Implementation of MPMM ballistic model for calculation of differential coefficients for TFTs according to NATO STANAG 4119

Ivan Katalinić

Abstract

MPMM ballistic model (modified point mass model) is used to calculate all necessary ballistic trajectory elements (range, time of flight, angle of fall, maximum ordinate, differential coefficients, etc.) for classical spin-stabilized artillery projectiles. In this paper, differential and algebraic equations that fully define MPMM ballistic model and its program solution (MPMM_D30) are presented. The MPMM_D30 enables calculation of differential coefficients for H122mmD30 howitzer (and other ballistic trajectory elements) that are an integral part of TFTs according to NATO STANAG\(^2\) 4119 (Adoption of a Standard Cannon Artillery Firing Table Format, Brussels, 2007), which should become HRVN STANAG\(^3\) 4119.

Keywords

external ballistics, ballistic model, standard atmosphere, projectile trajectory, meteorological report, differential equations, H122mmD30

---

1 The article was received by the Editorial Board on March 30, 2021 and accepted for publication on July 12, 2021.
2 STANAG (Official Gazette No. 88/2019, Article 6)
3 HRVN STANAG (Official Gazette No. 88/2019, Article 5)
Introduction

As a branch of the Croatian Army, artillery has adopted two NATO STANAGs as HRVN Croatian military standards that define the development of the TFTs according to NATO STANAG 4119. Those are HRVN STANAG 4355 (ballistic model MPMM) and HRVN STANAG 4044 (ICAO, International Civil Aviation Organization), respectively. The format of TFTs is established by STANAG 4119. It should soon be fully ratified and shall become HRVN STANAG 4119. The Croatian Armed Forces possess a variety of weapons from the so-called Eastern Country, where TFTs were developed according to “Russian” standards (M. Gajić and J. Viličić 1978, pp. 396-421). Those TFTs should be converted to NATO STANAG 4119 for compatibility with the artillery units of NATO member states. Differential coefficients or standard corrections according to which ballistic trajectory in the real atmosphere is calculated are the basic data in TFTs of any format. In this paper, a detailed description of the method for calculating differential coefficients for density and virtual temperature (humidity air feature included), which are the essential TFTs elements for the H122mm howitzer D30 according to NATO STANAG 4119, is given. The calculation is conducted based on the available TFTs according to the “Russian” standard. Other differential coefficients (muzzle velocity, projectile mass, wind, etc.) are calculated in the same manner. The MPMM ballistic model enables the realization of two main ballistic tasks, the direct and indirect one. The direct task of the external ballistics, based on the known initial elevation angle (elevation) in standard or real atmosphere calculates two main ballistic trajectory elements (as well as all other elements). Those are range and deflection from the firing plain. The direct task of the external ballistics is used for the development of TFTs by which the initial elements (elevation and deflection) for fire are calculated. Based on the known range in standard or real atmosphere, the indirect task of the external ballistics (INMPMM) calculates the optimal initial elevation angle and all other elements of the ballistic trajectory. It is used for the development of fire control systems for classical stabilized artillery projectiles. For the calculation of differential coefficients (and all other elements of the ballistic
trajectory), a program called **MPMM_D30** was developed according to the block diagram (Figure 3), which can solve direct (DMPMM) and indirect tasks of the external ballistics (INMPMM). The **MPMM_D30** enables the calculation and use (TC 3-09.81, 2016) of all tables (F(i), F(ii), G(i), H, etc.) that are an integral part of the TFTs according to NATO STANAG 4119, based on the "Russian" standard (M. Gajić and J. Viličić 1978, p 396-421) and their use (TC 3-09.81, 2016).

**MPMM ballistic model**

The MPMM is the basic ballistic model (HRVN STANAG 4355) that determines all necessary parameters of the ballistic trajectory for classical artillery spin-stabilized projectiles. The basic differential equations representing this model (Figure 1) are as follows:

![Figure 1. Graphic presentation of forces that influence the projectile mass m in the MPMM.](image-url)
Equation (1) (S. Janković 1998, pp. 234-235) shows which forces (Figure 1) affect the projectile of mass $m$, caliber $d$, projectile cross-sectional area $S$, and aerodynamic velocity that represents the projectile velocity with wind velocity.

- $\vec{F}_G = m \cdot \vec{g}$ (force of gravity),
- $\vec{D} = \frac{\rho \cdot V^2}{2} \cdot S \cdot C_D(\alpha) \cdot \frac{\vec{v}}{V}$ (aerodynamic drag force)
- $\vec{L} = \frac{\rho \cdot V^2}{2} \cdot S \cdot C_L \cdot \sigma \cdot \vec{v}_L$ (lift force)
- $\vec{F}_k = -2 \cdot m \cdot \vec{\Omega}_E \times \vec{V}_k$ (Coriolis force)

The flight velocity $\vec{V}_k$ shown by Equation (2) in the form of the vector and by Equation (3) in the windy atmosphere represents the projectile velocity (absolute) relative to the Earth. The force $\vec{F}_G$ is the intensity the gravity and is directed vertically downwards. The force $\vec{D}$ is a resistance drag force that lies on the velocity line $\vec{V}$ but is in the opposite direction. The force $\vec{L}$ is the lift force that appears due to the angle of attack $\sigma$ between the projectile axis and aerodynamic velocity. The lift force is proportional to the magnitude of the angle and lies in the plane formed by the projectile axis and aerodynamic velocity $\vec{V}$. It is vertical to the aerodynamic velocity and has a direction from the aerodynamic velocity toward the projectile axis. The total aerodynamic force applied to the projectile of mass $m$ is $\vec{R} = \vec{D} + \vec{L}$. The force $\vec{F}_k$ is Coriolis force due to the rotation of the Earth,
where $\Omega_E = 7.27 \cdot 10^{-5} \text{ rad/s}$ is angular velocity of the Earth. Dividing Equation (1) by the mass $m$ gives the equation that represents a total change in acceleration action on all the following forces:

$$\frac{d\vec{V}_k}{dt} = -\frac{\rho v^2}{2} \cdot \frac{s}{m} \cdot C_D \cdot \frac{\vec{v}_k-\vec{v}_W}{v} + \vec{g} + \frac{\rho v^2}{2} \cdot \frac{s}{m} \cdot C_L \sigma \cdot \vec{v}_L - 2 \cdot \vec{\Omega}_E \times \vec{V}_k. \quad (4)$$

