

The Capacitated Multiple Allocation Hub Location Problem under Demand Uncertainty and Excess Capacity with Possibilistic Programming Approach

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Abstract: Facility location is a factor of competitiveness and demand satisfaction. Using a hub on the network can facilitate communication across the network and reduce costs. In the current study, with regards to demand uncertainty, operational costs of the hub, and building extra capacity in the hub it has been aimed to develop a mathematical programming model for the middle hub location problem with a certain capacity. Due to the presence of the uncertainty in the problem's parameters, the possibilistic programming approach which is a subset of fuzzy programming has been used. The proposed model has been investigated via GAMS software and the CPLX solver. Finally, the proposed model has been validated by the dataset obtained from Iran Aviation Dataset (IAD) for a round-trip, and the proper locations for facilities in each level and allocation of the customers to the facilities, were determined by the obtained Pareto analysis answer.

Keywords: capacity; constraint; hub location; possibilistic programming; uncertainty

1 INTRODUCTION

The hubs are specific facilities that serve as the centers for transfer, coordination, and integration in the distribution systems. The research on locating and designing the hub network has been significantly improved during the last two decades. In the complex systems of transportation, communication, and computer systems, there are many points as the origin and destination among which the flow is exchanged. The hub is doing for such purpose [1]. Each of the network nodes is connected to one or several hubs and the hubs, by the use of these links as well as the links, which exist between them, can transfer the flow of information or goods. There is a very large set of points in the hub networks and the use of hubs has led the link between them to be established with a very low number of the links [2].

During recent years, the research on the hub points location has found an important position in the discussion of the location. It has been the result of the increasing use of hub points network in the modern systems of transportation and communication. Such systems try to meet the trip demands or connect the origin and destination in a way that large-scale economization is feasible. The use of the hub enables a higher number of origins and destinations to connect with fewer links. On the other hand, using fewer links in the network leads to the concentration of flow and makes the proper-scale economization viable [3]. The hub location problem was first introduced in the communication and transportation industries in which existed several transmitters and receivers. The system efficiency in transportation is increased by the use of hub points in which the products are collected from the origins and distributed in the destinations. Regarding the vast usages of the hub points networks as well as the product costs index in the competitive market, the study of hub points location is of great importance [4].

In the second section, the research on hub points location has been addressed and in the third section, a certain and uncertain model of hub location would be addressed. The analysis of the results based on the proposed model under the

fuzzy conditions has been done in the fourth section. The conclusion and suggestions are provided in the fifth section.

1.1 Review of the Related Literature

The hubs are special facilities used as the exchange, transfer, and classification points in a large number of distribution systems. The hub facilities, instead of serving between each origin-destination pair, concentrate the flows to use economizations generated by it. The flows are combined with a hub from the same origin with different destinations on their paths, and then, they are combined with the flows with different origins and same destination. In the hub location problems, the flows between the origins and destinations are indicative of the demand and the hubs act as the connection or integration points [5]. Many researchers have studied and compared the issues related to the hub location such as the different strategies of direct transfer or transfer by different terminals as well as the problems related to the route design [6]. Hekmatfar and Pishvaei [7] have dealt with the Hub Location Problems (HLP). The focus of the articles has been initially on the modeling and then its optimization under different conditions [8].

The research conducted on the hub location can be classified into three general categories of the simplification of the mathematical model and provision of a new model with new variables and constraints, provision of new solving methods, and changing the model and making it functional. The existing literature about the hub location problem indicates that the researchers' approach dealing with this subject has been a quantitative one and the qualitative discussions in this field have been rarely addressed. Different types of the problems investigated in the articles are categorized under three categories of problems as middle hub location (with the fixed costs of establishment and without them), the central hub, and the covering hub [7]. Among the features of these problems, the problem space (continuous, discrete, and combinatorial) and the type of the problem's certainty (certain, probable, and fuzzy) can be mentioned.

The hypotheses which are the main distinctions between most of the articles include the building and establishment costs (constant or variable), number of the hubs (definite or indefinite), the nodes and arcs capacity (limited or unlimited capacity), and the type of allocation of the non-hub to hub nodes (single-allocation or multiple allocation). The type of the objective function also, based on the problem modeling, can be single or multi-objective and the minimization (cost or time) and maximization (service or reliability) have been considered. After the establishment of the mathematical model, the used solving methods (accurate, innovative, meta-innovative) can create another classification for the problems [9].

Chen [10] proposed a linear whole number model for a single-allocation middle hub location with limited capacity and an indefinite number of hubs with a constant cost of expression hub establishment. He also provided an innovative solution for IT 5. De Sá et al. [11] proposed a linear whole number model for the location of a single-allocation p-middle hub with limited capacity. Taghipourian et al. [12] dealt with the investigation of hub location problems approach by the use of linear whole number programming. Mohammadi et al. [13] dealt with the investigation of the hub location problems by the use of the colonial competitive algorithm. Yang et al. [14] modeled the p-hub by the use of a genetic algorithm which included local search. Masaeli et al. [4] dealt with transportation programming in hub location problems. Paul et al. [15] investigated a multi-objective problem with the maximum covered problems to the hierarchical location change. Golestani et al. [33] presented a multi-objective green hub location problem with several different temperatures for perishable products.

