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Investigation of Pre-Service Primary School Teachers' Pedagogical Content Knowledge through Transition between Representations

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Abstract

The purpose of the study was to examine the pre-service teachers' transitions between representations and to make inferences about their pedagogical content knowledge. The participants in the study, which was designed as a case study, were 92 senior pre-service teachers attending a state university in Turkey. For data collection, two problems used for fourth-graders in the assessments held by TIMSS in 2007 and 2011 were applied. The participants solved these problems according to their own level and the level of fourth graders. Content analysis was conducted on the collected data. Pre-service teachers dominantly used symbolic (algebraic) representations in their solutions in line with their own level. On the other hand, in their solutions at the level of fourth-graders, a decrease was observed in their use of representations, particularly in the ticket problem in which they experienced difficulties. The rate of using suitable representations in the cake problem was found to be higher. Nearly 30 % of the pre-service teachers managed to solve both of the problems by using representation(s) suitable for the level of primary fourth-graders, as they had sufficient pedagogical content knowledge (PCK).

Keywords: *multiple representations; pedagogical content knowledge; pre-service primary school teachers.*

Introduction

Principles and Standards for School Mathematics (NCTM – National Council of Teachers of Mathematics, 2000) is an important reference that guides teachers in achieving quality and meaningful mathematics education in the pre-K-12 period. NCTM (2000) has set content and process standards to guide teachers under this title. Process standards refer to mathematical processes that require students to acquire and use mathematical knowledge. These standards are problem-solving, reasoning and proving, communication, connection and finally representations that are also the main subject of the current study. The representation standard emphasizes the use of symbols, charts, graphs, manipulatives and diagrams as an effective method of expressing mathematical ideas and relationships. The review of literature shows different definitions for the term of representation as states or structures effectively replacing or symbolizing the whole or parts of anything (Goldin & Kaput, 1996), or external concrete states or structures corresponding to students' mental concepts (Lesh, Post & Behr, 1987). The representation can be thought of as a general term for symbolic versions of the external world, which have counterparts in the mind and appear in more social contexts (Terwel et al., 2009). In mathematics education, the concept of representation can be defined as a tool that is needed/used to process mathematical facts in the mind and transfer them to another person (Delice & Sevimli, 2016).

Multiple representations refer to the repeated representation of a concept with different types of representation, such as verbal, graphical, mathematical representations, and the students being faced with the same concept several times (Prain & Waldrip, 2006). While with the representation process abstract concepts or symbols are modelled in the real world as concrete, relationships between objects and symbols are established, thus making it easier for individuals to understand mathematical situations (Kaput, 1998). As for the definition of the concept of multiple representations, there are different approaches to its classification. For example, Dufour-Janvier et al. (1987) examined the concept of representation under two general headings and named these headings as internal and external representations. When mathematical representations are considered in the form of mathematical ideas or cognitive schemes developed by the individual through experience, they can be defined as internal representations. On the other hand, representations such as numbers, algebraic equations, graphs, tables and diagrams are external manifestations that help us understand mathematical concepts (Janvier et al., 1993). The natural language used by a student for a fraction number, the meaning he/she assigns, his/her attitudes towards visual-spatial images or mathematics are examples of mental representation (Delice & Sevimli, 2016; Goldin, 1998). External representations are observable tools for understanding mathematical concepts and ideas. Examples of external representations include verbal representations (written words), symbolic representations (equations, formulas), and visual representations (tables, graphs and other drawings) (Pape & Tchoshanov, 2001).

Compared to internal representations, most of the studies in mathematics education have accepted external representations as a theoretical framework. Kendal and Stacey (2003) state that as at least one of the types of external representation is encountered in all subjects of mathematics and most concepts are more easily explained through these representations, greater emphasis has been put on external representations in studies. For example, Lesh et al. (1987) identified five key representations of concepts. These are real-life situations, manipulatives, pictures, written symbols and spoken language. Lesh, Post, Doerr et al. (2003) expanded this classification and added graphical and table representations. Lesh and Doerr (2003) mentioned eight types of representations in which both diagnosis and interpretation can be made. These are equation, table, graph, diagram, concrete models, spoken language, written symbol and metaphor. NCTM (2014), based on the classification by Lesh et al. (1987), addressed representations under five headings: visual, symbolic, verbal, contextual and physical. Contextual representations refer to situations in which mathematical concepts or ideas are situated and presented in a real-life context. Physical representations can be thought of as objects that students can touch and move. Illustrations, graphs, drawings and tables that students benefit from in mathematical processes can be given as examples for visual representations. Examples of verbal representations include the expression of mathematical concepts in spoken and written forms. Finally, the specific language of mathematics such as numbers, variables, equations, formulas and inequality can be handled within the scope of symbolic representation (Huinker, 2015).

Representation, problem-solving, connection or reasoning all represent important skills that must be used and developed in mathematics teaching processes. However, it should not be forgotten that the basic element that ensures the organization of all these skills in the process is the teacher. Therefore, a teacher who wants to design an effective teaching process for his/her students is expected to have a deep knowledge of the subject or concept which is at the center of the process (Fernandez, 2005). In this connection, teacher knowledge is thought to be the main factor in effective teaching and learning (Hill et al., 2008). Shulman (1986) stated that teachers should have three types of knowledge: **content knowledge**, **curriculum knowledge** and **pedagogical content knowledge**. In his study published only one year later, he explained the types of knowledge that teachers should have in his model called Knowledge base for teaching. There are seven types of knowledge in this model. These are: (1) content knowledge, (2) curriculum knowledge, (3) pedagogical content knowledge, (4) general pedagogical knowledge, (5) knowledge of learners, (6) knowledge of educational contexts, and (7) knowledge of educational ends. While the first three types of knowledge are classified as field-specific knowledge, the next fou types of knowledge are grouped as general knowledge that teachers should have regardless of the field (Rowland et al., 2009). Content knowledge includes knowledge about the concepts and phenomena related to the subject to be taught and the situations in which these concepts and phenomena can defend their validity. More briefly, what the teacher knows, how much he/she knows, and what he/she should know are addressed within the context of content knowledge (Ball et al., 2008; Işıksal-Bostan & Osmanoğlu, 2016; Shulman, 1987). According to

Shulman, "*The teacher need not only understand that something is so, the teacher must further understand why it is so*" (Shulman, 1986, p. 9). Thus, the students can learn the subject or concept in more depth thanks to the teacher's content knowledge (Krauss et al., 2008).

On the other hand, it has been stated in recent studies content knowledge alone is not sufficient for effective teaching and that teachers should have knowledge that will increase students' learning as well as content knowledge (Ball et al., 2008; Fennema & Franke, 1992; Grossman, 1990; Park & Oliver, 2008; Shulman, 1986). Shulman (1986) put forward the concept of pedagogical content knowledge (PCK) by stating the importance of how teachers transform content knowledge into a structure that can facilitate students' understanding. In this context, he defined the creation of the most useful representations of ideas, being able to make the most effective analogies, and providing the most appropriate examples and explanations for the subjects taught repeatedly in a field as the indicators of PCK. For example, the teacher's knowing the different meanings of fractions is a matter of content knowledge. On the other hand, deciding which meaning of the fraction at which level and through which method will be taught to students is an issue related to PCK (Aslan-Tutak & Köklü, 2016). According to Mishra and Koehler (2006), a teacher with good PCK has the competence needed to design and implement teaching. For example, a teacher who aims to teach fractions, which is a field in mathematics makes instructional planning, determines the teaching methods and takes students' prior knowledge into consideration, which are all indicators of his/her PCK competence (Ball et al., 2008; Erdem et al., 2015; Getenet & Callingham, 2019). PCK also includes understanding of what makes it easier or more difficult to learn a topic. The teacher should know the prior knowledge that may prevent students from understanding the content topic and should be able to organize the teaching most efficiently by considering this. In this respect, PCK is defined as a special blend of content and pedagogy, which is foremost within the teacher's area of expertise (Shulman, 1987).

The current study

Doing math is based on using representations because mathematical objects cannot be accessed without them (Duval, 2006). A mathematical object has more than one representation, and the relationships to be established between these representations are a requirement for conceptual understanding (Hiebert & Carpenter, 1992). Moving from one representation to another is an important way to add depth of understanding a newly created idea (NCTM, 2000). For example, within PISA (Programme for International Student Assessment) (2015), the ability to select and use appropriate types of representations, including graphs, tables, diagrams, images, equations, formulas and concrete materials, to address a situation, interact with a problem or to present a solution and to make transitions between these representations is an indication of representation competence (OECD – Organisation for Economic Co-operation and Development, 2017). In particular, the selection or design of the appropriate types of representation in the problem-solving process is one of the clear indicators of representation competence within the PISA framework (Turner et al., 2015). According to NCTM (2014), to provide a deep understanding of mathematical concepts and procedures and to carry out effective mathematics education, it is necessary to establish connections between mathematical representations, especially in problem-solving processes. Van der Meij and De Jong (2006) state that mathematics cannot be understood sufficiently in the conceptual dimension if transitions between multiple representations cannot be made. In this context, teachers are expected to encourage students to use multiple representations in the mathematics curriculum (NCTM, 2000). It is thought that teachers should have sufficient knowledge about multiple representations and transitions between representations to achieve this encouragement. Namely, as stated by Shulman (1986), teachers should be able to translate the information to be taught about a subject or concept into representations facilitate students' understanding. Only in this way can they meaningfully answer students' questions about the meanings behind the symbolic representations.

The use of multiple representations and PCK are two important concepts that are directly related to each other and have a significant impact on mathematics achievement (Ball et al., 2008; Dreher & Kuntze, 2015, Shulman, 1986). On the other hand, although many studies deal with the concepts of using multiple representations and PCK independent of each other and under different headings in mathematics teaching, the number of studies addressing this concept as intertwined with PCK by examining pre-service teachers' processes of using representations and transitions between representations is considerably small. For example, Adu-Gymfi et al. (2011) reveal the conceptual misunderstandings in examining students' errors in the transitions between representations. Furthermore, Dreher and Kuntze (2015) implemented a study on how the knowledge, awareness and thoughts of teachers and pre-service teachers about multiple representations were shaped around PCK. In this respect, it is thought that the current study, which investigates multiple representations and PCK together, will contribute to the literature. At the same time, investigation of how much pre-service teachers who will work in primary school where the abstract thinking skills of students are limited are ready for teaching in terms of PCK increases the importance of the current study. Therefore, the current study aims to make inferences about the pre-service teachers' PCK by examining the transitions performed by them between solutions formulated for two different levels. In this regard, answers to the following questions were sought:

1 Which representations did the pre-service teachers use while solving the problems according to their own level?
2 Which representations did the pre-service teachers use while solving the problems according to the level of primary school fourth graders?
3 What are the pre-service teachers' transitions between representations and their PCK in this regard?

Method

Research design

Aiming to examine the pre-service teachers' PCK based on their preferred representations in problem-solving processes, the current study was designed according to the case study design within the scope of qualitative research method. Case study, one of qualitative research designs, refers to the redefinition and examination of a case so as to explain this case, which has certain limitations, and to obtain in-depth data about the particular case (McMillan & Schumacher, 2014).

Study group

The current study was conducted on pre-service primary school teachers. The study group comprised 92 pre-service teachers attending the Department of Primary School Teacher Education at a state university in Turkey in the spring term of the 2018-2019 academic year. All the participating pre-service teachers were senior students. The study was conducted with the participation of senior students so that their degree of readiness to teach math in terms of PCK could be revealed. The participants had taken the Basic Mathematics courses in the 1st and 2nd term of their undergraduate education and the Mathematics Teaching courses in the 4th and 5th term of their undergraduate education. Moreover, they participated in classes in primary schools within the Teaching Practicum course in the 7th and 8th term of their education. Thus, it was assumed that pre-service classroom teachers, who formed the study group, had sufficient knowledge on issues such as problem-solving, representation and PCK. The participating pre-service teachers were included in the study on a voluntary basis.

