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Internal Motivation and Students' Knowledge of Math

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Abstract

The aim of this research was to determine whether teaching mathematics in an interesting way has an effect on increasing motivation and success of high school students. With this purpose, an experimental research was carried out with comparative groups in high schools in Čitluk, Ljubuški and Mostar on a sample of 300 students. Interesting teaching methods were utilised with students in the experimental group, while the control group worked in a traditional way. The survey questionnaire and the test of knowledge served as measuring instruments. Testing was performed at the beginning and at the end of the study. Lykert-type scale was used to test internal motivation while statistical significance of statistical significance of differences was measured using the mixed ANOVA model 2x2 and t- test. The results show a better effect of interesting teaching on the success of students, as well as an increase in students' motivation to learn mathematics. Therefore, it is recommended to apply interesting methods in teaching mathematics.

Key words: *education; learning; modern methods; teaching.*

Introduction

When discussing student's motivation for learning mathematics, and other subjects as well, we differentiate extrinsic and intrinsic motivation. "Intrinsic motivation is the response of inner student's needs, such as curiosity, need for knowledge, sense of competence, growth and development. Extrinsic motivation refers to the motivation that has the source outside the student, for example, a good test result, grades, commendation from the teacher, diplomas or medals and other awards" (Vizek-Vidović et al., 2003, p. 245). Therefore, intrinsic or internal motivation is what drives students to learn

from within. To increase intrinsic motivation, students need to be interested in the mathematical teaching content prescribed by the curriculum that cannot be changed by teachers, but which can be adapted to be more interesting to the students. According to Vizek et al., intrinsic motivation can be enhanced through the following: adapting teaching to students' interests, introducing novelty and diversity in teaching, active student participation and quick feedback, encouraging curiosity, linking teaching with personal life, and helping students to set their own goals. Middleton and Spanias (1999) described in their work the theoretical orientations that have been explored for motivation in teaching math. They also discussed the types of motivation in education and the role of teachers on students' motivation for learning math. According to the authors, there is internal and external motivation. "Internal motivation is the urge or desire of student to engage in learning for their own benefit" (Middleton & Spanias, 1999, p. 66). Students with inner motivation learn mathematics with pleasure, as opposed to those who are externally motivated and who learn to receive awards. According to Middleton and Spanias (1999), internal motivation refers to the students' perceptions of their competence in mathematics, whether they are motivated by curiosity or ratings. When students see that they can solve mathematical tasks, they will love math and will have a tendency for success, unlike those who think they are not talented in math (Eccles et al., 1987; Midgley et al., 1989). Students should feel comfortable learning math and doing the tasks and should expect to succeed before the development of internal motivation begins.

Middleton et al. (1992) have explored the influence of academic activity on students' internal motivation. "Their analysis has shown that children tend to organize their constructs into three general categories: excitement or cognitive stimulation provided by the activity; personal control, i.e. the degree to which an activity is considered free choice or appropriate difficulty; and interest (loosely defined category), i.e. the degree to which students liked their activity, the importance of their work and their ability to carry out their activities "(Middleton & Spanias, 1999, p. 75). If a student is interested in learning math, he will learn with pleasure regardless of the terms of task assessment.

Middleton (1993b) examines the impact of student motivation of a pilot curriculum that provides more learning opportunities to students, more choices of strategies and activities, and more challenging tasks than the traditional curriculum with activities placed in real situations, oplaced in real situations. Data show that teachers have started to notice the importance of a personalized curriculum to make math more meaningful to their students. The research results on the paradigms of personal constructs have shown that mathematical education motives are highly individual, linked to perception of ability, and are relatively stable with respect to success and failure. Some of the individual differences in motivation can be explained in relation to the identification of students with math teachers.

Research findings suggest that the decline in positive attitudes towards math can partly be explained by the lack of support of teachers and classroom environments.

"These results, along with the results of national evaluations (Dossey et al., 1988), indicate that motivational patterns have been taught, which is disturbing, as it shows that students mostly learn not to love math and that non-liking becomes an integral part of their mathematical self-concepts" (Middleton & Spanias, 1999, p 67).

Considering the subtle ways in which motivations are formed, modified and maintained, it becomes clear that there is no such thing as an unmotivated child. Children are motivated. "Motivations help guide children's activities; they provide a structure for evaluating the outcomes of the activity; and they help determine if children will participate in future mathematical activity or not "(Middleton & Spanias, 1999, p. 67).

Broussard and Garrison (2004) organize modern motivation research through three questions:

– Can I do this task?
– Do I want to do this task and why?
– Can I do this task?

As Broussard and Garrison noticed, those dealing with the first issue have developed several new theories about their own efficiency, attribution, and self-esteem. Bandura (1982) defines the perception of self-efficacy as a "judgment of how well the course of actions that are needed to resolve potential situations can be performed" (p. 122). Eccles and Wigfield (2002) elaborated Bandura's description, defining self-efficacy as an individual's trust in his "ability to organize and carry out a certain course of action to solve the problem or do the task" (p. 110).

According to Bandura's (1982) self-efficiency theory, efficiency is the main determinant of effort, persistence, and goal setting. The research carried out by Pintrich and DeGroot (1990) supports this idea. Self-efficiency is also associated with the use of cognitive strategies, and the perception of self-efficiency predicts success over and above the actual skill levels (Pintrich & DeGroot, 1990).

According to the second theory of motivation, the theory of control, individuals controlling their own successes and failures are more motivated (Eccles & Wigfield, 2002). Namely, in one formulation of control theory, autonomy is one of three basic psychological needs, alongisde competence and affiliation. Within this framework, individual differences, to the extent to which they meet the basic needs, correspond to variations in motivation levels (Connell & Wellborn 1991, acc. to Eccles & Wigfield, 2002). The control theory is closely related to the concept of attribution. There are several types of attributions including ability, effort, task, and happiness. There are several types of attributions depending on whether the cause of motivation can change and have personal control or not (Weiner, 1985, acc. to Eccles & Wigfield, 2002). For example, innate ability can be difficult to change, while personal effort is under a person's full control. "Empirical research show that those who hold the attribution of effort tend to be more exposed to positive learning behaviors, such as setting goals that focus on learning rather than appearing (Miller & Meece, 1997), using strategies, and persistence in the tough and challenging tasks " (Stipek, 1996, acc. to Lai, 2011, p. 8).

The theory of self-worth is related to self-efficiency and control theory. According to this theory, the basic human aspiration is to maintain a favorable image of oneself, which is based on self-esteem or awareness of one's own value (Covington, 1992). This line of research shows that students are trying to increase their sense of self-esteem and ability, which is the main factor determining the motivation for achievement. "A teacher who tries to acquire confidence in a student's own abilities and change an unfavorable image of himself will also stimulate the student's motivation for achievement" (Vizek et al., 2003, p. 219).

Do I want to do this task and why?

In this case, Broussard and Garrison (2004) have included the theory of expected-values, theory of intrinsic motivation and self-determination theory. "Values are incentives and reasons for participating in some activity" (Lai, 2011, p. 9). The value of a particular task or activity has four components: the value of achievement, which refers to personal values for goodness in a task; an internal value that relates to a subjective interest or pleasure in performing a task; the value of usefulness, which relates to the extent to which task completion is considered conducive to present or future purposes; and cost, which relates to negative aspects of the task, such as anxiety and fear of failure (Eccles & Wigfield, 2002; Stipek, 1996).

