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LINK TRAVEL TIME ESTIMATION IN DOUBLE-QUEUE-BASED TRAFFIC MODELS

ABSTRACT

Double queue concept has gained its popularity in dynamic user equilibrium (DUE) modeling because it can properly model real traffic dynamics. While directly solving such double-queue-based DUE problems is extremely challenging, an approximation scheme called first-order approximation was proposed to simplify the link travel time estimation of DUE problems in a recent study without evaluating its properties and performance. This paper focuses on directly investigating the First-In-First-Out property and the performance of the first-order approximation in link travel time estimation by designing and modeling dynamic network loading (DNL) on single-line stretch networks. After model formulation, we analyze the First-In-First-Out (FIFO) property of the first-order approximation. Then a series of numerical experiments is conducted to demonstrate the FIFO property of the first-order approximation, and to compare its performance with those using the second-order approximation, a point queue model, and the cumulative inflow and exit flow curves. The numerical results show that the first-order approximation does not guarantee FIFO and also suggest that the second-order approximation is recommended especially when the link exit flow is increasing. The study provides guidance for further study on proposing new methods to better estimate link travel times.

KEYWORDS

double queue model; dynamic user equilibrium; dynamic network loading; travel time estimation; first-in-first-out (FIFO).

1. INTRODUCTION

Dynamic traffic assignment (DTA), often broadly categorized into Dynamic User Equilibrium (DUE) and Dynamic System Optimum (DSO), is one of the most challenging problems in transportation engineering, which has been intensively studied for decades (Peeta and Ziliaskopoulos 2001). Being a specific type of DTA, DUE aims to simultaneously model user departure times and route choices by assuming that all users follow certain rational behavior. DUE ends at an equilibrium state where for any given origin-destination (OD) pair the generalized travel costs are identical for all route choices and departure time choices. Related literature reviews can be found in [1-3].

As queue spillbacks (i.e., downstream congestion propagates to the entrance of a link, thus restricting the inflow to the link) are prevalent on transportation networks, there have been some models that properly captured such phenomenon in literature. Developed by [4], the cell transmission model (CTM) is one of the earliest and most widely used models that can capture queue spillbacks [5, 6]. When applied in DUE models, the CTM needs to discretize each link over both space and time to model the flow dynamics, leading to a DUE model with a very large dimension especially for large networks with a long study time period. Furthermore, as DUE models are normally very complex, the CTM-based DUE models are highly challenging to solve for practical problems due to its large dimension. Thus, there is an increasing trend in the recent DUE literature focusing on balancing the computational efficiency and the level of details a DUE model can capture. Recent efforts have been focused on developing link-based traffic flow models such as the link transmission model (LTM) [7, 8], the double queue model [9-12], among others [13, 14]. The concept of double queue originated from LTM based on the triangular fundamental diagram of traffic flow [7]. It was first proposed and used in [9] and recently applied to model continuous-time DSO and DUE problems [11, 12, 15, 16] thanks to its capability of capturing queue spillbacks in macroscopic traffic modeling. Specifically, at time interval h the dynamics of link (i,j) in the double-queue model can be described by the changes of a downstream queue $q_{ij}^{d}(h)$ and an upstream queue $q_{ij}^{d}(h)$ as follows [15, 16].

$$q_{ij}^{u}(h) - q_{ij}^{u}(h-1) = \Delta \cdot \left(u_{ij}(h) - v_{ij}\left(h - n_{ij}^{\omega}\right)\right) \quad (1)$$

$$q_{ij}^{d}(h) - q_{ij}^{d}(h-1) = \Delta \cdot \left(u_{ij}(h - n_{ij}^{0}) - v_{ij}(h)\right) \quad (2)$$

where $u_{ij}(h)$ and $v_{ij}(h)$ are the inflow and exit flow rates of the link (i,j) at the time interval h; n_{ij}^{0} and n_{ij}^{ω} are the free-flow and shockwave travel times of the link (i,j) in terms of time intervals, respectively. To capture the flow withholdings caused by the double queues, two sets of complementary slack variables are defined, i.e., $\eta_{ij}(h)$ for the upstream queues and $\mu_{ij}(h)$ for the downstream queues as shown in *Equations 14 and 15*, respectively. Their physical meanings are described after the model formulation. Note that the continuous-time double-queue-based models are usually solved by discretization. Thus in this paper, we directly formulate the discrete-time model for simplification purposes.

Generally, the link-based DUE models can be categorized as instantaneous DUE models and predictive/ideal DUE models depending on whether travelers are assumed to make route choices based on the current prevailing travel times or predicted travel times [17]. The instantaneous DUE models assume that travelers are making route choice decisions based on the current prevailing travel times while the ideal DUE models assume that the travelers' route choice decisions are based on the predicted travel times. With the development of information technologies some new models are proposed considering a mixture of both. For example, when drivers get informed about traffic events via multiple information sources, the rerouting is modeled in [18]. This paper focuses on the ideal DUE problems that generally contain two basic components (or sub-models): a traffic flow model that describes traffic dynamics and evolutions in time and space, and a choice model that describes the behavior of traveler choices (such as departure-time choices and route choices). In theory, a DUE problem should integrate both components into an integrative formulation, which should then be solved simultaneously. One challenge, however, of solving such an integrated DUE model is that usually one cannot derive a closed-form expression for link (or path) travel times, especially when queue spillback occurs. Therefore, in literature most DUE models are solved via an iterative (and largely heuristic) procedure that solves the two components separately at each iteration. As shown in [11], this is just one way of solving DUE and one may choose to solve the integrative DUE model directly, i.e., solving the two components simultaneously. This, however, requires applying certain approximation schemes to the link or path travel time functions to derive certain closed-form approximations. In [11] the authors proposed the first-order approximation to simplify the computation of link travel times in order to solve their double-queue-based ideal DUE model. They did not, however, conduct a full investigation about the properties and performance of the first order approximation scheme, which means that the reliability of the estimation of link travel times remains questionable. The focus of this paper is to fill this gap by evaluating the properties and performance of the proposed first-order approximation in link travel time estimation, which will serve as reference for future studies on proposing a better method to estimate the link travel times for solving double-queue-based DUE models.