By projecting Equation (4) on the axis of the local coordinate system (S. Janković 1998, pp. 29-30) LKS $(X_L, Y_L, Z_L)$, the final system of differential equations of the MPMM ballistic model is obtained (S. Janković 1998, pp. 241-242) with the range and deflection fitting $C_1(\alpha_0)$ and the projectile direction $C_2(\alpha_0)$:

$$\frac{du_k}{dt} = -E \cdot \left( C_D \cdot \frac{u_k-u_W}{v} + C_L \cdot \beta \cdot \frac{w_k-w_W}{\sqrt{v^2-v_k^2}} \right) - 2 \cdot (\Omega_{EZ} \cdot v_k + \Omega_{EX} \cdot w_k), \quad (5)$$

$$\frac{dv_k}{dt} = -E \cdot \left( C_D \cdot \frac{v_k}{v} \right) - g - 2 \cdot (\Omega_{EZ} \cdot u_k - \Omega_{EX} \cdot w_k), \quad (6)$$

$$\frac{dw_k}{dt} = -E \cdot \left( C_D \cdot \frac{w_k-w_W}{v} - C_L \cdot \beta \cdot \frac{u_k-u_W}{\sqrt{v^2-v_k^2}} \right) - 2 \cdot (\Omega_{ey} \cdot u_k + \Omega_{EX} \cdot v_k), \quad (7)$$

$$\frac{dp}{dt} = \frac{s \cdot a^2}{4 \cdot l_x} \cdot \rho \cdot V \cdot C_{tp} \cdot p, \quad (8)$$

$$\frac{dx}{dt} = u_k, \quad (9)$$

$$\frac{dy}{dt} = v_k, \quad (10)$$

$$\frac{dz}{dt} = w_k. \quad (11)$$

where:

- $u_k$ (horizontal velocity component in LKS)
- $v_k$ (vertical velocity component in LKS)
- $w_k$ (side velocity component in LKS)
- $u_W$ (horizontal wind component in LKS)
- $w_W$ (side wind component in LKS)
- $E = \frac{\rho \cdot V^2}{2} \cdot \frac{S}{m}$ (parameter)
- $g$ (gravity constant)
- $V_W$ (wind velocity)
- $V = \sqrt{(u_k - u_W)^2 + v_k^2 + (w_k - w_W)^2}$ (aerodynamic velocity)
- $V_k$ (flight velocity)
- $V_0$ (standard muzzle velocity)
- $I_x$ (body axial moment of inertia)
- $m_n$ (standard projectile mass)
- $d$ (caliber)
- $S = \frac{d^2 \cdot \pi}{4}$ (projectile cross-sectional area)
- $A_\varepsilon$ (angle of twisting of the grooves)
- $p = \frac{2 \cdot V_k \cdot \tan A_\varepsilon}{d}$ (angular velocity)
- $\alpha_0$ (initial elevation angle)
- $\alpha$ (angle of inclination of the tangent)
- $\beta = -\frac{2 \cdot I_x \cdot g \cdot \cos \alpha \cdot \rho \cdot S \cdot d \cdot V^3 \cdot C_{m_{\sigma}}}{\rho \cdot S \cdot d \cdot V^3 \cdot C_{m_{\sigma}}}$ (sideslip)
- $A_0$ (azimuth of fire)
- $A_W$ (azimuth of wind)
- $\varphi_{LA}$ (weapon position latitude)
- $\Omega_E$ (angular velocity of the Earth)
- $\Omega_{EX} = \Omega_E \cdot \cos A_0 \cdot \cos \varphi_{LA}$ (horizontal component of angular velocity of the Earth in LKS)
- $\Omega_{EY} = \Omega_E \cdot \sin \varphi_{LA}$ (vertical component of angular velocity of the Earth in LKS)
- $\Omega_{EZ} = -\Omega_E \cdot \sin A_0 \cdot \cos \varphi_{LA}$ (side component of angular velocity of the Earth in LKS)
- \( a \) (speed of sound)
- \( M = \frac{v}{a} \) (Mach number)
- \( a(y) \) (change of sound of speed with altitude)
- \( T_v(y) \) (change of virtual with altitude)
- \( H(y) \) (change of pressure with altitude)
- \( \rho(y) = 0.3484 \cdot \frac{H(y)}{T_v(y)} \) (change of density with altitude)
- \( C_{D43}(M) \) (total axial force coefficient)
- \( C_1(\alpha_0) \) (fitting range factor), \( C_2(\alpha_0) \) (fitting deflection factor)
- \( C_D = C_1(\alpha_0) \cdot C_{D43}(M) \) (corrected total axial force coefficient)
- \( C_L = C_2(\alpha_0) \cdot C_{L\sigma}(M) \) (corrected lift force coefficient)
- \( C_{L\sigma}(M) \) (lift force coefficient derivative with an angle of attack)
- \( C_{m\sigma}(M) \) (pitching moment coefficient derivative with angle of attack)
- \( C_{lp}(M) \) (damp in roll coefficient derivative).