The concept of internal motivation is closely related to inner value. Internal motivation refers to motivation associated with personal pleasure, interest, or satisfaction, and is usually contrary to external motivation (Guay et al., 2010). Teachers usually find that internal motivation is more desirable than external motivation, and some researches show that the outcomes of learning with internal motivation are better than those obtained by external motivation (Ryan, Connell, & Plant, 1990).

What do I need to do to succeed in this task?

Broussard and Garrison (2004) claim that this research has led to the development of self-regulation and the theory of will, with which it shares the attempt to connect motivation to knowledge. "There are students who are extremely self-disciplined, i.e. those who do not need external learning incentives because they are motivated enough with learning something successfully" (Vizak et al., 2003, p. 257). Self-regulated students have shown that they use different strategies, have high self-efficiency, and set goals for themselves. Self-regulation theory implies that individuals can strengthen their motivation by engaging in numerous self-regulation strategies, such as setting appropriate and achievable goals, implementing learning strategies, and tracking and evaluating progress towards goals (Schunk & Zimmerman, 2007).

Corno (1993) claims that the effect of motivation on behavior is mediated by will. Motivation can lead to a decision to act, but will is what determines whether this decision will be implemented. "While motivation helps to set goals, the will supports the management and implementation of these goals" (Lai, 2011, p. 13).

There is little research on the relationship between factors that influence motivation. To build a wider spectrum of knowledge on motivational factors in mathematical education, McLeod et al. (1984) propose research designed to detect causal and

interactive relationships between motivational domains and students' achievements. Almost all mathematics researches use a model of math instruction that is not conceptually driven. Researchers who have studied the conceptual model of teaching have found that the effects of such teaching in students' motivation are quite different from the consequences of traditional teaching (e. g. Bransford et al., 1993b). In their research, conducted in Kenya's high schools, Githua and Mwang (2003) concluded that mathematical self-perception of students positively influences their motivation for learning math. Gottfried (1990) researched the relationship between internal motivation and students' achievement. Her results show that students with higher internal motivation achieve better success. The above-mentioned studies do not deal with the effect of different math teaching strategies on the development of students' motivation and achievement, and it is therefore considered that this research will contribute to a better and clearer understanding of the effects of interesting mathematics teaching on the development of mathematical achievements of high school students.

Interesting teaching is teaching in which students actively participate and at the same time create a strong sense of inner satisfaction due to the successful acquisition of new knowledge in the context of problem situations.

In this study, the influence of interesting math teaching on student internal motivation and achieved mathematical success has been tested. The first problem of our research foused on finding correlations between interesting ways of learning math and inner motivation for learning math. Our hypothesis was that interesting teaching would increase the internal motivation of students to learn math. The second research problem focused on the role of interesting teaching strategies on success in mathematics. We expected that students who learned how to do work in interesting ways would achieve better math success.

Research methodology

The subject of the present research is the effectiveness of interesting teaching methods on internal motivation and students' achievements in solving problems in order to raise the quality of students' knowledge and the teaching process to a higher level.

Respondent sample

The study involved 300 high school students from Čitluk, Ljubuški and Mostar. The subjects were divided into two groups, control ($N = 150$) and experimental ($N = 150$). The groups were evenly balanced on the scale of internal motivation and knowledge.

Procedure

Both groups of subjects learned the same contents contents as defined in the curriculum of Mathematics for high schools in Bosnia and Herzegovina, following the Croatian curriculum. The experiment was conducted over a period of three months. The experimental program was carried out by the teachers according to the ready-

made class preparation materials for the prescribed teaching units. The experimental groupworked via interesting teaching methods and forms of work, i.e. their teaching was enriched with computer animations using Geogebra, constructions of different geometric models as well as numerous examples from everyday life. Classes were carried out through individual work, group work, work in pairs and frontal work, with emphasis on goals and tasks and active teaching methods to motivate the learning of mathematics. For example, when processing a trigonometric circle, students had drawn on a cardboard a trigonometric circle to which they glued the platen strip (number direction). During practice, the students worked in pairs, set up tasks for each other and checked the results on the circle. This mode of operation did not allow passivity, and besides, those who did not immediately understand the exhibit were given help by a colleague. For processing quadratic and trigonometric graphs, they used the Geogebre dynamic program. For this purpose, the Geogebra program was installed in the computer science classroom and the teachers and students were trained for its use. These lessons were especially interesting to students because they immediately received feedback on their work. Group work was used for grouping complex numbers and solving quadratic equations. The students were divided into groups of 5 and 6. Each group got their task and at the end of the lesson the group's representative presented the results of their work. At the end of each class, the teacher declared the best group, which motivated the students to work. For homework, the students themselves prepared tasks that related to the subject. Applying Tale's teaching on the proportionality of length, they measured the height of the house, the church tower, the river's width, etc. All these mathematics classes were interesting to the students. An interesting math teaching that relies on the motivational teaching system was implemented, using active teaching methods and forms of work through differentiation, individualization, and application of modern IT tools.

The control group worked according to traditional organisation of classes. Methods of presentation, methods of conversation and methods of text work were used while the forms of work were frontal and individual.

Instruments

In this research, *survey and testing* techniques were used and as measuring instruments *informal knowledge tests* (*initial* - grouping and initial status statement, *final* - for comparing the final state) and *a survey* for the students who participated in the research.

The tasks for initial and final testing were taken, were taken from existing collections for the appropriate class and high school. The tests had five assignments, and the solving time was 45 minutes. The experimental group experienced classes with interesting learning methods (work on computer with Geogebra program, group work, collaborative learning, etc.) and the control group was taught in a traditional way. All respondents solved the same tests (students in the same class and the same kind of school), and all the tests were corrected by the same math teacher, to ensure the objectivity of the assessment.

The internal motivation test was carried out using the Likert type scale. The scale was modified for the needs of this research, adopted from the research by Lepper et al. (2005), originally construed by Harter in 1981. This scale was used in an unpublished master's thesis by Kalajdžić (Faculty of Philosophy, Pale, 2013).

The Likert type scale was in range of five degrees, 5 degrees, i. e. "I disagree" to "I completely agree". The scale is made up of three sub-scales that measure internal motivation, namely: *Challenge, Curiosity,* and *Independent Learning.*

In the Challenge subscale, there are six statements that relate to:

1. hard schoolwork,
2. learning at school,
3. new assignments,
4. school subjects that require more thinking and self-concluding,
5. difficult tasks and
6. difficult schoolwork.

The Curiosity subscale has six statements that relate to:

1. asking questions to learn new facts,
2. working on additional projects,
3. reading and how interesting the subject is,
4. work on school assignments,
5. learning new things,
6. working on problems to find their solution.

Claims in the third subscale include the ratio of respondents according to Independent Learning and there are five of them:

1. finding ways to independently solve school assignments,
2. self-discovering solutions,
3. self-correcting errors and finding the correct answer,
4. perseverance in solving problems,
5. independent work on school assignments.

The reliability of this scale was determined using the Cronbah alpha coefficient which is **α = 0.89**1 for the complete scale and indicates a satisfactory level of reliability.

The research goal was to compare the effects of applying interesting vs. traditional teaching methods on student motivation and knowledge.

Research hypotheses

Hypothesis 1: Innovative math instruction with specially selected work methods will increase the inner motivation of students to learn math.

Hypothesis 2: Innovative math instruction with specially selected work methods will increase the quality of knowledge of high school students.

Auxiliary hypotheses:

1. The experimental group will achieve better success with respect to the control group.
2. In the tests, the experimental group will achieve better results by applying modern methods of work in relation to the control group where classical teaching methods are applied.

