

# MATHEMATICAL FORMULATION AND THE PROBLEM SOLUTION OF CLUSTERING RECIPES OF CONCRETE MIXTURES USING TECHNOGENIC WASTE AND SLAGS OF METALLURGICAL ENTERPRISES

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Preliminary Note – Prethodno priopćenje

The relevance of the use of man-made waste and slags of metallurgical enterprises as a replacement for traditional fillers in concrete mixes has good potential due to producing high strength characteristics and reducing the cost of manufactured building products. The aim of the research is a mathematical formulation of the problem solution of clustering concrete mix recipes using man-made waste and slags of metallurgical enterprises, the criteria are the composition and strength of concrete mix recipes made experimentally. The statistical methods usage of analysis made it possible to determine the measures of the proximity of clusters, to produce clusters with optimal formulations of concrete mixtures.

**Keywords:** metallurgical slag, waste, mathematical formulation, clustering, formula

## INTRODUCTION

The term «cluster analysis» [1] was proposed and then widely used in the first half of the 20th century, thanks to R. Trion, who left a significant mark on the popularization and implementation of methods for classifying objects [2].

The use of mathematical procedures is most important when using cluster analysis methods.

The main task for a researcher in research using cluster analysis methods is the creation of clusters that are close in similarity or distance. The research used clustering as a method of analysis for quality determining of optimal formulations of concrete mixtures using metallurgical waste as criteria for objects the composition of the concrete mix and the and the strength are accepted.

Before starting the cluster analysis, each object under research or concrete mix is a separate cluster, and the proximity between the clusters is taken by the accepted metrics.

The most optimal way to solve the research problem is to determine the proximity (distances) between clusters in the space under research, or as they say in many sources of Euclidean distance. “Euclidean distance is a general type of distance used from ancient times to the present day, it is a geometric distance in multidimensional space” [3] and is used in various methods of cluster

analysis. The classical Euclidean distance is represented as:

$$(x,y)=\{\sum_i(x_i-y_i)^2\}^{1/2} \quad [4]$$

where:

$x, y$  - points of Euklidean space.

## OBJECT AND RESEARCH METHODS

Given:

$C_0$  – initial set of multidimensional objects, formulations of concrete mixtures using industrial waste and slags of metallurgical enterprises;

$C_0 = \{S_n\}, n = 1, ..N$

$M_p(M)$  – metric, concrete mix, strength.

$M_p(i)$  – weighting factor of the  $i$ -th characteristic,  $i = 1, .. M$

$X(n,i)$  –  $i$ -th condition characteristic  $i$ -th object  $n$ ,  $n=1, ..N, i=1, .. M$

it is required to distribute significantly different objects into clusters, formulations of concrete mixtures using man-made waste and slags of metallurgical enterprises.

Let's formulate the clustering problem:

There is data in the  $X$  array, it is necessary to divide the investigated objects about  $N$  into the required set of the resulting clusters  $m, K_1, K_2, ....., K_n$ , in this case, each object under study should belong to one, possibly only one, obtained subset:

The metric  $M_p(M)$  is normalized (see formula 1).

$$\sum_{i=1}^N M_p(i) = 1 \quad (1)$$

Solved mathematical formulation of the problem of clustering concrete mixes formulations into clusters,

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with the fulfillment of the condition that a certain peculiarity or dissimilarity of the input set of the produced objects to clusters is manifested.

It divide the input given set  $C_0$  by the admissible sets of clusters  $C_k$  (formulas 2 and 3) :

$$C_0 = \{ C_k \} \quad k = 1, \dots, k \tag{2}$$

$$C_k = \{ S_z \}, \quad z = 1, \dots, Nk \tag{3}$$

According to the requirements, each pair of the produced clusters cannot have common, coinciding elements, the object under research can be present only in 1 cluster (formula 4);

$$\forall C^k \in C^k \forall C_l \in C_0 : C^k \cap C_l = \emptyset \tag{4}$$

Where  $C_k$  – many investigated formulations of concrete mixes with an index k.