For the MPMM consisting of seven differential equations to be complete, it has to be supplemented with two interior ballistic differential coefficients \( l_t \) and \( l_q \) which are used to correct the initial speed because of the propellant temperature \( l_t \) and projectile mass \( l_q \) that is determined according to Sluhocki (M. Gajić and J. Viličić 1978, pp. 151-153). It is well known that the propellant temperature \( T_b \) °C, that is higher than the standard propellant temperature \( T_{tb} \) °C, gives a higher muzzle velocity \( V_0 \). The projectile mass \( m \) that is bigger than the standard projectile mass \( m_n \) causes a reduction of the muzzle velocity. The muzzle velocity deviation created due to the difference between the measured and standard propellant temperatures is calculated according to the equation:

\[
\Delta V_{0b} = (T_b - T_{tb}) \cdot l_t \cdot V_0 \tag{12}
\]
The influence of the projectile mass on the muzzle velocity deviation is calculated according to the equation:

$$\Delta V_{0m} = -\frac{m-m_n}{m_n} \cdot l_q \cdot V_0$$  \hspace{1cm} (13)

An additional decrease of the muzzle velocity $\Delta V_0$ causes the gunpowder chamber to lengthen and the pipe to wear out. This muzzle velocity reduction is recorded in a technical weapon booklet. Now, muzzle velocity $V_k$ can be expressed by the equation:

$$V_k = V_0 + \Delta V_{0b} + \Delta V_{0m} + \Delta V_{0c}$$  \hspace{1cm} (14)

Differential and algebraic equations containing the complete MPMM ballistic model are presented here. By solving them, the ballistic trajectory of the classical spin-stabilized projectile is completely determined in the standard or real atmosphere. The real atmosphere is obtained by radio sounding observation, i.e. by measuring the parameters of the atmosphere by altitude (temperature, humidity, azimuth of wind, wind velocity, pressure, etc.) which are measured in the CAF by the meteo station "Vaisala" (K. Šilinger, M. Blaha, 2017, pp. 196-197). The differential equations of such a ballistic model are solved by numerical integration most often using the well-known Runge-Kutta Method. This method is used and programmed in this paper (MPMM_D30).

**Differential coefficients**

All TFTs, regardless of whether they were made according to the “Russian” or NATO standards, with the most important elements of ballistic trajectories (range, flight time, trajectory, maximum ordinate, etc.), which are calculated for certain initial angles in the adopted standard atmosphere (ICAO) also contain differential coefficients or standard corrections by which the standard ballistic trajectory is “corrected” to the current meteorological-ballistic conditions. It is evident from the differential and algebraic equations of the complete MPMM ballistic model that each element of the ballistic trajectory “B” depends on a series of meteorological-
ballistic parameters $p_i$ (density, speed of sound, wind, propellent temperature, muzzle velocity deviation, projectile mass deviation, etc.), i.e., there is a functional dependence $B = f(p_i)$. The standard meteorological-ballistic parameters $B_0$ correspond to some standard ballistic trajectory elements $p_{i0}$. If they change by the amount of deviation $\delta p_i$ i.e. $p_i = p_{i0} + \delta p_i$, the ballistic trajectory element will also change by $\delta B$. The new value of the ballistic element $B_1$ will be $B_1 = B_0 + \delta B$. The amount of the correction (or deviation) of ballistic trajectory elements(s) $\delta B$ can be calculated using the Taylor equation:

$$\delta B = B_1 - B_0 = \frac{\partial B_0}{\partial p_1} \cdot \delta p_1 + \frac{\partial B_0}{\partial p_2} \cdot \delta p_2 + \ldots = \sum_{i=1}^{n} \frac{\partial B_0}{\partial p_i} \cdot \delta p_i . \quad (15)$$

in which higher order members are neglected:

$$\frac{\partial^2 B_0}{\partial p_1 \partial p_2} \cdot \delta p_1 \cdot \delta p_2, \frac{\partial^2 B_0}{\partial p_1 \partial p_3} \cdot \delta p_1 \cdot \delta p_3, \frac{\partial^2 B_0}{\partial p_2 \partial p_3} \cdot \delta p_2 \cdot \delta p_3$$

because $\delta p_i$ are small. The values $\frac{\partial B_0}{\partial p_1}, \frac{\partial B_0}{\partial p_2}, \frac{\partial B_0}{\partial p_3}$ are called differential coefficients for the standard ballistic trajectory element $B_0$. Each perturbation $p_i$ causes a non-linear deviation of the element(s) of the ballistic trajectory because the differential equations are by their very nature non-linear. Under certain conditions, some linear deviations can have a linear change, but it significantly depends on the magnitude of the deviation and of ballistic element sensitivity to that deviation. Considering that those small deviations are usually observed in external ballistics, as they are in this case, the deviation of the ballistic trajectory element has an almost linear change. The linearity of the deviation of the ballistic trajectory element(s) in the event of the influence of a small deviation is proportional to the magnitude of that deviation. The influence of several different deviations is the sum of the influence of individual deviations, as shown by Equation (15). Thus, for example, to correct the range $\delta X$ as the ballistic trajectory element that depends on the virtual temperature $T_v$, density $\rho$, range wind $W_X$, and projectile mass $m$, it can be written:

$$\delta X = \frac{\partial X}{\partial T_v} \cdot \delta T_v + \frac{\partial X}{\partial \rho} \cdot \delta \rho + \frac{\partial X}{\partial W_X} \cdot \delta W_X + \frac{\partial X}{\partial m} \cdot \delta m . \quad (16)$$
The values $\frac{\partial x}{\partial T_v}, \frac{\partial x}{\partial \rho}, \frac{\partial x}{\partial W_X}, \frac{\partial x}{\partial m}$ are called differential coefficients for temperature, density, range wind, and projectile mass. In TFTs, they are used for a certain deviation value, e.g. for $\Delta T_v = 1\% (1^\circ C)$, $\Delta \rho = \Delta \rho = 1\%$, $\Delta W_X = \Delta W_X = 1 m/s$, $\Delta m = \Delta m = 1$ weight mark. They are called range corrections or deviations $\delta X$, and are marked by $\Delta X_{T_v}, \Delta X_{\rho}, \Delta X_{W_X}, \Delta X_m$. It should be noted that artillery live firing is carried out in all weather conditions. Therefore, the deviation magnitudes $\delta p_i$ for virtual temperature, wind speed, and propellant temperature affecting domestic projectile range can be several times higher than the values for which their respective differential coefficients $\frac{\partial x}{\partial p_i}$ were calculated. In that case, the range deviations $\delta X_{p_i}$ for each of these deviations can exceed the allowable linearity limit which applies only to small deviations, while the trend of change in nature is always non-linear. It can be easily explained with Figure 2, where the change in the range $X$ depending on a particular parameter $p$ is shown.

![Figure 2. Change of range X in proportion to the deviation magnitude \( \delta p \).](image)
In Figure 2, a certain parameter $p$ corresponds to a particular standard range $X_o$. The differential coefficient $\frac{\partial X_o}{\partial p_o} = \tan(\alpha)$ in point $N$ is represented by the tangent $t_N$. The range $X_1$ can be calculated using the differential coefficient and positive deviation value $+\delta p$ based on the equation $X_1 = X_o + \frac{\partial X_o}{\partial p_o} \cdot \delta p$. However, Figure 2 clearly shows that this equation gives us the range $X_{01}$ but not the real range $X_1$. In that way, a small error is made which is $\Delta X_1 = X_1 - X_{01}$. When determining the range $X_2$ based on twice the deviation $2 \cdot \delta p$ using the differential coefficient $\frac{\partial X_o}{\partial p_o}$ i.e. $X_2 = X_o + \frac{\partial X_o}{\partial p_o} \cdot 2\delta p$ (the range $X_{01}$ is obtained but not the real range $X_1$), a significant error occurs in determining the actual range $X_2$, which is $\Delta X_2 = X_2 - X_{12}$. It follows that the initial elements of artillery live firing are more accurately determined by fire control systems based on the MPMM ballistic model (which correctly calculates the real ranges), than by the “manual” method using TFTs, where standard ranges are “corrected” to real ones by the means of differential coefficients. Due to their non-linearity, those coefficients give an error in determining the real ranges. It should be noted here (Figure 2) that when the deviation value is negative $-\delta p$, and the value of the real range $X_{-1}$ is determined based on the differential coefficient $\frac{\partial X_o}{\partial p_o}$ by the equation $X_{-1} = X_o - \frac{\partial X_o}{\partial p_o} \cdot \delta p$, the value of the real range $X_{-1}$ will not be obtained. Instead, the real range value will be obtained. In other words, the error $\Delta X_{-1} = X_{-1} - X_{o-1}$ will appear which is greater than the error $\Delta X_1$ obtained for the positive deviation value $+\delta p$. It is also clear that this can be reversed depending on the type of differential coefficient and its change in a particular parameter $p$. Due to this fact, TFTs by NATO STANAG 4119 have two differential coefficients for one parameter. The first differential coefficient $\frac{\partial X_o}{\partial p_o+}$ is used for the area of positive deviations $+\delta p$, while the second differential coefficient $\frac{\partial X_o}{\partial p_o+}$ is used for the area of negative deviations $-\delta p$. In this way, the interval of linear deviation applicability is extended, and real changes are reached more precisely. Differential coefficients are calculated for the “Russian” TFTs standard using the equation:
\[
\frac{\partial B_o}{\partial p_o} = \frac{B(p_o + \Delta p) - B(p_o - \Delta p)}{2\Delta p}.
\]  (17)

The procedure of calculating differential coefficients using Equation (17) is known as a differential method. It is performed using the \texttt{MPMM_D30} program by calculating the ballistic trajectory twice, i.e., calculating two values of the ballistic trajectory element to obtain one differential coefficient. The first time the parameter \( p_o \) to which the standard element of trajectory \( B_o \) belongs is increased by \( \Delta p \), and the trajectory element \( B(p_o + \Delta p) \) is calculated. The second time the parameter \( p_o \) is decreased by \( \Delta p \) and the trajectory element \( B(p_o - \Delta p) \) is calculated. In both cases, all other parameters \( p_{oi} \) should remain unchanged. The obtained results are included in Equation (17) and the differential coefficient \( \frac{\partial B_o}{\partial p_o} \) for the trajectory element \( B_o \) and the parameter \( p_o \) are calculated. Differential coefficients for TFTs according to NATO STANAG 4119 are calculated for the positive deviation value \( +\delta p \) by the equation:

\[
\frac{\partial B_o}{\partial p_{o+}} = \frac{B(p_o + \Delta p) - B(p_o)}{\Delta p}.
\]  (18)

While for the negative deviation value \( -\delta p \) they are calculated by using the equation:

\[
\frac{\partial B_o}{\partial p_{o-}} = \frac{B(p_o - \Delta p) - B(p_o)}{\Delta p}.
\]  (19)

The procedure of calculating differential coefficients for positive \( +\delta p \) and negative \( -\delta p \) deviation values for TFTs according to NATO STANAG 4119 is also performed using the \texttt{MPMM_D30} program and differential method except that in this case, the ballistic trajectory for one differential coefficient should be calculated only once. The differential coefficients, regardless of whether they are calculated according to the “Russian” or NATO STANAG 4119, are constant for one ballistic trajectory.
Procedure for calculating differential coefficients