Data collection

In the current study, two problems from the 4^{th} grade level in TIMSS 2007 and TIMSS 2011 assessments were used to collect the data. The main rationale in the selection of these problems was their suitability for the use of different representations. The pre-service teachers were asked to solve these two problems in two different ways: in the first they were asked to solve them according to their own level, in the second they were asked to solve them as if they were explaining their solutions to fourth graders. The two problems used in data collection are presented in Figures 1 and 2.

The opinions of a scholar specialized in the field of mathematics education were obtained about the suitability of the two problems serving as the data collection tool for transition between representations in the problem-solving process. The opinion of the scholar and a primary school teacher working in the city where the current study was conducted about the suitability of these two problems for primary school level were also sought.

1. Bir baba üç çocuğunu fuara götürmüştür. Bir yetişkinin bilet ücreti bir çocuğun bilet ücretinin iki katıdır. Baba 4 bilet için 50 TL ödemiştir. Her bir çocuğun bilet fiyatı ne kadardır?

a) Siz nasıl çözersiniz?

b) İlkokul 4. sınıf öğrencilerine anlatırken nasıl çözersiniz? (Eğer a seçeneğindeki yolu kullanırsanız tekrar çözmenize gerek yok. "Aynı yolu kullanırım." ifadesini yazmanız yeterlidir.)

Figure 1. The ticket problem asked in the TIMSS 2007 assessment[1]

2. Tunç, bir pastanın $\frac{1}{2}$ ini, Jale de $\frac{1}{4}$ ini yemiştir. Tunç ve Jale birlikte pastanın ne kadarını yemişlerdir?

a) Siz nasıl çözersiniz?

b) İlkokul 4. sınıf öğrencilerine anlatırken nasıl çözersiniz? (Eğer a seçeneğindeki yolu kullanırsanız tekrar çözmenize gerek yok. "Aynı yolu kullanırım." ifadesini yazmanız yeterlidir.)

Figure 2. The cake problem asked in the TIMSS 2011 assessment[2]

Data Analysis

In the analysis of the collected data, the content analysis approach was adopted in line with the nature of the qualitative research approach. First, the pre-service teachers' solutions to the problems presented to them were analysed in terms of the types of representations they used. Relating to this, the representations were separately analysed for the pre-service teachers' solutions at their own level (a) and the fourth grade level (b). Then, the pre-service teachers' transitions between representations (a-b) were analysed and thus attempted to make inferences about their PCK. To establish reliability in the current study, the collected data were first analysed by the researcher and thus codes were formed. The scholar specialized in the field of math teaching was asked to analyse the same data and then interrater agreement was calculated. To do so, the formula [Reliability = Agreement / (Agreement + Disagreement)] proposed by Miles and Huberman (1994) was used, and the rate of agreement was found to be higher than 90 % in all the analysis categories. The raters discussed the categories on which they could not agree and finally reached an agreement.

[1] Translation of the ticket problem:
A father took his 3 children to a fair. Tickets cost twice as much for adults as for children. The father paid a total of 50 zeds for the 4 tickets. How many zeds did each child's ticket cost?

a) How would you solve this problem?

b How would you solve it while explaining its solution to 4th graders? (If you follow the same path you used in (a), then there is no need to solve it again. Just write the statement "I followed the same path")

[2] Translation of the cake problem:
Tunç ate ½ of a cake and Jale ate ¼ of the cake. How much of the cake did they eat altogether?

a) How would you solve this problem?

b) How would you solve it while explaining its solution to 4th graders? (If you follow the same way you used in (a), then there is no need to solve it again. Just write the statement "I followed the same way")

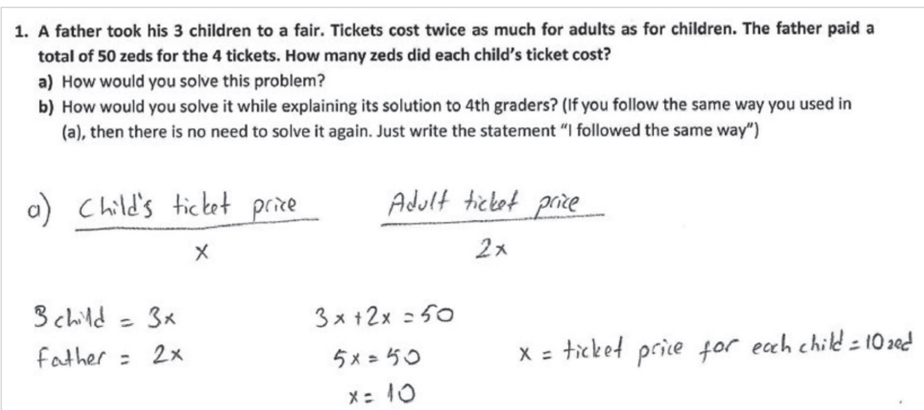
Results

In the current study, in which the pre-service teachers' PCK was analysed based on the representations they used and the transitions between these presentations, firstly the representations they used while solving the problems according to their own level were analysed. The obtained findings are presented in Table 1.

Table 1
Types of representations preferred by the pre-service teachers in the solutions formulated according to their own level

	Ticket problem		Cake problem	
	f	%	f	%
Verbal	4	4.3	2	2.2
Symbolic	80	87	80	87
Visual	3	3.3	8	8.7
Verbal+Symbolic	5	5.4	2	2.2

As can be seen in Table 1, the pre-service teachers mostly preferred symbolic representations for solving both problems (87 %). Also, they rarely preferred verbal and visual representations. In the solution for the ticket problem, 5.4 % of the pre-service teachers used verbal and symbolic representations together, while 4.3 % used verbal representations and 3.3 % used visual representations. An example of the symbolic representation used by the pre-service teachers in the solution of the ticket problem according to their own level is given below.


1. A father took his 3 children to a fair. Tickets cost twice as much for adults as for children. The father paid a total of 50 zeds for the 4 tickets. How many zeds did each child's ticket cost?
a) How would you solve this problem?
b) How would you solve it while explaining its solution to 4th graders? (If you follow the same way you used in (a), then there is no need to solve it again. Just write the statement "I followed the same way")

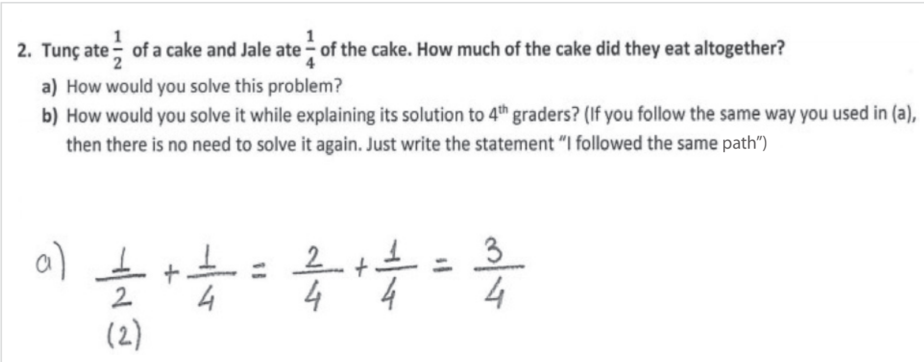
a) Child's ticket price: x   Adult ticket price: 2x

3 child = 3x   3x + 2x = 50
Father = 2x   5x = 50
x = 10
x = ticket price for each child = 10 zed

Figure 3. Symbolic representations used in the solution of the ticket problem

A great majority of the pre-service teachers, as can be seen in Figure 3, preferred the establishment of an equation by using one unknown (such as x, a or k) while solving the ticket problem according to their own level (symbolic representation). When the solutions proposed to the cake problem are examined, it is seen that similar findings were obtained. A great majority of the pre-service teachers (87 %) operated addition by using the symbolic representation by equating denominators utilizing expansion.

In addition to this, 8.7 % of the pre-service teachers used visual representations for the cake problem, while the use of verbal (2.2 %) and verbal+symbolic (2.2 %) representations remained quite low. An example of symbolic representation that the pre-service teachers used according to their level concerning the cake problem are presented in Figure 4.


2. Tunç ate $\frac{1}{2}$ of a cake and Jale ate $\frac{1}{4}$ of the cake. How much of the cake did they eat altogether?

a) How would you solve this problem?

b) How would you solve it while explaining its solution to 4th graders? (If you follow the same way you used in (a), then there is no need to solve it again. Just write the statement "I followed the same path")

a) $\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$
(2)

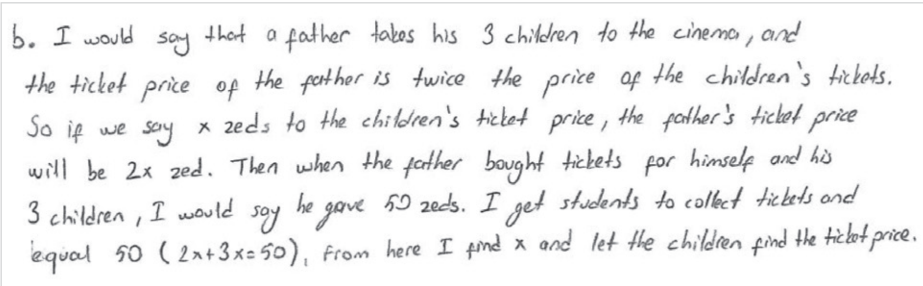
Figure 4. The symbolic representation used in the cake problem

As can be seen in Figure 4, the pre-service teacher solved the problem according to his/her own level by equating the denominators utilizing the expansion of the fraction without using any visual or verbal explanations. In the next stage of the current study, the pre-service teachers were asked to solve the two problems presented to them not according to their own level but according to the level of primary school fourth-graders as a classroom teacher would. In this context, the pre-service teachers' solutions were analysed through the representations they used. The obtained findings are presented in Table 2.

Table 2
The types of representations preferred by pre-service teachers in the solutions formulated according to the level of primary school fourth-graders

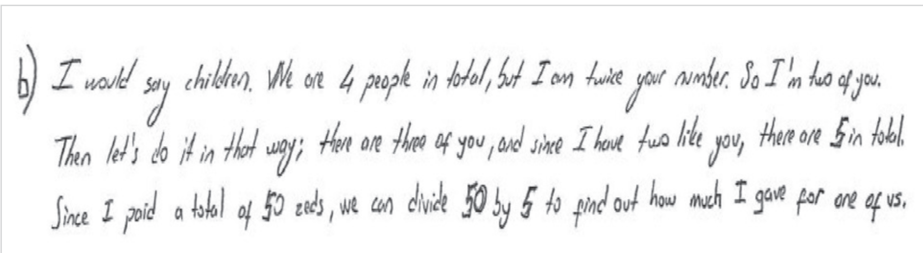
	Ticket problem		Cake problem	
	f	%	F	%
Verbal	23	25	16	17.4
Symbolic	18	19.5	22	23.9
Visual	11	12	18	19.6
Verbal+Symbolic	26	28.3	10	10.9
Verbal+Visual	14	15.2	22	23.9
Visual+Symbolic	-	-	4	4.3

As can be seen in Table 2, it is highly remarkable that the variety and distribution of the representations changed a lot compared to Table 1. When the solutions proposed for the ticket problem by the pre-service teachers according to the level of primary school fourth-graders were analysed, 28.3 % of the pre-service teachers found the use of the type of representation in which verbal and symbolic representations are used together as sufficient for the solution, while 25 % found the use of only the verbal type of representation sufficient. They were followed by symbolic representations with 19.5 % and visual representations with 12 %, while 15 % of the pre-service teachers found the type of representation in which verbal and visual representations are used together sufficient for the solution. Below are examples of preferred representations of pre-service teachers in their solutions to the ticket problem at the primary school level.



b. I would say that a father takes his 3 children to the cinema, and the ticket price of the father is twice the price of the children's tickets. So if we say x zeds to the children's ticket price, the father's ticket price will be 2x zed. Then when the father bought tickets for himself and his 3 children, I would say he gave 50 zeds. I get students to collect tickets and equal 50 (2x+3x=50), from here I find x and let the children find the ticket price.

