}{dt} = v_k, \quad (10)$$

$$\frac{dz}{dt} = w_k. \quad (11)$$

where:

- u_k (horizontal velocity component in LKS)
- v_k (vertical velocity component) in LKS)
- w_k (side velocity component in LKS)
- u_W (horizontal wind component in LKS)
- w_W (side wind component in LKS)

- $E = \frac{\rho \cdot V^2}{2} \cdot \frac{S}{m}$ (parameter)
- g (gravity constant)
- V_W (wind velocity)
- $V = \sqrt{(u_k - u_W)^2 + v_k{}^2 + (w_k - w_W)^2}$ (aerodynamic velocity)
- V_k (flight velocity)
- V_0 (standard muzzle velocity)
- I_x (body axial moment of inertia)
- m_n (standard projectile mass)
- d (caliber)
- $S = \frac{d^2 \cdot \pi}{4}$ (projectile cross-sectional area)
- A_ε (angle of twisting of the grooves)
- $p = \frac{2 \cdot V_k \cdot \tan A_\varepsilon}{d}$ (angular velocity)
- α_0 (initial elevation angle)
- α (angle of inclination of the tangent)
- $\beta = -\frac{2 \cdot I_x \cdot g \cdot \cos\alpha \cdot p}{\rho \cdot S \cdot d \cdot V^3 \cdot C_{\overline{m}\sigma}}$ (sideslip)
- A_0 (azimuth of fire)
- A_W (azimuth of wind)
- φ_{LA} (weapon position latitude)
- Ω_E (angular velocity of the Earth)
- $\Omega_{EX} = \Omega_E \cdot \cos A_0 \cdot \cos\varphi_{LA}$ (horizontal component of angular velocity of the Earth in LKS)
- $\Omega_{EY} = \Omega_E \cdot \sin\varphi_{LA}$ (vertical component of angular velocity of the Earth in LKS)
- $\Omega_{EZ} = -\Omega_E \cdot \sin A_0 \cdot \cos\varphi_{LA}$ (side component of angular velocity of the Earth in LKS)

- a (speed of sound)
- $\mathrm{M} = \frac{\mathrm{V}}{\mathrm{a}}$ (Mach number)
- $\mathrm{a(y)}$ (change of sound of speed with altitude)
- $\mathrm{T_v(y)}$ (change of virtual with altitude)
- $\mathrm{H(y)}$ (change of pressure with altitude
- $\rho(\mathrm{y}) = 0{,}3484 \cdot \frac{\mathrm{H(y)}}{\mathrm{T_v(y)}}$ (change of density with altitude)
- $\mathrm{C_{D43}(M)}$ (total axial force coefficient)
- $\mathrm{C_1}(\alpha_0)$ (fitting range factor), $\mathrm{C_2}(\alpha_0)$ (fitting deflection factor)
- $\mathrm{C_D} = \mathrm{C_1}(\alpha_0) \cdot \mathrm{C_{D43}(M)}$ (corrected total axial force coefficient)
- $\mathrm{C_L} = \mathrm{C_2}(\alpha_0) \cdot \mathrm{C_{L\sigma}(M)}$ (corrected lift force coefficient)
- $\mathrm{C_{L\sigma}(M)}$ (lift force coefficient derivative with an angle of attack)
- $\mathrm{C_{\overline{m}\sigma}(M)}$ (pitching moment coefficient derivative with angle of attack)
- $\mathrm{C_{lp}(M)}$ (damp in roll coefficient derivative).

For the MPMM consisting of seven differential equations to be complete, it has to be supplemented with two interior ballistic differential coefficients l_t and l_q which are used to correct the initial speed because of the propellant temperature l_t and projectile mass l_q that is determined according to Sluhocki (M. Gajić and J. Viličić 1978, pp. 151-153). It is well known that the propellant temperature T_b°C, that is higher than the standard propellant temperature T_{tb}°C, gives a higher muzzle velocity V_0. The projectile mass m that is bigger than the standard projectile mass m_n causes a reduction of the muzzle velocity. The muzzle velocity deviation created due to the difference between the measured and standard propellant temperatures is calculated according to the equation:

$$\Delta V_{0b} = (T_b - T_{tb}) \cdot l_t \cdot V_0 \tag{12}$$

The influence of the projectile mass on the muzzle velocity deviation is calculated according to the equation:

$$\Delta V_{0m} = -\frac{m - m_n}{m_n} \cdot l_q \cdot V_0 \tag{13}$$

An additional decrease of the muzzle velocity ΔV_{0c} causes the gunpowder chamber to lengthen and the pipe to wear out. This muzzle velocity reduction is recorded in a technical weapon booklet. Now, muzzle velocity V_k can be expressed by the eqation:

$$V_k = V_0 + \Delta V_{0b} + \Delta V_{0m} + \Delta V_{0c} \tag{14}$$

Differential and algebraic equations containing the complete MPMM ballistic model are presented here. By solving them, the ballistic trajectory of the classical spin-stabilized projectile is completely determined in the standard or real atmosphere. The real atmosphere is obtained by radio sounding observation, i.e. by measuring the parameters of the atmosphere by altitude (temperature, humidity, azimuth of wind, wind velocity, pressure, etc.) which are measured in the CAF by the meteo station "Vaisala" (K. Šilinger, M. Blaha, 2017, pp. 196-197). The differential equations of such a ballistic model are solved by numerical integration most often using the well-known *Runge-Kutta Method*. This method is used and programmed in this paper **(MPMM_D30)**.