In this paper, we narrow our study down to Dynamic Network Loading (DNL) on single-line stretch networks and the reasons are as follows. First, the DNL on a single-line stretch network eliminates travelers' route choices and thus eliminates the necessity to estimate link travel times to obtain the predictive route travel times as in an ideal DUE. Second, the predefined time-dependent demands in DNL also avoid the computational complexity from travelers' departure time choices. Therefore, we can solve the DNL problem directly and then compute the link travel times based on the cumulative inflow and exit flow curves, which are defined as the 'real link travel times' in this paper. By comparing with the real link travel times, we can study the performances of the first-order approximation and other approximation methods.

In particular, we first formulate the discretized DNL model for single-line stretch networks as a linear programming problem. Note that we omit the continuous-time DNL model to save space [19, 20]. Then, we investigate the FIFO property of the first-order approximation and show that FIFO is guaranteed if there are no downstream queues on any links of the network anytime as presented later. After that, we conduct some numerical experiments on a simple single-line stretch network with only two links and compare the performances of the travel times estimated using the first-order approximation, the second-order approximation, the point-queue model [3], and the cumulative inflow and exit flow curves, respectively. Here the second-order approximation is an extension of the first-order approximation. We also define the link travel times estimated from the cumulative link inflow and exit flow curves using linear interpolation as the real link travel times, which are intuitive deductions of real travel times and can be obtained after the DNL problem is solved. Detailed performance results can be found in the numerical analysis section. The results show that the first-order approximation could overestimate the link travel times under certain circumstances when demand is larger than link capacities, whereas most of the time it underestimates the link travel times. Overall, the link travel times from the first-order and second-order approximations are more reliable compared with the point queue model. The numerical results also suggest that the second-order approximation is recommended when the link exit flow is increasing. The study can provide guidance for future research on deriving a closed-form expression for link travel time approximations in ideal DUE models.

This paper is organized as follows. The second section is about the discretized DNL model formulation on single-line networks followed by the first-order and second-order link travel time approximation functions. In the beginning, there will be a brief introduction on the double queue dynamics and the slack variables defined for reflecting the flow withholdings at road link entrances and exits. The third section presents the numerical experiments and discussions. The final part is the conclusion.

2. MODAL FORMULATION AND LINK TRAVEL TIME

The paper mainly concentrates on the double-queue-based DNL on single-line stretch networks in order to study the performance of the first-order approximation proposed by [11] in a direct manner. The DNL model is to assign a given time-dependent demand profile onto the network, which is an important component for DUE. *Figure 1* displays the structure of a single-line stretch network on which the DNL model will be built, followed by the notation used throughout the paper. In *Figure 1*, we use red and green bars to represent the upstream and downstream queues at the two ends of each link, respectively. Note that it is assumed that all travel time parameters are integral multipliers of the discretizing time interval Δ.

Notation

N – the node set;
L – the link set;
O – the only origin $O \in N$;
$\tilde{O}$ – the dummy origin;
$(\tilde{O},O)$ – the dummy link;
S – the destination;
(x,S) – the last link connecting the destination S;
Δ – the selected discrete time interval;

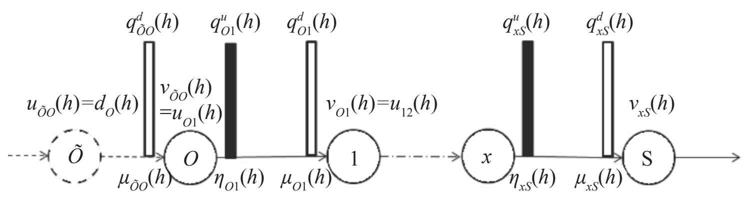


Figure 1 – Single-Line Network Structure with Double Queues on Links

$\tilde{T}$ – the number of time intervals required for all demand arriving at the destination S;
h – $h \in [0, \tilde{T}]$, h is the ordered ID of a time interval;
f_{ij} – the free-flow travel speed of link $(i,j) \in L$;
ω_{ij} – the shockwave travel speed of link $(i,j) \in L$;
$d_O(h)$ – the demand at origin O at discrete time interval h;
$q^u_{ij}(h)$ – the upstream queue of link $(i,j) \in L$ at discrete time interval h;
$q^d_{ij}(h)$ – the downstream queue of link $(i,j) \in L \cup (\tilde{O}, O)$ at discrete time interval h;
$\mu_{ij}(h)$ – the slack variable related to the downstream queue of link $(i,j) \in L$ at discrete time interval h;
$\eta_{ij}(h)$ – the slack variable related to the upstream queue of link $(i,j) \in L$ at discrete time interval h;
$v_{ij}(h)$ – the exit flow rate of link $(i,j) \in L \cup (\tilde{O}, O)$ at discrete time interval h;
$u_{ij}(h)$ – the inflow rate of link $(i,j) \in L$ at discrete time interval h;
n^0_{ij} – $n^0_i = \tau^0_{ij}/\Delta$ (note: $n^0_{\tilde{O}O}=0$) where τ^0_{ij} is the free flow travel time on link $(i,j) \in L$;
n^ω_{ij} – $n^\omega_{ij} = \tau^\omega_{ij}/\Delta$(note: $n^\omega_{\tilde{O}O}=0$) where τ^ω_{ij} is the shockwave travel time on link $(i,j) \in L$;
$n_{ij}(h)$ – $n_{ij}(h) = \tau_{ij}(h)/\Delta$ where $\tau_{ij}(h)$ is the travel time on link $(i,j) \in L$ at discrete time interval h;
C_{ij} – the exit flow capacity of link $(i,j) \in L$;
Q_{ij} – the queue storage capacity of link $(i,j) \in L$ (note: $Q_{\tilde{O}O}=\infty$).

