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Stability analysis of network-controlled DC position servo system with time-delay

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ABSTRACT

This paper presents a stability analysis of network-controlled direct current (DC) position servo system with time-delays (TDs). The usage of communication links and measurement devices in the network-controlled system involves a considerable amount of TDs. Those TDs would degrade the dynamic performance and cause instability to the DC position servo system. A less conservative delay-dependent stability (DDS) criterion is presented by using Lyapunov–Kravoskii functional and linear matrix inequality techniques. The maximum allowable TD called delay margin (DM) is computed, which can be used to tune a proportional-integral (PI) controller of the DC position servo system. Two cases of TD are considered in this analysis: time-invariant and time-varying delays. The DDS approach is used to study the trade-off between the PI controller gains and the DMs. This approach together with numerical simulation gives the insightful trade-off between DM and dynamic performance of the system. Finally, the stability of the DC position servo system with PI controller is analysed in the presence of TD through the experimental setup.

1. Introduction

The direct current (DC) position servo system is used in industry for many applications such as robotics, process control, antenna positioning, etc. The usage of network-controlled techniques to the DC position servo system provides the application of long-distance control [1]. The conventional network-controlled schemes involve dedicated communication channels for transmitting data between the controller and the plant. The network-controlled DC position servo system involves transmitting measured data from the controller to the plant and vice-versa. Although there is a time-delay (TD) introduced in the system, such a TD is usually ignored during design and operation of the network-controlled DC position servo system. The future usage of open communication channel for network-controlled systems involves both time-invariant and time-varying delays (TVDs) [2,3].

TDs become a significant issue in network control systems (NCs). These TDs are due to the geological distance between controller and plant or may due to network traffic or network congestion [4]. Further, the usage of sensors, actuators, and other relevant instrumentation which naturally leads to the existence of the TD in the feedback as well as the forward path of the control loop. It is well known that the presence of TD in dynamical systems may often lead to instability [5].

The DC motor installed with communication channel is a typical TD system. However, unavoidable TD will affect the stability when the system is operated through the network. Hence, TD affects the stability of the overall closed-loop control system. This naturally leads to doing an investigation of the effect of TD on the system's stability [6–13].

For stability analysis and design of the controller for TD system, it is mandatory to compute the maximum value of TD up to which the system can be retained stable is denoted as delay margin (DM) [14,15]. There are two methods for determining the stability and computing the DM of the TD systems: namely time-domain methods and frequency domain methods. In [16], frequency-domain method is used to compute the DM of the load frequency control system with constant delay. The results obtained from this method are exact delay bounds. However, this method is not a generalized one and it is system-specific and restricted with constant delays. Time-domain methods are used to develop a generalized method for TD systems to compute the DMs for various delay effects such as time-invariant delay, TVD, and multiple delays [17–19]. The most existing approaches on TD systems are delay-dependent stability (DDS) approach and delay-independent stability approach. From literature, it is evident that results derived from DDS approach are less conservative than the delay-independent approach for considering the additional information of the TD [4,20–24].
In the past few decades, research works have been carried out on reducing the conservativeness of the DDS results by constructing Lyapunov–Krasovskii functional (LKF) and choosing appropriate bounding technique in time-derivative of LKF [3,14,15,17,18]. In [9], the usage of Jenson inequality induces some conservativeness to the DDS results. Therefore, the performance of the DDS criterion is improved by using free-weighting matrices techniques, but it also increases the complexity by introducing additional slack variables [4]. In [25], reciprocally convex combination lemma was used to reduce the conservativeness of the results with less number of decision variables. In this paper, DDS criterion is derived by constructing the LKF and the integral term in time derivative of LKF is bounded using reciprocally convex combination lemma and free-weighting matrices and finally expressed in linear matrix inequalities (LMIs). The DDS criterion is solved to determine the DM of the DC position servo system. 

Various stability results for TD systems under different delay effects have been reported in [1–13,29–37]. To the best of the authors’ knowledge, only a few kinds of literature discuss the effect of TD in stability analysis of DC position servo systems [1,26–28]. This has motivated us to carry out the present research work. This paper investigates the DDS analysis of DC position servo systems with constant TD and TVDs. After deriving the DDS criterion and DMs are computed for TD system with constant TD and TVDs. Finally, conclusions are drawn in Section 7.

The notations used for DC position servo system are given in Table 1.