Mokhtar et al. [2] proposed a model for solving the hub location problems. Eghbali-Zarch et al. [16] investigated the problem of locating the hub by considering the M/G/C queue system. Considering congestion in hubs under uncertainty, they presented a new mixed-integer linear programming model for an HLP with a ring-structured hub network. Wasner and Zäpfel [17] dealt with the programming and optimization of a hub routing model for post-service vehicles. Fernández and Sgalambro [34] examined hub location problem by considering several telecommunications companies.

Mohammadi et al. [18] dealt with the hub location problems by the use of game theory and fuzzy numbers. Zhalehchian et al. [19] dealt with the hub location investigation by the use of an evolutionary algorithm. Ernandes Real et al. [20] dealt with the hub location problems in the aerial networks. Wu et al. [35] selected the market to examine the hub location problem.

Gelareh et al. [21] dealt with the investigation of the hub location problems in transportation. Sadeghi et al. [22], using an artificial bee colony algorithm, investigated the problem of p-hub coverage with reliable travel time. Mohammadi et al. [1] dealt with hub location problems using a metaheuristic hybrid.

Regarding the mentioned research in the field of middle hub location, the objective function has been the

minimization of the transportation and hub establishment costs. In the current study, in addition to these costs, the costs for building extra capacity and the operational costs inside the hub have been also considered, and the demand parameters, operational costs, and costs for creating extra capacity inside the hub in the uncertainty and fuzzy conditions have been investigated. In the current study, a mathematical programming model for middle hub location problem with the definite capacity and uncertainty in demand, operational costs of the hub, and costs for building extra capacity have been developed. To encounter the problem uncertainty, a possibilistic programming approach which is a subset of fuzzy programming has been used.

2 METHODOLOGY

In terms of the type of allocation, the hub location minimization problems can be divided into single-allocation and multi-allocation hub locations. These two models have approximately the same objective with different constraints. In the current study, the development of a fuzzy model for multi-allocation middle hub location problems with limited capacity has been investigated. The model proposed in this section is developed from the Skorin-Kapov et al. [23] reference model.

Sets and indices

Nset of nodes

nset of hub type

inode number index $N \in$

jnode number index $N \in$

khub number index $N \in$

lhub number index $N \in$

Decision-making variables:

W_{ij} The demand rate with the flow between the i and j groups

C_{ijkl} The cost of good transfer from group i to group j by hubs k and l

F_{kn} The constant cost of hub establishment from the n -type in node k

g_{kn} The variable cost of establishment of each unit of hub capacity from the n -type in node k

\bar{Q}_{kn} The maximum annual capacity of goods crossing in the hub from the type n in node k

\underline{Q}_{kn} The minimum annual capacity of goods crossing in the hub from the type n in node k

Cp_{kn} The operational cost including the separation and preparation costs in the hub from the type n in the k^{th}

Y_{kn} If in the k node, an n -type hub is established, the value will be 1 and if not, the value will be 0.

X_{ijkl} The shortage from the sent flow from i to j which is transferred by k and l hubs.

Q_{kn} The quantity of existing goods from the n -type in the k hub

Parameter

2.1 The Certain Model of the Problem

The allocation in the current study is multi-type and any non-hub point can be linked to more than one hub. The number of hubs is not predetermined and the direct link between the hubs is not allowable, and the hub network is complete. The problem space is discrete. The objective function is minimization and single-objective and the arcs capacity is infinite.

The costs for building extra capacity as well as the operational costs inside the hub have been also considered. The parameters and variables added to the basic model are as follows:

- There is a different operational cost in each hub.
- There is a different extra capacity building cost in each hub.
- The operational and extra hub capacity building costs, as well as the demand costs, have been considered as uncertain.
- There are several hub type options for building the hub in a location.

The decision-making variables and indices are initially introduced for mathematical modeling of the problem under study. The objective function is the minimization of total costs including the transportation, hub establishment, extra hub capacity, and operational costs inside the hub. The constraint (2) guarantees that all W_{ij} demand would be supplied. Constraint (3) means that in each site, a maximum of one type of hub can be established. Constraints (4) and (5) indicate that X_{ijkl} would have a value when it is developed in nodes k and l . Constraint (6) is indicative of the quantity of hubs in the hub k . The first part is the total number of goods input to the hub and the second part is the total number of goods output from it. Constraint (7) is a lower limit and suggests a maximum limit for the number of the goods inside hub k . Constraints (8) indicate the non-negative and without shortage flow and the quantity of the existing goods in the hub. Constraint (10) also shows the binary state of the hub building variable.