As shown in *Figure 1*, a dummy origin $\tilde{O}$ and a dummy link $(\tilde{O}, O)$ are created for modeling purposes. By setting the free-flow and shockwave travel times of the dummy link $(\tilde{O}, O)$ as zeros and the queue storage capacity as infinite, we can always replace the time-dependent demand at origin O as the time-dependent inflow to the dummy link $(\tilde{O}, O)$. The following is a brief description of the double queue concept and the slack variables defined for reflecting the flow withholdings.

As the DNL model for single-line stretch networks does not have the route and departure time choice behavior as in a DUE model, the formulation is much simpler as presented below. Again, the inflow rate to the dummy link, namely the time-dependent demand, is pre-defined. By solving the DNL model, we can obtain the time-varying inflow and exit flow rates, the upstream and downstream queues, and the values of the slack variables, with which we can estimate the link travel times using different approximation schemes and compare their performances.

Initial conditions:

$$q^d_{ij}(h) = 0, \ \forall (i,j) \in L \cup (\widetilde{O}, O), \ \forall h \in [0, \widetilde{T}] \tag{3}$$

$$q^u_{ij}(h) = 0, \ \forall (i,j) \in L \cup (\widetilde{O}, O) \tag{4}$$

$$n^0_{\tilde{O}O} = 0; \ n^\omega_{\tilde{O}O} = 0 \tag{5}$$

$$u_{\tilde{O}O}(h) = d_O(h), \ \forall h \in [0, \widetilde{T}] \tag{6}$$

Traffic dynamics:

$$v_{\tilde{O}O}(h) = u_{O1}(h), \ \forall h \in [0, \widetilde{T}] \tag{7}$$

$$v_{ij}(h) = u_{jk}(h), \ (i,j) \in L, \ (j,k) \in L, \ h \in [0, \widetilde{T}] \tag{8}$$

$$\begin{aligned} q^d_{ij}(h) &= q^d_{ij}(0) + \Delta \cdot \sum_h \left(u_{ij}\left(h - n^0_{ij}\right) - v_{ij}(h) \right), \\ &(i,j) \in L \cup (\widetilde{O}, O), \ h \in [0, \widetilde{T}] \end{aligned} \tag{9}$$

$$\begin{aligned} q^u_{ij}(h) &= q^u_{ij}(0) + \Delta \cdot \sum_h \left(u_{ij}(h) - v_{ij}\left(h - n^\omega_{ij}\right) \right), \\ &(i,j) \in L, \ h \in [0, \widetilde{T}] \end{aligned} \tag{10}$$

$$\begin{aligned} &0 \le \eta_{ij}(h) \perp Q_{ij} - q^u_{ij}(h) \ge 0 \text{ namely,} \\ &0 \le \eta_{ij}(h) \perp Q_{ij} - \Delta \cdot \sum_h \left(u_{ij}(h) - v_{ij}\left(h - n^\omega_{ij}\right) \right) \ge 0, \\ &(i,j) \in L, \ h \cup [0, \widetilde{T}] \end{aligned} \tag{11}$$

$$\begin{aligned} &0 \le \eta_{ij}(h) \perp q^{d,h}_{ij} \ge 0, \text{namely,} \\ &0 \le \eta_{ij}(h) \perp \Delta \cdot \sum_h \left(u_{ij}(h - n^0_{ij}) - v_{ij}(h) \right) \ge 0, \\ &(i,j) \in L, \ h \in [0, \widetilde{T}] \end{aligned} \tag{12}$$

$$\begin{aligned} &0 \le v_{ij}(h) \perp \eta_{jk}(h) - \left(C_{ij} - \mu_{ij}(h) - v_{ij}(h) \right) \ge 0, \\ &(i,j) \in L \cup (\widetilde{O}, O), \ (j,k) \in L, \ h \in [0, \widetilde{T}] \end{aligned} \tag{13}$$

$$\begin{aligned} &\delta_{ij}(h) = min\left(C_{ij} - \mu_{ij}(h), \eta_{jk}(h) \right), \ (i,j) \in L \cup (\widetilde{O}, O), \\ &(j,k) \in L, \ h \in [0, \widetilde{T}] \end{aligned} \tag{14}$$

$$v_{ij}(h) = C_{ij} - \mu_{ij}(h) - \delta_{ij}(h) \tag{15}$$

$$v_{xS}(h) = C_{xS} - \mu_{xS}(h), \ h \in [0, \widetilde{T}] \tag{16}$$

Equations 3–6 give the initial conditions; *Equations 7 and 8* define the flow conservation at each node; *Equations 9 and 10* describe the upstream and downstream queue dynamics, respectively; *Equations 11 and 12* are the complementarity problems for the upstream and downstream queues; *Equations 13-16* define the complementarity problems on exit flow rates and the computation of exit flow rates. In particular, $\eta_{ij}(h)$ in *Equation 11* describes the flow withholding at the entrance of the link (i,j). If Q_{ij}-$q_{ij}^u(h)$>0, then $\eta_{ij}(h)$=0, i.e. no withholding at the link entrance if the upstream queue is less than the link queue storage capacity. Otherwise the inflow rate to the link (which is also the exit flow of the upstream link) may be restrained, i.e., $\eta_{ij}(h)\geq 0$. $\mu_{ij}(h)$ is the slack variable associated with the downstream queue of the link (i,j), which collectively restrains the exit flow of the link (i,j) with the inflow withholding of its downstream link (j,k), i.e., $\eta_{jk}(h)$. To be specific, if $q_{ij}^d(h)$>0, $\mu_{ij}(h)$ will be zero, indicating $v_{ij}(h)=C_{ij}-\mu_{ij}(h)=C_{ij}$ if there is no withholding on the inflow to the downstream link (j,k), i.e., $\eta_{jk}(h)=0$. *Equations 13–15* are designed to restrict $v_{ij}(h)$ to zero if $\eta_{jk}(h)\geq C_{ij}-\mu_{ij}(h)$. Similar formulations for the double-queue-based DUE model can be found in [11].