### Table 1. Notations.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_m$</td>
<td>Motor armature resistance</td>
</tr>
<tr>
<td>$L_m$</td>
<td>Motor armature inductance</td>
</tr>
<tr>
<td>$K_m$</td>
<td>Motor back EMF</td>
</tr>
<tr>
<td>$J_m$</td>
<td>Moment of inertia</td>
</tr>
<tr>
<td>$b_m$</td>
<td>Viscous friction</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angular position</td>
</tr>
<tr>
<td>$V_m$</td>
<td>Input voltage</td>
</tr>
<tr>
<td>$\alpha_m$</td>
<td>Speed</td>
</tr>
<tr>
<td>$\tau(t)$</td>
<td>Time-varying delay</td>
</tr>
<tr>
<td>$K_p$</td>
<td>Pulse width modulation gain</td>
</tr>
<tr>
<td>$K_i$</td>
<td>Integral gain</td>
</tr>
<tr>
<td>$K_m$</td>
<td>Proportional gain</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Time-delay</td>
</tr>
</tbody>
</table>

### 2. Model of DC position servo system with time-delay

The conventional DC position servo system is modified with TD in the control loop. The PI controller is used in this model. It is assumed that TVD in the forward loop is $e^{-\tau(t)}$ and feedback loop is $e^{-\tau_2(t)}$ are considered as the single delay component ($\tau_1(t) + \tau_2(t) = \tau(t)$) [2–4,14,15]. Figure 1 shows the block diagram of DC position servo system.

The state-space equation of the TD system is given as follows:

$$
\dot{x}(t) = A x(t) + A_d x(t - \tau(t)),
$$
$$
x(t) = \psi(t), t \in [-\bar{\tau}, 0],
$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $A \in \mathbb{R}^{n \times n}$ and $A_d \in \mathbb{R}^{n \times n}$ are the system matrices. $\tau(t)$ satisfies the following conditions

$$
0 \leq \tau(t) \leq \bar{\tau},
$$
$$
\dot{\tau}(t) \leq \bar{\delta} < 1,
$$

where $\bar{\tau}$ and $\bar{\delta}$ are represent DM and delay derivative, respectively. It is assumed to be $\delta < 1$ for NCS through a delayed communication channel [2,4,15,22,25,37]. The controller is described as

$$
u(t) = -K_p \dot{\theta}(t) - K_I \int \theta(t) \, dt.
$$

The state-space equation of the DC position servo system shown in Figure 1 is given as

$$
\begin{bmatrix}
\dot{x}_1(t) \\
\dot{x}_2(t) \\
\dot{x}_3(t) \\
\dot{x}_4(t)
\end{bmatrix} = 
\begin{bmatrix}
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
-(R_m B_m + K_m^2) & 0 & 0 & -(J_m R_m + L_m B_m) \\
\end{bmatrix}
\begin{bmatrix}
x_1(t) \\
x_2(t) \\
x_3(t) \\
x_4(t)
\end{bmatrix} + 
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\frac{-K_E K_m K_p}{J_m L_m} & \frac{-K_E K_m K_I}{J_m L_m} & 0
\end{bmatrix}
\begin{bmatrix}
x_1(t - \tau(t)) \\
x_2(t - \tau(t)) \\
x_3(t - \tau(t)) \\
x_4(t - \tau(t))
\end{bmatrix},
$$

(6)
where $x(t)$ is given as

$$x(t) = \begin{bmatrix} \omega_m(t) & \theta(t) & \int \theta(t) dt & \dot{\omega}_m(t) \end{bmatrix}^T.$$  (7)

In the following section, the stability results for DC position servo system is presented.

3. Main result

The DDS criterion for the system (1) is stated below:

**Theorem 1:** For scalars $\delta < 1$ and $\tau > 0$, the system (1) is stable if there exist symmetric positive-definite matrices $U$, $V_1$, $V_2$, $M$, and $N$ is a free matrix of appropriate dimensions such that the following LMIs hold true

$$\begin{bmatrix} M & N \\ * & M \end{bmatrix} \geq 0,$$  (8)

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & N^T \\ \Pi_{12}^T & \Pi_{22} & M - N^T \\ * & * & -M - V_2 \end{bmatrix} < 0,$$  (9)

where

$$\Pi_{11} = A^T U + UA + V_1 + V_2 + \bar{\tau}^2 A^T MA - M,$$  (10)

$$\Pi_{12} = UA_d + \bar{\tau}^2 A^T MA_d + M - N^T,$$  (11)

$$\Pi_{22} = -(1 - \delta) V_1 + \bar{\tau}^2 A_d^T MA_d - 2M + N + N^T.$$  (12)

The proof of the Theorem 1 is given in [26]. The reciprocal convex combination lemma is applied to the Theorem 1 to bound the integral term in the time-derivative of the LKF [25].