Differential coefficients

All TFTs, regardless of whether they were made according to the "Russian" or NATO standards, with the most important elements of ballistic trajectories (range, flight time, trajectory, maximum ordinate, etc.), which are calculated for certain initial angles in the adopted standard atmosphere (ICAO) also contain differential coefficients or standard corrections by which the standard ballistic trajectory is "corrected" to the current meteorological-ballistic conditions. It is evident from the differential and algebraic equations of the complete MPMM ballistic model that each element of the ballistic trajectory "B" depends on a series of meteorological-ballistic parameters p_i (density, speed of sound, wind, propellent temperature, muzzle velocity deviation, projectile mass deviation, etc.), i.e., there is a functional dependence $B = f(p_i)$. The standard meteorological-ballistic parameters B_0 correspond to some standard ballistic trajectory elements p_{i0}. If they change by the amount of deviation δ_{pi} i.e. $p_i = p_{i0} + \delta_{pi}$, the ballistic trajectory element will also change by δB. The new value of the ballistic element B_1 will be $B_1 = B_0 + \delta B$. The amount of the correction (or deviation) of ballistic trajectory elements(s) δB can be calculated using the Taylor equation:

$$\delta B = B_1 - B_0 = \frac{\partial B_0}{\partial p_1} \cdot \delta p_1 + \frac{\partial B_0}{\partial p_2} \cdot \delta p_2 + \ldots. = \sum_{i=1}^{n} \frac{\partial B_0}{\partial p_i} \cdot \delta p_i\,. \qquad (15)$$

in which higher order members are neglected:

$\frac{\partial^2 \cdot B_0}{\partial p_1 \cdot \partial p_2} \cdot \delta p_1 \cdot \delta p_2$, $\frac{\partial^2 \cdot B_0}{\partial p_1 \cdot \partial p_3} \cdot \delta p_1 \cdot \delta p_3$, $\frac{\partial^2 \cdot B_0}{\partial p_2 \cdot \partial p_3} \cdot \delta p_2 \cdot \delta p_3$ because δp_i are small. The values $\frac{\partial B_0}{\partial p_1}, \frac{\partial B_0}{\partial p_2}, \frac{\partial B_0}{\partial p_3}$ are called differential coefficients for the standard ballistic trajectory element B_0. Each perturbation p_i causes a non-linear deviation of the element(s) of the ballistic trajectory because the differential equations are by their very nature non-linear. Under certain conditions, some linear deviations can have a linear change, but it significantly depends on the magnitude of the deviation and of ballistic element sensitivity to that deviation. Considering that those small deviations are usually observed in external ballistics, as they are in this case, the deviation of the ballistic trajectory element has an almost linear change. The linearity of the deviation of the ballistic trajectory element(s) in the event of the influence of a small deviation is proportional to the magnitude of that deviation. The influence of several different deviations is the sum of the influence of individual deviations, as shown by Equation (15). Thus, for example, to correct the range δX as the ballistic trajectory element that depends on the virtual temperature T_v, density ρ, range wind W_X, and projectile mass m, it can be written:

$$\delta X = \frac{\partial X}{\partial T_v} \cdot \delta T_v + \frac{\partial X}{\partial \rho} \cdot \delta \rho + \frac{\partial X}{\partial W_X} \cdot \delta W_X + \frac{\partial X}{\partial m} \cdot \delta m\,. \qquad (16)$$

The values $\frac{\partial X}{\partial T_v}, \frac{\partial X}{\partial \rho}, \frac{\partial X}{\partial W_X}, \frac{\partial X}{\partial m}$ are called differential coefficients for temperature, density, range wind, and projectile mass. In TFTs, they are used for a certain deviation value, e.g. for $\delta T_v = \Delta T_v = 1\%(1°C)$, $\delta\rho = \Delta\rho = 1\%$, $\delta W_X = \Delta W_X = 1m/s, \delta m = \Delta m = 1\ weight\ mark$. They are called range corrections or deviations δX, and are marked by $\Delta X_{Tv}, \Delta X_\rho$, $\Delta X_{W_X}, \Delta X_m$. It should be noted that artillery live firing is carried out in all weather conditions. Therefore, the deviation magnitudes δp_i for virtual temperature, wind speed, and propellant temperature affecting domestic projectile range can be several times higher than the values for which their respective differential coefficients $\frac{\partial X}{\partial p_i}$ were calculated. In that case, the range deviations δX_{pi} for each of these deviations can exceed the allowable linearity limit which applies only to small deviations, while the trend of change in nature is always non-linear. It can be easily explained with Figure 2, where the change in the range X depending on a particular parameter p is shown.

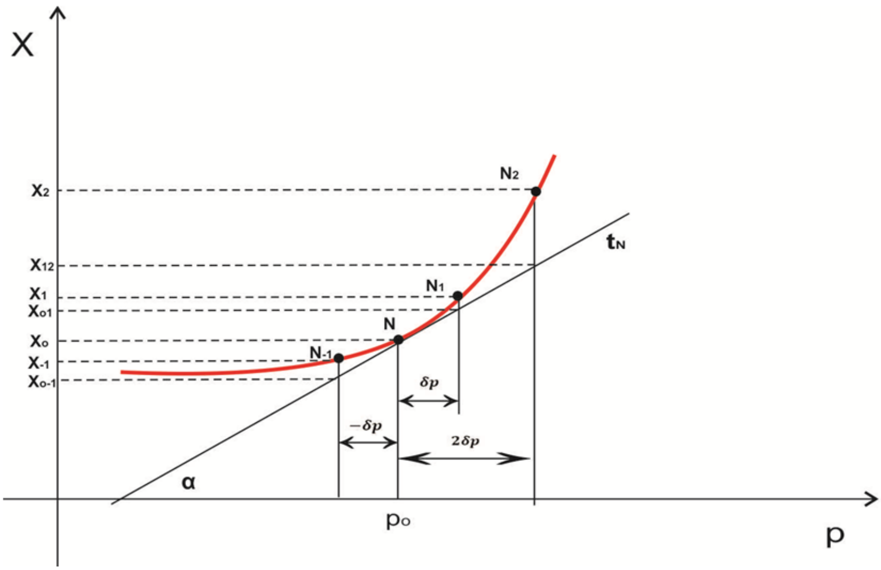


Figure 2. Change of range X in proportion to the deviation magnitude δp.