The calculation of differential coefficients for density and virtual temperature as components of TFTs according to NATO STANAG 4119 is in this paper performed for the HI22mmD30 howitzer for several standard ranges in the interval from 11000m to a maximum range of 15221m for low and high angles for the full charge with OF-462 projectile and RGM-2 fuze. In order for such a calculation to be performed, it is necessary to determine the dependence of the of the fitting range coefficients $C_1(\alpha_0)$ and deflections $C_2(\alpha_0)$ on the initial elevation angle $\alpha_0$. from TFTs according to the “Russian” standard. Range and deflection fitting coefficients are also determined by using the MPMM_D30 program for standard ANA atmosphere (artillery normal atmosphere) and standard ballistic conditions according to which TFTs were developed by the “Russian” standard (M. Gajić, and J. Viličić 1978, pp. 396-421) but they are valid for all standard or real atmospheres, including the ICAO (HRVN STANAG 4044) atmosphere on the basis of which TFTs are developed by NATO STANAG 4119. Using the Least Squares Method (R. Scitovski, 1993, pp. 39-42), the obtained fitting coefficients, which depend on the initial angles $\alpha_0$, are approximated by certain function-models, most often in the form of polynomials $C = b_0 + b_1 \cdot \alpha_0^1 + b_2 \cdot \alpha_0^2 + \ldots$. This method is used to determine the required coefficients $b_0, b_1, b_2$, etc. The function-model in the form of a polynomial of a higher degree will be a smaller approximation error. The calculated fitting range $C_1(\alpha_0)$ and the deflection $C_2(\alpha_0)$ coefficients of the projectiles are in the MPMM_D30 program code. For the MPMM_D30 program to solve the indirect INMPMM model, i.e., to determine the initial angle $\alpha_0$ based on the known range in the real or standard atmosphere, ICAO must contain the initial approximation angles $\alpha_0$ that correspond to these ranges. In this paper, the initial angles are determined based on standard ranges $X_T$ from TFTs according to the “Russian” standard using the Least Squares Method. The functional dependence $\alpha_0(X_T)$ of the initial angle $\alpha_0$ on the corresponding standard range $X_T$ is determined. The approximation of the initial angle $\alpha_0$ proved to be sufficiently accurate because the standard
ANA and ICAO atmospheres differ slightly. (ICAO is 1.6% “heavier” than ANA). The functional dependences $\alpha_0(X_\tau)$ of the initial angle $\alpha_0$ on the standard range $X_\tau$ for low angles are found in Table 1 and for the high angles in Table 2, according to the “Russian” standard.

### Table 1. Initial angle dependence $\alpha_0$ on range $X_\tau$ for low angles from TFTs according to the “Russian” standard

<table>
<thead>
<tr>
<th>$\alpha_0$</th>
<th>18.36°</th>
<th>26.04°</th>
<th>38.46°</th>
<th>45°</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_\tau$</td>
<td>11000m</td>
<td>13000m</td>
<td>15000m</td>
<td>15286m</td>
</tr>
<tr>
<td>$\alpha_0(X_\tau)$</td>
<td>$-184.45532 + 43.838276 \cdot \alpha_0 - 3.436705 \cdot \alpha_0^2 + 0.09009344 \cdot \alpha_0^3$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 2. Initial angle dependence $\alpha_0$ on range $X_\tau$ for high angles from TFTs according to the “Russian” standard

<table>
<thead>
<tr>
<th>$\alpha_0$</th>
<th>66.84°</th>
<th>61.02°</th>
<th>50.7°</th>
<th>45°</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_\tau$</td>
<td>11000m</td>
<td>13000m</td>
<td>15000m</td>
<td>15286m</td>
</tr>
<tr>
<td>$\alpha_0(X_\tau)$</td>
<td>$37.77706 - 7.7175736 \cdot \alpha_0 + 0.6537563 \cdot \alpha_0^2 - 0.01902082 \cdot \alpha_0^3$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The procedure for calculating the differential coefficients for virtual temperature $T_v$ and density $\rho$ for the TFT format by NATO STANAG 4119 using **MPMM_D30** is shown on the standard range of 11000m for low angles. The procedure is the same for all ranges and angles. Since the original TFTs according to NATO STANAG are developed for standard propellant temperature $T_{tb} = 21°C$, while the TFTs according to the “Russian” standard are made for standard propellant temperature $T_{tb} = 15°C$ it is necessary to calculate the increase of the standard muzzle velocity $V_0$ on the basis of which the calculation procedure will be conducted because higher propellant temperature increases the muzzle velocity. The interior ballistic correction coefficient for propellant temperature (Sluhocki) for the standard muzzle velocity of $V_{01} = 690 m/s$ is $l_t = 0.0008$. The required increase of standard muzzle velocity is now calculated on the basis of Equation (12) $\Delta V_{0b} = (T_b - T_{tb}) \cdot l_t \cdot V_{01} = (21°C - 15°C) \cdot 0.0008 \cdot 690 = 3.312 m/s$. Now the the standard muzzle
velocity for TFTs according to NATO STANAG 4119 used by \textbf{MPMM\_D30} is $V_{02} = 693.3\text{m/s}$.