**Remark 1:** The use of reciprocal convex combination lemma in the Theorem 1 has reduced the number of decision variables comparable to those based on Jensen’s inequality [4]. Since, there is a further scope to reduce the conservativeness of proposed results in Theorem 1, a new DDS criterion will be derived by constructing LKF and choosing appropriate bounding technique. Similar to Theorem 1, the following result are presented using free-weighting matrices and Leibniz-Newton formula [24].

**Theorem 2:** For scalars $\delta < 1$ and $\tau > 0$, the system (1) with $0 \leq \tau(t) \leq \bar{\tau}$ is asymptotically stable if there exist $U > 0$, $V_1 > 0$, $V_2 > 0$, $M > 0$, $D = [D_1^T D_2^T 0]^T$ and $F = [F_1^T F_2^T 0]^T$ such that

$$\begin{bmatrix} \phi & \bar{\tau} D \\ * & -\bar{\tau} M \end{bmatrix} < 0,$$  (13)

$$\begin{bmatrix} \phi & \bar{\tau} F \\ * & -\bar{\tau} M \end{bmatrix} < 0,$$  (14)

where

$$\phi = \begin{bmatrix} \tilde{\phi}_{11} + \bar{\tau} A_d^T MA_d & \tilde{\phi}_{12} + \bar{\tau} A_d^T MA_d & -F_1 \\ * & \tilde{\phi}_{22} + \bar{\tau} A_d^T MA_d & -F_2 \\ * & * & -V_2 \end{bmatrix},$$  (15)

$$\tilde{\phi}_{11} = A^T U + UA + V_1 + V_2 + D_1 + D_1^T,$$  (16)

$$\tilde{\phi}_{12} = UA_d + F_1 + D_2^T - D_1,$$  (17)

$$\tilde{\phi}_{22} = -(1 - \delta) V_1 + F_2 + F_2^T - D_2 - D_2^T.$$  (18)

**Proof:** Construct the LKF

$$E(t) = x^T(t) U x(t) + \int_{t-\tau(t)}^t x^T(z) V_1 x(z) dz + \int_{t-\tau(t)}^t x^T(z) V_2 x(z) dz + \int_{t-\bar{\tau}}^{t-\theta} x^T(z) M x(z) dz d\theta.$$  (19)
The time-derivative of $\dot{E}(t)$ is given by

$$\dot{E}(t) = 2x^T(t)Ux(t) + x^T(t)(V_1 + V_2)x(t)$$

$$- x^T(t - \tau)V_2x(t - \bar{\tau})$$

$$- (1 - \delta)x^T(t - \tau(t))$$

$$V_1x(t - \tau(t)) + \bar{\tau}(t)x(t)M\dot{x}(t) - \int_{t - \bar{\tau}}^{t} \dot{x}^T(s)M\dot{x}(s) \, ds.$$  

(20)

The integral term in (20) is bounded by the method reported in [24]. Finally, equation (20) can be expressed in quadratic form as follows

$$\dot{E}(t) = \Xi^T(t)\pi \Xi(t),$$

(21)

where

$$\Xi(t) = [x^T(t) \ x^T(t - \tau(t)) \ x^T(t - \bar{\tau})]^T,$$  

(22)

if $\pi$ is negative definite, then $\dot{E}(t) < 0$ and, therefore, in the sense of Lyapunov the given system is asymptotically stable. The maximum value of delay for which $\pi < 0$ will be the DM for the given system. Hence, this completes the proof of Theorem 2.

**Remark 2:** When the rate of change of TD is constant, by setting $V_1 = 0$ in Theorem 2, the DDS criterion can be used to determine the stability of the system in the presence of constant delay. The usage of free-weighting matrices to bound the integral terms in time-derivative of LKF (20) increases the number of decision variables in Theorem 2.

### 4. Results and discussion

In this section, DMs are computed for different sets of the PI controller gains by using Theorem 2. The DC position servo system parameters used in this paper are taken from the laboratory motor as follows: $K_p = 1$, $K_m = 0.0287 \text{ V/s/rad}$, $R_m = 3.3 \Omega$, $L_m = 1.16 \text{ mH}$, $J_m = 9.64 \times 10^{-6} \text{ Kgm}^2$ and $\mu_0 = 0.01 \text{ N.m.s}$. The results presented in Tables 2 and 3 are obtained by solving Theorem 2 for different values of $K_p$ and $K_i$. Results from Tables 2 and 3 show that DM of the system is quite higher for the constant TD as compared with the TVDs.