Methods. The following methods were used in the research: *experiment method with comparative groups*, *survey method, method of theoretical analysis and comparison method.*

The basic method in this study is *the experiment method with comparative groups* (experimental and control group). In the experimental method, we distinguished an independent variable (sample) and a dependent variable (consequence).

The independent variable was characterized by *different forms of interactive learning such as making cardboard models, Geogebra program, group work and collaborative learning.*

The dependent variable comprised *internal motivation*, *the quality of the students' knowledge of mathematics and the students' achievements in math.*

The Success in mathematics variable was created as the average mathematics grade from terms in the previous and current school year.

The quality of mathematics knowledge was tested by math achievement tests. Test assignments were taken from the existing workbooks approved by the respective Ministry of Education.

Inner motivation was measured via a survey based on a Likert-type scale.

The *survey method* is highly appropriate for empirical research and field work since it allows the application of various measuring instruments and different data processing procedures.

The method of *theoretical analysis* served as a complement to the previous method, especially in collecting and selecting relevant information from previous methodological and similar subject research, thus contributing to a more comprehensive view of students' motivation to learn math as a factor of the quality of students' knowledge in Čitluk, Mostar and Ljubuški. This method enables theoretical overview of these phenomena and an adequate interpretation of the obtained research results

The *comparison method* was also indispensable in this research in determining enable the determination of the similarities and differences between the obtained results based on which reliable and objective conclusions and generalizations can be made.

Statistical data drocessing

The data were processed using the STATISTICA software package. For this purpose, the test of statistical significance of differences was applied using mixed ANOVA 2x2 model and *t*- test.

Research results

Table 1 shows differences in the results between the students of experimental and control group in motivation and success in math on initial and final measurement.

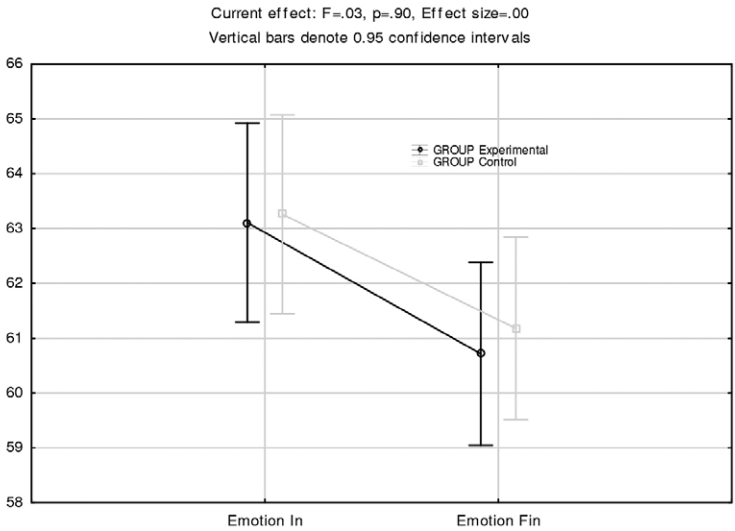
Table 1

Differences between the groups in default variables in initial and final measure

Variables	Mean Control	t-value	p	Std. Dev. Experimental	Std. Dev. Control
Emotion In	63.26	-0.12	0.91	11.36	11.21
Emotion Fin	61.18	-0.39	0.67	9.73	10.97
Motiv In	89.38	0.01	0.99	9.75	9.74
Motiv Fin	88.78	2.74	0.016	9.88	9.62
Task In	1.78	-3.14	0.00	1,36	1.36
Task Fin	2.25	0.26	0.80	1.39	1.31
Success In	3.16	0.78	0.44	0.84	0.79
Success Fin	3.45	3.09	0.00	0.74	0.79

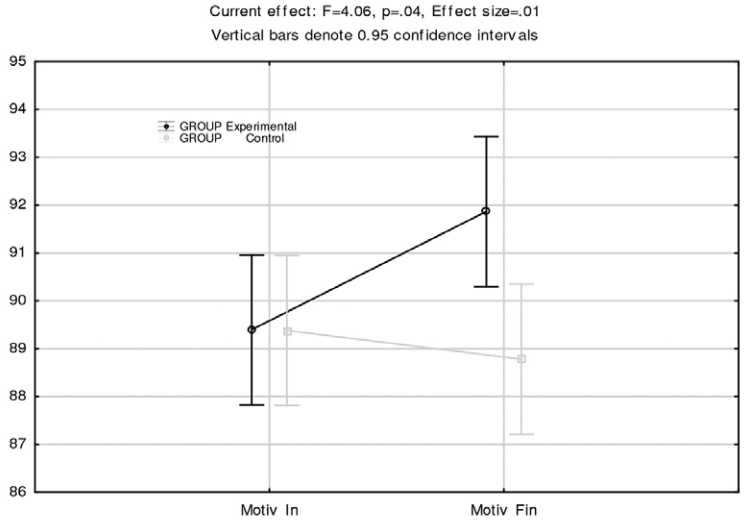
The above table shows arithmetic means and *t*-test for all these variables in initial and final measurements. We see that both groups have succeeded in increasing success and solving tasks, but this increase is more notable in the experimental group.

The following graphs are the result of 2x2 mixed ANOVA.



Graph 1. Emotions interaction between groups in initial and final measurement

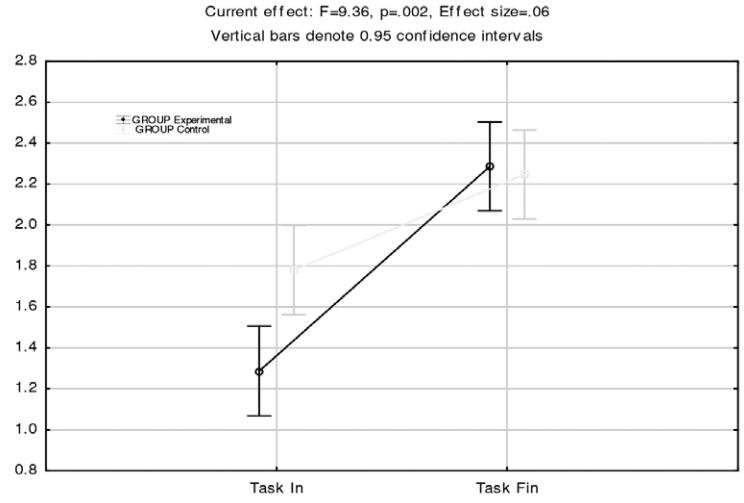
From the given p value, it is evident that the groups do not have statistically significantly different progression because the students of both groups have the same emotions both at the beginning and end of the measurement.



Graph 2. Interaction of motivation between groups in initial and final measurement

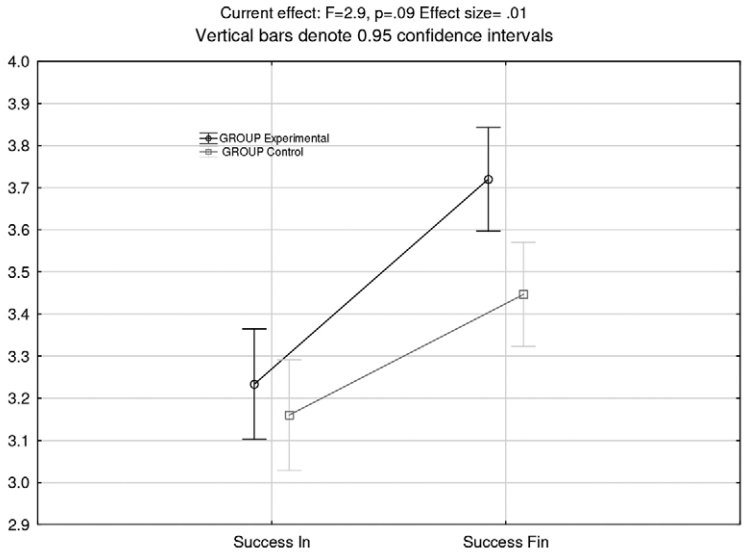
As for motivation, there is a statistically significant difference in the initial and final measurements between the groups, and because $p < 0.05$, we can conclude that the experimental group was more motivated, due to the influence of the experimental factor.