In Figure 2, a certain parameter p corresponds to a particular standard range X_o. The differential coefficient $\frac{\partial X_o}{\partial p_o} = \mathrm{tg}(\alpha)$ in point N is represented by the tangent t_N. The range X_1 can be calculated using the differential coefficient and positive deviation value $+\delta p$ based on the equation $X_1 = X_o + \frac{\partial X_o}{\partial p_o} \cdot \delta p$. However, Figure 2 clearly shows that this equation gives us the range X_{01} but not the real range X_1. In that way, a small error is made which is $\Delta X_1 = X_1 - X_{o1}$. When determining the range X_2 based on twice the deviation $2 \cdot \delta p$ using the differential coefficient $\frac{\partial X_o}{\partial p_o}$ i.e. $X_2 = X_o + \frac{\partial X_o}{\partial p_o} \cdot 2\delta p$ (the range X_{01} is obtained but not the real range X_1), a significant error occurs in determining the actual range X_2 which is $\Delta X_2 = X_2 - X_{12}$. It follows that the initial elements of artillery live firing are more accurately determined by fire control systems based on the MPMM ballistic model (which correctly calculates the real ranges), than by the "manual" method using TFTs, where standard ranges are "corrected" to real ones by the means of differential coefficients. Due to their non-linearity, those coefficients give an error in determining the real ranges. It should be noted here (Figure 2) that when the deviation value is negative $-\delta p$, and the value of the real range X_{-1} is determined based on the differential coefficient $\frac{\partial X_o}{\partial p_o}$ by the equation $X_{-1} = X_o - \frac{\partial X_o}{\partial p_o} \cdot \delta p$, the value of the real range X_{-1} will not be obtained. Instead, the real range value will be obtained. In other words, the error $\Delta X_{-1} = X_{-1} - X_{o-1}$ will appear which is greater than the error ΔX_1 obtained for the positive deviation value $+\delta p$. It is also clear that this can be reversed depending on the type of differential coefficient and its change in a particular parameter p. Due to this fact, TFTs by NATO STANAG 4119 have two differential coefficients for one parameter. The first differential coefficient $\frac{\partial X_o}{\partial p_{o+}}$ is used for the area of positive deviations $+\delta p$, while the second differential coefficient $\frac{\partial X_o}{\partial p_{o+}}$ is used for the area of negative deviations $-\delta p$. In this way, the interval of linear deviation applicability is extended, and real changes are reached more precisely. Differential coefficients are calculated for the "Russian" TFTs standard using the equation:

$$\frac{\partial B_o}{\partial p_o} = \frac{B(p_o+\Delta p)-B(p_o-\Delta p)}{2\cdot\Delta p}\ . \tag{17}$$

The procedure of calculating differential coefficients using Equation (17) is known as a *differential method*. It is performed using the **MPMM_D30** program by calculating the ballistic trajectory twice, i.e., calculating two values of the ballistic trajectory element to obtain one differential coefficient. The first time the parameter p_o to which the standard element of trajectory B_o belongs is increased by Δp, and the trajectory element $B(p_o + \Delta p)$ is calculated. The second time the parameter p_o is decreased by Δp and the trajectory element $B(p_o - \Delta p)$ is calculated. In both cases, all other parameters p_{oi} should remain unchanged. The obtained results are included in Equation (17) and the differential coefficient $\frac{\partial B_o}{\partial p_o}$ for the trajectory element B_o and the parameter p_o are calculated. Differential coefficients for TFTs according to NATO STANAG 4119 are calculated for the positive deviation value $+\delta p$ by the equation:

$$\frac{\partial B_o}{\partial p_{o+}} = \frac{B(p_o+\Delta p)-B(p_o)}{\Delta p} \tag{18}$$

While for the negative deviation value $(-\delta p)$ they are calculated by using the equation:

$$\frac{\partial B_o}{\partial p_{o-}} = \frac{B(p_o-\Delta p)-B(p_o)}{\Delta p}. \tag{19}$$

The procedure of calculating differential coefficients for positive $+\delta p$ and negative $-\delta p$ deviation values for TFTs according to NATO STANAG 4119 is also performed using the **MPMM_D30** program and differential method except that in this case, the ballistic trajectory for one differential coefficient should be calculated only once. The differential coefficients, regardless of whether they are calculated according to the “Russian” or NATO STANAG 4119, are constant for one ballistic trajectory.

Procedure for calculating differential coefficients

The calculation of differential coefficients for density and virtual temperature as components of TFTs according to NATO STANAG 4119 is in this paper performed for the HI22mmD30 howitzer for several standard ranges in the interval from 11000m to a maximum range of 15221m for low and high angles for the full charge with OF-462 projectile and RGM-2 fuze. In order for such a calculation to be performed, it is necessary to determine the dependence of the of the fitting range coefficients $C_1(\alpha_0)$ and deflections $C_2(\alpha_0)$ on the initial elevation angle α_0. from TFTs according to the "Russian" standard. Range and deflection fitting coefficients are also determined by using the **MPMM_D30** program for standard ANA atmosphere (artillery normal atmosphere) and standard ballistic conditions according to which TFTs were developed by the "Russian" standard (M. Gajić, and J. Viličić 1978, pp. 396-421) but they are valid for all standard or real atmospheres, including the ICAO (HRVN STANAG 4044) atmosphere on the basis of which TFTs are developed by NATO STANAG 4119. Using the *Least Squares Method* (R. Scitovski, 1993, pp. 39-42), the obtained fitting coefficients, which depend on the initial angles α_0, are approximated by certain function-models, most often in the form of polynomials $C = b_0 + b_1 \cdot \alpha_0^1 + b_2 \cdot \alpha_0^2 + \cdots$. This method is used to determine the required coefficients b_0 , b_1 , b_2, etc. The function-model in the form of a polynomial of a higher degree will be a smaller approximation error. The calculated fitting range $C_1(\alpha_0)$ and the deflection $C_2(\alpha_0)$ coefficients of the projectiles are in the **MPMM_D30** program code. For the **MPMM_D30** program to solve the indirect INMPMM model, i.e., to determine the initial angle α_0 based on the known range in the real or standard atmosphere, ICAO must contain the initial approximation angles α_0 that correspond to these ranges. In this paper, the initial angles are determined based on standard ranges X_T from TFTs according to the "Russian" standard using the *Least Squares Method*. The functional dependence $\alpha_0(X_T)$ of the initial angle α_0 on the corresponding standard range X_T is determined. The approximation of the initial angle α_0 proved to be sufficiently accurate because the standard