Figure 2 shows the relation between $K_p$ and DM. The values of $K_i$ are chosen to be $K_i = 0.05$ and $K_i = 0.10$. In this graph, the DM increases for an increase in $K_p$ value from 0.05 to 0.5 then, further increase in $K_p$ decreases the DM. Figure 3 shows the relation between $K_j$ and DM. The value of $K_p$ is chosen to be $K_p = 0.5$ and $K_p = 0.75$. This graph shows that DM decreases as $K_i$ increases. For TVD and constant delay, DM decreases for an increase in $K_i$ value. These finding can be used to tune the PI controller and maintain system stability in NCSs.

### 5. Simulation validation of the Obtained results

The simulation studies are accomplished using MATLAB to verify the accuracy of the obtained results from Theorem 2. To illustrate the verification, the gain of the PI controller chosen from Table 2 are $K_p = 1.5$ and $K_i = 0.2$ and corresponding DM is $\bar{\tau} = 1.045$ s. This theoretical value of DM implies a marginally stable system at $\bar{\tau} = 1.045$ s. The response of the DC position servo system is plotted for the step input. When $\bar{\tau} = 0$ s system is asymptotically stable as shown in Figure 4. The system response with oscillations as compared to the delay-free system for DM $\bar{\tau} = 1.045$ s ensures that system is under stable condition. Further, increase in DM for $\bar{\tau} = 1.143$ s, system becomes marginally stable with sustained oscillations. However, the theoretical results show that DM for sustained oscillation is 1.045 s. The difference between the theoretical values and simulation values show the conservativeness of the proposed criterion. The system response results in increasing oscillations at $\bar{\tau} = 1.150$ s hence, the system is under unstable condition.

The simulations studies are carried out on the bench mark DC position servo system having TVD specified with maximum delay bound and delay derivative. The evolution of state variable $\theta(t)$ is monitored in DC position servo system when system is disturbed from the equilibrium position ($\theta_e(t) = 0$) at $t = 0$. From Table 3, the controller parameters employed for simulation study are $K_p = 1.5$ and $K_i = 0.2$ and their corresponding DM determined through Theorem 2 is 0.999 s. The model of TVD is given by $t = \frac{\mu}{2} \sin(\frac{2\mu}{\tau} t + \bar{\tau})$ [38]. In Figure 5, the TVD is assumed to be sinusoidal varying between $0 \leq t(t) \leq 1$ and satisfying $t(t) = 0.4995 \sin(t(t) + 0.4995)$. If the TD is reduced from 0.999 s retaining $\mu = 0.5$, deviation variable $\theta(t)$ converges to the equilibrium point ensures stable operation as shown in Figure 5 for $\bar{\tau} = 0.999$ s. On the other hand, increase in the magnitude of TVD

| Table 3. Delay margin vs $(K_p, K_i)$ (Theorem 2: $\delta = 0$). |
|---|---|---|---|---|
| $K_p$ | $K_i$ | 0.05 | 0.75 | 1 | 1.25 | 1.5 |
| 0.05 | 2.861 | 2.091 | 1.613 | 1.306 | 1.095 |
| 0.1 | 2.374 | 1.944 | 1.555 | 1.277 | 1.079 |
| 0.15 | 1.979 | 1.795 | 1.493 | 1.247 | 1.062 |
| 0.2 | 1.668 | 1.650 | 1.430 | 1.216 | 1.045 |

| Table 2. Delay margin vs $(K_p, K_i)$ (Theorem 2: $\delta = 0$). |
|---|---|---|---|---|
| $K_p$ | $K_i$ | 0.5 | 0.75 | 1 | 1.25 | 1.5 |
| 0.05 | 2.861 | 2.091 | 1.613 | 1.306 | 1.095 |
| 0.1 | 2.374 | 1.944 | 1.555 | 1.277 | 1.079 |
| 0.15 | 1.979 | 1.795 | 1.493 | 1.247 | 1.062 |
| 0.2 | 1.668 | 1.650 | 1.430 | 1.216 | 1.045 |
Figure 2. $K_P$ vs Delay margin.

Figure 3. $K_I$ vs Delay margin.

Figure 4. Evolution of $\theta(t)$ for different DMs ($K_P = 1.5, K_I = 0.2$).