$$\text{Min } z = \sum_{i \in N} \sum_{j \in N} \sum_{k \in N} \sum_{l \in N} W_{ij} C_{ijkl} X_{ijkl} + \sum_n \sum_{k \in N} F_{kn} Y_{kn} +$$

$$+ \sum_n \sum_{k \in N} F_{kn} Y_{kn} + \sum_n \sum_{k \in N} g_{kn} Q_{kn} + \sum_n \sum_{k \in N} C p_{kn} Q_{kn} \quad (1)$$

$$\sum_{k \in N} \sum_{l \in N} X_{ijkl} = 1 \quad \forall i, j \quad (2)$$

$$\sum_n Y_{kn} \leq 1 \quad \forall k \quad (3)$$

$$\sum_{l \in N} X_{ijkl} \leq \sum_n Y_{kn} \quad \forall i, j, k \quad (4)$$

$$\sum_{k \in N} X_{ijkl} \leq \sum_n Y_{ln} \quad \forall i, j, k \quad (5)$$

$$\sum_{i \in N} \sum_{j \in N} \sum_{l \in N} (W_{ij} X_{ijkl} + W_{ji} X_{jikl}) \leq \sum_n Q_{kn} \quad \forall k, n \quad (6)$$

$$Q_{kn} Y_{kn} \leq Q_{kn} \leq \bar{Q}_{kn} Y_{kn} \quad \forall k, n \quad (7)$$

$$Q_{kn} \geq 0 \quad \forall k, n \quad (8)$$

$$X_{ijkl} \geq 0 \quad \forall i, j, k, l \quad (9)$$

$$Y_{kn} \in \{0, 1\} \quad \forall k, n \quad (10)$$

2.2 The Uncertain Model of the Problem

There are several methods to encounter uncertainties including the mathematical optimization methods, AI-based methods, simulation-based methods, and combined methods [24].

In the current study, the mathematical optimization approach has been used for the application of the uncertain conditions of the parameters. This method of encountering the uncertainty has been developed by the researchers [25].

The possibilistic programming method is used when there is not enough knowledge about the exact values of the input data or parameters due to a lack of access to the needed data or their inadequacy.

In this regard, proper possibilistic distributions based on the existing objective data and the subjective preferences of the decision-maker are introduced for modeling the vague data in the form of fuzzy numbers. These possibilistic distributions are indicative of the possibility of occurrence of different values of parameters based on the existing historical data or the experts' opinions, or both. The possibility distribution is indicative of the possibility rate of possible values occurrence for each parameter in the uncertain state. It is usually determined based on experts' opinions and the existing data on the intended parameter [26].

Fig. 1 shows a trapezoidal possibility distribution in the form of $\varepsilon_1, \varepsilon_2, \varepsilon_3$ and ε_4 . The points ε_1 and ε_4 are indicative of the most pessimistic and most optimistic possible values, respectively. The points ε_2 and ε_3 , and their distance, are indicative of the most possible values which are determined by the decision-makers and the experts based on the existing data and personal knowledge. In the current study and based on the type of the problem being investigated, a trapezoidal fuzzy number has been used for modeling the parameters with uncertainty.

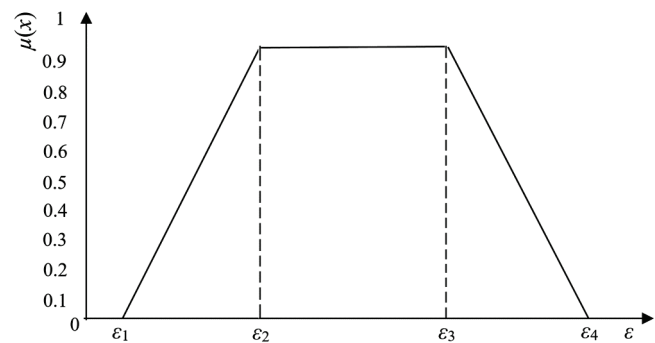


Figure 1 Trapezoidal possibility distribution of uncertain parameter $\bar{\varepsilon}$

In the possibilistic programming problems, there is one possibilistic distribution function considered for each of the uncertain parameters. In the literature related to the fuzzy models, several methods have been presented to encounter

the fuzzy models with uncertain coefficients in constraints and objective functions. The method proposed by Liu has been used in the current study to change the proposed models into their definite counterparts.

Generally, the credit-based chance-constrained programming method is one of the sufficient possibilistic programming methods which has the following advantages [27]:

- 1) It is based on credible mathematical concepts such as the expected distances and mathematical expectation of the fuzzy numbers. Therefore, it has a strong mathematical ground.
- 2) It is designed based on the general rating method proposed by Liu [27] and can be applied to uncertain parameters with different fuzzy membership functions such as the triangular and trapezoidal functions. It can be also applied to non-linear membership functions in both symmetric and asymmetric states.
- 3) It enables the decision-maker to adjust the confidence level of constraints satisfaction and helps him to satisfy some chance constraints in the minimum confidence level.
- 4) The most significant advantage of this method is the use of credit measure which, opposed to the location and necessity measure that lacks the self-duality feature, is a self-dual measure [28]. In other words, when the credit measure is equal to 1, the decision-maker believes that the fuzzy event would occur. In addition, when the credit measure is equal to 0, he would believe that the fuzzy event will not occur (theoretically). However, when the fuzzy event possibility is equal to 1, there is still the possibility that the event won't occur. Besides, when the necessary measure of a fuzzy event is equal to 0, theoretically, there is no guarantee that the event will not occur.