The effect size with motivation 0.01 shows that the effect of different programs on the differences between the groups is small.



Graph 3. Interaction in the completed tasks between groups in initial and final measurement

According to Graph 3, there is no statistically significant difference between the groups in solving the tasks, but due to the low p, the probability that the experimental group has progressed more than the control was 94 %. The effect size shows that the effect of the experimental factor is moderate.



Graph 4. Interaction in success between groups in initial and final measurement

Graph 4 shows that both groups achieved a certain success and no statistically significant difference was obtained. The *p* is low and the probability that the experimental group progressed more than the control was 91 %. The effect size in success is 0.01, indicating little effect of different programs.

Differences by the gender are not significant in any of the variables.

Discussion

Based on the statistical analysis of the results obtained in this research, verification of the set hypotheses was performed:

Hypothesis 1, which assumed that monitoring innovative math teaching increases inner motivation of students, is confirmed. After the initial uniformity of the students in control and experimental group, confirmed by the initial testing, and exposure of the experimental group students to innovative teaching methods in mathematics class, the final tests established a statistically significant difference between the groups in the final measurement of motivation.

Hypothesis 2, which assumed that monitoring innovative math teaching increases *the quality of knowledge of high school students*, is confirmed. The final tests found that there is no statistical significance, although the probability of 91 % shows that the experimental group has progressed more.

Auxiliary Hypothesis 1, which assumed that the experimental group would achieve more success than the control group, is confirmed. Specifically, Graph 4 shows that the progress of students in the experimental group is greater than the progress of students in the control group, although there is no statistically significant difference

between the initial and final measurements.The *t*-test showed a statistically significant difference between the groups in the final measurement.

Auxiliary Hypothesis 2, which assumed that the experimental group would achieve better results by applying modern methods of work in relation to the control group that would follow classes by classical methods, is confirmed. According to Graph 3, the effects of different programs on the differences in the groups is medium, and there is no statistically significant difference between the initial and final measurement. However, due to the low *p* with 94% probability, we can argue that the experimental group is better at solving tasks.

Based on the data presented above, it can be concluded that interesting mathematics teaching had a positive impact on the students' success, as opposed to traditional teaching, as well as on increasing internal motivation for learning math. Namely, the application of innovative teaching methods has had a positive effect on the changes in student motivation toward mathematics learning, which is consistent with the findings of some other research (Branford et al., 1988; Cobb et al., 1992; Middleton, 1993b).

When solving math tests in the initial measurement, the approximate results were achieved by the students in the control and experimental group (Graph3). Students in the control group achieved better results in the initial measurement than students in the experimental group when no experimental factor was applied. However, when using the experimental factor, i.e. innovative math teaching, in the final measurement, greater progress was made by the experimental group compared to the control group where students learned in a traditional way. This finding could be explained with the evaluation, which was not anonymous and served as a form of motivation for learning mathematics. In addition, this test was conducted in the first half of the year when most students struggle for better grades, so the next survey should be conducted anonymously.

Conclusion

Based on the results of the conducted research, it can be concluded that interesting math teaching has a positive effect on student achievement and that its effects are greater than those of traditional teaching. Students who have learned mathematics content through interesting learning methods have achieved greater success and they have been more successful in solving tasks.

This research shows a higher motivation in students to learn math when interesting teaching methods are applied, which is in line with previous research on younger students (Githua& Mwang 2003). The research confirmed the positive effects of interesting lessons on achievements in math teaching, which is consistent with previous research (Gottfried, 1990). Therefore, ultimately, it can be concluded that mathematics teaching in an interesting way in relation to traditional teaching yields better results in achieving success in mathematics and that it should be included as such in mathematics classes.

Since research on the effectiveness of interesting and different programs of math teaching is of a rather small scope and lacking adequate guidance for its implementation in practice, the results of this research represent a significant contribution to improving the educational work in high schools. That is why this research is a stimulus to further study of the effects of interesting teaching on motivation and achievements in math classes. Applying interesting ways of teaching goes in favour of this hypothesis. However, this research was implemented only for a period of one school term. For this reason, further research should be carried out over a longer period of time.

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Unutrašnja motivacija i znanje učenika iz matematike

Sažetak

Cilj je ovoga istraživanja utvrditi postoji li povezanost zanimljive nastave matematike i povećanja motivacije i uspjeha učenika srednjih škola. S tom je svrhom provedeno eksperimentalno istraživanje s usporednim skupinama u srednjim školama u Čitluku, Ljubuškom i Mostaru na uzorku od 300 učenika. U radu s učenicima eksperimentalne skupine primjenjene su zanimljive nastavne metode i oblici rada, dok je kontrolna skupina radila na tradicionalan način. Kao mjerne instrumente koristili smo anketni upitnik i test znanja. Testiranje smo proveli na početku i na kraju istraživanja. Ispitivanje unutarnje motivacije izvršeno je putem skale Likertova tipa, a testiranje statističke značajnosti razlika pomoću Anove mixed model 2 x 2 i t-testa. Rezultati pokazuju bolji učinak zanimljive nastave na uspjeh učenika, kao i na povećanje motivacije učenika za učenje matematike. Zbog toga se preporučuje primjena zanimljivih načina rada u nastavi matematike.

Ključne riječi: *nastava; obrazovanje; suvremene metode; učenje.*

Uvod

Kada govorimo o motivaciji učenika za učenje, kako matematike tako i drugih nastavnih predmeta, razlikujemo ekstrinzičnu i intrinzičnu motivaciju. „Intrinzična motivacija odgovor je na unutarnje učenikove potrebe kao što su radoznalost, potreba za znanjem, osjećaj kompetencije te rasta i razvoja. Ekstrinzična motivacija odnosi se na motivaciju koja svoj izvor ima izvan učenika, primjerice, dobar rezultat na testu, ocjene, nastavnikova pohvala, diploma ili medalja i druge nagrade" (Vizek Vidović, Vlahović-Štetić, Rijavec, Miljković, 2003, str. 245). Dakle, intrinzična ili unutarnja motivacija je ono što iznutra pokreće učenika na učenje. Da bi povećali intrinzičnu motivaciju, potrebno je učenike zainteresirati za matematičke nastavne sadržaje koji su propisani nastavnim planom i programom pa ga nastavnici ne mogu mijenjati, ali ga mogu prilagoditi da budu zanimljiviji učenicima. Prema Vizek Vidović i sur. (2003), intrinzičnu motivaciju moguće je povećati: prilagođavanjem onoga što se uči učeničkim interesima, uvođenjem novosti i raznolikosti u nastavu, aktivnim sudjelovanjem učenika i brzim povratnim informacijama, poticanjem radoznalosti, povezivanjem onoga što se uči s osobnim životom i pomaganjem učenicima da sami

postave svoje ciljeve. Middleton i Spanias (1999) u svojem radu opisali su teorijske orijentacije u motivaciji u nastavi matematike. Raspravljali su i o vrstama motivacije u obrazovanju i ulozi nastavnika na motivaciju učenika za učenje matematike. Imamo unutrašnju i vanjsku motivaciju. „Unutrašnja motivacija je nagon ili želja učenika da se uključe u učenje za vlastitu korist" (Middleton i Spanias, 1999., str. 66). Učenici s unutrašnjom motivacijom uče matematiku iz zadovoljstva, za razliku od onih koji su vanjski motivirani i koji uče zbog dobivanja nagrada. Prema Middleton i Spanias (1999), unutrašnja motivacija odnosi se na percepcije učenika, njihove kompetentnosti u matematici, jesu li motivirani znatiželjom ili ocjenama. Kad učenici uvide da su sposobni rješavati matematičke zadatke, oni će zavoljeti matematiku i težit će za uspjehom, za razliku od onih učenika koji smatraju da nisu talentirani za matematiku (Eccles, Wigfield i Reuman, 1987; Midgley, Feldlaufer i Eccles, 1989). Učenici se moraju osjećati ugodno kad uče matematiku, moraju uraditi zadatak, i moraju očekivati da će uspjeti prije nego razvoj unutarnje motivacije započne.