Figure 5. Evolution of $\theta(t)$ ($K_P = 1.5, K_I = 0.2, \bar{\tau} = 0.999$ s).
Figure 6. Evolution of $\theta(t)$ ($K_P = 1.5, K_I = 0.2, \bar{\tau} = 1.2 \text{ s}$).

Figure 7. Experimental setup.

$(0 \leq \tau(t) \leq 4)$ increases the oscillations of the deviation variable $\theta(t)$ about the equilibrium point as shown in Figure 6.

6. Experimental setup for DC position servo system

The experimental setup as shown in Figure 7, which includes the following devices: Computer with LabVIEW software, QNET DC motor hardware, National Instruments (NI) hardware with data acquisition system for transfer motor parameters to the personal computer (PC) and to send appropriate control signal to the DC motor. The data acquisition system provided in NI acquires the speed data from DC motor using tachogenerator and transferred to computer and a proposed controller algorithm developed in Labview generates the appropriate pulse-width-modulated signal to the converter driver.

To confirm the effectiveness of the proposed results, the position control of DC motor with TD is done through the experimental setup. Table 4 shows comparison between analytical and experimental results. Two special instances are taken into consideration in which the PI controller parameters are chosen to be $K_P = 1.5$ and $K_I = 0.2$. In Case A, the stability of DC position servo system is analysed with constant TD. The DM determined from Theorem 2 for the controller parameter $K_P = 1.5$ and $K_I = 0.2$ is $\bar{\tau} = 1.045 \text{ s}$. In Case B, the stability of DC position servo system is analysed with TVD. The DM determined from Theorem 2 for the controller parameter $K_P = 1.5$ and $K_I = 0.2$ is $\bar{\tau} = 0.999 \text{ s}$.

**Case A: Constant TD:** The control circuit for DC motor is given in Figure 8. The PI controller is chosen for overall closed-loop operation. The response of DC motor is observed for the step input with different DMs.

<table>
<thead>
<tr>
<th>$K_I$</th>
<th>Delay margin</th>
<th>Hardware result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.7157</td>
<td>0.7472</td>
</tr>
<tr>
<td>1.2</td>
<td>0.6260</td>
<td>0.6535</td>
</tr>
<tr>
<td>1.5</td>
<td>0.5263</td>
<td>0.5473</td>
</tr>
<tr>
<td>1.7</td>
<td>0.4757</td>
<td>0.4954</td>
</tr>
<tr>
<td>2</td>
<td>0.4157</td>
<td>0.4323</td>
</tr>
</tbody>
</table>

Table 4. Delay margin obtained via Theorem 2 for $K_P = 1$. 


The controller task is to maintain the stability of DC position servo system in the presence of the TD in the control loop. The chosen value of controller gain are $K_P = 1.5$ and $K_I = 0.2$. In Figure 10, the response of the DC motor is observed for delay-free system. It shows that system response reaches the steady state, which ensures that it is under stable condition. If TD increases from $\tau > 1.200$ s, then the system becomes unstable as shown in Figure 11. Hence the effect of constant TD on stability of the system is experimentally tested.

Case B: The control circuit for position control of DC motor with TVD is shown in Figure 9.

Figure 12 shows that system becomes unstable for increase in DM ($\bar{\tau} > 1.081$) for $K_P = 1.5$ and $K_I = 0.2$. Hence, the simulation and experimental result clearly bringing out the effect of TD and TVD on stability and performance of the DC position servo system.

Remark 3: The difference between the DMs of analytical and experimental value shows the conservativeness of the proposed DDS criterion and numerical approximations considered during the analysis.

7. Conclusion

In this paper, the DDS of network-controlled DC position servo system with the TD and TVD has been investigated. The chosen LKF and appropriate
bounding methods for integral terms on the time-derivative of LKF decide the conservativeness of the proposed criterion. The usage of free-weighting matrices approach increases the estimation of DM. The stability of network-controlled DC position servo system is maintained by choosing the controller gains with concern to the TD. The results shows that gain of PI controller has influence on the DMs of the closed-loop system. For a fixed $K_I$ value, DM increases at first and then decreases, with the increase in $K_P$ value. For fixed $K_P$ value, DM decreases for increases in $K_I$ value. A small reduction in controller gains have significant improvement in the DM. The DM decreases for increases in rate of change of TVD. Since there is a difference in the DM between theoretical and experimental results, it shows that there is a further scope to construct an LKF and choose an appropriate bounding technique. Thus, the proposed method provides set of tuning to the controller for NCS.

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Disclosure statement

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