Assume that $\bar{\varepsilon}$ is a fuzzy variable with the membership function of $\mu(x)$, and r is a real number, the credit measure is defined as Eq. (11) [29].

$$C_r \{ \bar{\varepsilon} \leq r \} = \frac{1}{2} \begin{cases} \sup \mu(x) + 1 - \sup \mu(x) & x \leq r \\ \sup \mu(x) & x > r \end{cases} \quad (11)$$

Since the possibility measure is equal to $pos \{ \bar{\varepsilon} \leq r \} = \sup \mu(x)$, $x \leq r$ and necessity measure is equal to $Nec \{ \bar{\varepsilon} \leq r \} = 1 - \sup \mu(x)$, $x > r$, the credit measure can be defined as Eq. (12):

$$C_r \{ \bar{\varepsilon} \leq r \} = \frac{1}{2} [pos \{ \bar{\varepsilon} \leq r \} + Nec \{ \bar{\varepsilon} \leq r \}]. \quad (12)$$

Eq. (12) indicates that the credit measure is, in fact, the mean of the possibility and necessity measures. In addition, concerning the proposed definitions, Eq. (13) expresses the mathematical expectation of variable $\bar{\varepsilon}$.

$$E[\bar{\varepsilon}] = \int_0^\infty C_r \frac{1}{2} \{ \bar{\varepsilon} \leq r \} dr - \int_{-\infty}^0 C_r \frac{1}{2} \{ \bar{\varepsilon} \leq r \} dr \quad (13)$$

Now assume that $\bar{\varepsilon}$ is a trapezoidal fuzzy number as $\bar{\varepsilon} = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4)$, then based on the Eqs. (12) and (13), the mathematical expectation of this fuzzy number is $(\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4)/4$ and its credit measure is in the form of Eqs. (14) and (15).

$$C_r \{ \bar{\varepsilon} \leq r \} = \begin{cases} 1 & r \in (-\infty, \varepsilon_3) \\ \frac{r - \varepsilon_1}{2(\varepsilon_2 - \varepsilon_1)} & r \in (\varepsilon_1, \varepsilon_2] \\ \frac{1}{2} & r \in (\varepsilon_2, \varepsilon_3] \\ \frac{r - 2\varepsilon_3 + \varepsilon_4}{2(\varepsilon_4 - \varepsilon_3)} & r \in (\varepsilon_3, \varepsilon_4] \\ 1 & r \in (\varepsilon_4, \infty) \end{cases} \quad (14)$$

$$C_r \{ \bar{\varepsilon} \leq r \} = \begin{cases} 1 & r \in (-\infty, \varepsilon_3) \\ \frac{2\varepsilon_2 - \varepsilon_1 - r}{2(\varepsilon_2 - \varepsilon_1)} & r \in (\varepsilon_1, \varepsilon_2] \\ \frac{1}{2} & r \in (\varepsilon_2, \varepsilon_3] \\ \frac{\varepsilon_4 - r}{2(\varepsilon_4 - \varepsilon_3)} & r \in (\varepsilon_3, \varepsilon_4] \\ 0 & r \in (\varepsilon_4, \infty) \end{cases} \quad (15)$$

Based on the Eqs. (14) and (15), for each $0.5 > \alpha$, the following expressions are provable [30]:

$$C_r \{ \bar{\varepsilon} \leq r \} \geq \alpha \leftrightarrow r \geq (2 - 2\alpha)\varepsilon_3 + (2\alpha - 1)\varepsilon_4 \quad (16)$$

$$C_r \{ \bar{\varepsilon} \geq r \} \geq \alpha \leftrightarrow r \geq (2\alpha - 1)\varepsilon_1 + (2 - 2\alpha)\varepsilon_2 \quad (17)$$

$$C_r \{ \bar{\varepsilon} = r \} \geq \alpha \leftrightarrow r \geq \left[2 \left(\frac{\alpha}{2} \right) - 1 \right] \varepsilon_1 + \left[2 - 2 \left(\frac{\alpha}{2} \right) \right] \varepsilon_2 \quad (18)$$

$$C_r \{ \bar{\varepsilon} = r \} \geq \alpha \leftrightarrow r \geq \left[2 - 2 \left(\frac{\alpha}{2} \right) \right] \varepsilon_2 + \left[1 - 2 \left(\frac{\alpha}{2} \right) \right] \varepsilon_3 \quad (19)$$

The Eqs. (16) and (17) can be directly used for changing the fuzzy chance constraints into their definite counterparts. It should be noted that Liu [27] has also proposed expressions like what was mentioned under the title of α critical values for changing the fuzzy chance constraints into their definite counterparts.