\textit{Necessary input parameters for differential coefficient calculation}

- PeP \rightarrow full charge
- OF-462 projectile with RGM-2 fuze
- $\alpha_0 \rightarrow$ initial angle for DMPMM ballistic model
- $C_1(\alpha_0), C_2(\alpha_0) \rightarrow$ fitting coefficients (integrated in \textbf{MPMM\_D30})
- $\alpha_0(X_T) \rightarrow$ (Tables 1 and 2)
- $V_{01} = 690\text{ m/s}$ \rightarrow standard muzzle velocity for “Russian” standard for TFTs
- $V_{02} = 693.3\text{ m/s}$ \rightarrow standard muzzle velocity for NATO STANAG 4119
- $m_n = 21.76\text{ kg}$ \rightarrow standard projectile mass
- 1 weight mark (+ or -) on OF-462 projectile = 0.145 kg
- $I_x = 0.0414\text{ kg} \cdot \text{m}^2$ \rightarrow body axial moment of inertia
- $\alpha_{ec} = 7.15^\circ \rightarrow$ angle of twisting of the grooves
- $l_t = 0.0008 \rightarrow$ interior ballistic coefficient for propellant temperature
- $l_q = 0.31 \rightarrow$ interior ballistic coefficient for projectile mass
- $\phi_{LA} = 45^\circ \rightarrow$ weapon position latitude (FP)
- $\alpha_k = 1^\circ \rightarrow$ initial angle correction $\alpha_0(0)$ for INMPMM ballistic model
- \textit{mil} = one thousandth 1/6400
- $td$ \rightarrow one thousandth 1/6000
Aerodynamic coefficients for OF-462 (S. Janković 1998, CD) are shown in Table 3.

Table 3. Aerodynamic coefficients for OF-462

<table>
<thead>
<tr>
<th>M</th>
<th>$C_D$</th>
<th>$C_{L\alpha}$</th>
<th>$C_l\beta$</th>
<th>$C_{m\alpha}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.157</td>
<td>1.635</td>
<td>-0.038</td>
<td>2.831</td>
</tr>
<tr>
<td>0.4</td>
<td>0.157</td>
<td>1.610</td>
<td>-0.034</td>
<td>2.874</td>
</tr>
<tr>
<td>0.6</td>
<td>0.158</td>
<td>1.566</td>
<td>-0.031</td>
<td>2.950</td>
</tr>
<tr>
<td>0.8</td>
<td>0.160</td>
<td>1.470</td>
<td>-0.029</td>
<td>3.126</td>
</tr>
<tr>
<td>0.9</td>
<td>0.190</td>
<td>1.355</td>
<td>-0.028</td>
<td>3.339</td>
</tr>
<tr>
<td>0.94</td>
<td>0.244</td>
<td>1.311</td>
<td>-0.027</td>
<td>3.507</td>
</tr>
<tr>
<td>0.96</td>
<td>0.271</td>
<td>1.299</td>
<td>-0.027</td>
<td>3.579</td>
</tr>
<tr>
<td>0.98</td>
<td>0.298</td>
<td>1.325</td>
<td>-0.027</td>
<td>3.591</td>
</tr>
<tr>
<td>1.05</td>
<td>0.360</td>
<td>1.597</td>
<td>-0.026</td>
<td>3.651</td>
</tr>
<tr>
<td>1.1</td>
<td>0.382</td>
<td>1.787</td>
<td>-0.026</td>
<td>3.455</td>
</tr>
<tr>
<td>1.2</td>
<td>0.385</td>
<td>2.044</td>
<td>-0.025</td>
<td>3.189</td>
</tr>
<tr>
<td>1.3</td>
<td>0.382</td>
<td>2.218</td>
<td>-0.024</td>
<td>3.039</td>
</tr>
<tr>
<td>1.5</td>
<td>0.361</td>
<td>2.440</td>
<td>-0.023</td>
<td>2.840</td>
</tr>
<tr>
<td>2</td>
<td>0.316</td>
<td>2.702</td>
<td>-0.020</td>
<td>2.551</td>
</tr>
<tr>
<td>2.5</td>
<td>0.287</td>
<td>2.707</td>
<td>-0.016</td>
<td>2.322</td>
</tr>
</tbody>
</table>
Implementation of MPMM ballistic model for calculation of differential coefficients for TFTs according to NATO STANAG 4119.
Figure 3: Block diagram for MPMM_D30

Output parameters

\[ X_M, \alpha_0(0), \alpha_0(1), Y_m, t_M, V_o, A_x, p_x, A_s, A_{CS}, \Delta A_{CS} \]

\[ A_s = \arctg \left( \frac{Y_T - Y_P}{X_M} \right), A_{CS} = \alpha(1) - \alpha(0) - A_s \]

\[ \Delta A_{CS} = \frac{A_{CS}}{A_s} \]

Computing the complementary range \( X_{CS} \)

\[ \alpha_0(0) = \alpha_0(0) + A_{CS} \]

\[ \alpha_0(1) = 0, Y_P = 0, Y_T = 0 \]

Ballistic model = DMPMM

Calculating a new initial angle \( \alpha_0 \) using the correction angle \( \alpha_k \) to "limit" the range \( X_T \):

\[ \alpha_0(0) = \alpha_0(0) \pm \alpha_k \]

Calculating a new initial angle \( \alpha_0 \) using the correction angle \( \alpha_k \) after "limiting" the range \( X_T \):

\[ \alpha_k = \frac{\alpha_k}{2} \]

\[ \alpha_0(0) = \alpha_0(0) \pm \alpha_k \]
All standard ballistic and meteorological conditions and standard range $X_T = 11000m$ are entered into MPMM_D30 and using the indirect ballistic INMPMMP model, the following output results are obtained. They are shown in Figure 4.