Figure 5. The verbal+symbolic representation used in the solution of the ticket problem at the primary school level



b) I would say children. We are 4 people in total, but I am twice your number. So I'm two of you. Then let's do it in that way; there are three of you, and since I have two like you, there are 5 in total. Since I paid a total of 50 zeds, we can divide 50 by 5 to find out how much I gave for one of us.

Figure 6. The verbal representation used in the solution of the ticket problem at the primary school level

When the two examples given in Figure 5 and Figure 6 are examined, it is seen that the pre-service teachers frequently used verbal explanations in their solutions at the primary school level. The pre-service teacher whose solution is presented in Figure 5 thought that he/she managed to simplify the solution produced at his/her own level by expanding the symbolic representation he/she used in this solution through verbal explanations to the primary school level. The PCK of this pre-service teacher who continued to use the unknown variable (like x, a or k) can be claimed to be insufficient. When the example presented in Figure 6 is examined, it is seen that the pre-service teacher refrained from using an unknown variable and completely relied on verbal representation. Although he/she not preferring the use of unknown variables at primary school level was considered positive in terms of PCK, his/her complete reliance on verbal representation was considered as his/her level of content knowledge was not good enough.

The analysis of the solutions to the cake problem showed that while nearly 24 % of the pre-service teachers preferred symbolic representation for the primary school level, 24 % of them found the type of representation using visual and verbal representations together more appropriate for the primary school level. In addition to these, while 19.6 % of the pre-service teachers focused on visual representations and 17.4 % of them on verbal representations for their solutions at the primary school level, 10.9 % preferred the type of representation where verbal and symbolic representations are used together. Few of the pre-service teachers (4.3 %) found the use of symbolic and visual representations together appropriate. Below are examples of the representations preferred by the pre-service teachers for the solutions produced at the primary school level.

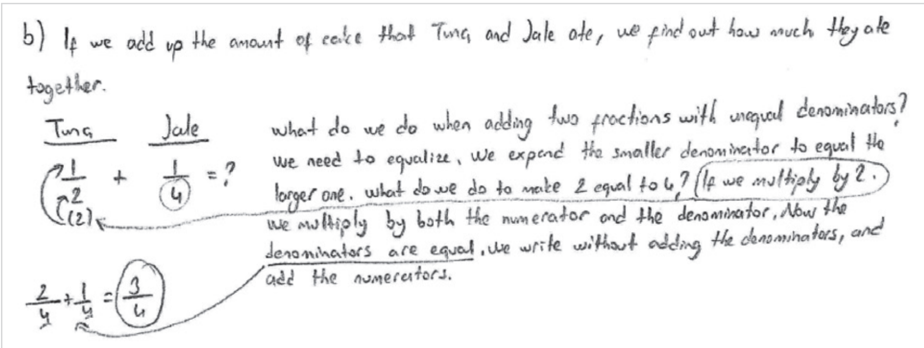


Figure 7. The verbal+symbolic representation used in the solution to the cake problem at the primary school level

The pre-service teacher whose solution is presented in Figure 7, as in the solution he/she formulated at his/her own level, reached the conclusion by equating the denominators using fraction expansion. The only difference was that he/she supported his/her solution with verbal explanations. Here, the pre-service teacher thought that he/she made the solution suitable for the primary school level by using verbal representation. Given that teaching fractions at primary school level strongly requires the use of visual representations, pre-service teachers exhibiting similar approaches can be said to have insufficient PCK based on this example.

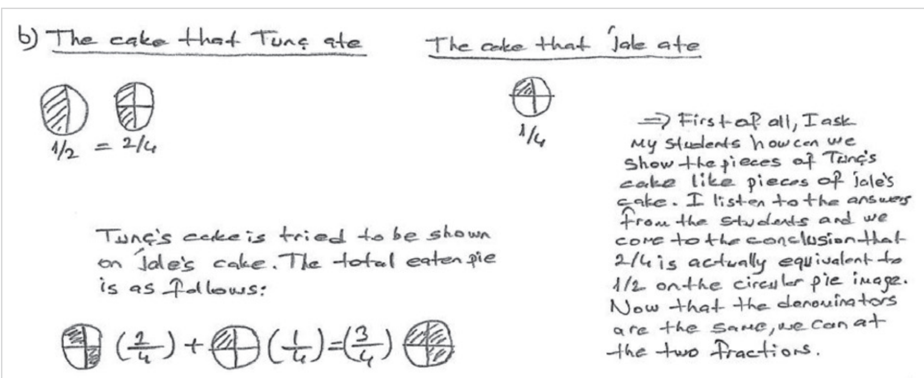


Figure 8. The verbal+visual representation used in the solution to the cake problem at the primary school level

As can be seen in Figure 8, the pre-service teacher effectively used particularly visual representations supported by verbal explanations in his/her solution to the cake problem. This pre-service teacher carried out the fraction expansion operation, not over the arithmetic operation but over the visuals to be suitable for the primary school level. Based on this example, the PCK of the pre-service teachers exhibiting similar approaches can be said to be sufficient. In the final stage of the present study, the types of representations preferred by pre-service teachers in their solutions formulated according to their own level and the types of representations preferred by them in their solutions at the primary school level were comparatively analysed. In this way, transitions between the representations were analysed and the obtained findings are presented in Table 3.

Table 3

Transitions between the representations preferred by the pre-service teachers

		Ticket problem		Cake problem	
From - To		f	%	f	%
Symbolic	Verbal	19	20.6	14	15.2
Symbolic	Visual	8	8.7	10	10.9
Symbolic	Verbal+Visual	14	15.2	22	23.9
Symbolic	Verbal+Symbolic	21	22.8	8	8.7
Symbolic	Visual+Symbolic	-		4	4.3
Total		62	67.4	58	63
Unchanged					
Symbolic	Symbolic	18	19.6	22	23.9
Verbal	Verbal	4	4.3	2	2.2
Visual	Visual	3	3.3	8	8.7
Symbolic+Verbal	Symbolic+Verbal	5	5.4	2	2.2
Total		30	32.6	34	37

The results of the analysis presented in Table 3 concerning the ticket problem show that 67.4 % of the pre-service teachers tended to change the type(s) of representations used at their own level when they were required to formulate a solution for the same problem at the primary school level. On the other hand, 32.6 % of the pre-service teachers thought that the type of representation they used in the solution formulated at their own level was also suitable for a solution at the primary school level and that they would not need any other representation for the solution. The examination of transitions of the pre-service teachers thinking that it is necessary to change the type of representation used in the solution to the ticket problem between representations showed that the highest amount of transitions occurred from symbolic representation to the type of representation using verbal and symbolic representation together (22.8 %) and from symbolic representation to verbal representation (20.6 %). Also, some pre-service teachers displayed transitions from symbolic to verbal+visual (15.2 %) and from symbolic to visual (8.7 %). On the other hand, 20 % of pre-service teachers who used the representation preferred in the solution at their own level at the primary school level as well did not conduct any transition and used symbolic representation at the primary school level thinking that it would also be suitable for the primary school level. Below are examples of pre-service teachers' transitions between representations within the context of the ticket problem.

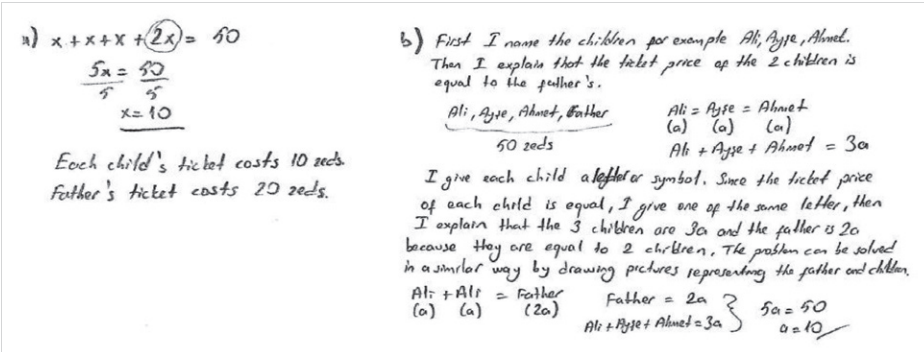


Figure 9. The transition from symbolic representation to verbal+symbolic representation within the context of the ticket problem

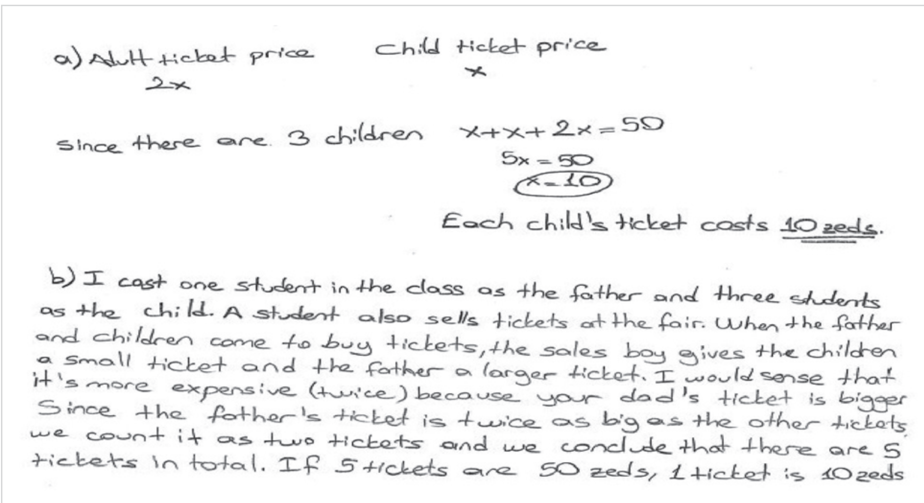


Figure 10. The transition from symbolic representation to verbal representation within the context of the ticket problem

As can be seen in Figure 9, pre-service teacher who used symbolic representation in the solution formulated at his/her own level demonstrated a transition from symbolic to the type of representation using verbal and symbolic representations together in his/her solution formulated to be suitable for the primary school level. To make his/her own solution suitable for the primary school level, the pre-service teachers used the symbol "a" instead of the symbol "x" in his/her representation transition and also used verbal representation to explain the process. Based on this transition, it can be argued that the pre-service teacher does not have sufficient PCK at the primary school level. In Figure 10, the transition of another pre-service teacher from symbolic representation to verbal representation is shown. It can be argued that as the pre-service teacher thought that formulating an equation by using an unknown would not be suitable for the primary school level, he/she was led to a different type of representation. Although it is seen as positive that the pre-service teacher did not use symbolic representation including unknowns at the primary school level, the fact that he/she did not include suitable visuals in his/her solution led to the conviction that he/she did not have sufficient PCK.