The uncertainty parameters in the model include the followings:

W_{ij} is the demand rate or the sent flow between the groups i and j . The following parameters are indicative of the trapezoidal fuzzy numbers of the demand rate:

$$(W_{ij}^1, W_{ij}^2, W_{ij}^3, W_{ij}^4)$$

Cp_{kn} is the operational cost. The following parameters are indicative of the trapezoidal fuzzy numbers for operational as follows:

$$(Cp_{kn}^1, Cp_{kn}^2, Cp_{kn}^3, Cp_{kn}^4)$$

g_{kn} is the variable cost of building each extra unit of capacity in the hub from the type n in node k . The following parameters are indicative of trapezoidal fuzzy numbers of costs of building each extra unit of capacity in the hub:

$$(g_{kn}^1, g_{kn}^2, g_{kn}^3, g_{kn}^4)$$

In the current study, a combined approach of credit-based possibilistic programming which uses the mathematical expectation for modeling the objective function and a chance programming approach for modeling the constraints has been applied. This model, unlike the chance-dependent programming approaches [31, 32], has not increased the number of the basic model's constraints and does not need extra information about the objective function (such as the confidence level or ideal value). In addition, this model lacks the disadvantages of chance programming and chance-dependent programming methods. It is also possible to use these methods' advantages in controlling the constraints [26]. Regarding the mentioned approach, the uncertain model would be as follows:

$$\begin{aligned} \text{Min } z = & \sum_{i \in N} \sum_{j \in N} \sum_{k \in N} \sum_{l \in N} \left(\frac{W_{ij}^1 + W_{ij}^2 + W_{ij}^3 + W_{ij}^4}{4} \right) C_{ijkl} X_{ijkl} + \\ & + \sum_n \sum_{k \in N} F_{kn} Y_{kn} + \sum_n \sum_{k \in N} \left(\frac{g_{kn}^1 + g_{kn}^2 + g_{kn}^3 + g_{kn}^4}{4} \right) Q_{kn} + \end{aligned} \quad (20)$$

$$\begin{aligned} & + \sum_n \sum_{k \in N} \left(\frac{Cp_{kn}^1 + Cp_{kn}^2 + Cp_{kn}^3 + Cp_{kn}^4}{4} \right) Q_{kn} \\ & \sum_{k \in N} \sum_{l \in N} X_{ijkl} = 1 \quad \forall i, j \end{aligned} \quad (21)$$

$$\sum_n Y_{kn} \leq 1 \quad \forall k \quad (22)$$

$$\sum_{l \in N} X_{ijkl} \leq \sum_n Y_{kn} \quad \forall i, j, k \quad (23)$$

$$\sum_{k \in N} X_{ijkl} \leq \sum_n Y_{ln} \quad \forall i, j, k \quad (24)$$

$$\begin{aligned} & \sum_{i \in N} \sum_{j \in N} \sum_{l \in N} \left[(2 - 2a_{kn})W_{ij}^3 + (2a_{kn} - 1)W_{ij}^4 \right] x_{ijkl} + \\ & + \sum_{i \in N} \sum_{j \in N} \sum_{l \in N} \left[(2 - 2a_{kn})W_{ij}^3 + (2a_{kn} - 1)W_{ij}^4 \right] x_{ijkl} \leq \sum_n Q_{kn} \end{aligned} \quad (25)$$

$$Q_{kn} Y_{kn} \leq Q_{kn} \leq \bar{Q}_{kn} Y_{kn} \quad \forall k, n \quad (26)$$

$$Q_{kn} \geq 0 \quad \forall k, n \quad (27)$$

$$X_{ijkl} \geq 0 \quad \forall i, j, k, l \quad (28)$$

$$Y_{kn} \in \{0, 1\} \quad \forall k, n \quad (29)$$

$$0.5 \leq \alpha \leq 1 \quad \forall k \quad (30)$$

3 RESULTS AND DISCUSSION

In this section, the data obtained from the IAD (considering 20 nodes) have been used via the proposed model, so that the performance and efficiency of this model will be investigated. The data in the references are the data on the demands for a trip from each province to another. Therefore, for the calculation of the transfer costs between the origin and destination, the discount factor for transfers between the hubs have been generated. In the classic problems of hub point location, the costs for hub-to-hub arcs transfer are decreased with a constant discount factor generically. In this problem, the discount factor has been considered to be 0.7. With the assumption that in each center, three types of hubs with different costs and capacities can be built, the data which are not available (Tab. 1) have been generated in invariable intervals, each in three levels.

The GAMS software Ver.24.1.3 has been used on a computer with 4 gigabytes of RAM and Core i5 2.6 GHz CPU.