![Figure 4. Ballistic trajectory parameters for standard range $X_T = 11000m$ by NATO STANAG 4119](image)

The symbols used for elevation angles and deflection according to NATO STANAG 4119 are:

$\alpha_Q$ (quadrant elevation), $\alpha_E$ (standard angle of elevation), $\alpha_S$ (angle of site), $\alpha_{CS}$ (complementary angle of site), $\Delta\alpha_{CS}$ (complementary angle of site for 1 mil), $\alpha_\omega$ (angle of fall), $\alpha_D$ (drift), $X_{CS}$ (complementary range). The complementary range $X_{CS}$ (Elizabeth R. Dickinson, 1967, pp. 57-58) is the range with the complementary correction $\Delta_cX_{CS}$ that is present due to the difference between the weapon altitude and the target. The total elevation for shooting a target if there is an altitude difference between the weapon position $Y_p$ and the target $Y_T$ is $\alpha_Q = \alpha_E + \alpha_S + \alpha_{CS}$. In MPMM_D30, the total elevation $\alpha_Q$ is marked by $\alpha_0(1)$, and the standard initial angle $\alpha_E$ by $\alpha_0(0)$ due to the adjustment of the notation to the program code structure. Table F(ii) (Figure 9) in TFTs developed by NATO STANAG 4119 is employed to determine differential coefficients (range corrections) that are used to calculate the ballistic trajectory of the projectile in the real atmosphere. The direct DMPMM model in MPMM_D30 is used to
calculate Table F(ii). In TFTs according to NATO STANAG 4119, the state of the atmosphere is determined by differential coefficients for the ballistic deviation of density $\Delta \rho_B$ and ballistic deviation of virtual temperature $\Delta T_{Bv}$. According to the “Russian” standard, the state of the atmosphere is determined by differential coefficients for ballistic air pressure deviation $\Delta H_B$ and ballistic deviation of virtual temperature $\Delta T_{Bv}$. The differential coefficient for the ballistic deviation of the virtual temperature $\Delta T_{Bv}$ in TFTs according to NATO STANAG 4119 is the differential coefficient for the ballistic deviation of the speed of sound $\Delta a_B$ because the change in the virtual temperature is already included in the density. Therefore, in TFTs according to NATO STANAG, instead of the differential coefficient for ballistic deviation of the speed of sound $\Delta a_B = 0.5\%$, the equivalent differential coefficient for ballistic deviation of virtual temperature $\Delta T_{Bv} = 1\%$ is entered (temperature change of 1\% changes the speed of sound by 0.5\%). The influence of the virtual temperature $T_v$ in TFTs according to “Russian” standard is different from the virtual temperature in TFTs according to NATO STANAG 4119. According to the “Russian” standard, temperature has dual effect, i.e. virtual temperature $T_v$ changes the function of pressure $H$ with an altitude (according to the vertical balance of the atmosphere), which immediately affects the change of density and the speed of the sound. Thus, in TFTs according to the “Russian” standard, the influence of virtual temperature $T_v$ is shown as a sum of both effects. It is clear that the effect of the virtual temperature $T_v$ for both standards is only the result of the MPMM ballistic model programming method (for both standards the differential coefficients for the effect of the wind are calculated in the same manner using the MPMM D30. When meteorological reports METB3 (HRVN STANAG 4061, M. Blaha, L. Potužák, 2013, pp. 291-292) that is compatible with NATO STANAG 4199 is not available, the relative ballistic density deviation $\Delta \rho_B$ (%) and virtual temperature $\Delta T_{Bv}$ (%) are manually calculated using the equations:

$$\Delta \rho_B(\%) = \frac{\rho_i-\rho_o}{\rho_o} \cdot 100 \ .$$

(20)
Implementation of MPMM ballistic model for calculation of differential coefficients for TFTs according to NATO STANAG 4119

\[ \Delta T_{BV}(\%) = \frac{T_{iv} - T_0}{T_0} \cdot 100, \]  

(21)

Where \( \rho_i \) is (real density), \( \rho_0 = 1.225\text{kg/m}^3 \) (standard density for ICAO atmosphere at sea level), \( T_{iv} \) °K (measured virtual temperature), \( T_0 = 273.15 + 15 = 288.15 \) °K (standard virtual temperature for ICAO at sea level). The output results of the DMPMM direct model of MPMM_D30 for the standard range \( X_T = 11000\text{m} \) and the value of the ballistic deviation of the virtual temperature \( \Delta T_{BV}(\%) = +1\% \) are shown in Figure 5. All other standard meteorological ballistic conditions remain unchanged. The ballistic deviation of virtual temperature \( \Delta T_{BV}(\%) = +1\% \) affected the range increase \( \Delta X_{M-T} = +3.7\text{m} \). Therefore, the differential coefficient entered in column 15 in Table F(ii) is \( \Delta X_{M-T} = -3.7\text{m} \).

The output results of the DMPMM direct model within MPMM_D30 for the standard range \( X_T = 11000\text{m} \) and the ballistic deviation value of virtual temperature \( \Delta T_{BV}(\%) = -1\% \) are shown in Figure 6. All other standard meteorological-ballistic conditions remain unchanged. The ballistic deviation of the virtual temperature \( \Delta T_{BV}(\%) = -1\% \) affected the range decrease \( \Delta X_{M-T} = -4.8\text{m} \). Therefore, the differential coefficient entered in column 14 in Table F(ii) is \( \Delta X_{M-T} = +4.8\text{m} \).
Figure 6. Correction of range due to the ballistic deviation of virtual temperature $\Delta T_v = -1\%$

The output results of the DMPMM direct model within MPMM_D30 for the standard range $X_T = 11000\text{m}$ and the ballistic deviation value of density $\Delta \rho_B (\%) = +1\%$ are shown in Figure 7. All other standard meteorological-ballistic conditions remain unchanged. The ballistic deviation of density $\Delta \rho_B (\%) = +1\%$ affected the range decrease $\Delta X_{M-T} = -53.9\text{m}$. Therefore, the differential coefficient entered in column 17 in Table F(ii) is $\Delta X_{M-T} = -53.9\text{m}$.