Reference [11] proposed the first-order approximation to estimate link travel times using the first-order Taylor expansion theory to solve their double-queue-based DUE. The approximation first assumes the exit flow rate remains constant as $v_{ij}(h+n_{ij}^0)$ from h+n_{ij}^0 to $h+n_{ij}(h)$ as shown in *Equation 17* and then further approximates the link travel time using first-order Taylor expansion theory as presented in *Equation 18*.

$$\begin{aligned} q_{ij}^d\left(h+n_{ij}^0\right) &= \sum_{r=h+n_{ij}^0}^{h+n_{ij}(h)} v_{ij}(r) \\ &\approx v_{ij}\left(h+n_{ij}^0\right)\left[n_{ij}(h)-n_{ij}^0\right] \end{aligned} \tag{17}$$

$$\begin{aligned} n_{ij}(h) &\approx n_{ij}^0+\frac{q_{ij}^d\left(h+n_{ij}^0\right)}{v_{ij}\left(h+n_{ij}^0\right)}=n_{ij}^0+\frac{q_{ij}^d\left(h+n_{ij}^0\right)}{C_{ij}-\mu_{ij}\left(h+n_{ij}^0\right)-\delta_{ij}\left(h+n_{ij}^0\right)} \\ &= n_{ij}^0+\frac{q_{ij}^d\left(h+n_{ij}^0\right)/C_{ij}}{\left[C_{ij}-\mu_{ij}\left(h+n_{ij}^0\right)-\delta_{ij}\left(h+n_{ij}^0\right)\right]/C_{ij}} \\ &= n_{ij}^0+\frac{q_{ij}^d\left(h+n_{ij}^0\right)}{C_{ij}}\cdot\frac{1}{1-\left[\mu_{ij}\left(h+n_{ij}^0\right)+\delta_{ij}\left(h+n_{ij}^0\right)\right]/C_{ij}} \\ &\approx n_{ij}^0+\frac{q_{ij}^d\left(h+n_{ij}^0\right)}{C_{ij}}\cdot\left(1+\frac{\mu_{ij}\left(h+n_{ij}^0\right)+\delta_{ij}\left(h+n_{ij}^0\right)}{C_{ij}}\right) \end{aligned} \tag{18}$$

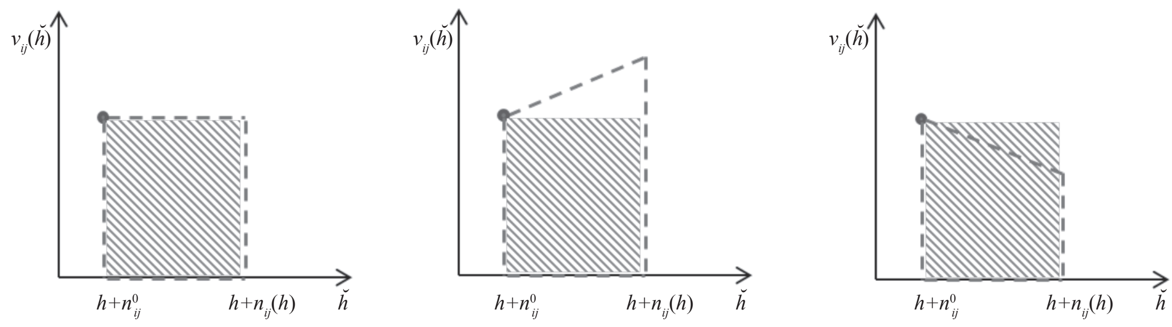
It can be seen from *Equations 17 and 18* that the approximated link travel time $n_{ij}(h)$ is estimated by assuming that the link exit flow rate is constant from $h+n_{ij}^0$ to $h+n_{ij}(h)$, which could either underestimate or overestimate the link downstream queue in *Equation 17* and thus the link travel time in *Equation 18*. The first-order Taylor expansion is applied in *Equation 18* to deal with the non-zero requirement for the term 1 - $((\mu_{ij}(t+\tau_{ij}^0)+\delta_{ij}(t+\tau_{ij}^0))/C_{ij}$. *Figure 12* below is created for demonstration purposes, where the area surrounded by the dashed lines represents the actual link downstream queue while the area marked by pattern lines is the estimated link downstream queue assuming the exit flow rate is approximated as $v_{ij}(h+n_{ij}^0)$ from $h+n_{ij}^0$ to $h+n_{ij}(h)$ in *Equation 17*.