ANA and ICAO atmospheres differ slightly. (ICAO is 1.6% "heavier" than ANA). The functional dependences $\alpha_0(X_T)$ of the initial angle α_0 on the standard range X_T for low angles are found in Table 1 and for the high angles in Table 2. according to the "Russian" standard

Table 1 . Initial angle dependence α_0 on range X_T for low angles from TFTs according to the "Russian" standard

α_0	18.36°	26.04°	38.46°	45°
X_T	11000m	13000m	15000m	15286m
$\alpha_0(X_T) = -184.45532 + 43.838276 \cdot \alpha_0^1 - 3.4366705 \cdot \alpha_0^2 + 0.09009344 \cdot \alpha_0^3$				

Table 2. Initial angle dependence α_0 on range X_T for high angles from TFTs according to the "Russian" standard

α_0	66.84°	61.02°	50.7°	45°
X_T	11000m	13000m	15000m	15286m
$\alpha_0(X_T) = 37.77706 - 7.7175736 \cdot \alpha_0^1 + 0.6537563 \cdot \alpha_0^2 - 0.01902082 \cdot \alpha_0^3$				

The procedure for calculating the differential coefficients for virtual temperature T_v and density ρ for the TFT format by NATO STANAG 4119 using **MPMM_D30** is shown on the standard range of 11000m for low angles. The procedure is the same for all ranges and angles. Since the original TFTs according to NATO STANAG are developed for standard propellant temperature $T_{tb} = 21°C$, while the TFTs according to the "Russian" standard are made for standard propellant temperature $T_{tb} = 15°C$ it is necessary to calculate the increase of the standard muzzle velocity V_0 on the basis of which the calculation procedure will be conducted because higher propellant temperature increases the muzzle velocity. The interior ballistic correction coefficient for propellant temperature (Sluhocki) for the standard muzzle velocity of $V_{01} = 690\, m/s$ is $l_t = 0.0008$. The required increase of standard muzzle velocity is now calculated on the basis of Equation (12) $\Delta V_{0b} = (T_b - T_{tb}) \cdot l_t \cdot V_{01} = (21°C - 15°C) \cdot 0.0008 \cdot 690 = 3.312\, m/s$. Now the the standard muzzle velocity for TFTs according to NATO STANAG 4119 used by **MPMM_D30** is $V_{02} = 693.3 m/s$.

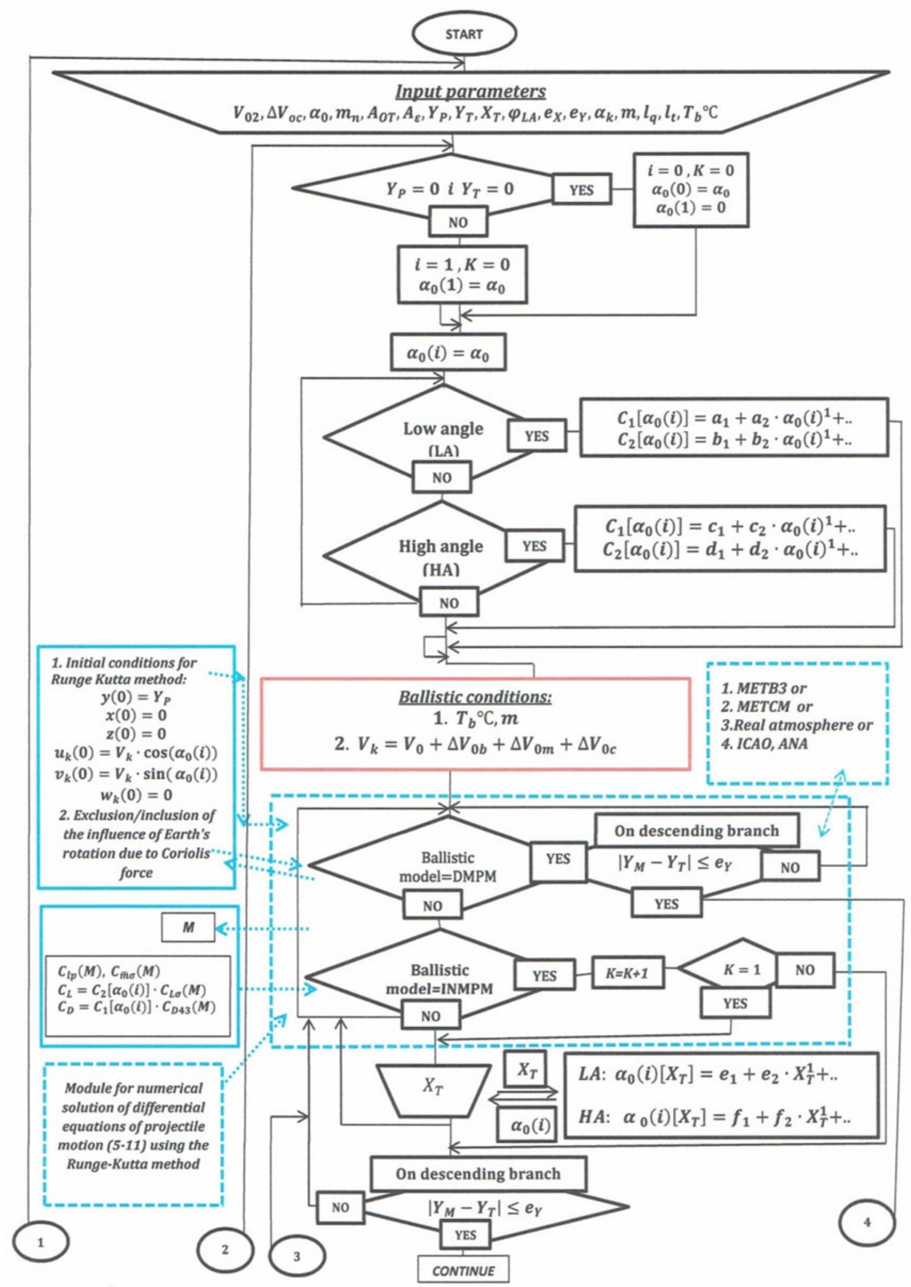
Necessary input parameters for differential coefficient calculation