Table 1 The data used in the problem

Row	Data type	Data value
1	Demand flow	Available
2	Distance	Available
3	Building cost	An invariable interval [400 and 750]
4	Capacity building cost	An invariable interval [65 and 35]
5	Maximum capacity	An invariable interval [9000 and 125000]
6	Minimum capacity	An invariable interval [900 and 2100]
7	Operational cost	An invariable interval [45 and 75]

As was mentioned in section three, α is the confidence level value, which is the uncertain parameter constraint per α percent of the feasible events. On the other hand, this coefficient can take 0.5 to 1 values. The α factor is analyzed twice in the current study in a way that once it is considered as the variable to determine the value the model itself obtains, and once it has been taken as a parameter. In the latter state, different values are allocated to the parameter and the results have been analyzed. In fact, with the increase in α value, risk-taking of the decision-maker is also increased and in fact, it indicates that how much we are willing to cost and risk so that the constraint with the uncertain parameter be feasible with the higher confidence level and satisfy the decision-maker's confidence level. To respond to the research questions, the model solving results are demonstrated in tables and figures in a way that the hub points in each phase of problem-solving, and the allocated nodes, as well as the objective costs, have been investigated.

3.1 Factor α as the Variable

In this mode, the factor α is treated as a variable and the model itself would obtain its value. The results are shown in Tab. 2 and Fig. 2. The results indicate that the model since

the objective function has been minimized has obtained the minimum α value to have the lowest risk and cost.

Table 2 The obtained results with taking α as a variable

The value obtained from model solving α	Objective function value	The intended points for building the hubs	Type of hub	Model solving time (s)
0.5	2.515522×10^8	2, 19	1, 1	8781/352

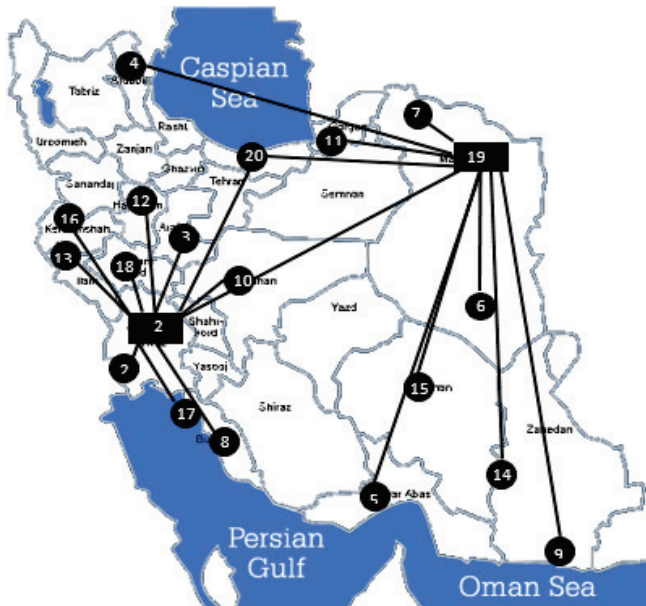


Figure 2 The hub points and nodes allocation to the hub with the assumption of variable being of factor

3.2 Factor α as the Parameter

In this mode, this factor has been taken as a parameter and different values in the allowable range have been allocated to it. The selection of this value depends on the rate of risk-taking by the decision-maker and with the increase in this rate; the intensity of constraint with the uncertain parameter is increased. The results are shown in the following, in the form of Tab. 3 and Fig. 3. In this mode, the results are the same as the mode in which the confidence level was taken as a variable (Tabs. 4-8, Figs. 4-8).

Table 3 The results for $\alpha = 0.5$

Parameter's value	Value of objective function	The points intended for hub building	Type of the hub	Solving time (s)
0.5	3.515522×10^8	2, 19	1, 1	6.521

Table 4 The results for the hub points and allocated nodes with $\alpha = 0.6$

Parameter's value	Value of objective function	The points intended for hub building	Type of the hub	Solving time (s)
0.6	3.536406×10^8	2, 19	1, 2	3.556

Table 5 The results for the hub points and allocated nodes with $\alpha = 0.7$

Parameter's value	Value of objective function	The points intended for hub building	Type of the hub	Solving time (s)
0.7	3.647931×10^8	19, 17, 3	2, 1, 3	3.453

Table 6 The results for the hub points and allocated nodes with $\alpha = 0.8$

Parameter's value	Value of objective function	The points intended for hub building	Type of the hub	Solving time (s)
0.8	3.756350×10^8	15, 17, 12	2, 2, 3	3.582

Table 7 The results for the hub points and allocated nodes with $\alpha = 0.9$

Parameter's value	Value of objective function	The points intended for hub building	Type of the hub	Solving time (s)
0.9	3.886346×10^8	20, 19, 17	2, 2, 3	3.591

Table 8 The results for the hub points and allocated nodes with $\alpha = 1$

Parameter's value	Value of objective function	The points intended for hub building	Type of the hub	Solving time (s)
1	4.212080×10^8	15, 19, 17, 12	1, 2, 2, 3	3.623