As shown in *Figure 12*, if the link exit flow rate is increasing from $h+n_{ij}^0$ to $h+n_{ij}(h)$, the link downstream queue will be underestimated, leading to the underestimation of link travel time in *Equation 18*; it is the opposite for the case with decreasing link exit flow rate. Since the first-order Taylor expansion leads to additional underestimation of link travel times, the second-order Taylor expansion as expressed by *Equation 19* is proposed as an alternative method to improve the approximation performance. In this study, we define the approximations used in *Equations 18 and 19* as the first-order and second-order approximation, respectively.

$$\begin{aligned} n_{ij}(h) &\approx n_{ij}^0+q_{ij}^d\left(h+n_{ij}^0\right)/C_{ij}\cdot \\ &\begin{pmatrix} 1+\left(\mu_{ij}\left(h+n_{ij}^0\right)+\delta_{ij}\left(h+n_{ij}^0\right)\right)/C_{ij}+ \\ \left(\left(\mu_{ij}\left(h+n_{ij}^0\right)+\delta_{ij}(h+n_{ij}^0)/C_{ij}\right)^2\right. \end{pmatrix} \end{aligned} \tag{19}$$

As FIFO is an important requirement in DTA, it has been investigated in many DTA models (Ran and Boyce 1996). It requires the derivative of travel times to be no less than -1, namely:

$$\partial\tau_{ij}(t)/\partial t \geq -1 \text{ or } \frac{n_{ij}(h+1)-n_{ij}(h)}{\Delta} \geq -1 \tag{20}$$



a) Accurate *b) Underestimate* *c) Overestimate*

Figure 2 – Link Downstream Queue Approximation

FIFO Property: If the link travel times are approximated by the first-order approximation in *Equation 21*, FIFO is guaranteed if there are no downstream queues on any links of the network anytime.

Proof: If there is no downstream queue on the network, i.e., $q_{ij}^{d}(h+n_{ij}^{0})=0$ for any h, then $n_{ij}(h)=n_{ij}^{0}$ for any h based on *Equation 19*, resulting in $\partial n_{ij}(h)/\partial h=0$. Therefore, FIFO is guaranteed for first-order approximation.

It is worth to mention that there will be no queue on the network if the inflow and exit flow capacities of all links on a single-line stretch network are the same, which guarantees FIFO. On the other hand, if downstream queues exist on a road network because of large demands or various link inflow and exit flow rate capacities on routes, FIFO will not be guaranteed. Counter examples can be found in the numerical tests.

3. NUMERICAL ANALYSIS

The numerical analysis is conducted on a two-link stretch network as shown in *Figure 3*. A dummy node $\tilde{O}$ and a dummy link $(\tilde{O},O)$ are added to the network. Different pre-defined inflow rates (i.e., demand profiles) are tested with all experiments assuming that the inflow lasts for 15 minutes. In addition, it is assumed that the capacity of the link $(1,S)$ is time-varying, which could be caused by road maintenance, accidents, or disasters. Related parameters are given in *Table 1*. Note that in all numerical experiments the inflow rate capacity of the link $(O,1)$ is set the same as the inflow rate to the dummy link, while the inflow rate capacity of the link $(1,S)$ is set the same as its exit flow rate capacity.

The discrete time interval is selected as 0.2 minutes, i.e., Δ=0.2 minutes, so that the free flow travel time τ_{ij}^{0}, the shockwave travel time τ_{ij}^{ω}, and the two capacity-changing time points of C_{1S} (2 minutes and 5 minutes) will all be integral multiples of Δ. Again, as previously mentioned, the time-dependent inflow rates to the dummy link are given and the queue storage capacity of the dummy link is defined as infinite. Further, we assume that the Link O-1 and the Link 1-S are three-lane road links with a normal flow capacity of 4200 vph while the capacity of the Link 1-S is decreased to 2100 vph during time intervals 10 to 25 due to an incident. We also assume that the inflow rate capacity of the first link is unrestricted to test various scenarios. In order to measure the performance of the first-order approximation, the point queue model denoted by *Equation 21* is used for comparison purposes [11].

$$n_{ij}(h) = n_{ij}^{0} + q_{ij}^{d}\frac{h+n_{ij}^{0}}{C_{ij}} \tag{21}$$

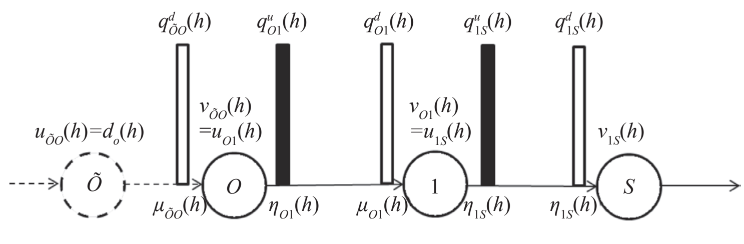


Figure 3 – A Simple Stretch Network for Numerical Analysis

Table 1 – Parameters for the Three-node, Two-Link Stretch Network

Parameters	Values of parameters
f_{ij}, ω_{ij}	$O1=f_{1S}$=60 mph; $\omega_{O1}=\omega_{1S}$=20 mph
Q_{ij}	$Q_{\tilde{O}O}=\infty$; $Q_{O1}=Q_{1S}$=(4200/60+4200/20) · 0.2=112 vph
Δ	Discretized time interval, which is 0.2 minutes
τ_{ij}^{0}, n_{ij}^{0}	$\tau_{O1}^{0}=\tau_{1S}^{0}$=0.4 minutes; $\tau_{\tilde{O}O}^{0}$=0; $n_{O1}^{0}=n_{1S}^{0}$=2; $n_{\tilde{O}O}^{0}$=0
τ_{ij}^{ω}, n_{ij}^{ω}	$\tau_{O1}^{\omega}=\tau_{1S}^{\omega}$=1.2 minutes; $\tau_{\tilde{O}O}^{\omega}$=0; $n_{O1}^{\omega}=n_{1S}^{\omega}$=6; $n_{\tilde{O}O}^{\omega}$=0
$u_{\tilde{O}O}(h)$	A fixed value for the first 75 time intervals, then becomes zero
$C_{ij}(h)$	$C_{O1}(h)$ = 4200 vph; $C_{1S}(h)=\begin{cases}4200\text{ vph if } h\leq 10\\ 2100\text{ vph if} > 10 \text{ and } h\leq 25\\ 4200 \text{ vph if } h>25\end{cases}$

Considering the inflow and exit flow rate capacities of the link (O,1) and (1,S), different demand profiles ranging from 2100 vph to 8000 vph are tested in the numerical analysis. For each demand profile, the real link travel times based on the cumulative inflow and exit flow curves, the first-order and the second-order approximated travel times, and the travel times based on the point queue model will all be computed for performance comparison. First, we test with an inflow rate of 4200 vph with results shown in *Figures 3 and 4*.