- PeP → full charge
- OF-462 projectile with RGM-2 fuze
- α_0 → initial angle for DMPMM ballistic model
- $C_1(\alpha_0)$, $C_2(\alpha_0)$ → fitting coefficients (integrated in **MPMM_D30**)
- $\alpha_0(X_T)$ → (Tables 1 and 2)
- $V_{01} = 690$ m/s → standard muzzle velocity for "Russian" standard for TFTs
- $V_{02} = 693.3$ m/s → standard muzzle velocity for NATO STANAG 4119
- $m_n = 21.76$ kg → standard projectile mass
- 1 weight mark (+ or -) on OF-462 projectile = 0.145 kg
- $I_x = 0.0414\ kg \cdot m^2$ → body axial moment of inertia
- $A_\varepsilon = 7.15\ °$ → angle of twisting of the grooves
- $l_t = 0.0008$ → interior ballistic coefficient for propellant temperature
- $l_q = 0.31$ → interior ballistic coefficient for projectile mass
- $\varphi_{LA} = 45°$ → weapon position latitude (FP)
- $\alpha_k = 1°$ → initial angle correction $\alpha_0(0)$ for INMPMM ballistic model
- mil = one thousandth $1/6400$
- td → one thousandth $1/6000$

Aerodynamic coefficients for OF-462 (S. Janković 1998, CD) are shown in Table 3.

Table 3. Aerodynamic coefficients for OF-462

M	C_{D43}	$C_{L\sigma}$	C_{lp}	$C_{\bar{m}\sigma}$
0.1	0.157	1.635	-0.038	2.831
0.4	0.157	1.610	-0.034	2.874
0.6	0.158	1.566	-0.031	2.950
0.8	0.160	1.470	-0.029	3.126
0.9	0.190	1.355	-0.028	3.339
0.94	0.244	1.311	-0.027	3.507
0.96	0.271	1.299	-0.027	3.579
0.98	0.298	1.325	-0.027	3.591
1.05	0.360	1.597	-0.026	3.651
1.1	0.382	1.787	-0.026	3.455
1.2	0.385	2.044	-0.025	3.189
1.3	0.382	2.218	-0.024	3.039
1.5	0.361	2.440	-0.023	2.840
2	0.316	2.702	-0.020	2.551
2.5	0.287	2.707	-0.016	2.322


START
Input parameters
$V_{02}, \Delta V_{oc}, \alpha_0, m_n, A_{OT}, A_\varepsilon, Y_P, Y_T, X_T, \varphi_{LA}, e_X, e_Y, \alpha_k, m, l_q, l_t, T_b$°C
$Y_P = 0 \ i \ Y_T = 0$
YES
NO
$i = 0, K = 0$
$\alpha_0(0) = \alpha_0$
$\alpha_0(1) = 0$
$i = 1, K = 0$
$\alpha_0(1) = \alpha_0$
$\alpha_0(i) = \alpha_0$
Low angle (LA)
YES
NO
$C_1[\alpha_0(i)] = a_1 + a_2 \cdot \alpha_0(i)^1 + ..$
$C_2[\alpha_0(i)] = b_1 + b_2 \cdot \alpha_0(i)^1 + ..$
High angle (HA)
YES
NO
$C_1[\alpha_0(i)] = c_1 + c_2 \cdot \alpha_0(i)^1 + ..$
$C_2[\alpha_0(i)] = d_1 + d_2 \cdot \alpha_0(i)^1 + ..$
1. Initial conditions for Runge Kutta method:
$y(0) = Y_P$
$x(0) = 0$
$z(0) = 0$
$u_k(0) = V_k \cdot \cos(\alpha_0(i))$
$v_k(0) = V_k \cdot \sin(\alpha_0(i))$
$w_k(0) = 0$
2. Exclusion/inclusion of the influence of Earth's rotation due to Coriolis force
Ballistic conditions:
1. T_b°C, m
2. $V_k = V_0 + \Delta V_{0b} + \Delta V_{0m} + \Delta V_{0c}$
1. METB3 or
2. METCM or
3.Real atmosphere or
4. ICAO, ANA
Ballistic model=DMPM
YES
NO
On descending branch
$|Y_M - Y_T| \le e_Y$
NO
YES
M
$C_{lp}(M)$, $C_{m\alpha}(M)$
$C_L = C_2[\alpha_0(i)] \cdot C_{L\alpha}(M)$
$C_D = C_1[\alpha_0(i)] \cdot C_{D43}(M)$
Ballistic model=INMPM
YES
NO
K=K+1
$K = 1$
NO
YES
Module for numerical solution of differential equations of projectile motion (5-11) using the Runge-Kutta method
X_T
X_T
$\alpha_0(i)$
LA: $\alpha_0(i)[X_T] = e_1 + e_2 \cdot X_T^1 + ..$
HA: $\alpha_0(i)[X_T] = f_1 + f_2 \cdot X_T^1 + ..$
On descending branch
$|Y_M - Y_T| \le e_Y$
NO
YES
CONTINUE
1
2
3
4