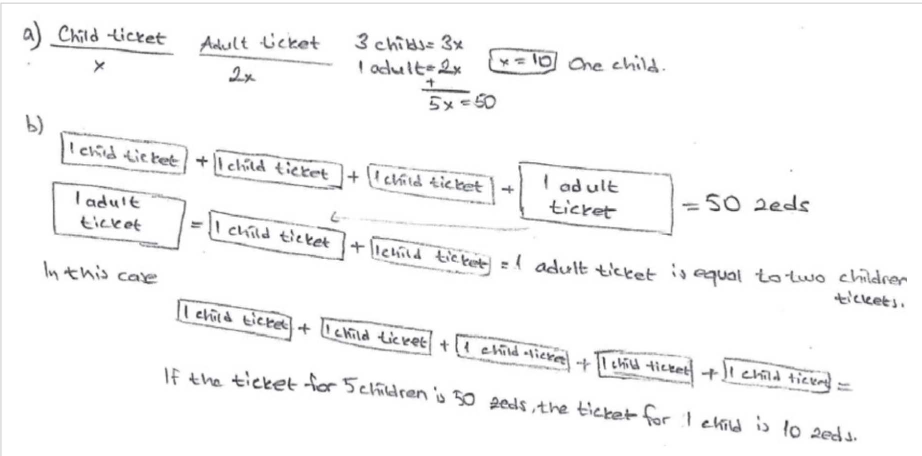


Figure 11. The transition from symbolic representation to visual representation within the context of the ticket problem

As can be seen in Figure 11, the pre-service teacher exhibited a transition from symbolic representation to visual representation. The pre-service teacher solved the problem at the primary school level by using the tickets as the way of representation. By drawing small boxes to represent the tickets depicted the father's ticket as much bigger than those of the children, the teacher indicated that a big ticket is equal to two small tickets. As the pre-service teacher did not prefer the use of the unknown for the solution to the problem and he/she designed an appropriate process of representation with suitable visuals, it can be said that the pre-service teacher's PCK is sufficient.

Similar results have been obtained in the solutions to the cake problem. As for this problem, 63 % of the pre-service teachers changed the type(s) of representations used at their own level when they were required to formulate a solution for the same problem at the primary school level. On the other hand, 37 % of the pre-service teachers thought that the type of representation they used in the solution formulated at their own level was also suitable for the solution at the primary school level and that they would not need any other representation for the solution. The examination of the transitions of the pre-service teachers' thinking that it is necessary to change the type of representation used in the solution to the cake problem between representations showed that the highest number of transitions occurred from symbolic representation to the type of representation using verbal and symbolic representation together (nearly 24 %). Also, 15.2 % of the pre-service teachers preferred transition from symbolic to verbal while 10.9 % of them from symbolic to visual. Fewer pre-service teachers preferred transition from verbal+symbolic (8.7 %) and from symbolic to visual+symbolic (4.3 %). As in the ticket problem, nearly 24 % of the pre-service teachers who used the representation preferred in the solution at their own level for the primary school level did not carry out any transition and used symbolic representation at the primary school level thinking that it would be suitable for the primary school level. Finally, 8.7 % of the pre-service teachers did not use transitions between representations stating that the type of representation they used at their own level would also be suitable for the primary school level. Below are examples of pre-service teachers' transitions between representations within the context of the cake problem.

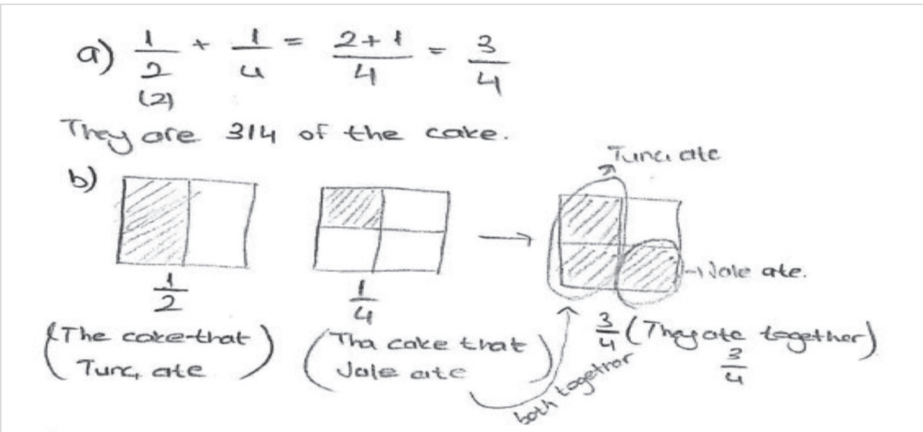


Figure 12. The transition from symbolic representation to visual representation within the context of the cake problem

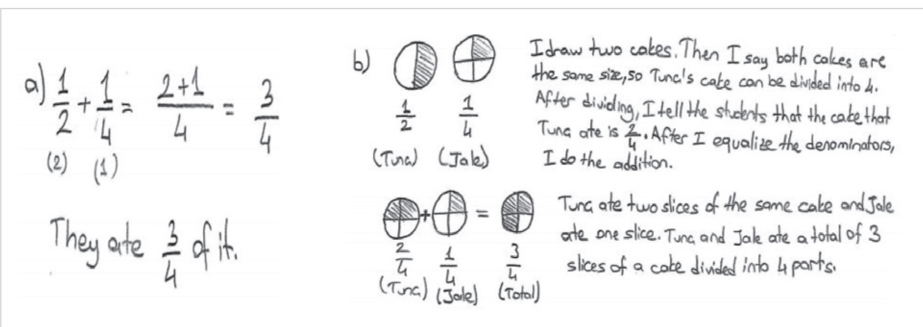


Figure 13. The transition from symbolic representation to verbal-visual representation within the context of the cake problem

According to Figure 12, while the pre-service teacher solved the cake problem at his/her own level by operating denominator equation through fraction expansion, he/she preferred to use visual representation instead of symbolic representation in his/her solution at the primary school level. The pre-service teacher solved the problem at the primary school level by correctly showing the amount of cake the children ate on the square model. Similarly, as can be seen in Figure 13, the pre-service teacher did not prefer symbolic representation based on denominator equalization for the primary school level. In this example, the pre-service teacher performed the denominator equalization operation visually on identical models, not symbolically. Unlike the pre-service teacher in Figure 10, this pre-service teacher showed the amount of cake on the circle model and also preferred to use visual representation together with verbal representation. Based on the two examples of transition, it can be said that the pre-service teachers preferring similar transitions between representations have sufficient PCK.

Discussion and Conclusion

In the current study, representations and transitions between representations preferred by pre-service classroom teachers in their problem-solving process were examined. Firstly, the pre-service teachers were asked to solve the given ticket and cake problems according to their own level. The pre-service teachers preferred to use symbolic representation in their solutions since they did not have any teaching concerns. When their solution processes were examined within the context of the ticket problem, the results showed that pre-service teachers carried out the symbolic representation process by creating equations using the unknown (x, a or k). The analysis of their solutions to the cake problem showed that the symbolic representation process was mostly based on denominator equalization through fraction expansion operation. Similarly, Delice and Sevimli (2010) stated that algebraic (symbolic) representations are remarkably more preferred by pre-service teachers in solving problems with different characteristics in terms of the use of representations. Although most pre-service teachers preferred symbolic representations for the solutions formulated at their own level, some pre-service teachers used visual, verbal and verbal + symbolic representation types. As stated above, it can be said that pre-service teachers tend to prefer symbolic representations more because these representations make it possible for them to solve problems in a shorter period when they do not have any teaching concern.

Secondly, the pre-service teachers were asked to solve the two problems they had initially solved at their own level, but this time at the level of primary school fourth-graders. Here, no directive was given to the pre-service teachers about using a different solution or changing the previous solution. On the contrary, they were told that they could use the same solution if they thought it was appropriate for the primary school level. The results of the analyses revealed that the use of symbolic representations in solutions to both problems decreased. The decreased use of symbolic representations in the solutions formulated at the primary school level can be said to be positive.

Although there was a decrease in the rate of using symbolic representations, the pre-service teachers mostly used verbal + symbolic representation, verbal representation and symbolic representation in the case of the ticket problem. The least used types of representations were visual representation and verbal + visual representation. This shows that the pre-service teachers have the habit and desire to use the algebraic (symbolic) representation, even if the expected gain from the problem is different. Kaldrimidou and Ikonomou (1992) concluded that both students and teachers tend to prefer algebraic use (symbolic representation) to graphical representations (visual representation). Although the decrease in the use of symbolic representation is seen as a positive situation, less use of visual representation compared to other types of representation may be seen as a negative situation for the primary school level. The examination of solutions to the cake problem showed that almost half of the pre-service teachers preferred visual representation and verbal + visual representation. The high rate of use of visual representation and verbal+visual representation at the primary school level is regarded as positive for pre-service teachers. On the other hand, the fact that the other half of the pre-service teachers relied on verbal and symbolic representations in a problem related to fractions is considered to be negative.

In the final stage of the present study, a comparative analysis was made on the representations used by the pre-service teachers in the solutions formulated at their own level and their representations used in the solutions formulated at the primary school level; that is, transitions between representations. Hitt (1998) and Gagatsis and Elia (2004) stated that the ability of making transition between representations is an important factor that plays a role in problem-solving processes. As a result of examining the transitions between representations, inferences were made about the pre-service teachers' PCK. Harries and Barmby (2008) and Heinze et al. (2009) emphasize that the use of multiple representations and the ability to make transitions between different types of representations are an important sign for conceptual learning. The use of multiple representations in mathematics teaching is an important part of PCK and it has come to the fore in mathematics teaching for many reasons (Aslan-Tutak & Köklü, 2016). Representation of concepts with different representations is within the scope of PCK and is very important for effective teaching (Ball et al., 2008). In their solutions to both of the problems, more than 60 % of the pre-service teachers preferred to change their representations. Thus, it can be said that pre-service classroom teachers have moderate flexibility in the transition between representations. This result shows that the pre-service teachers questioned the appropriateness of the types of representation used in the solutions at their own level to the level of primary school fourth-graders and that they felt the need for a change. When these transitions, which are perceived as a positive situation at first glance, are examined in detail, it can be said that they do not meet the expectations adequately, especially in terms of PCK.

In the problem-solving processes for the primary school level, concrete materials and visual representations should be preferred more than verbal and symbolic representations since the students have not yet entered the abstract operations period. Shulman (1987) emphasized that there is not only one strong representation to be used in subject teaching, but teachers also need to know alternative representations related to the subject or concept to be taught. On the other hand, the examination of solutions to the ticket problem showed that the representation transitions of the pre-service teachers were mostly from symbolic to verbal and from symbolic to verbal + symbolic. Bal (2015) concluded that pre-service teachers were more successful in their transition between symbolic and verbal representations than in their transitions between other types of representations. In the transition from symbolic representation to verbal representation, deficiencies in representation skills rather than deficiencies in PCK can be observed. When pre-service teachers switched from symbolic representation to verbal representation, they felt that symbolic representation was not sufficient for the primary school level. They felt the need to apply verbal explanations as they could not think of a more appropriate representation. On the other hand, it can be said that the PCK of pre-service teachers who switched from symbolic representation to verbal + symbolic representation is more insufficient as they did not give up the use of symbolic representation. Here, the pre-service teachers thought that they could simplify the process of creating and solving equations including unknowns with the help of verbal explanations in a way which is suitable for the primary school level. Math classes instructed under the dominance of algebraic (symbolic) representations (Kendal & Stacey, 2003) can be shown as an important cause of this result. Approximately 24 % of the pre-service teachers symbolically represented the ticket problem according to their own level; yet, they made the transition to visual and verbal + visual representations for the solutions formulated at the primary school level. It can be said that the PCK and representation skills of these pre-service teachers are sufficient. Kar et al. (2011) concluded that the rate of transition from symbolic representation to visual representation was low in the pre-service math teachers' transitions between representations. Again, within the context of the ticket problem, 20 % of the pre-service teachers did not feel the need to change symbolic representation and continued to form equations using unknown (x, a, k…) in their solutions at the primary school level. This result can be seen as an indication that approximately 20 % of the pre-service teachers participating in the current study are in a very bad situation in terms of PCK.