Determination of the quality of concrete mix recipes should ensure the selection of optimal recipes that provided iteration, as a result of which the best (optimal) value of the quality of the formulations of concrete mixtures included in one class (cluster) is determined

**There is a task in the calculation** such  $C_k$ , capable of maximizing such a criterion U (formula 5):

$$U(K_0) = \max \{ (U_1(K) - U_2(K)) \} \tag{5}$$

$$K = N - 2$$

$$\text{for } K = N, N - 1, \dots, 2$$

Where  $U(K_0)$  – the optimal value of the quality of the concrete mixture included in the cluster;

$U_1(K)$  – compactness of the quality of the concrete mixture included in the class for the obtained K clusters;

$U_2(K)$  – minimum distance, recipe quality for each class K of the cluster.

The minimum distance between the qualities of two concrete mix recipes, which is calculated using the function  $f(S_i, S_j)$ :

$$f(S_i, S_j) = 1 / (1 + \rho^2(S_i, S_j))$$

$$\rho(S_i, S_j) = \sqrt{\sum_{m=1}^N (Mp(m) \cdot (X_{i,m} - X_{j,m}))^2}$$

Euclidean distance between  $S_i$  and  $S_j$ »;

When calculating for clustering, it makes sense for the reliability of calculations and the qualitative result of clustering, to carry out calculations that ensure the compactness (brevity) of the quality classes of the concrete mix formulation  $U_1(K)$ , as well as the minimum distance, the quality of concrete mix formulations for each class  $U_2(K)$ :

$$U_1(K) = \frac{1}{K} \sum_{k=1}^K 2 / (N_k \cdot (N_k - 1)) \cdot \sum_{S_i \in C_k} \sum_{S_j \in C_k} f(S_i, S_j) \quad i \neq j$$

where K are the formulations of optimal concrete mixtures combined into classes at a certain iteration step.

$C_k$  – formulations of concrete mixes of the k-class;

$Nk$  – total number of concrete mix recipes for each class  $C_k$ ;

$f(S_i, S_j)$  – function of two formulations of concrete mixes  $S_i$  and  $S_j$ , class spacing  $S_i$  and  $S_j$  with given properties X and metrics;

$$U_2(K) = 2 / (K(K - 1)) \cdot \sum_{C_k \in C_p} \sum_{C_l \in C_p} F(C_k, C_l) \quad k \neq l$$

where  $C_p$  – the number of clusters made from the results of solving the mathematical formulation of the problem of determining the optimal formulations of concrete mixtures. Considering all calculations, we get a measure of similarity between two classes of formulations (objects) of concrete mixtures (formula 6).

$$F(C_k, C_l) = 1 / (N_k \cdot N_l) \cdot \sum_{S_i \in C_k} \sum_{S_j \in C_l} f(S_j, S_i) \tag{6}$$

The final formula 6 determines the mathematical formulation of the problem of solving the problem of clustering the optimal formulations of concrete mixtures using man-made waste and slags of metallurgical enterprises.

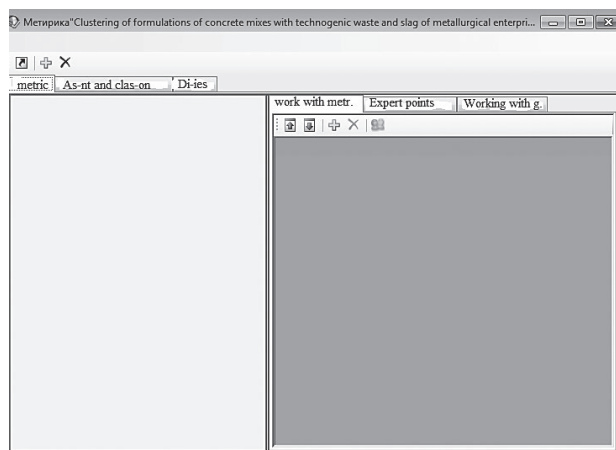
## RESEARCH RESULTS AND DISCUSSION

For the software implementation of the mathematical formulation of the solution of the problem of clustering concrete mixtures using industrial waste and slags of metallurgical enterprises, the program «Integrated quality assessment and classification of multidimensional objects» in Russian was used [5].