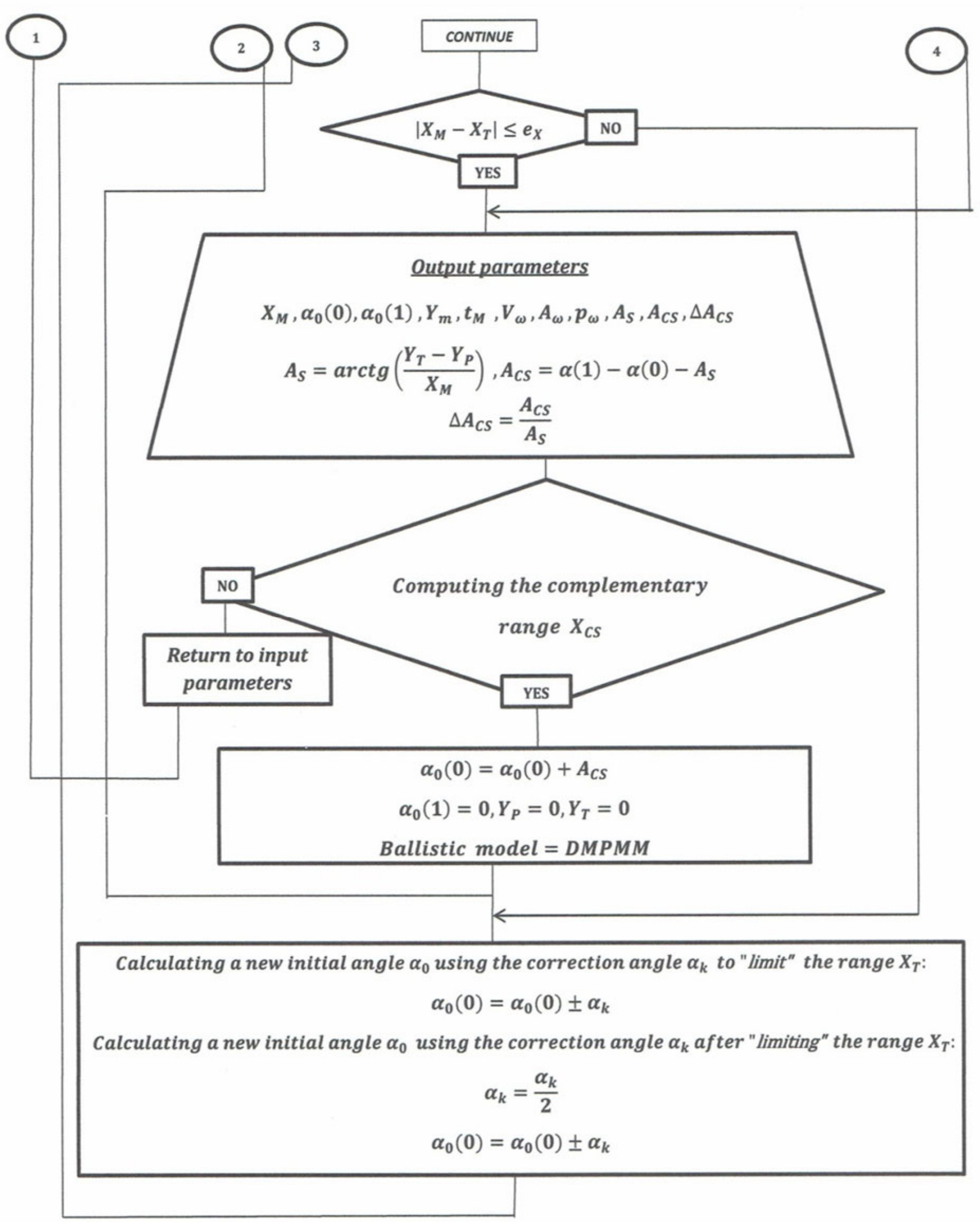


Figure 3: Block diagram for MPMM_D30

All standard ballistic and meteorological conditions and standard range $X_T = 11000$m are entered into **MPMM_D30** and using the indirect ballistic INMPMM model, the following output results are obtained. They are shown in Figure 4.

Standard initial angle A_E from INMPMM model $\alpha_0(0)=$	18.47°
Standard initial angle A_E from INMPMM model (1/6400) $\alpha_0(0)=$	328.3 mils
Quadrant elevation from INMPMM model (1/6400) $\alpha_0(1)=$	328.3 mils
Angle of site from INMPMM model (1/6400) A_S = ...	0 mils
Complementary angle of site from INMPMM model (1/6400) A_{CS} = ...	0 mils
Correction A_S for 1 mil from INMPMM model (1/6400) ΔA_{CS} = ...	0 mils
Computational range from the INMPMM model X_M =..	10999.5 m
Altitude weapon from INMPMM model Y_P = ...	0 m
Altitude target from INMPMM model Y_T = ...	0 m
Standard deflection (drift 1/6400) from INMPMM model A_D = ...	8.3 mils
Standard muzzle velocity V_{02}=...	693.3 m/s
Maximum ordinate from INMPMM model Y_m = ...	1310.4m
Time of flight from INMPMM model t_M = ...	31.43 s
Angle of fall from INMPMM model (1/6400) A_ω = ...	-563 mils
Remaining velocity from INMPMM model V_ω =..	279.7 m/s
Remaining angular velocity from INMPMM model p_ω =..	994.0 rad/s

Figure 4. Ballistic trajectory parameters for standard range X_T = 11000m by NATO STANAG 4119

The symbols used for elevation angles and deflection according to NATO STANAG 4119 are:

A_{QE} (quadrant elevation), A_E (standard angle of elevation), A_S (angle of site), A_{CS} (complementary angle of site), ΔA_{CS} (complementary angle of site for 1 mil), A_ω (angle of fall), A_D (drift), X_{CS} (complementary range). The complementary range X_{CS} (Elizabeth R. Dickinson, 1967, pp. 57-58) is the range with the complementary correction $\Delta_c X_{cs}$ that is present due to the difference between the weapon altitude and the target. The total elevation for shooting a target if there is an altitude difference between the weapon position Y_P and the target Y_T is $A_{QE} = A_E + A_S + A_{CS}$. In **MPMM_D30**, the total elevation A_{QE} is marked by $\alpha_0(1)$, and the standard initial angle A_E by $\alpha_0(0)$ due to the adjustment of the notation to the program code structure. Table F(ii) (Figure 9) in TFTs developed by NATO STANAG 4119 is employed to determine differential coefficients (range corrections) that are used to calculate the ballistic trajectory of the projectile in the real atmosphere. The direct DMPMM model in **MPMM_D30** is used to calculate Table F(ii). In TFTs according to NATO STANAG 4119, the state of the atmosphere is determined by differential coefficients for the ballistic deviation of density $\Delta\rho_B$ and ballistic deviation of virtual temperature ΔT_{Bv}. According to the "Russian" standard, the state of the atmosphere is determined by differential coefficients for ballistic air pressure deviation ΔH_B and ballistic deviation of virtual temperature ΔT_{Bv}. The differential coefficient for the ballistic deviation of the virtual temperature ΔT_{Bv} in TFTs according to NATO STANAG 4119 is the differential coefficient for the ballistic deviation of the speed of sound Δa_B because the change in the virtual temperature is already included in the density. Therefore, in TFTs according to NATO STANAG, instead of the differential coefficient for ballistic deviation of the speed of sound $\Delta a_B = 0.5\%$, the equivalent differential coefficient for ballistic deviation of virtual temperature $\Delta T_{Bv} = 1\%$ is entered (temperature change of 1% changes the speed of sound by 0.5%. The influence of the virtual temperature T_v in TFTs according to "Russian" standard is different from the virtual temperature in TFTs according to NATO STANAG 4119. According to the "Russian" standard, temperature has dual effect, i.e. virtual temperature T_v changes the function of pressure H with an altitude (according to the vertical balance of the atmosphere), which immediately affects the change of density and the speed of the sound. Thus, in TFTs according to the "Russian" standard, the influence of virtual temperature T_v is shown as a sum of both effects. It is clear that the effect of the virtual temperature T_v for both standards is only the result of the MPMM ballistic model programming method (for both standards the differential coefficients for the effect of the wind are calculated in the same manner using the **MPMM_D30**. When meteorological reports METB3 (HRVN STANAG 4061, M. Blaha, L. Potužák, 2013, pp. 291-292) that is compatible with NATO STANAG 4199 is not available, the relative ballistic density deviation $\Delta\rho_B$ (%) and virtual temperature ΔT_{Bv} (%) are manually calculated using the equations:

$$\Delta\rho_B(\%) = \frac{\rho_i - \rho_o}{\rho_o} \cdot 100 \,, \tag{20}$$

$$\Delta T_{Bv}(\%) = \frac{T_{iv}-T_o}{T_o} \cdot 100, \qquad (21)$$

Where ρ_{i} is (real density), $\rho_{\mathrm{o}} = 1.225\mathrm{kg/m^3}$ (standard density for ICAO atmosphere at sea level), $\mathrm{T_{iv}}$ °K (measured virtual temperature), $\mathrm{T_o} = 273.15 + 15 = 288.15$ °K (standard virtual temperature for ICAO at sea level). The output results of the DMPMM direct model of **MPMM_D30** for the standard range $\mathrm{X_T} = 11000\mathrm{m}$ and the value of the ballistic deviation of the virtual temperature $\mathbf{\Delta T_{Bv}}$ $(\%) = \mathbf{+1}\%$ are shown in Figure 5. All other standard meteorological ballistic conditions remain unchanged. The ballistic deviation of virtual temperature $\mathbf{\Delta T_{Bv}}(\%) = \mathbf{+1}\%$ affected the range increase $\Delta\mathrm{X_{M-T}} = +3.7\mathrm{m}$. Therefore, the differential coefficient entered in column 15 in Table F(ii) is $\Delta\mathrm{X_{M-T}} = -3.7\mathrm{m}$.

Standard initial angle A_S from DMPMM model $\alpha_0(0) =$ ....18.47°
Standard range from DMPMM model $\mathrm{X_T}$ =..11000 m
Computational range from DMPMM model $\mathrm{X_M}$ =...11003.7 m
Correction range due to $\Delta\mathbf{T_{Bv}}$ $(\%) = \mathbf{+1}\%$ $\Delta\mathrm{X_{M-T}}$ =...-3.7m
Standard muzzle velocity $\mathrm{V_{02}}$ =...693.3 m/s
Maximum ordinate from DMPMM model $\mathrm{Y_m}$ = ...1308.4 m
Time of flight from DMPMM model $\mathrm{t_M}$ = ...31.42 s

Figure 5. Correction of range due to the ballistic deviation of virtual temperature $\Delta\mathrm{T_{Bv}}$ = +1%

The output results of the DMPMM direct model within **MPMM_D30** for the standard range $\mathrm{X_T} = 11000\mathrm{m}$ and the ballistic deviation value of virtual temperature $\mathbf{\Delta T_{Bv}}$ $(\%) = \mathbf{-1}\%$ are shown in Figure 6. All other standard meteorological-ballistic conditions remain unchanged. The ballistic deviation of the virtual temperature $\mathbf{\Delta T_{Bv}}$ $(\%) = \mathbf{-1}\%$ affected the range decrease $\Delta\mathrm{X_{M-T}} = -4.8\mathrm{m}$. Therefore, the differential coefficient entered in column 14 in Table F(ii) is $\Delta\mathrm{X_{M-T}} = +4.8\mathrm{m}$.

Standard initial angle A_E from DMPMM model $\alpha_0(0)$ = ...18.47°
Standard range from DMPMM model X_T =...11000 m
Computational range from DMPMM model X_M =...10995.2 m
Correction range due to ΔT_{Bv} (%) = -1% ΔX_{M-T} =..+4.8 m
Standard muzzle velocity V_{02} =...693.3 m/s
Maximum ordinate from DMPMM model Y_m = ...1310.6 m
Time of flight from DMPMM model t_M = ...31.45 s

Figure 6. Correction of range due to the ballistic deviation of virtual temperature $\Delta T_{Bv} = -1\%$

The output results of the DMPMM direct model within **MPMM_D30** for the standard range $X_T = 11000$m and the ballistic deviation value of density $\Delta\rho_B$ (%) = $+\mathbf{1}\%$ are shown in Figure 7. All other standard meteorological-ballistic conditions remain unchanged. The ballistic deviation of density $\Delta\rho_B$ (%) = $+\mathbf{1}\%$ affected the range decrease $\Delta X_{M-T} = -53.9$m. Therefore, the differential coefficient entered in column 17 in Table F(ii) is $\Delta X_{M-T} = +53.9$m.

Standard initial angle A_E from DMPMM model $\alpha_0(0)$ = ...18.47°
Standard range from DMPMM model X_T =...11000 m
Computational range from DMPMM model X_M =...10946.1 m
Correction range due to $\Delta\rho_B$ (%) = $+1\%$ ΔX_{M-T} =..+53.9 m
Standard muzzle velocity V_{02} =...693.3 m/s
Maximum ordinate from DMPMM model Y_m = ...1303.5 m
Time of flight from DMPMM model t_M = ...31.36 s

Figure 7. Correction of range due to the ballistic deviation of density $\Delta\rho_B = +1\%$

The output results of the DMPMM direct model within MPMM_D30 for the standard range $X_T = 11000$m and the ballistic deviation value of density $\Delta\rho_B$ (%) = $-\mathbf{1}\%$ are shown in Figure 8. All other standard meteorological-ballistic conditions remain unchanged. The ballistic deviation of density $\Delta\rho_B$ (%) = $-\mathbf{1}\%$ affected the range increase $\Delta X_{M-T} = +52.7$m. Therefore, the differential coefficient entered in column 16 in Table F(ii) is $\Delta X_{M-T} = -52.7$m.