When the transitions related to the cake problem were examined, it was observed that the rate of transition from symbolic representation to visual representation (visual and verbal + visual representation) is higher than that found concerning the ticket problem. Chahine (2011) stated that the use of multiple representations and the transitions between representations increase student achievement especially in solving problems related to fractions and thus they can use their reasoning skills more effectively. The teacher's knowledge of the different meanings of fractions is related to content knowledge. On the other hand, teaching which meaning of the fraction at which level and through which method to students is an issue related to PCK (Aslan-Tutak & Köklü, 2016). While the rate of using visual representations in the primary school level solutions to the ticket problem remained around 24 %, this rate increased to about 35 % in the cake problem. Based on this result, it can be argued that the pre-service teachers' PCK about the subjects of the fraction is better. Another remarkable point is that transition from symbolic representation to verbal representation was less in the fraction problem. Here, the pre-service teachers found the appropriate representation more easily in the fraction problem. Therefore, it can be said that the pre-service teachers' ability to represent fraction problems is better than their ability to represent ticket problems. Approximately 24 % of the pre-service teachers considered the symbolic representations they preferred in the solutions at their level as appropriate for the primary school level and carried out denominator equalization operations based on fraction expansion there. Based on this result, it can be said that pre-service teachers who insisted on symbolic representations have insufficient PCK about fractions. A general evaluation of the results of the current study shows that about 30 % of the pre-service teachers were able to solve both problems by using appropriate representation(s) at the primary school level. The PCK of these pre-service teachers can be said to be sufficient. However, given that the participating students are senior students, the rate of these teachers seems to be very low. Dreher and Kuntze (2015) stated that pre-service teachers as well as teachers do not fully understand the key role of the use of multiple representations and PCK directly related to it in mathematics teaching processes.

In the current study, transitions between the representations used by the pre-service teachers were examined and thus inferences were made about their PCK. To improve pre-service teachers' multi-representation skills and PCK in this regard, it can be suggested that the mathematics course contents in the undergraduate program should be more PCK-centred. Similarly, within the scope of practicum teaching courses, activities can be conducted to increase the use of multiple representations and PCK of pre-service teachers. Also, future research can investigate the PCK of in-service teachers and compare it with that of pre-service teachers.

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Istraživanje metodičkoga znanja budućih učitelja na primjeru prijelaza između dvaju prikaza

Sažetak

Svrha ove studije je istražiti prijelaz između prikaza budućih učitelja te doći do zaključaka o njihovom metodičkom znanju. U istraživanju, provedenom kao studiji slučaja sudjelovalo je 92 budućih učitelja, studenata viših godina državnoga fakulteta u Turskoj. Za prikupljanje podataka koristila su se dva zadatka koji su u vrednovanju TIMSS upotrijebljeni za ocjenjivanje učenika četvrtih razreda 2007. i 2011. godine. Sudionici su te zadatke rješavali na osobnoj razini te na razini učenika četvrtih razreda. Provedena je anliza sadržaja na prikupljenim podatcima. Budući učitelji u svojim rješenjima većinom su se koristili simboličkim (algebarskim) prikazima koji su bili u skladu s njihovom razinom. S druge strane, kada su rješavali zadatke na razini učenika četvrtih razreda, primijećena je smanjena uporaba prikaza, osobito u zadatku s ulaznicom. Postotak uporabe prikladnih prikaza u zadatku s tortom bio je viši. Gotovo 30 % budućih učitelja uspjelo je riješiti oba zadatka koristeći se prikazima prikladnima za razinu učenika četvrtih razreda jer su imali dovoljno metodičkoga znanja.

Ključne riječi: *budući učitelji; metodičko znanje (PCK); višestruki prikazi.*

Uvod

Načela i standardi školske matematike (NCTM [Nacionalno vijeće učitelja matematike], 2000) važna su odrednica koja učitelje vodi prema postizanju kvalitetnoga i smislenoga matematičkog obrazovanja u predškolskom, osnovnoškolskom i srednjoškolskom razdoblju. NCTM (2000) je pod tim naslovom postavilo standarde sadržaja i postupaka za vođenje učitelja. Standardi postupaka odnose se na matematičke postupke koji od učenika zahtijevaju stjecanje i uporabu matematičkoga znanja. Ti standardi su rješavanje problema, obrazloženje i dokazivanje, komunikacija, povezivanje i, konačno, prikazi koji su također glavni predmet ovoga istraživanja. Standard prikaza naglašava uporabu simbola, grafikona, konkretnih predmeta i dijagrama kao učinkovitu metodu izražavanja matematičkih ideja i odnosa. Pregled literature prikazuje različite definicije pojma prikaza kao stanja ili struktura koje učinkovito zamjenjuju ili simboliziraju cjelinu ili dijelove nečega (Goldin i Kaput, 1996) ili kao vanjska konkretna stanja ili strukture koje odgovaraju učenikovim mentalnim konceptima (Lesh, Post i Behr, 1987). Prikaz se može promatrati kao opći izraz za simboličke inačice vanjskoga svijeta, koje imaju svoje ekvivalente u umu te se pojavljuju u društvenim kontekstima (Terwel, van OERs, van Dijk i van den Eeden, 2009). U matematičkom obrazovanju pojam prikaza može se definirati kao sredstvo koje je potrebno/ korišteno za obradu matematičkih činjenica i njihovo prenošenje drugoj osobi (Delice i Sevimli, 2016).

Višestruki prikazi odnose se na ponovljeno prikazivanje koncepta putem različitih vrsta prikaza, kao što su verbalni, grafički, matematički prikazi, a učenik se nekoliko puta suočava s istim konceptom (Prain i Waldrip, 2006). U procesu prikaza apstraktni koncepti ili simboli u stvarnom svijetu modeliraju se kao konkretni, stvaraju se odnosi između objekata i simbola, čime se pojedincima olakšava razumijevanje matematičkih situacija (Kaput, 1998). Kada je riječ o definiciji pojma višestrukih prikaza, postoje različiti pristupi njegovoj klasifikaciji. Primjerice, Dufour-Janvier, Bednarz i Belanger (1987) svrstali su pojam prikaza u dvije opće skupine i te skupine nazvali unutarnjim i vanjskim prikazima. Kada se matematički prikazi razmatraju kao matematičke ideje ili kognitivne sheme koje je pojedinac razvio iskustvom, mogu se definirati kao unutarnji. S druge strane, prikazi kao što su brojevi, algebarske jednadžbe, grafovi, tablice, dijagrami su vanjske manifestacije koje nam pomažu razumjeti matematičke koncepte (Janvier, Girardon i Morand, 1993). Prirodni jezik kojim se učenik koristi za razlomak, značenje koje dodjeljuje za svoje stavove prema vizualno-prostornim slikama ili matematici, primjeri su mentalne reprezentacije (Delice i Sevimli, 2016; Goldin, 1998). Vanjski prikazi su vidljivi alati za razumijevanje matematičkih koncepata i ideja. Primjeri vanjskih prikaza uključuju verbalne prikaze (pisane riječi), simboličke prikaze (jednadžbe, formule) i vizualne prikaze (tablice, grafove i druge crteže) (Pape i Tchoshanov, 2001).

U usporedbi s unutarnjim prikazima, većina studija iz metodike matematike prihvatile su vanjske prikaze kao teorijski okvir. Kendal i Stacey (2003) navode da je, s obzirom na to da se barem jedna od vrsta vanjskih prikaza susreće u svim područjima matematike, a većina koncepata se njima lakše objašnjava, veći naglasak u studijama stavljen na vanjske prikaze. Na primjer, Lesh i sur. (1987.) identificirali su pet ključnih prikaza koncepata. To su situacije iz stvarnoga života, realija, slike, pisani simboli i govorni jezik. Lesh, Cramer, Doerr, Post i Zawojewski (2003) proširili su ovu podjelu i dodali grafičke i tablične prikaze. Lesh i Doerr (2003) spominju osam vrsta prikaza u kojima se mogu postaviti dijagnoza i interpretacija. To su jednadžbe, tablice, grafikoni, dijagrami, konkretni modeli, govorni jezik, pisani simboli i metafore. Na temelju klasifikacije koju su ponudili Lesh i dr. (1987), NCTM (2014), svrstao je prikaze u pet skupina: vizualne, simboličke, verbalne, kontekstualne i fizičke. Kontekstualni prikazi odnose se na situacije u kojima su matematički koncepti ili ideje smješteni i prikazani u kontekstu stvarnoga života. Fizičkim prikazima mogu se smatrati predmeti koje učenici mogu dodirnuti i pomaknuti. Ilustracije, grafovi, crteži i tablice od kojih učenici imaju koristi u matematičkim procesima mogu poslužiti kao primjeri vizualnih prikaza. Primjeri verbalnih prikaza uključuju izražavanje matematičkih koncepata u govornim i pisanim oblicima. Konačno, specifičan jezik matematike kao što su brojevi, varijable, jednadžbe, formule, nejednakosti pripada simboličkom prikazu (Huinker, 2015).

Prikaz, rješavanje problema, povezivanje ili apstraktno mišljenje važne su vještine koje se moraju rabiti i razvijati u matematičkim nastavnim procesima. Međutim, ne treba zaboraviti da je učitelj taj osnovni element koji osigurava organizaciju svih tih vještina u nastavnom procesu. Stoga se očekuje da će učitelj koji želi osmisliti učinkovit nastavni proces za svoje učenike imati temeljito znanje o predmetu ili konceptu koji je u središtu procesa (Fernandez, 2005). U tom smislu, znanja učitelja smatraju se glavnim čimbenikom u učinkovitom poučavanju i učenju (Hill, Ball i Schilling, 2008). Shulman (1986) smatra da bi učitelji trebali imati tri vrste znanja: **znanje sadržaja, znanje o kurikulu** i **metodičko znanje**. U studiji objavljenoj godinu dana kasnije, pojasnio je vrste znanja koje bi učitelji trebali imati prema njegovu modelu nazvanom baza znanja za poučavanje. U tom modelu postoji sedam vrsta znanja. To su: (1) znanje sadržaja, (2) kurikulsko znanje, (3) metodičko znanje, (4) opće pedagoško znanje, (5) znanje o učenicima, (6) znanje obrazovnih konteksta i (7) znanje obrazovnih ciljeva. Dok su prva tri navedena znanja klasificirana kao vrste znanja specifičnih za područje, preostale četiri vrste znanja grupirane su kao opće vrste znanja koje bi učitelji trebali imati, bez obzira na područje (Rowland, Turner, Thwaites, i Huckstep, 2009). Znanje sadržaja uključuje spoznaje o pojmovima i pojavama vezanim uz predmet koji se poučava te situacije u kojima ti pojmovi i pojave mogu braniti svoju valjanost. Ukratko, ono što učitelj zna, koliko zna i što bi trebao znati obrađuje se u kontekstu znanja sadržaja (Ball, Thames i Phelps, 2008; Işıksal-Bostan i Osmanoğlu, 2016; Shulman, 1987). Prema Shulmanu, „učitelj ne treba samo shvatiti da je nešto tako, učitelj mora razumjeti i zašto je to tako" (Shulman, 1986, str. 9). Tako će učenici školski predmet ili koncept naučiti dublje zahvaljujući učiteljevu znanju sadržaja (Krauss i sur., 2008).