Figure 3 The hub points and allocated nodes with $\alpha = 0.5$



Figure 4 The hub points and allocated nodes with $\alpha = 0.6$

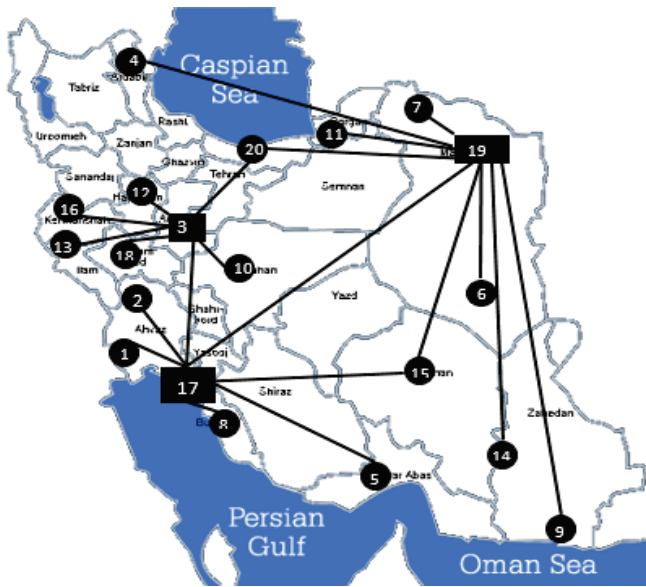


Figure 5 The hub points and allocated nodes with $\alpha = 0.7$

minimized and through obtaining the minimum confidence level, lower costs would be incurred.

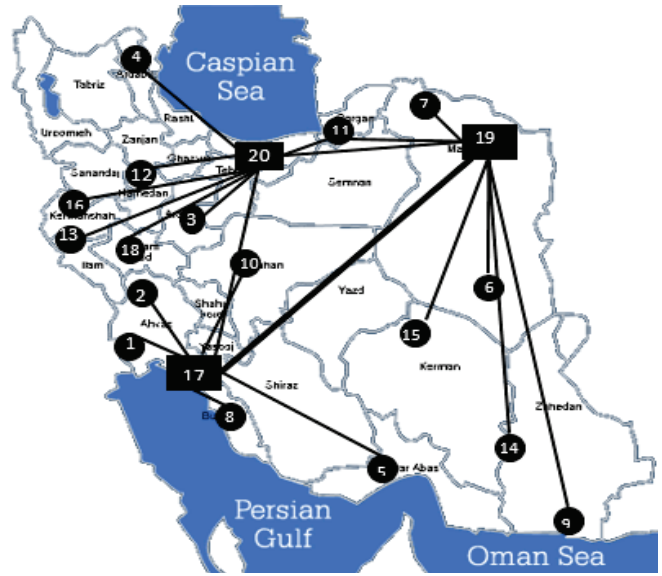


Figure 7 The hub points and allocated nodes with $\alpha = 0.9$

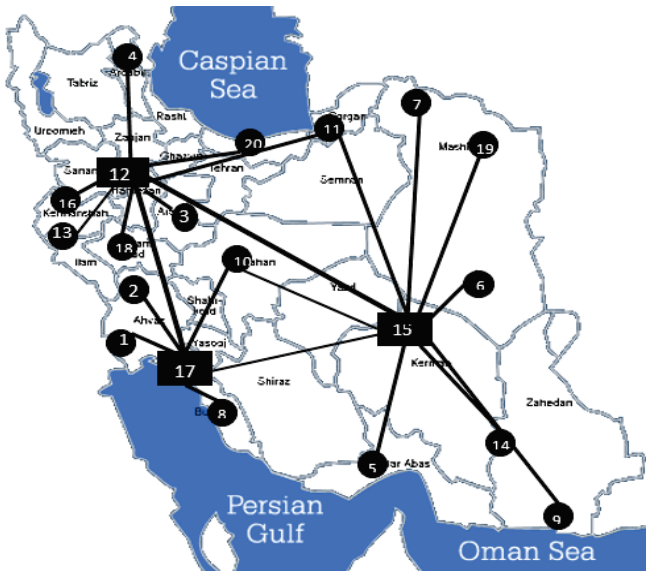


Figure 6 The hub points and allocated nodes with $\alpha = 0.8$

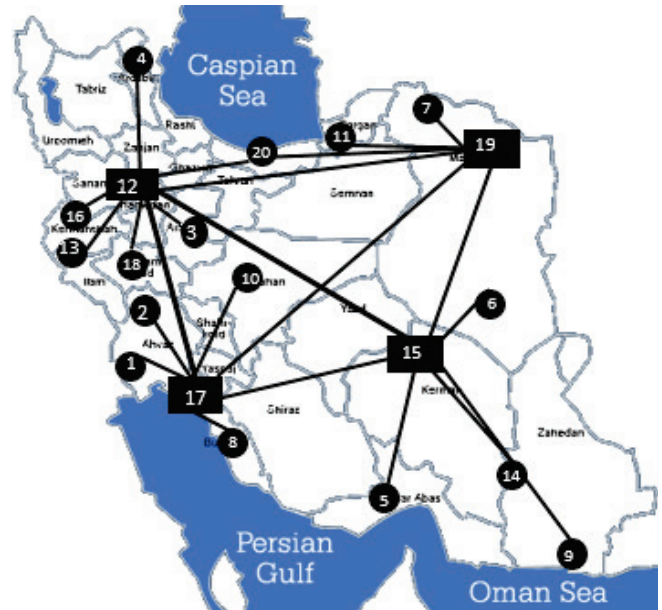


Figure 8 The hub points and allocated nodes with $\alpha = 1$

Table 9 The model solving results with consideration of the confidence level as a variable