Standard initial angle A_E from DMPMM model $\alpha_0(0) =$ ....18.47°
Standard range from DMPMM model $X_T =$...11000 m
Computational range from DMPMM model $X_M =$.................................11052.7 m
Correction range due to $\Delta\rho_B$ (%) $= -1\%$ $\Delta X_{M-T} =$.......................-52.7 m
Standard muzzle velocity $V_{02} =$...693.3 m/s
Maximum ordinate from DMPMM model $Y_m =$ ....1315.4 m
Time of flight from DMPMM model $t_M =$ ....31.51 s

Figure 8. Correction of range due to the ballistic deviation of density $\Delta\rho_B = -1\%$

In addition to the standard range $X_T = 11000$m, Table F(ii) in Figure 9 contains several other standard ranges in the interval from 11000m to the maximum range of 15221m for low and high angles using the **MPMM_D30** program.

TFTs for H122mmD30 by NATO STANAG 4119

charge **FULL** projectile **OF462/462Ž** fuze **RGM-2** V_0=693,3m/s

TABLE F(ii)

CORRECTIONS OF RANGE FOR NON-STANDARD CONDITIONS
$\Delta_C X$

1	10	11	12	13	14	15	16	17	18	19
	Range corrections $\Delta_C X$									
Range (X)	1 M/S muzzle velocity (1 m/s V_0)		1 KT range wind (1KT W_X)		1% ballistic air temperature (1% T_B)		1% ballistic air density (1% ρ_B)		1 mark masss projectile ($m_n \pm 1$ *mark*)	
	Dec (-)	Inc. (+)	Head $\overline{W}$	Tail $\underline{W}$	Dec (-)	Inc (+)	Dec. (-)	Inc (+)	Dec (-)	Inc (+)
m	m	m	m	m	m	m	m	m	m	m
.	.	.	.	.	.	.	.	.	.	.
11000					4.8	-3.7	-52.7	53.9		
11200					5.1	-3.9	-54.2	55.4		
.	.	.	.	.	.	.	.	.	.	.
15200					11.7	-12.0	-88.5	87.1		
15221					12.1	-12.2	-90.2	88.3		
HIGH ANGLE										
15200					12.5	-12.3	-90.1	88.5		
.	.	.	.	.	.	.	.	.	.	.
11200					10.6	-10.2	-71.3	69.5		
11000					10.5	-10.0	-70.1	68.3		
.	.	.	.	.	.	.	.	.	.	.

Figure 9. Table F(ii) according to NATO STANAG 4119

In this paper, Table F(ii) is completed with two differential coefficients (density and temperature). However, using the MPMM_D30 program, other differential coefficients can be calculated in the same way, and Table F(ii) can be filled out.

Conclusion

The MPMM ballistic model enables determination of all necessary parameters of the ballistic trajectory (range, time of flight, differential coefficients, maximum ordinate etc.) and is therefore very suitable for making firing tables according to various standards and design of computer fire control systems for classical spin stabilized artillery projectiles. As it is shown in this paper, with the help of MPMM model, a modern and complete process of conversion of any standardized firing tables to another standard can be performed if standard meteo-ballistic conditions, projectile data, its aerodynamic coefficients and other possible technical conditions according to which they were made are known. It should also be mentioned that the MPMM ballistic model is suitable for calculating all elements of a classical projectile (or mortar mine) trajectory and it uses a relatively small number of aerodynamic coefficients that define projectile aerodynamics with an accuracy close to the six-degree (6DOF) model.

Marks

A_0 azimuth of fire	[mil]
A_w azimuth of wind	[mil]
A_ε angle of twisting of the grooves	[°]
d caliber	[mm]
H air pressure	[hPa]
I_X body axial moment of inertia	[kg·m²]
KT knot	[m/s]
m mass	[kg]
p angular velocity (parameter)	[rad/s]
V_O muzzle velocity	[m/s]
V_W speed of wind	[m/s]
u horizontal velocity component	[m/s]
v vertical velocity component	[m/s]
w side velocity component	[m/s]
T_V virtual air temperature	[° K]
T_b propellant temperature	[° C]
X_T standard range	[m]

X_M MPMM model range [m]

X_{CS} complementary range [m]

$\Delta_c X_{CS}$ complementary correction of range [m]

Y_T target altitude [m]

Y_M MPMM model target altitude [m]

Y_P weapon altitude [m]

Greek letters

α_0 initial angle [°]

α angle of inclination of the tangent [°]

β sideslip [°]

ρ air density [kg/m^3]

φ_{LA} weapon latitude [°]

σ angle of attack [°]

Ω_E angular velocity of the Earth [rad/s]

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Pravilnik o normizaciji u MO i OSRH, izdanje NN 88/2019, broj dokumenta u izdanju 1759, (*članak 5, članak 6*)

About the author

Major Ivan Katalinić (ivankatal@net.hr) joined the CAF on June 30, 1991 in volunteer national protection detachments. On August 28, 1991, he became a member of Company "A", 2nd Battalion, 132nd Brigade, Našice, from which on August 10, 1992, he transferred to the Artillery Rocket Launcher Battalion, 3rd Guards Brigade (today's Armored Mechanized Guard Brigade) and remained there until January 31, 2016. Since then, he has been working as an artillery teacher at CDA "Dr. Franjo Tuđman" (Deanery, Department of Tactics, and Section of Combat Arms Support), where he teaches military training (Basic Officer Course and Advanced Officer Course) and military study programs in Military Engineering, Artillery. He is a member of the CAF GS Team for the adoption of fire management software for the needs of the CAF. Areas of interest and work include theory and firing rules of field artillery, artillery fire control systems and implementation of specialist capability objectives, and artillery STANAGs in artillery curricula.