Middleton, Littlefield i Lehrer (1992) istraživali su utjecaj akademske aktivnosti na unutrašnju motivaciju. Njihova je analiza pokazala da su djeca sklona organizirati svoje konstrukte u tri opće kategorije: uzbuđenje ili kognitivne stimulacije koju pruža djelatnost; osobna kontrola, odnosno stupanj u kojem se aktivnost smatra slobodnim izborom ili odgovarajuće težine i interesi (nepouzdano definirana kategorija), odnosno stupanj u kojem se učenicima svidjela aktivnost, važnost djelovanja te njihova sposobnost u obavljanju djelatnosti (Middleton i Spanias, 1999, str.75). Ako je učenik zainteresiran za učenje matematike, on će učiti sa zadovoljstvom bez obzira na uvjete procjene zadataka.

Middleton (1993 b) ispituje utjecaj na motivaciju učenika pomoću pokusnoga nastavnog plana i programa koji pruža učenicima više mogućnosti za učenje, više izbora strategija i aktivnosti i više izazovnih zadataka od tradicionalnoga nastavnog plana; njegove aktivnosti su smještene u stvarnim situacijama. Podatci pokazuju da su učitelji počeli uočavati važnost personaliziranoga kurikula kako bi matematika imala više smisla za njihove učenike. Rezultati istraživanja o paradigmama osobnih konstrukta pokazali su da su motivi iz matematičkoga obrazovanja vrlo individualni, vezani su uz percipiranje sposobnosti te da su relativno stabilni s obzirom na uspjeh i neuspjeh. Neke od individualnih razlika u motivaciji mogu se objasniti u odnosu na identifikaciju učenika s nastavnicima matematike.

Rezultati istraživanja ukazuju na to da se pad pozitivnoga stava prema matematici može dijelom objasniti funkcijom nedostatka podrške nastavnika i razrednoga okruženja. „Ovi rezultati, zajedno s rezultatima nacionalnih procjena (Dossey i sur., 1988), ukazuju na to da su motivacijski obrasci naučeni i, što je posebno uznemirujuće, da učenici uglavnom uče ne voljeti matematiku i da to nesviđanje postaje sastavni dio njihovih matematičkih samo-koncepata"(Middleton i Spanias, 1999, str. 67).

Kad se pogleda na suptilne načine na koji su motivacije formirane, modificirane i održavane, postaje jasno da ne postoji nemotivirano dijete. Djeca su motivirana.

„Motivacije pomažu voditi dječje aktivnosti; one pružaju strukturu za vrednovanje ishoda aktivnosti; i one pomažu odrediti hoće li ili ne djeca sudjelovati u budućoj matematičkoj aktivnosti" (Middleton i Spanias, 1999, str. 67).

Broussard i Garrison (2004) istraživanje suvremene motivacije organiziraju kroz tri pitanja:

– Mogu li uraditi ovaj zadatak?
– Želim li uraditi ovaj zadatak i zašto?
– Što trebam učiniti kako bih uspio u ovom zadatku?

Mogu li uraditi ovaj zadatak?

Kao što Broussard i Garrison (2004) primjećuju, oni koji se bave prvim pitanjem razvili su niz novih teorija u vezi s vlastitom učinkovitošću, atribucije i samopoštovanja. Bandura (1982) definira percipiranje samoučinkovitosti kao „presudu kako dobro može izvršiti tijek akcija koje su potrebne za rješavanje potencijalnih situacija" (str. 122). Eccles i Wigfield (2002) razradili su Bandurin opis, definiranjem samoučinkovitosti kao povjerenja pojedinca u svoju „sposobnost da organizira i izvrši određeni tijek akcije kako bi riješio problem ili obavio zadatak" (str. 110).

Prema Bandurinoj (1982) teoriji samoučinkovitosti, učinkovitost je glavna odrednica truda, upornosti i postavljanja ciljeva. Istraživanja što su ih proveli Pintrich i DeGroot (1990.) podupiru ovu ideju. Samoučinkovitost je također povezana s upotrebom kognitivnih strategija, a percepcija samoučinkovitosti predviđa uspjeh preko i iznad stvarnih razina sposobnosti (Pintrich i DeGroot, 1990).

Prema drugoj teoriji motivacije, teoriji kontrole, pojedinci koji kontroliraju svojim vlastitim uspjesima i neuspjesima više su motivirani (Eccles i Wigfield, 2002). Naime, u jednom formuliranju teorija kontrole, autonomija je jedna od tri osnovne psihološke potrebe, uz nadležnost i povezanost. U tom okviru, individualne razlike u mjeri u kojoj su te osnovne potrebe ispunjene odgovaraju varijacijama u razinama motivacije (Connell i Wellborn, 1991, preuzeto od Eccles i Wigfield, 2002). Teorija kontrole usko je povezana s pojmom atribucije. Postoji nekoliko vrsta atribucije, uključujući sposobnost, trud, zadatak i sreću. Postoji nekoliko vrsta atribucije ovisno o tome može li se uzrok motivacije mijenjati i imati pod osobnom kontrolom ili ne (Weiner, 1985, preuzeto od Eccles i Wigfield, 2002). Na primjer, urođena sposobnost teško se može mijenjati, dok je osobni napor pod potpunom kontrolom osobe. „Empirijska istraživanja pokazuju da oni koji drže atribuciju truda imaju tendenciju da se više izlažu pozitivnim ponašanjima učenja, kao što su postavljanja ciljeva koji se fokusiraju na učenje, a ne nastup (Miller i Meece, 1997), korištenje strategija, i upornosti u teškim i izazovnim zadacima" (Stipek, 1996, preuzeto od Lai, 2011, str. 8).

Teorija vlastite vrijednosti je u vezi sa samoučinkovitosti i teorijom kontrole. Prema toj teoriji osnovna ljudska težnja je održavanje povoljne slike o sebi u čijoj se osnovi nalazi samopoštovanje ili svijest o vlastitoj vrijednosti (Covington, 1992). Ova linija istraživanja pokazuje da učenici pokušavaju povećati svoj osjećaj samopoštovanja i

sposobnosti, što je glavni činitelj koji određuje motivaciju za postignućem. Primjerice, „učiteljica koja se trudi da učenik stekne povjerenje u vlastite sposobnosti i promijeni nepovoljnu sliku o sebi, potaknut će i učenikovu motivaciju za postignućem" (Vizek Vidović i sur., 2003, str. 219).

Želim li uraditi ovaj zadatak i zašto?