As it can be seen from *Figure 4*, the upstream queues are always larger than or equal to the downstream queues on both links, which is consistent with *Equations 12 and 13*. The cumulative inflow and exit flow curves are plotted in *Figure 5a*, based on which the real link travel times can be obtained using linear interpolation. Since link (O,1) is the link with complete double-queue constraints on both ends, it is chosen as the study link. *Figure 5b* shows its link travel times obtained from different approaches and its exit flow rates.

From *Figure 5b*, the second-order approximation technique always produces higher link travel times than the first-order approximation and the point queue model, which is consistent with *Equations 18, 19, and 21*. In addition, when congestion presents the point queue model always underestimates the link travel times while the first-/second-order approximation could either underestimate or overestimate the link travel times. Furthermore, the second-order approximation always produces smaller errors in the case of underestimation but greater errors in the case of overestimation. The numerical results show that: (1) if the inflow rates are lower than or equal to 2100 vph for 15 minutes, there are no queues and vehicles are traveling at a free flow speed; FIFO is satisfied by all estimation schemes; (2) if the inflow rates are higher than 2100 vph and lower than 4200 vph for 15 minutes, the travel time patterns are similar to that in *Figure 5b*; (3) if the demand is much higher, the travel time pattern will be different from *Figure 5b* as shown in *Figure 6* with an inflow rate of 8000 vph for 15 minutes.

Figure 6 presents two peaks of link travel times, reflecting two different bottleneck capacity levels during the study time period. The first bottleneck capacity level is 4200 vph (i.e., exit flow rate capacity of the link O-1) while its inflow rate capacity is 8000 vph, and the second is the time-varying exit flow rate capacity of the link 1-S. The sudden decrease of travel time on the link (O,1), i.e., from $\tau_{O1}(28)$=1.6 min to $\tau_{O1}(29)$=1.1 min, produces $\nabla\tau(28)$=-(1.6-1.1)/0.2=-2.5<-1. Obviously, FIFO is violated here. It is similar for the cases with inflow rates higher than 2100 vph and lower than or equal to 4200 vph.

From the above numerical analysis, it is also observed that (1) the point queue model could produce an underestimation but never an overestimation; (2) as shown in *Figures 5b and 6*, when the real link travel time (i.e., the blue solid line) is increasing, the first order approximation (i.e., the green solid line), the second order approximation (i.e., the pink dashed line), and the point queue model (i.e., the blue dash-dot line) all give underestimations, whereas the second-order approximation gives the best estimation; (3) when the real link travel time is decreasing, both the first-order and the second-order approximation first give an underestimation and then an