S druge strane, u novijim se studijama navodi da samo znanje sadržaja nije dovoljno za učinkovito poučavanje te da bi učitelji trebali imati znanja koja će povećati učenikovo učenje i znanje sadržaja (Ball i sur., 2008; Fennema i Franke, 1992; Grossman, 1990; Park i Oliver, 2008; Shulman, 1986). Shulman (1986) je iznio koncept metodičkoga znanja (engl. *pedagogical content knowledge* ili *PCK*) navodeći važnost načina na koji učitelj pretvara znanja sadržaja u strukturu koja će olakšati učenikovo razumijevanje. U tom je kontekstu definirao stvaranje najkorisnijih prikaza ideja, mogućnost stvaranja najučinkovitijih analogija te davanje najprikladnijih primjera i objašnjenja u temama koje se u nekom području više puta poučavaju kao pokazatelje metodičkoga znanja. Primjerice, učiteljevo znanje o različitim značenjima razlomaka pripada znanju sadržaja. S druge strane, odluka o tome koje će značenje razlomka poučavati, na kojoj razini i pomoću koje metode pitanje je koje se odnosi na PCK (Aslan-Tutak i Köklü, 2016). Prema Mishri i Koehleru (2006), učitelj s dobrim metodičkim znanjem posjeduje kompetencije potrebne za planiranje i provedbu nastave predmeta. Primjerice, učitelj kojemu je cilj poučavati tematiku razlomaka, jednu od matematičkih tema, provodi planiranje nastave, određuje metode poučavanja i uzima u obzir predznanje učenika; sve su to pokazatelji njegove metodičke (PCK) kompetencije (Ball i sur., 2008; Erdem i sur., 2015; Getenet i Callingham, 2019). Metodičko znanje također uključuje razumijevanje onoga što olakšava ili otežava učenje neke teme. Učitelj bi trebao biti svjestan predznanja učenika koje bi moglo spriječiti njihovo razumijevanje predmeta i trebao bi, uzimajući to u obzir, znati organizirati poučavanje predmeta na najučinkovitiji način. U tom smislu, metodičko znanje definira se kao poseban spoj sadržaja i pedagogije koja se nalazi samo unutar područja učiteljeve struke (Shulman, 1987).

Istraživanje

Matematika se temelji na prikazima jer bez njih nije moguće pristupiti matematičkim objektima (Duval, 2006). Matematički objekt ima više od jednoga prikaza, a odnosi koje treba uspostaviti između tih prikaza preduvjet su za konceptualno razumijevanje (Hiebert i Carpenter, 1992). Prijelaz iz jednoga prikaza u drugi važan je način da se produbi razumijevanje novostvorene ideje (NCTM, 2000). Na primjer, u okviru PISA-e [Programa za međunarodnu procjenu znanja i vještina učenika] (2015) sposobnost odabira i uporabe odgovarajućih vrsta prikaza uključujući grafove, tablice, dijagrame, slike, jednadžbe, formule i konkretne materijale, za pristup situaciji, interakciju s problemom ili za predstavljanje rješenja te za prijelaze između tih prikaza, pokazatelj su kompetencije o prikazima. (OECD [Organizacija za ekonomsku suradnju i razvoj], 2017.). Konkretno, odabir ili oblikovanje odgovarajućih vrsta prikaza u procesu rješavanja problema jedan je od jasnih pokazatelja znanja o prikazima u okviru PISA-e (Turner, Blum i Niss, 2015). Prema NCTM (2014), kako bi se omogućilo detaljno razumijevanje matematičkih koncepata i postupaka te kako bi matematičko obrazovanje bilo učinkovito, potrebno je uspostaviti vezu između matematičkih prikaza, posebno u procesima rješavanja problema. Van der Meij i De Jong (2006) tvrde da matematiku nije moguće dovoljno razumjeti u konceptualnoj dimenziji ako se ne mogu napraviti prijelazi između višestrukih prikaza. U tom kontekstu, od učitelja se očekuje da potiču učenike na uporabu višestrukih prikaza u nastavnom programu matematike (NCTM, 2000). Kako bi se to postiglo, smatra se da bi učitelji trebali imati dovoljno znanja o višestrukim prikazima i prijelazima između njih. Jer, kako kaže Shulman (1986), učitelji bi trebali informacije koje će se naučiti o predmetu ili konceptu znati prevesti u prikaze koji će učenicima olakšati razumijevanje. Samo će tako moći smisleno odgovoriti na učenička pitanja o značenjima simboličkih prikaza.

Uporaba višestrukih prikaza i metodičko znanje dva su važna pojma koja su izravno međusobno povezana i imaju značajan utjecaj na postignuća u matematici (Ball i sur., 2008; Dreher i Kuntze, 2015, Shulman, 1986). S druge strane, iako se u mnogim istraživanjima razmatraju koncepti višestrukih prikaza i metodičkoga znanja neovisno jedan o drugome i u različitim poglavljima u nastavi matematike, vrlo je mali broj istraživanja koja se odnose na taj koncept isprepleten s metodičkim znanjem provjeravanjem koriste li se budući učitelji prikazima i prijelazima između njih. Primjerice, Adu-Gyamfi, Stiff i Bosse (2011), u ispitivanju pogrešaka studenata u prijelazima između prikaza pokušali su otkriti konceptualne nesporazume. U drugom su komparativnom istraživanju Dreher i Kuntze (2015) ispitali kako su se oko metodike oblikovala znanja, osviještenost i razmišljanja učitelja i budućih učitelja o višestrukim prikazima. U tom smislu, smatra se da će trenutačno istraživanje, kojim se istražuje višestruki prikazi i PCK, doprinijeti literaturi. Istodobno, važnost ovoga istraživanja povećava se ispitivanjem o tome koliko su budući učitelji, koji će raditi u osnovnoj školi u kojoj su apstraktne vještine razmišljanja učenika ograničene, spremni za poučavanje u smislu metodičkoga znanja. Stoga je cilj istraživanja ustanoviti spoznaje o metodičkom znanju budućih učitelja ispitivanjem prijelaza koje oni izvode između rješenja koja su formulirali za dvije različite razine. U tom pogledu potražili smo odgovore na sljedeća pitanja:

1. Koje su prikaze koristili budući učitelji dok su rješavali zadatke na vlastitoj razini?
2. Koje su prikaze koristili budući učitelji pri rješavanju zadataka na razini učenika četvrtog razreda osnovne škole?
3. Kakvi su prijelazi između prikaza kod budućih učitelja te kakvo je njihovo metodičko znanje (PCK) u tom pogledu?

Metoda

Nacrt istraživanja

Cilj je ovoga istraživanja ispitati metodičko znanje budućih učitelja na temelju prikaza koje preferiraju u procesima rješavanja problema, a osmišljeno je kao studija slučaja u okviru kvalitativne metode istraživanja. Studija slučaja, kao kvalitativna metoda istraživanja, odnosi se na redefiniciju i ispitivanje određenoga slučaja kako bi se isti objasnio unutar određenih granica i kako bi se dobili detaljni podatci o konkretnom slučaju (McMillan i Schumacher, 2014).

Uzorak

Istraživanje je provedeno je na uzorku studenata-budućih učitelja. Skupina je obuhvaćala 92 buduća učitelja koji su pohađali Odjel za obrazovanje učitelja u osnovnim školama na državnom sveučilištu u Turskoj u ljetnom semestru akademske godine 2018./2019. Svi uključeni budući učitelji bili su studenti viših godina studija. Istraživanje je provedeno na studentima viših godina studija kako bi se mogao otkriti njihov stupanj spremnosti za poučavanje matematike u smislu metodičkoga znanja. Polaznici su pohađali kolegije Osnove matematike u 1. i 2. semestru preddiplomskoga obrazovanja te kolegije Poučavanje matematike u 4. i 5. semestru preddiplomskoga obrazovanja. Također su sudjelovali u nastavi u osnovnim školama u sklopu Nastavne prakse u 7. i 8. semestru školovanja. Stoga se pretpostavilo da budući učitelji, koji su bili dio skupine, imaju dovoljno znanja o pitanjima kao što su rješavanje problema, prikazi i metodičko znanje. Budući učitelji dobrovoljno su sudjelovali u istraživanju.

Prikupljanje podataka

U ovom su istraživanju za prikupljanje podataka korištena dva zadatka na razini 4. razreda osnovne škole s procjena TIMSS 2007 i TIMSS 2011. Glavni razlog za odabir tih zadataka bila je njihova prikladnost za uporabu različitih prikaza. Od budućih učitelja zatraženo je da riješe ta dva zadatka na dva različita načina. Prvo (a) su ih rješavali na vlastitoj razini, a zatim (b) se od njih se tražilo da ih riješe kao da svoja rješenja objašnjavaju učenicima četvrtog razreda. Na Slici 1 i Slici 2 prikazana su dva zadatka korištena za prikupljanje podataka.

Slika 1.

Slika 2.

Od znanstvenika specijaliziranoga za matematičko obrazovanje tražilo se mišljenja o prikladnosti tih dvaju zadatka koji su poslužili kao alat za prikupljanje podataka za prijelaze između prikaza u procesu rješavanja problema, a od znanstvenika i učitelja u osnovnoj školi zaposlenih u gradu u kojem je provedeno istraživanje tražilo se mišljenje o prikladnosti tih dvaju zadataka za razinu osnovne škole.

Analiza podataka

U analizi prikupljenih podataka korištena je analiza sadržaja u skladu s prirodom kvalitativnoga istraživačkog pristupa. Prvo su analizirana rješenja zadataka koje su ponudili budući učitelji s obzirom na vrste prikaza koje su koristili. S tim u vezi, prikazi su zasebno analizirani za ponuđena rješenja budućih učitelja na vlastitoj razini (a) i na razini četvrtog razreda (b). Zatim su analizirani njihovi prijelazi između prikaza (a-b) te su se prema tome pokušali donijeti zaključci o njihovom metodičkom znanju. Kako bi se utvrdila pouzdanost ovoga istraživanja, prikupljene podatke najprije je analizirao istraživač i prema tome su kodirani. Od znanstvenika specijaliziranoga za poučavanje matematike zatraženo je da analizira te iste podatke, a zatim je izračunat stupanj slaganja među ocjenjivačima. U tu svrhu primijenjena je formula [pouzdanost = slaganje/(slaganje + odstupanje)] koju su predložili Miles i Huberman (1994) i utvrđeno je da je stopa slaganja veća od 90 % u svim kategorijama analize. Ocjenjivači su raspravljali o kategorijama oko kojih se nisu mogli složiti i konačno su postigli sporazum.

Rezultati

U ovom istraživanju, u kojem je analizirano metodičko znanje budućih učitelja na temelju prikaza koje su koristili i prijelaza između tih prikaza, prvo su analizirani prikazi koje su koristili pri rješavanju problema na njihovoj vlastitoj razini. Dobiveni nalazi prikazani su u Tablici 1.

Tablica 1.

Kao što je vidljivo iz Tablice 1, budući učitelji uglavnom su rabili simboličke prikaze za rješavanje oba problemska zadatka (87 %). Također, rijetko su se služili verbalnim i vizualnim prikazima. U rješenju zadatka s ulaznicom 5,4 % budućih učitelja rabilo je i verbalne i simboličke prikaze, dok je 4,3 % rabilo samo verbalne prikaze, a 3,3 % služilo se samo vizualnim prikazima. Primjer simboličkoga prikaza koji su budući učitelji rabili u rješavanju zadatka s ulaznicom na vlastitoj razini nalazi se u nastavku.

Slika 3.

Kao što se može vidjeti na Slici 3, većina je budućih učitelja pri rješavanju zadatka s ulaznicom na vlastitoj razini najradije rabila postavljanje jednadžbe s jednom nepoznanicom (kao što je x, a ili k) (simbolički prikaz). Pri usporedbi predloženih rješenja sa zadatkom s tortom uočava se da su dobiveni nalazi slični. Velika većina budućih učitelja (87 %) upotrijebila je zbrajanje koristeći simbolički prikaz izjednačavajući nazivnike proširivanjem. Osim toga, 8,7 % budućih učitelja rabilo je vizualne prikaze u zadatku s tortom, dok je uporaba verbalnih (2,2 %) i verbalnih + simboličkih (2,2 %) prikaza ostala prilično niska. Na Slici 4 nalazi se primjer simboličnoga prikaza koji su budući učitelji koristili za zadatak s tortom rješavajući ga na vlastitoj razini.

Slika 4.