Broussard i Garrison (2004) u ovo su uključili teorije očekivane vrijednosti, teorije intrinzične motivacije i teorije samoodređenja. „Vrijednosti su poticaji i razlozi za sudjelovanje u nekoj aktivnosti" (Lai, 2011, str. 9.). Vrijednost određenoga zadatka ili aktivnosti ima četiri komponente: vrijednost dostignuća, koja se odnosi na osobne vrijednosti kvalitete rada na zadatku; unutarnja vrijednost, koja se odnosi na subjektivni interes ili užitak obavljanja zadatka; vrijednost korisnosti, koja se odnosi na mjere u kojima se završetak zadatka smatra olakšavajući za sadašnje ili buduće ciljeve; i trošak, koji se odnosi na negativne aspekte bavljenja zadatkom, kao što su anksioznost i strah od neuspjeha (Eccles i Wigfield, 2002; Stipek, 1996).

Pojam unutrašnje motivacije usko je povezan s unutarašnjom vrijednosti. Unutrašnja motivacija odnosi se na motivaciju povezanu s osobnim užitkom, interesom ili zadovoljstvom, a obično je u suprotnosti s vanjskom motivacijom (Guay i sur., 2010). Nastavnici obično smatraju da je unutrašnja motivacija poželjnija od vanjske motivacije, a neka istraživanja pokazuju da su ishodi učenja unutrašnje motivacije bolji od onih dobivenih vanjskom motivacijom (Ryan, Connell i Plant, 1990).

Što trebam učiniti kako bih uspio u ovome zadatku?

Broussard i Garrison (2004) tvrde da je ovo istraživanje dovelo do razvoja samoregulacije i teorije volje, s kojom dijeli pokušaj povezivanja motivacije sa spoznajom. „Postoje učenici koji su iznimno samodisciplinirani, tj. takvi kojima nisu potrebni vanjski poticaji za učenje, jer im je kao motivacija dovoljno to što su nešto uspješno naučili" (Vizek Vidović i sur., 2003, str. 257). Samoregulirani učenici pokazali su da se koriste različitim strategijama, imaju visoku samoučinkovitost i postavljaju ciljeve za sebe. Teorija samoregulacije postulira da pojedinci mogu učvrstiti svoju motivaciju angažiranjem u brojnim samoregulacijskim strategijama, kao što je postavljanje odgovarajućih i ostvarivih ciljeva, primjena strategije učenja te praćenje i evaluacija napretka prema ciljevima (Schunk i Zimmerman, 2007).

Corno (1993) tvrdi da je učinak motivacije na ponašanje posredovan voljom. Motivacija može dovesti do odluke da se djeluje, ali volja je ono što određuje hoće li se ta odluka provesti. „Dok motivacija pomaže utvrditi ciljeve, volja podržava upravljanje i provedbu tih ciljeva"(Lai, 2011, str. 13).

Malo je istraživanja provedeno o odnosima između činitelja koji utječu na motivaciju. Za izgradnju šireg tijela znanja o motivacijskim čimbenicima u matematičkom obrazovanju, McLeod, Reyes, Fennema i Surber (1984) predlažu istraživanja koja su osmišljena tako da utvrde kauzalne i interaktivne odnose između motivacijskih domena i postignuća učenika. Gotovo sva istraživanja provedena na području matematike

koriste model nastave matematike koja se ne vodi konceptualno. Istraživači koji su proučavali konceptualni model nastave otkrili su da su učinci takvog poučavanja u učeničkoj motivaciji sasvim drugačiji od posljedica tradicionalne nastave (npr. Bransford, Hasselbring, Barron, Kulewicz, Littlefield I Goin, 1988; Cobb, Wood, Yackel i Perlwitz, 1992; Middleton, 1993b). Githua i Mwang (2003) u svojem su istraživanju, provedenom u kenijskim srednjim školama, došli do zaključka da matematičko samopoimanje učenika pozitivno utječe na njihovu motivaciju za učenje matematike. Gottfried (1990) je istraživala odnos unutarnje motivacije i postignuća učenika razredne nastave. Njezini rezultati pokazuju da učenici s višom unuutrašnjom motivacijom postižu bolji uspjeh. Navedene se studije ne dotiču pitanja kakav je učinak različitih nastavnih strategija matematike na razvoj motivacije i postignuća učenika te se stoga smatra da će ovo istraživanje pridonijeti kvalitetnijem i jasnijem razumijevanju učinaka zanimljive nastave matematike na razvoj matematičkih postignuća učenika srednjih škola.

Zanimljiva nastava je nastava u kojoj učenici aktivno sudjeluju i pri tom se kod njih stvara snažan osjećaj unutrašnjega zadovoljstva zbog uspješnoga usvajanja novih spoznaja u kontekstu problemskih situacija.

U ovom smo istraživanju testirali utjecaj zanimljive nastave matematike na unutarnju motivaciju učenika i postignuti uspjeh iz matematike. Prvi problem našega istraživanja odnosio se na to postoje li korelacije između zanimljivih načina učenja matematike i unutrašnje motivacije za njezino učenje. Naša je hipoteza bila da će zanimljiva nastava povećati unutrašnju motivaciju učenika za učenje matematike.

Drugi problem istraživanja odnosio se na ulogu zanimljivih nastavnih strategija na uspjeh u matematici. Očekivali smo da će učenici koji uče zanimljivim načinima rada postići bolji uspjeh u matematici.

Metodologija istraživanja

Predmet istraživanja je učinkovitost zanimljivih metoda nastave na unutarnašnju motivaciju i na postignuća učenika pri rješavanju problema u cilju podizanja kvalitete znanja učenika i samog nastavnoga procesa na jednu višu razinu.

Uzorak ispitanika

U istraživanju je sudjelovalo 300 učenika srednjih škola iz Čitluka, Ljubuškog i Mostara. Ispitanici su bili podijeljeni u dvije skupine, kontrolnu (N = 150) i eksperimentalnu (N = 150). Skupine su bile ujednačene i po skali unutrašnje motivacije i po znanju.

Postupak

Obje skupine ispitanika usvajale su iste nastavne sadržaje koje su određene Nastavnim planom i programom Matematike za srednje škole u BiH koje rade po hrvatskom planu. Eksperiment je proveden u vremenu od tri mjeseca. Eksperimentalni program realizirali su nastavnici prema potpuno izrađenim pripremama sati za izvođenje propisanih nastavnih jedinica. Eksperimentalna skupina radila je uz primjenu zanimljivih nastavnih metoda i oblika rada, tj. njihova nastava je bila obogaćena računalnim animacijama korištenjem Geogebre, konstrukcijama različitih geometrijskih modela kao i brojnim

primjerima iz svakodnevnoga života. Sati su realizirani kroz individualni, grupni, rad u paru i frontalni rad s naglašavanjem ciljeva i zadataka te aktivnim nastavnim metodama kako bi se motivirali na učenje matematike. Primjerice, prilikom obrađivanja trigonometrijske kružnice učenici su nacrtali na kartonu trigonometrijsku kružnicu na koju su prilijepili platnenu traku (brojevni pravac). Prilikom vježbanja učenici su radili u paru, postavljali jedan drugome zadatke i provjeravali rezultate na kružnici. Ovakav način rada nije dozvoljavao pasivizaciju, a osim toga onaj tko nije odmah shvatio izloženo gradivo, pomogao mu je kolega. Prilikom obrađivanja grafova kvadratnih i trigonometrijskih funkcija koristili su dinamički program Geogebre. U tu svrhu instaliran je program Geogebre u informatičkoj učionici te su i nastavnici i učenici osposbljeni za korištenje njime. Ovi nastavni sati učenicima su posebno bili zanimljivi jer su odmah dobili povratne informacije o svojem radu. Prilikom obrade kompleksnih brojeva i rješavanja kvadratnih jednadžbi korišten je grupni rad. Učenici su bili podijeljeni u grupe po 5 i po 6 učenika. Svaka grupa dobila je svoj zadatak i na kraju sata predstavnik grupe izvijestio je o rezultatima njihova rada. Na kraju svakog sata nastavnik je proglašavao najbolju grupu što je učenike dodatno motiviralo za rad. Za domaću zadaću učenici su sami sastavljali zadatke koji su bili vezani za određeno nastavno gradivo. Primjenjujući Talesov poučak o proporcionalnosti dužina mjerili su visinu kuće, crkvenoga tornja, širinu rijeke i sl. Sve su to njima bili zanimljivi sati matematike. Znači, izvođena je zanimljiva nastava matematike koja se oslanja na motivacijski nastavni sustav, primjenom aktivnih nastavnih metoda i oblika rada kroz diferencijaciju i individualizaciju te primjenu suvremenih informatičkih sredstava.