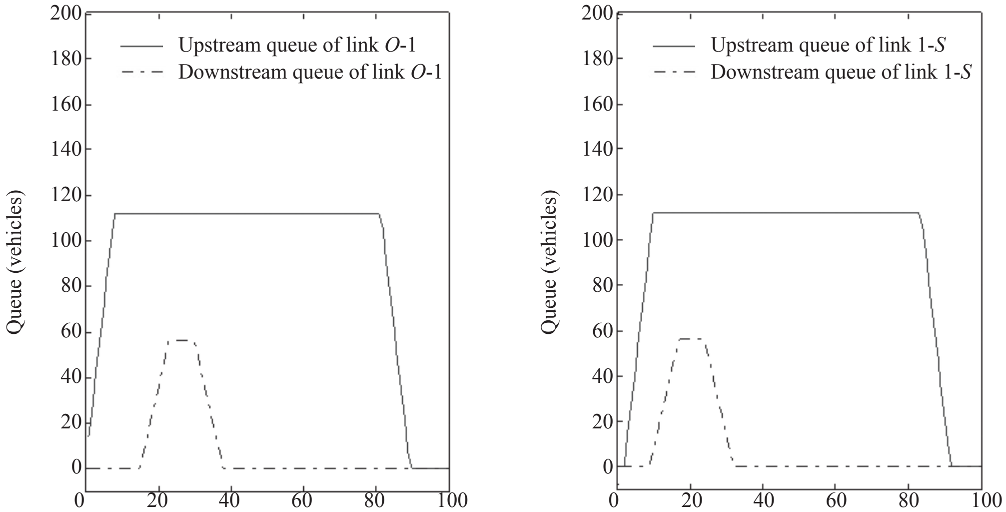


Figure 4 – Upstream and Downstream Queues with Inflow Rate of 4200 vph

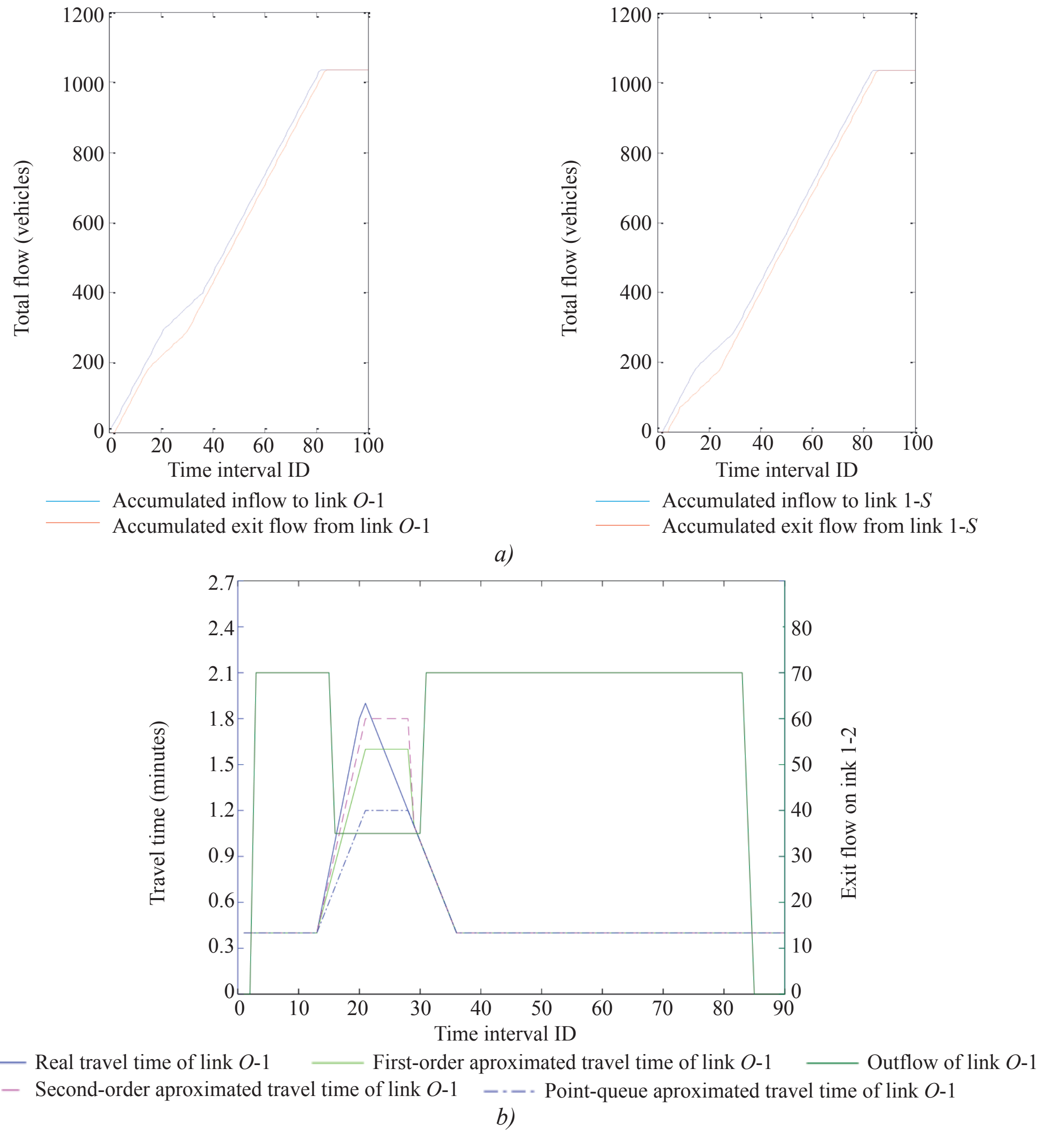


Figure 5 – a) Cumulative Inflow of O-1 and Exit Flow Curves 1-S; b) Travel Times and Exit Flow Rate of Link O-1 with Inflow Rate of 4200 vph

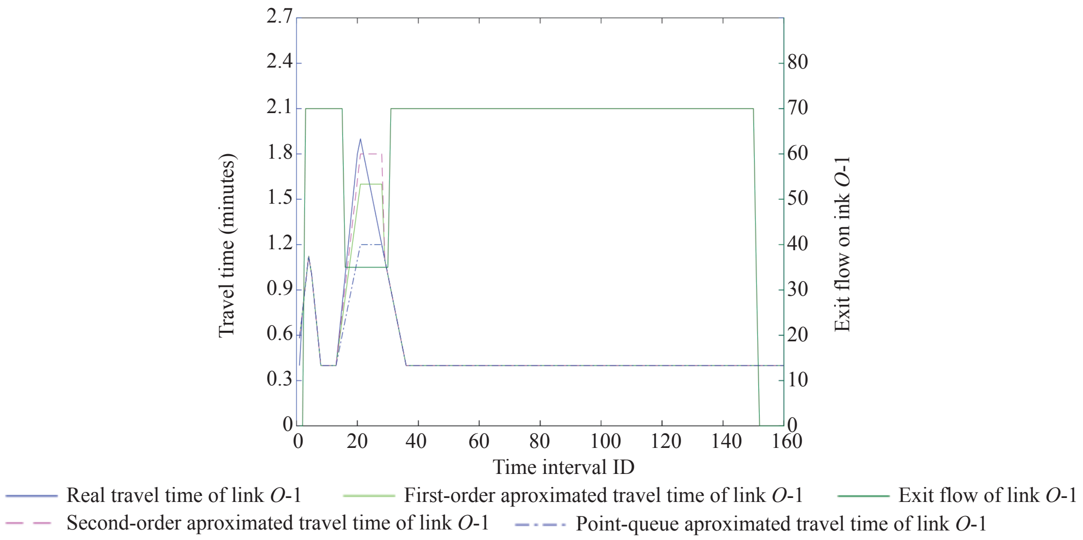


Figure 6 – Travel Times and Exit Flow Rate of the Link (O,1) with an Inflow Rate of 8000 vph

Table 2 – Link Travel Times on the Link Computed with Different Approximation Methods