As metrics (criteria) when solving the problem, the composition and strength values of concrete mixtures using man-made waste and slags of metallurgical enterprises, tables 1 and 2, were taken, the data were obtained experimentally for control samples  $10 \times 10 \times 10$ .

Figures 1-5 shows the data entry procedure and the result of clustering the formulations of concrete mixtures using man-made waste and slags from metallurgical enterprises..

From the produced clustering result, it can be seen that the formulations of concrete mixtures are distributed over 6 clusters. Each cluster contains formulations of concrete mixtures with the closest characteristics in composition and strength. Concrete recipes with low strength indicators are located in clusters 1-4. Table 2



**Figure 1** Preparation of the initial file for the clustering of concrete mixes recipes

Table 1 **Composition of the formulation of the developed concrete mixtures using man-made waste and slags of metallurgical enterprises**

№ Recipes RBS	Composition of the concrete mix			
	Ravr. Compr./ MPa	Ash/ gr/%	Metall. Slag/ gr/%	Bauxite sludge/ gr/%
1	3,11			337/3
2	3,7			505/4
3	8,32			674/5
4	3,5			842/7
5	3,6			1 011/8
6	2,84			944/7
7	2,56			910/7
8	2,73	574/4,4		
9	2,3	246/2		337/3
10	4,4	574/4,4		
11	2,5	656/5		
12	4,5	3370/26		
13	2,51	492/4		
14	2,8	574/4,4		
15	3,45			410/3,2
16	4,54			410/3,2
17	3,9	246/2	798/6	
18	2,55	328/2,5	798/6	
19	3,5	328/2,5	640/5	
20-15	1,47	107/1,3	1 066/13	
21	9,74	328/2,5	798/6	246/2
22	3,18		399/3	328/2,5
23	3,23	164/1,3	798/6	164/1,3
24-15	1,6		1 020/12	357/4,2
25	4,77	328/2,5	399/3	337/2,5
26	4,33	164/1,2	798/6	337/2,5
27	3,3	337/2,5	798/6	164/1,2
28	9	337/2,5	798/6	505/4
29	10	337/2,5	1 596/12	
30	3,22	328/5	798/6	674/5,2
31	4,62	246/2		505/3,2
32	2,3			1 685/13
33	9,1	164/1,3		1 685/13
34	8,18	164/1,3		2 022/16
35	4,37	1 011/8		
36	1,35	7 798/80		
37	4,43	337/1,5	798/6	674/5,2
38	4,13	337/2,5	798/6	337/2,5
39	22,3	505/4	1 197/9,2	337/2,5