Kontrolna skupina radila je po tradicionalnoj nastavi. Korištene su metode izlaganja, metoda razgovora i metoda rada na tekstu, a od oblika rada - frontalni i individualni.

Instrumenti

U ovom istraživanju koriste se tehnike *anketiranja i testiranja*, a kao mjerni instrument *neformalni testovi znanja* (*inicijalni* –za ujednačavanje skupina i konstataciju inicijalnoga stanja, *finalni*– za uspoređivanje finalnoga stanja) i *anketa* za učenike koji su sudjelovali u realizaciji eksperimenta.

Zadatci, i za inicijalno i za finalno testiranje, uzeti su iz postojećih zbirki za odgovarajući razred i odgovarajuću srednju školu. Testovi su imali po pet zadataka, a vrijeme rješavanja bio je jedan školski sat. Eksperimentalna skupina radila je uz primjenu zanimljivih elemenata učenja (rad na računalu programom Geogebra, grupni rad, suradničko učenje i sl.), a kontrolna prema tradicionalnoj nastavi. Svi ispitanici rješavaju iste testove (misli se na učenike istoga razreda i iste vrste škole) i sve je testove ispravljala ista nastavnica matematike, kako bi se osigurala objektivnost ocjenjivanja.

Ispitivanje unutrašnje motivacije izvršeno je putem skale Likertova tipa. Skala je preuzeta iz istraživanja Merk. R. Lepper, Jenifer H. Corpus i SheenaIyengar (2005), koju je konstruirala S. Harter 1981. godine i prilagođena je za potrebe ovoga istraživanja. Ova skala korištena je u neobjavljenom magistarskom radu Kalajdžić (Filozofski fakultet Pale, 2013).

Skala Likertova tipa pruža mogućnost ispitanicima da razine usuglašavanja s tvrdnjama iskažu kroz jedan od 5 stupnjeva, tj. od „uopće se ne slažem" do „potpuno se slažem". Skala je sastavljena od tri podskale koje mjere unutrašnju motivaciju, a to su : *Izazov, Radoznalost i Neovisno učenje*.

U podskali *Izazov* postoji šest tvrdnji koje se odnose na:

1. naporan školski rad
2. učenje u školi
3. nove zadatke
4. školske predmete koji zahtijevaju više razmišljanja i samostalno zaključivanje
5. teške zadatke i
6. težak školski rad.

Podskala *Radoznalost* ima šest tvrdnji koje se odnose na:

1. postavljanje pitanja u cilju spoznaje novih činjenica
2. rad na dodatnim projektima
3. čitanje literature i zainteresiranost za predmet
4. rad na školskim zadatcima
5. učenje novih sadržaja i
6. rad na problemima u cilju pronalaženja njihova rješenja.

Tvrdnje u trećoj podskali obuhvaćaju odnos ispitanika prema neovisnom učenju i ima ih pet:

1. traženje načina za samostalno rješavanje školskih zadataka
2. samostalno otkrivanje rješenja
3. samostalno ispravljanje pogreške i traženje točnoga odgovora
4. ustrajnost u samostalnom rješavanju problema i
5. samostalan rad na školskim zadatcima.

Pouzdanost ove skale utvrđena je pomoću Cronbachova alpha koeficijenta i za kompletnu skalu iznosi $\alpha = 0{,}891$ što ukazuje na zadovoljavajuću razinu pouzdanosti.

Cilj istraživanja je eksperimentalno uspoređivanje učinaka zanimljivoga načina rada i klasičnoga oblika rada na motiviranje i znanje učenika.

Hipoteze istraživanja

Hipoteza 1: Inovativno praćenje nastave matematike s posebno odabranim metodama rada povećat će unutrašnju *motivaciju učenika za učenje matematike.*

Hipoteza 2: Inovativno praćenje nastave matematike s posebno odabranim metodama rada povećat će kvalitetu znanja učenika srednjih škola.

Pomoćne hipoteze:

Eksperimentalna skupina postići će bolji uspjeh u odnosu na kontrolnu skupinu.

Na testovima će eksperimentalna skupina postići bolje rezultate primjenom suvremenih metoda rada u odnosu na kontrolnu skupinu koja će pratiti nastavu klasičnim metodama.

Metode. Tijekom rada korištene su: *metoda eksperimenta s usporednim skupinama, survey metoda, metoda teorijske analize i metoda komparacije.*

Osnovna metoda u ovom istraživanju je *metoda eksperimenta s usporednim skupinama* (eksperimentalna i kontrolna skupina). Kod eksperimentalne metode razlikujemo nezavisnu varijablu (uzorak) i zavisnu varijablu (posljedicu).

Nezavisna varijabla - *različiti oblici interaktivnoga* učenja kao što su pravljenje modela od kartona, program Geogebra, grupni rad, suradničko učenje.

Zavisna je varijabla sastavljena od *unutarnje motivacije, kvalitete znanja učenika iz matematike i postignutim uspjehom učenika iz matematike.*

Varijabla *uspjeh iz matematike* kreirana je kao prosjek školskih ocjena iz matematike s polugodišta prethodne i polugodišta tekuće godine.

Kvaliteta znanja iz matematike ispitana je testovima postignuća iz matematike. Zadatci za test uzeti su iz postojećih zbirki zadataka za odgovarajuću školu, a koje je odobrilo nadležno Ministarstvo prosvjete.

Unutrašnja motivacija ispitana je skalom Likertova tipa, a ispitivanje je provedeno anketom

Survey metoda čijom će primjenom biti izvršeno istraživanje vrlo je povoljna za empirijsko istraživanje i terenski rad s obzirom na to da omogućuje primjenu raznih mjernih instrumenata i različitih postupaka obrade podataka.

Metoda teorijske analize poslužit će kao dopuna prethodnoj metodi, posebno u prikupljanju i selekciji relevantnih informacija iz dosadašnjih metodoloških i predmetno sličnih istraživanja i na taj način doprinijeti potpunijem sagledavanju motivacije učenika za učenje matematike kao faktora kvalitete znanja učenika na području Čitluka, Mostara i Ljubuškog. Ovom metodom omogućeno je teorijsko sagledavanje pojava koje su predmet istraživanja kao i adekvatno tumačenje rezultata dobivenih tijekom istraživanja.

Metoda komparacije također je nezaobilazna metoda u ovom istraživanju koja će omogućiti utvrđivanje sličnosti i razlika dobivenih rezultata te na osnovi toga izvođenje pouzdanih i objektivnih zaključaka i generalizacija.