Kao što se može vidjeti na Slici 4, budući učitelj riješio je zadatak na vlastitoj razini izjednačavajući nazivnike proširivanjem razlomka bez uporabe vizualnih ili verbalnih objašnjenja. U idućoj fazi ovoga istraživanja od budućih učitelja zatraženo je da riješe dva zadana zadatka ne na njihovoj vlastitoj razini, već na razini četvrtog razreda osnovne škole, na način na koji bi učitelj postavio zadatak. U tom kontekstu, rješenja budućih učitelja analizirana su kroz prikaze kojima su se koristili. Dobiveni nalazi prikazani su u Tablici 2.

Tablica 2.

Kao što se vidi iz Tablice 2, prilično je uočljivo/neobično da su se raznolikost i raspodjela prikaza znatno promijenili u usporedbi s Tablicom 1. Kada su analizirana rješenja koja su budući učitelji predložili za zadatak s ulaznicom na razini četvrtog razreda osnovne škole, 28,3 % budućih učitelja smatralo je da je uporaba vrste prikaza u kojoj se verbalni i simbolički prikazi upotrebljavaju zajedno dovoljna za rješenje, dok je 25 % tvrdilo da je dovoljna uporaba samo verbalnoga oblika prikaza. Slijede simbolički prikazi s 19,5 % i vizualni prikazi s 12 %, dok je 15 % budućih nastavnika tvrdilo da je oblik prikaza u kojem se verbalni i vizualni prikazi koriste zajedno dovoljan za rješenje. U nastavku su primjeri prikaza kojima su budući učitelji dali prednost pri rješavanju zadatka s ulaznicom na razini osnovne škole.

Slika 5.

Slika 6.

Pri pregledu dvaju primjera na Slici 5 i 6, vidljivo je da su budući učitelji često rabili verbalna objašnjenja u svojim rješenjima na razini osnovne škole. Budući učitelj čije je rješenje prikazano na Slici 5 mislio je da je uspio pojednostavniti rješenje do kojega je došao rješavajući zadatak na vlastitoj razini proširivanjem simboličkoga prikaza kojim se koristio u ovom rješenju uporabom verbalnih objašnjenja na razini osnovne škole. Metodičko znanje ovoga učitelja koji se nastavio koristiti nepoznanicom (kao što su x, A ili k) može se smatrati nedostatnim. Uvid u primjer prikazan na Slici 6, pokazuje da se budući učitelj suzdržao od korištenja nepoznanicom i u potpunosti se oslonio na verbalni prikaz. Iako se njegovo nekorištenje nepoznanicama na razini osnovne škole smatralo pozitivnim u smislu metodičkoga znanja, zbog njegova oslanjanja samo na verbalne izraze smatra se da njegovo poznavanje sadržaja nije bilo dovoljno dobro.

Analiza ponuđenih rješenja zadatka s tortom pokazala je da, iako je gotovo 24 % budućih učitelja radije rabilo simboličke prikaze za razinu osnovne škole, njih 24 % smatra da je za razinu osnovne škole prikladnija vrsta prikaza pomoću zajedničkih vizualnih i verbalnih prikaza. Osim toga, 19,6 % budućih učitelja usredotočilo se na vizualne prikaze, a 17,4 % na verbalne prikaze za svoja rješenja na razini osnovne škole, dok je 10,9 % dalo prednost tipu prikaza u kojem se verbalni i simbolički prikazi upotrebljavaju zajedno. Malo je budućih učitelja (4,3 %) smatralo kako je zajednička uporaba simboličkih i vizualnih prikaza primjerena. U nastavku su primjeri prikaza koje budući učitelji preferiraju za rješenja do kojih dolaze na razini osnovne škole.

Slika 7.

Budući učitelj čije je rješenje prikazano na Slici 7 kao rješenje koje je izrazio na vlastitoj razini, došao je do rješenja izjednačavanjem nazivnika primjenom proširivanja razlomaka. Jedina razlika bila je u tome što je svoje rješenje potkrijepio verbalnim objašnjenjima. Ovdje je budući učitelj mislio da je uporabom verbalnih prikaza rješenje učinio prikladnim za razinu osnovne škole. Budući da poučavanje razlomaka na razini osnovne škole strogo zahtijeva uporabu vizualnih prikaza, može se na temelju ovoga primjera reći da budući učitelji koji imaju slične pristupe nemaju dovoljno metodičkoga znanja.

Slika 8.

Kao što se može vidjeti na Slici 8, budući učitelj osobito je učinkovito koristio vizualne prikaze potkrijepljene verbalnim objašnjenjima u rješenju zadatka s tortom. Ovaj budući učitelj izveo je operaciju proširivanja razlomka, ne preko aritmetičke operacije, već vizualnim prikazom kako bi bio pogodan za razinu osnovne škole. Na temelju tog primjera, metodičko znanje predškolskih učitelja koji pokazuju slične pristupe može se smatrati dovoljnim. U završnoj fazi ovoga istraživanja uspoređene su vrste prikaza kojima se radije služe budući učitelji u svojim rješenjima izraženim na vlastitoj razini i vrste prikaza koje preferiraju u svojim rješenjima na razini osnovne škole. Na taj su način analizirani prijelazi između prikaza i dobiveni nalazi prikazani su u Tablici 3.

Tablica 3.

Rezultati analize koji se odnose na zadatak s ulaznicom prikazani u Tablici 3, pokazuju da je 67,4 % budućih učitelja obično mijenjalo tip (tipove) prikaza kojima su se koristili na vlastitoj razini kada se od njih tražilo da formuliraju rješenje za isti zadatak na razini osnovne škole. S druge strane, 32,6 % budućih učitelja smatralo je da je vrsta prikaza koju su koristili u rješenju formuliranom na vlastitoj razini također prikladna za rješenje na razini osnovne škole i da im za rješenje neće biti potrebni nikakvi drugi prikazi. Analiza prijelaza budućih učitelja koji su smatrali da je potrebno promijeniti vrstu prikaza kojim se koriste u rješavanju zadatka s ulaznicom pokazala je da je najveći broj prijelaza nastao od simboličkoga prikaza prema tipu prikaza u kojima se zajedno koriste verbalni i simbolički prikazi (22,8 %) i od simboličkoga prikaza prema verbalnom prikazu (20,6 %). Također, neki budući učitelji pokazali su prijelaze sa simboličkoga na verbalni + vizualni (15,2 %) i sa simboličkoga na vizualni (8,7 %). S druge strane, 20 % budućih učitelja koji su isti prikaz koji je poželjan u rješenju na njihovoj vlastitoj razini i koristili na razini osnovne škole, nije provodilo prijelaze i upotrebljavalo je simboličke prikaze na razini osnovne škole smatrajući da bi to bilo prikladno i za razinu osnovne škole. U nastavku su primjeri prijelaza između prikaza budućih učitelja u kontekstu zadatka s ulaznicom.

Slika 9.

Slika 10.

Kao što se vidi na Slici 9, budući učitelj koji se koristio simboličkim prikazom u rješenju formuliranom na vlastitoj razini, u svojem rješenju ponuđenom kao prikladnim za razinu osnovne škole pokazao je prijelaz sa simboličkoga na tip prikaza u kojem se zajedno koristi verbalnim i simboličkim prikazom. Kako bi vlastito rješenje učinili prikladnim za razinu osnovne škole, budući učitelji su rabili simbol „A" umjesto simbola „x" u prijelazu između prikaza te su također koristili verbalni prikaz kako bi objasnili proces. Na temelju tog prijelaza može se utvrditi da budući učitelj nema dovoljno metodičkoga znanja na razini osnovne škole. Slika 10 prikazuje prijelaz drugog budućega učitelja sa simboličkoga na verbalni prikaz. Može se utvrditi da je, s obzirom na to da je budući učitelj smatrao da formuliranje jednadžbe s jednom nepoznanicom ne bi bilo prikladno za razinu osnovne škole, došao do druge vrste prikaza. Iako se smatra pozitivnim to što budući učitelj na razini osnovne škole nije koristio simbolički prikaz koji uključuje nepoznanice, činjenica da u svoje rješenje nije uključio odgovarajuće vizualne prikaze navodi na uvjerenje da nema dovoljno metodičkoga znanja.

Slika 11.

Kao što se vidi na Slici 11, budući je učitelj predočio prijelaz sa simboličkoga na vizualni prikaz. Budući je učitelj riješio zadatak na razini osnovne škole upotrijebivši ulaznice kao način prikaza. Nacrtavši male pravokutnike koji su predstavljali ulaznice i prikazavši očevu ulaznicu puno većom od dječjih ulaznica, učitelj je ukazao na to da je velika ulaznica jednaka dvjema malim ulaznicama. S obzirom na to da budući učitelj nije izabrao uporabu nepoznanice pri rješavanju zadatka te je osmislio odgovarajući proces prikaza s prikladnom vizualnom podrškom, može se reći da je njegovo metodičko znanje dovoljno.

Slični rezultati dobiveni su i u rješenjima za zadatak s tortom. Što se tiče ovoga problemskoga zadatka, 63 % budućih učitelja promijenilo je vrstu (vrste) prikaza kojima su se koristili na svojoj vlastitoj razini kada su rješenje za isti zadatak trebali formulirati za razinu osnovne škole. S druge strane, 37 % budućih učitelja smatralo je da je vrsta prikaza koju su koristili u rješenju formuliranom na vlastitoj razini prikladna i za rješenje na razini osnovne škole te da im za rješenje nije potreban nikakvi drugi prikaz. Provjerom prijelaza onih budućih učitelja koji su smatrali da je potrebno promijeniti vrstu prikaza u rješavanju zadatka s tortom ustanovljeno je da se najveći broj prijelaza dogodio od simboličkoga prikaza prema tipu prikaza u kojima se zajedno koriste verbalni i simbolički prikazi (gotovo 24 %). Također, udio od 15,2 % budućih učitelja dalo je prednost prijelazu sa simboličkoga na verbalni prikaz, dok ih je 10,9 % odabralo prijelaz sa simboličkoga na vizualni. Manji je broj budućih učitelja bio za prijelaz s verbalnoga + simboličkoga (8,7 %) i sa simboličkoga prikaza na vizualni + simbolički (4,3 %). Kao i u zadatku s ulaznicama, gotovo 24 % budućih učitelja koji su rabili prikaz koji je poželjan u rješenju na vlastitoj razini i za razinu osnovne škole, nije provodilo prijelaze i upotrijebilo je simboličke prikaze na razini osnovne škole smatrajući da bi to bilo prikladno i za tu razinu. Konačno, 8,7 % budućih učitelja nije se koristilo prijelazima između prikaza smatrajući da će vrsta prikaza kojom su se služili na vlastitoj razini također biti prikladna za razinu osnovne škole. U nastavku su primjeri prijelaza između prikaza budućih učitelja u kontekstu zadatka s tortom.

Slika 12.

Slika 13.

Prema Slici 12, dok je budući učitelj riješio zadatak s tortom na vlastitoj razini rješavajući izjednačavanje nazivnika proširivanjem razlomka, u svojem rješenju na razini osnovne škole radije je upotrijebio vizualni prikaz umjesto simboličnoga prikaza. Budući je učitelj riješio zadatak na razini osnovne škole ispravnim prikazivanjem količine torte koju su djeca pojela na modelu kvadrata. Slično tome, kao što je vidljivo na Slici 13, budući učitelj nije se odlučio za simbolički prikaz temeljen na izjednačavanju nazivnika za prikaz na razini osnovne škole. U ovom je primjeru budući učitelj izveo postupak izjednačavanja nazivnika vizualno na identičnim modelima, a ne simbolično. Za razliku od budućeg učitelja sa slike 10, ovaj budući učitelj pokazao je količinu torte na modelu kruga i radije je upotrijebio vizualni prikaz zajedno s verbalnim prikazom. Na temelju dvaju primjera prijelaza može se reći da budući učitelji koji se radije koriste sličnim prijelazima između prikaza imaju dostatno metodičko znanje.