The value of α obtained from model solving	Value of objective function	The points intended for hub building	Type of the hub	Solving time (s)
0.5	3.515522×10^8	2,19	1,1	8781.352

α is the confidence level value which is the uncertain parameter constraint per α percent of the feasible events. On the other hand, this coefficient can take 0.5 to 1 values. The α factor is analyzed twice in the current study in a way that once it is considered as the variable to determine the value the model itself obtains, and once it has been taken as a parameter. In the latter state, different values are allocated to the parameter and the results have been analyzed. According to the results obtained in Tab. 9, when we consider the confidence level of feasibility of the constraint as a variable, the model solving obtains the minimum value, which is 0.5, for the confidence level factor since the objective function is

Table 10 The results obtained from the model solving with the change in confidence level parameter in the allowable range

Row	Value α	Value of objective function	The points intended for hub building	Type of the hub	Solving time (s)
1	0.5	3.515522×10^8	2,19	1,1	6.521
2	0.6	3.536406×10^8	2,19	1,2	3.556
3	0.7	3.647931×10^8	19,7,3	2,1,3	3.453
4	0.8	3.756350×10^8	15,17,12	2,2,3	3.582
5	0.9	3.886346×10^8	20,19,17	2,2,3	3.591
6	1	4.212082×10^8	20,19,17,12	1,2,2,3	3.623

Then, the confidence level factor has been taken as a parameter and different values in the allowable range have been allocated to it so that the model's behavior can be investigated. Besides, each of the change modes of this

factor, the number of hub points, and allocated nodes to the hub were determined. Tab. 10 shows a summary of the results.

Tab. 10 shows the sensitivity analysis on the values of parameter α and each case of a parameter change, the costs, problem-solving time, and the intended hub points, as well as their types, have been determined. As seen in the table, with the increase in parameter α , the objective function value has been also increased, which is shown in Fig. 9. The higher the confidence level rate, the more conservative the decision will be in a way that the number of hub points is increased and the hub type is also changed. Therefore, regarding the conducted validation, this model can be used for hub location in the presence of uncertainty through the use of experts' opinions for the determination of the range of the uncertain parameters. By this possibility, the decision-maker would be able to satisfy some chance constraints in the lowest confidence level.

The numbers in the proposed model analysis have been obtained by the GAMS software.

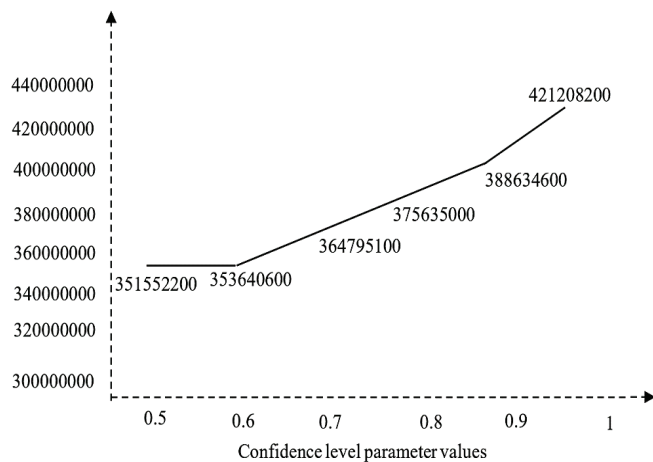


Figure 9 The graph of the changes in costs per increase in the confidence level parameter

4 CONCLUSIONS

The main objective of the hub location models is the selection of the proper hub points from the potential points and allocation of the non-hub point to selected hub points by the model. Its objective function is the minimization of the total costs of transportation between the nodes and the hub building costs. A single-objective linear model has been developed in the current study which is, in fact, a more advanced variant of hub models. In this model, in addition to the hub building costs and the transportation costs, the extra capacity building inside the hub and the operational costs have been also considered. On the other hand, regarding the fact that the objective is to apply uncertain conditions, the demand parameters, operational costs, and extra capacity building costs have been also considered to be uncertain and these uncertain parameters have been represented by trapezoidal fuzzy numbers. In the following, the introduced fuzzy programming was used and the model was rewritten in the form of a linear programming model. The proposed model

was solved by standard optimization software and the hubs and the nodes allocated to them were determined. The results indicated that with the change in the feasibility confidence level of the constraint with the uncertain parameter, the costs are also increased and the number of the type of hubs is changed as well. It is also tangible in the real world since we incur additional costs as decisions become more stringent, so it can be mentioned that the more the uncertainty, the higher the costs. Therefore, with consideration for the experts' opinions about the range of uncertain parameters this model can be used for middle hub location problems under uncertainty. Through this model, the decision-maker, with the lowest confidence level, would be able to satisfy some constraints with uncertain parameters. It is suggested that future studies use an innovative or meta-innovative approach for solving the problem at larger scales. Also, proportionate to the increase in hub capacity, the operational costs can be increased step-by-step.

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