Statistička obrada podataka

Podatci su obrađeni pomoću programskoga paketa Statistica. Za testiranje statističke značajnosti razlika koristili smo mixed ANOVU model 2 x 2 i t-test.

Rezultati istraživanja

U Tablici 1 prikazane su razlike u rezultatima između učenika eksperimentalne i kontrolne skupine u motivaciji i uspjehu iz matematike na inicijalnom i finalnom mjerenju.

Tablica 1.

U tablici su predstavljene aritmetičke sredine i t-test za sve navedene varijable u inicijalnom i u finalnom mjerenju. Vidimo da je i kod jedne i druge skupine došlo do

povećanja uspjeha i riješenih zadataka, ali je to povećanje značajnije kod eksperimentalne skupine.

Sljedeći grafikoni rezultat su 2 x 2 mixed ANOVE.

Grafikon 1.

Iz dane p vrijednosti za testiranje jesu li grupe statistički značajno različito napredovale pokazuje da nisu jer su učenici obiju skupina zadržali gotovo iste emocije i na početku i na kraju mjerenja.

Grafikon 2

Što se tiče motivacije postoje statistički značajne razlike u finalnom mjerenju između skupina, a i zbog $p < 0,05$ možemo zaključiti da je pod utjecajem eksperimentalnoga faktora, eksperimentalna skupina motiviranija.

Veličina efekta kod motivacije 0,01 pokazuje da je djelovanje različitih programa na razlike po skupinama malo.

Grafikon 3.

Iz Grafikona 3 vidimo da ne postoji statistički značajna razlika između skupina u rješavanju zadataka, ali zbog male vrijednosti p vjerojatnost da je eksperimentalna skupina više napredovala od kontrolne je 94 %. Veličina efekta pokazuje da je djelovanje eksperimentalnoga faktora umjereno.

Grafikon 4.

Grafikon 4 pokazuje da su obje skupine postigle određeni uspjeh i nije dobivena statistički značajna razlika, ali je p nizak pa vjerojatnost da je eksperimentalna skupina više napredovala od kontrolne je 91 %.

Veličina efekta kod uspjeha je 0,01, što pokazuje da je malo djelovanje različitih programa po skupinama.

Razlike po spolu nisu značajne ni prema jednoj od navedenih varijabli.

Rasprava

Na temelju statističke analize rezultata dobivenih ovim istraživanjem izvršena je verifikacija postavljenih hipoteza:

Hipoteza 1 kojom je pretpostavljeno da inovativno praćenje nastave matematike povećava unutrašnju motivaciju učenika je potvrđena. Naime, nakon početne ujednačenosti učenika kontrolne i eksperimentalne skupine utvrđene početnim ispitivanjem te izloženosti učenika eksperimentalne skupine inovativnim nastavnim metodama u nastavi matematike, završnim je ispitivanjem utvrđeno da postoji statistički značajna razlika između skupina u završnom mjerenju kod motivacije.

Hipoteza 2 kojom je pretpostavljeno da inovativno praćenje nastave matematike povećava kvalitetu znanja učenika je potvrđena. Iako je završnim ispitivanjem utvrđeno

da kod uspjeha nema statističke značajnosti, vjerojatnost od 91 % pokazuje da je eksperimentalna skupina bolje napredovala.

Pomoćna hipoteza 1 kojom je pretpostavljeno da će eksperimentalna skupina postići bolji uspjeh u odnosu na kontrolnu je potvrđena. Naime, iz Grafikona 4 vidi se da je napredovanje učenika u eksperimentalnoj skupini veće nego u kontrolnoj, mada ni ovdje ne postoji statistički značajna razlika između početnoga i završnoga mjerenja.

Pompćna hipoteza 2 kojom je pretpostavljeno da će eksperimentalna skupina postići bolje rezultate primjenom suvremenih metoda rada u odnosu na kontrolnu skupinu koja će pratiti nastavu klasičnim metodama, je potvrđena. Iz Grafikona 3 vidi se da je djelovanje različitih programa na razlike po skupinama srednje veličine, a ne postoji statistički značajna razlika između početnoga i završnoga mjerenja. Međutim, zbog malog p s 94 % vjerojatnosti možemo tvrditi da je eksperimentalna skupina bolja u rješavanju zadataka.

Na osnovi navedenih podataka izvodi se zaključak da je zanimljiva nastava matematike djelovala pozitivno i na uspjeh učenika u odnosu na tradicionalnu nastavu i na povećanje unutrašnje motivacije za učenje matematike. Naime, primjena inovativnih nastavnih metoda utjecala je pozitivno na promjene u motivaciji učenika za učenje matematike što je u skladu s istraživanjima Bransford, Hasselbring, Barron, Kulewicz, Littlefield i Goin, 1988; Cobb, Wood, Yackel i Perlwitz, 1992; Middleton, 1993b.

Pri rješavanju testova iz matematike u početnom mjerenju približne rezultate ostvarili su i učenici kontrolne i eksperimentalne skupine (Grafikon 3). Učenici kontrolne skupine u početnom mjerenju postigli su bolje rezultate od učenika eksperimentalne skupine kada nije bio primijenjen eksperimentalni čimbenik. Međutim, primjenom eksperimentalnoga čimbenika, inovativne nastave matematike, u završnom mjerenju veći je napredak ostvarila eksperimentalna skupina u odnosu na kontrolnu skupinu, gdje su učenici usvajali znanja na tradicionalan način. Predpostavljamo da je razlog za određeni napredak, koji je ostvarila kontrolna skupina, ocjenjivanje. Ocjenjivanje nije bilo anonimno, a ono je ipak jedan od oblika motivacije za učenje matematike. Osim toga, ovo testiranje provedeno je pred kraj polugodišta kada se većina učenika bori za bolje ocjene, pa bi iduće istraživanje trebalo provesti anonimno.

Zaključak

Na temelju rezultata provedenoga istraživanja može se zaključiti da zanimljiva nastava matematike pozitivno djeluje na postignuće učenika te da su njezini učinci značajniji u odnosu na tradicionalnu nastavu. Učenici koji su nastavne sadržaje matematike spoznavali zanimljivim učenjem, ostvarili su bolji uspjeh i bili su uspješniji u rješavanju zadataka.

Ovo istraživanje pokazuje da zanimljiva nastava matematike povećava motiviranost učenika za učenje matematike, što je u s kladu s dosadašnjim istraživanjima među učenicima mlađe dobi (Githua i Mwang 2003). Istraživanje je potvrdilo pozitivne učinke

zanimljive nastave na postignuća u nastavi matematike, što je u skladu s dosadašnjim istraživanjima (Gottfried 1990). Stoga se u konačnici može zaključiti da zanimljiva nastava matematike u odnosu na tradicionalnu nastavu, daje bolje rezultate u postizanju uspjeha iz matematike i da ju je kao takvu dobro uključiti u nastavu matematike.

Budući da su istraživanja o učinkovitosti zanimljivih i različitih programa u nastavi matematike malobrojna i da nedostaju odgovarajuće smjernice za njezinu provedbu u praksi, rezultati ovoga istraživanja predstavljaju značajan doprinos za poboljšanje odgojno-obrazovnoga rada u srednjim školama. Zato ovo istraživanje predstavlja poticaj za daljnja ispitivanja učinaka zanimljive nastave na motivaciju i postignuća u nastavi matematike. Primjena zanimljivih načina rada ide u prilog ovoj hipotezi, ali je vremenski trajalo jedno polugodište. Zbog toga bi iduće istraživanje valjalo provesti tijekom dužega razdoblja.