Figure 7. Correction of range due to the ballistic deviation of density $\Delta \rho_B = +1\%$

The output results of the DMPMM direct model within MPMM_D30 for the standard range $X_T = 11000\text{m}$ and the ballistic deviation value of density $\Delta \rho_B (\%) = -1\%$ are shown in Figure 8. All other standard meteorological-ballistic conditions remain unchanged. The ballistic deviation of density $\Delta \rho_B (\%) = -1\%$ affected the range increase $\Delta X_{M-T} = +52.7\text{m}$. Therefore, the differential coefficient entered in column 16 in Table F(ii) is $\Delta X_{M-T} = -52.7\text{m}$.
In addition to the standard range $X_T = 11000\,\text{m}$, Table F(ii) in Figure 9 contains several other standard ranges in the interval from 11000m to the maximum range of 15221m for low and high angles using the MPMM_D30 program.

**Figure 9. Table F(ii) according to NATO STANAG 4119**

In this paper, Table F(ii) is completed with two differential coefficients (density and temperature). However, using the MPMM_D30 program,
other differential coefficients can be calculated in the same way, and Table F(ii) can be filled out.

Conclusion

The MPMM ballistic model enables determination of all necessary parameters of the ballistic trajectory (range, time of flight, differential coefficients, maximum ordinate etc.) and is therefore very suitable for making firing tables according to various standards and design of computer fire control systems for classical spin stabilized artillery projectiles. As it is shown in this paper, with the help of MPMM model, a modern and complete process of conversion of any standardized firing tables to another standard can be performed if standard meteo-ballistic conditions, projectile data, its aerodynamic coefficients and other possible technical conditions according to which they were made are known. It should also be mentioned that the MPMM ballistic model is suitable for calculating all elements of a classical projectile (or mortar mine) trajectory and it uses a relatively small number of aerodynamic coefficients that define projectile aerodynamics with an accuracy close to the six-degree (6DOF) model.
Marks

$A_0$ azimuth of fire [mil]

$A_w$ azimuth of wind [mil]

$A_e$ angle of twisting of the grooves [$^\circ$]

$d$ caliber [mm]

$H$ air pressure [hPa]

$I_X$ body axial moment of inertia [kg·m²]

$KT$ knot [m/s]

$m$ mass [kg]

$p$ angular velocity (parameter) [rad/s]

$V_0$ muzzle velocity [m/s]

$V_W$ speed of wind [m/s]

$u$ horizontal velocity component [m/s]

$v$ vertical velocity component [m/s]

$w$ side velocity component [m/s]

$T_V$ virtual air temperature [$^\circ$ K]

$T_b$ propellant temperature [$^\circ$ C]

$X_T$ standard range [m]
\( X_M \) MPMM model range \([\text{m}]\)

\( X_{CS} \) complementary range \([\text{m}]\)

\( \Delta_c X_{CS} \) complementary correction of range \([\text{m}]\)

\( Y_T \) target altitude \([\text{m}]\)

\( Y_M \) MPMM model target altitude \([\text{m}]\)

\( Y_F \) weapon altitude \([\text{m}]\)

**Greek letters**

\( \alpha_0 \) initial angle \([^\circ]\)

\( \alpha \) angle of inclination of the tangent \([^\circ]\)

\( \beta \) sideslip \([^\circ]\)

\( \rho \) air density \([\text{kg/m}^3]\)

\( \varphi_{LA} \) weapon latitude \([^\circ]\)

\( \sigma \) angle of attack \([^\circ]\)

\( \Omega_E \) angular velocity of the Earth \([\text{rad/s}]\)
References

S. Janković (Zagreb 1998), *Mehanika leta projektila*, Fakultet strojarstva i brodogradnje

M. Gajić, i J. Viličić (Beograd 1978), *Balistika*, Vojnoizdavački zavod

Adoption of a standard cannon artillery firing table format (Brussels, 2007, STANAG 4119

TC 3-09.81 (Washington DC, 2016), *Field Artillery Manual Cannon Gunnery*, Headquarters, Department of the Army


Elizabeth Dickinson (Maryland, 1967), *The production of firing tables for cannon artillery*, Ballistic research laboratories, report no.1371

R. Scitovski (Osijek 1993), *Problemni najmanjih kvadrata i financijska matematika*, Elektrotehnički fakultet Osijek

Pravilnik o normizaciji u MO i OSRH, izdanje NN 88/2019, broj dokumenta u izdanju 1759, (članak 5, članak 6)
About the author

Major Ivan Katalinić (ivankatal@net.hr) joined the CAF on June 30, 1991 in volunteer national protection detachments. On August 28, 1991, he became a member of Company “A”, 2nd Battalion, 132nd Brigade, Našice, from which on August 10, 1992, he transferred to the Artillery Rocket Launcher Battalion, 3rd Guards Brigade (today’s Armored Mechanized Guard Brigade) and remained there until January 31, 2016. Since then, he has been working as an artillery teacher at CDA “Dr. Franjo Tuđman” (Deanery, Department of Tactics, and Section of Combat Arms Support), where he teaches military training (Basic Officer Course and Advanced Officer Course) and military study programs in Military Engineering, Artillery. He is a member of the CAF GS Team for the adoption of fire management software for the needs of the CAF. Areas of interest and work include theory and firing rules of field artillery, artillery fire control systems and implementation of specialist capability objectives, and artillery STANAGs in artillery curricula.