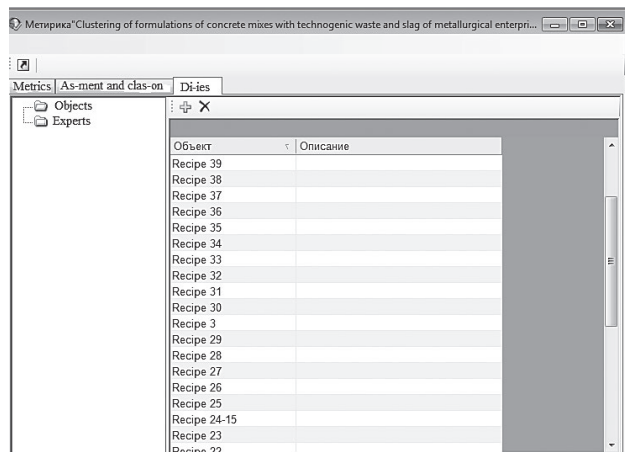


Figure 2 Entering concrete mix recipes

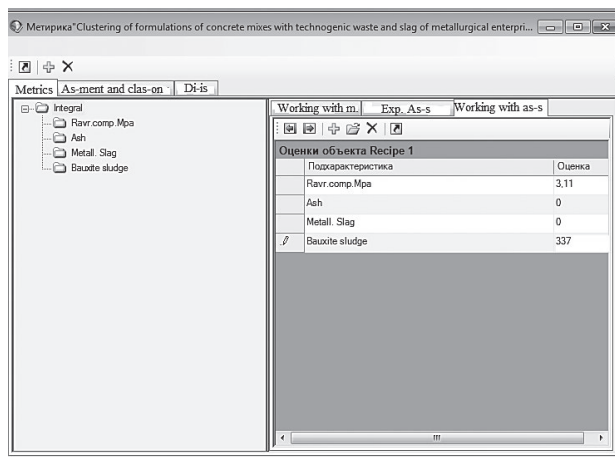


Figure 3 Entering criteria

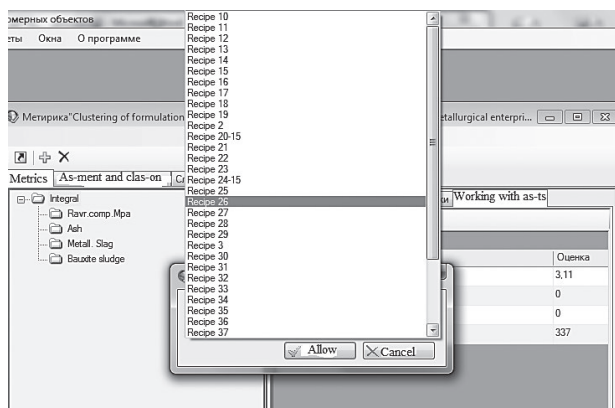


Figure 4 Ready data for clustering

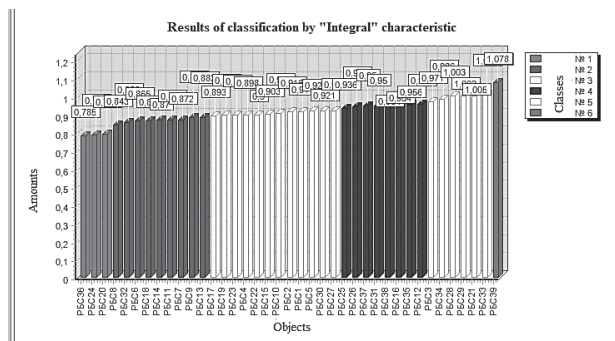


Figure 5 Clustering of concrete mixes formulations using industrial waste and slags of metallurgical enterprises

Table 2 **Recipes, composition of concrete mixtures in terms of strength 5-6 cluster**

Cls. №	Recip. №	Ravr. Comp/MPa	Ash/gr/%	Metall. Slag/gr/%	Bauxite sludge/gr/%
5	RBS3	8,32			674/5
	RBS34	8,18			842/7
	RBS28	9	337/2,5	798/6	505/4
	RBS29	10	337/2,5	196/12	
	RBS33	9,1	164/1,3		1 685 /13
6	RBS21	9,74	328/2,5	798/6	246/2
	RBS39	22,3	505/4	1 197/9,2	337/2,5

shows the recipes, the composition of concrete mixtures with the highest strength values of 5 and 6 clusters.

## CONCLUSION

The software implementation of the developed mathematical formulation of the problem of clustering the optimal formulations of concrete mixtures using man-made waste and slags of metallurgical enterprises made it possible to classify objects, objects are divided into clusters according to proximity criteria.

Concrete mixes 5-6 of the cluster corresponds to the requirements [6-7] and used in the building materials production at the Ekostronii-PV LLP.

The use of man-made waste and slags of metallurgical enterprises as additives and fillers in the production of construction products, allowed to reduce the cost of construction products, save consumption of traditional raw materials, solve environmental problems of the Pavlodar region [8].

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**Note:** A. Kyandykov is responsible for English language, Pavlodar, Kazakhstan