Rasprava i zaključak

U istraživanju su analizirani prikazi i prijelazi između prikaza za koje se radije odlučuju budući učitelji u postupku rješavanja problema. Prvo je od budućih učitelja zatraženo je da riješe zadane zadatke s ulaznicom i tortom na vlastitoj razini. Budući su učitelji u svojim rješenjima radije koristili simboličke prikaze jer nisu trebali razmišljati kako ih prezentirati u nastavi. Kada su njihovi postupci rješavanja analizirani u kontekstu zadatka s ulaznicom, rezultati su pokazali da su budući učitelji provodili postupak simboličkoga prikaza pomoću jednadžbi s jednom nepoznanicom (x, a ili k). Analiza njihovih rješenja zadatka s tortom pokazala je da se proces simboličkoga prikaza uglavnom temeljio na izjednačavanju nazivnika postupkom proširivanja razlomka. Slično tome, Delice i Sevimli (2010) smatraju da, što se uporabe prikaza tiče, budući učitelji znatno više rabe algebarske (simboličke) prikaze u rješavanju problemskih zadataka različitih obilježja. Iako se većina budućih učitelja radije koristila simboličkim prikazima za rješenja formulirana na vlastitoj razini, neki su od njih rabili vizualne, verbalne i verbalne + simbolične prikaze. Kako je prethodno navedeno, može se reći da budući učitelji preferiraju simboličke prikaze jer im omogućavaju rješavanje zadataka u kraćem vremenskom roku kada ne moraju razmišljati o tome kako ih prezentirati u nastavi.

Drugo, od budućih je učitelja zatraženo da riješe dva zadatka koja su prvobitno rješavali na vlastitoj razini, ali ovaj put na razini četvrtog razreda osnovne škole. Ovdje se budućim učiteljima nisu davale nikakve upute o uporabi drugačijega puta do rješenja ili izmjeni prethodnoga rješenja. Naprotiv, rečeno im je da mogu upotrijebiti jednak prikaz ako ga smatraju primjerenim za razinu osnovne škole. Rezultati analiza pokazali su da se uporaba simboličkih prikaza u rješenjima oba zadatka smanjila. Smanjena uporaba simboličkih prikaza u rješenjima koja su formulirana na razini osnovne škole može se smatrati pozitivnom. Iako je došlo do smanjenoga udjela uporabe simboličkih prikaza, budući učitelji su se u slučaju zadatka s ulaznicom uglavnom koristili verbalnim + simboličkim, verbalnim i simboličkim prikazima. Najmanje korišteni tipovi prikaza bili su vizualni prikaz i verbalni + vizualni prikaz. To pokazuje kako budući učitelji imaju naviku i želju rabiti algebarske (simboličke) prikaze, čak i ako je očekivana dobit od zadatka drugačija. Kaldrimidou i Ikonomou (1992) zaključili su da i učenici i učitelji radije koriste algebarski pristup (simbolički prikaz) od grafičkih prikaza (vizualnih prikaza). Iako se smanjena uporaba simboličkoga prikaza smatra pozitivnom situacijom, manja uporaba vizualnih prikaza u usporedbi s drugim vrstama prikaza može se smatrati negativnom situacijom za razinu osnovne škole. Pregled rješenja zadatka s tortom pokazao je da je gotovo polovica budućih učitelja radije rabila vizualni i verbalni + vizualni prikaz. Visok postotak uporabe vizualnoga prikaza i verbalnoga + vizualnoga prikaza na razini osnovne škole smatra se pozitivnim za buduće učitelje. S druge strane, činjenica da se druga polovica budućih učitelja oslanjala na verbalne i simboličke prikaze u zadatku koji se odnosio na razlomke smatra se negativnom.

U završnoj fazi istraživanja napravljena je komparativna analiza prikaza koje rabe budući učitelji u rješenjima koja su formulirali na vlastitoj razini i njihovih prikaza kojima se koriste u rješenjima koja su formulirali na razini osnovne škole; analizirani su prijelazi između prikaza. Hitt (1998), Gagatsis i Elia (2004) smatraju da je sposobnost prijelaza između prikaza važan čimbenik u procesima rješavanja problema. Na temelju rezultata ispitivanja prijelaza između prikaza izneseni su zaključci o metodičkom znanju budućih učitelja. Harries i Barmby (2008), Heinze, Star i Verschaffel (2009) ističu da je uporaba višestrukih prikaza i mogućnost prijelaza između različitih vrsta prikaza važna značajka konceptualnoga učenja. Uporaba višestrukih prikaza u poučavanju matematike važan je dio metodičkoga znanja i zbog mnogih se razloga istaknula u poučavanju matematike (Aslan-Tutak i Köklü, 2016). Zastupljenost koncepata s različitim prikazima spada u metodičko znanje i vrlo je važna za učinkovito poučavanje (Ball i sur., 2008). U svojim rješenjima za oba zadatka više od 60 % budućih učitelja željelo je promijeniti svoje prikaze. Stoga se može reći da budući osnovnoškolski učitelji posjeduju umjerenu fleksibilnost u prijelazu između prikaza. Ovaj rezultat pokazuje da su budući učitelji preispitivali prikladnost vrsta prikaza koje su rabili u rješenjima na vlastitoj razini te na razini četvrtog razreda osnovne škole i da su osjetili potrebu za promjenom. Kada se ti prijelazi, koji se na prvi pogled čine kao pozitivne situacije, detaljnije istraže, može se reći da nedovoljno ispunjavaju očekivanja, posebno u pogledu metodičkoga znanja.

U procesima rješavanja zadataka za razinu osnovne škole, trebalo bi dati prednost konkretnim materijalima i vizualnim prikazima nad verbalnim i simboličkim prikazima budući da učenici još nisu ušli u razdoblje apstraktnih operacija. Shulman (1987) je istaknuo kako ne postoji samo jedan dominantni prikaz koji bi se rabio u predmetnoj nastavi, već učitelji također trebaju poznavati alternativne prikaze vezane za predmet ili koncept koji će se poučavati. S druge strane, pregled rješenja zadatka s ulaznicom pokazao je da su prijelazi između prikaza budućih učitelja uglavnom bili od simboličkoga k verbalnom i od simboličkoga k verbalnom + simboličnom. Bal (2015) zaključuje da su budući učitelji bili uspješniji u prijelazu između simboličkih i verbalnih prikaza nego u prijelazu između drugih vrsta prikaza. U prijelazu sa simboličkoga prikaza na verbalni prikaz mogu se uočiti nedostaci u vještini prikaza, a ne nedostaci u metodičkom znanju. Kada su budući učitelji prešli sa simboličkoga prikaza na verbalni prikaz, smatrali su kako simbolički prikaz nije dovoljan za razinu osnovne škole. Osjetili su potrebu primjene verbalnih objašnjenja budući da se nisu mogli dosjetiti prikladnijega prikaza. S druge strane, može se reći da je metodičko znanje budućih učitelja koji su prešli sa simboličkoga prikaza na verbalni + simbolički prikaz nedostatnije jer nisu odustali od uporabe simboličkoga prikaza. Ovdje su budući učitelji mislili da mogu pojednostavniti proces postavljanja i rješavanja jednadžbi s nepoznanicama uz pomoć verbalnih objašnjenja, na način koji je prikladan za razinu osnovne škole. Nastava matematike u kojoj u poučavanju prevladavaju algebarski (simbolički) prikazi (Kendal i Stacey, 2003) može biti važan uzrok ovoga rezultata. Oko 24 % budućih učitelja simbolički je predstavilo zadatak s ulaznicom na vlastitoj razini, no prešli su na vizualne i verbalne + vizualne prikaze za rješenja koja su formulirana za razinu osnovne škole. Može se reći da su vještine uporabe prikaza i metodičko znanje tih budućih učitelja dovoljni. Kar i sur. (2011) zaključili su da je postotak prijelaza sa simboličkoga na vizualni prikaz u prijelazu između prikaza budućih učitelja matematike nizak. Opet, u kontekstu zadatka s ulaznicom, 20 % budućih učitelja u svojim rješenjima na razini osnovne škole nije osjećalo potrebu za promjenom simboličkoga prikaza i nastavilo je postavljati jednadžbe koristeći nepoznanice (x, a, k …). Taj se rezultat može uzeti kao pokazatelj da oko 20 % budućih učitelja koji su sudjelovali u ovom istraživanju ima vrlo loše metodičko znanje.

Pri ispitivanju prijelaza koji se odnose na zadatak s tortom uočeno je da je stopa prijelaza sa simboličkoga prikaza na vizualni prikaz (vizualni i verbalni + vizualni prikaz) veća u odnosu na zadatak s ulaznicom. Chahine (2011) je ustanovila da uporaba višestrukih prikaza i prijelaza između prikaza povećava postignuća učenika, osobito u rješavanju problemskih zadataka povezanih s razlomcima te oni tako mogu učinkovitije iskoristiti svoje vještine promišljanja. Znanje učitelja o različitim značenjima razlomaka povezano je s poznavanjem sadržaja. S druge strane, odluka o tome koja značenja razlomka, na kojoj razini i uporabom koje metode poučavati učenicima pitanje je vezano za metodičko znanje (Aslan-Tutak i Köklü, 2016). Dok je stopa uporabe vizualnih prikaza u rješavanju zadatka s ulaznicom na razini osnovne škole ostala na oko 24 %, ta se stopa u zadatku s tortom povećala na oko 35 %. Na temelju tog rezultata može se ustvrditi da je metodičko znanje budućih učitelja vezano uz temu razlomaka bolje. Drugi je neobičan detalj da je prijelaz sa simboličkoga prikaza na verbalni prikaz bio manji u zadatku s razlomkom. Ovdje su budući učitelji u zadatku s razlomkom lakše pronašli odgovarajući prikaz. Stoga se može reći da je njihova sposobnost da postave zadatke s razlomcima bolja od njihove sposobnosti da postave zadatak s ulaznicom. Oko 24 % budućih učitelja smatralo je da su simbolički prikazi koje su radije koristili u rješenjima na vlastitoj razini primjereni i za razinu osnovne škole i u njima su se koristili operacijama izjednačavanja nazivnika temeljenim na proširivanju razlomaka. Na temelju tog rezultata, može se reći kako budući učitelji koji su ustrajali na uporabi simboličkih prikaza nemaju dovoljno metodičkoga znanja o razlomcima. Opća procjena rezultata ovoga istraživanja pokazuje da je oko 30 % budućih učitelja uspjelo riješiti oba zadatka uporabom odgovarajućih prikaza na razini osnovne škole. Metodičko znanje ovih budućih učitelja može se smatrati dovoljnim. Međutim, s obzirom da su studenti koji sudjeluju u istraživanju studenti viših godina studija, postotak tih učitelja čini se vrlo niskim. Dreher i Kuntze (2015) smatraju da učitelji i budući učitelji u potpunosti ne razumiju ključnu ulogu uporabe višestrukih prikaza i metodičkoga znanja koje je izravno vezano uz njih u procesima poučavanja matematike.

U ovom su istraživanju ispitani prijelazi između prikaza koje su rabili budući učitelji te su na temelju toga izneseni zaključci o njihovom metodičkom znanju. Kako bi se u tom pogledu unaprijedile vještine budućih učitelja za uporabu višestrukih prikaza i njihovo metodičko znanje, moglo bi se predložiti da sadržaji matematičkih kolegija u preddiplomskom programu budu usmjereniji na metodičko znanje. Slično tome, u okviru praktičnih nastavnih kolegija mogle bi se provoditi aktivnosti u kojima bi budući učitelji više rabili višestruke prikaze i metodičko znanje. Također, buduća istraživanja mogu istražiti metodičko znanje učitelja koji su već u službi te ga usporediti s metodičkim znanjem budućih učitelja.