Demand $d_o(h)$/ Capacity $C_{O1}(h)$ $h \leq 75$	Maximum real travel time $\tau_{O1}^{\max} = \max_h \tau_{O1}^{real}(h)$ [min]	Maximum absolute difference compared with real travel time $dif_m = \max_h \left\| \tau_{O1}^{x}(h) - \tau_{O1}^{real}(h) \right\|$			Total absolute difference compared with real travel time $total = \mathrm{Sum}_h \left\| \tau_{O1}^{x}(h) - \tau_{O1}^{real}(h) \right\|$		
		point queue	first-order	second-order	point queue	first-order	second-order
0.24	0.4	0	0	0	0	0	0
0.48	0.4	0	0	0	0	0	0
0.50	0.4	0	0	0	0	0	0
0.71	0.64	0.10	0.09	0.14	0.28	0.21	0.20
0.95	1.74	0.65	0.40	0.60	4.75	2.66	2.82
1.00	1.90	0.70	0.40	0.60	5.60	3.00	2.90
1.19	1.90	0.70	0.40	0.60	5.64	3.04	2.94
1.43	1.90	0.70	0.40	0.60	5.69	3.09	2.99
1.67	1.90	0.70	0.40	0.60	5.73	3.13	3.03
1.90	1.90	0.70	0.40	0.60	5.78	3.18	3.08
2.14	1.90	0.70	0.40	0.60	5.83	3.23	3.13
2.38	1.90	0.70	0.40	0.60	5.88	3.28	3.18

overestimation before reaching the real link travel time, while the point queue model gives an underestimation before the real link travel time. These findings from the numerical results give us insight into how to design a more accurate approximation scheme.

To further compare the performances of different link travel time approximation methods, *Table 2* summarizes some statistics on the travel times on the link $(O,1)$ estimated using different approximation methods given a series of demand profiles.

In *Table 2*, the maximum real travel time is the maximum travel time on the link $(O,1)$ estimated with the cumulative inflow and exit flow curves among all the discrete time intervals; the maximum absolute difference compared with the real time as defined by $dif_m = \max_h \left| \tau_{O1}^{x}(h) - \tau_{O1}^{real}(h) \right|$ means that, among all the time intervals, the maximum absolute difference between the travel times on the link $(O,1)$ computed by the approximation technique x (where x refers to the point queue model, the first-order approximation, or the second-order approximation) and its real travel time based on the cumulative inflow and exit flow curves; the total absolute difference compared with real travel time is defined as $total = \mathrm{Sum}_h \left| \tau_{O1}^{x}(h) - \tau_{O1}^{real}(h) \right|$.

Table 2 shows that with the increase of the demand, the maximum travel time on the link $(O,1)$ and the maximum travel time differences with different approximation methods increase first and then become stable after the demand arrives at the exit flow capacity of the link $(O,1)$. Total absolute travel time differences, on the other hand, keep increasing with the increase of demand. In terms of total absolute travel time differences, as plotted in *Figure 7*, compared to the real link travel times, the first-order approximation is more reliable than the point queue mode; the second-order approximation is slightly better than the first-order approximation. To sum up, the point queue model, the first-order, and the second-order approximations increase in both the performance and the computational complexity.

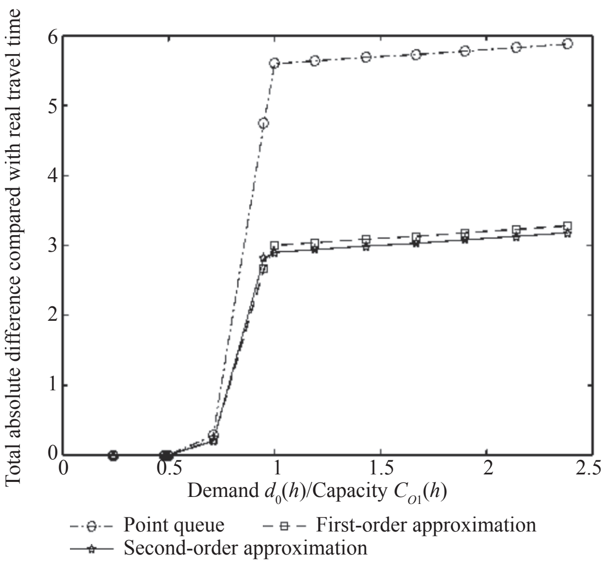


Figure 7 – Total Absolute Difference Compared with Real Travel Time

4. CONCLUSION

This paper focused on investigating the properties and performance of the first-order and the second-order approximation methods for the link travel time estimation in the double-queue-based ideal DUE models. By analyzing the approximation function, it was found that the first-order approximation could result in either overestimated or underestimated link travel times when the demand is larger than the link capacity. In addition, FIFO is guaranteed if there is no downstream queue on any link of the single-line stretch network over time but not guaranteed if there is downstream queue on any link.

In order to directly study the performances of the first-order approximation, the discretized double-queue-based DNL model was designed and formulated for single-line stretch networks. Then we proposed and compared the link travel times using the first-order approximation with those using the second-order approximation, the point queue model, and the cumulative inflow and exit flow curves.

Numerical analysis was conducted on a two-link stretch network to demonstrate the FIFO violation of the first-order approximation and compare its performance with other estimation schemes. Overall the point queue model, the first-order approximation, and the second-order approximation increase in both the performance and the computational complexity. To be specific, the results showed that all these estimation approaches produced the exact real link travel times if the demand was always no larger than the bottleneck link exit flow rate capacity. However, if the demand exceeded the bottleneck capacity, queues would build up. The point queue model could underestimate the link travel times while the first-order and the second-order approximation may either underestimate or overestimate the link travel times. When the real link travel time is increasing, the link travel time from either the first-order or the second-order approximation is always underestimated. When the real link travel time is decreasing, the link travel time from either the first-order or the second-order approximation is first underestimated and then overestimated before reaching the real link travel time. These give us insight into how to design a more accurate approximation scheme. For instance, it is suggested that the second-order approximation should always be preferred when the real link travel time is increasing and the computational complexity is not an issue. In addition, with the increase of the total demand, the link travel time estimation error would increase with the first-order and the second-order approximations. The numerical results show that the first-order approximation produces more reliable results compared with the point queue model, while the second-order approximation may be used as an alternative to further increase the travel time estimation performance.

This paper specifically studied the performance and properties of the link travel times estimated with the first-order approximation proposed and applied in the previous study on the ideal double-queue-based DUE modeling [11]. Future study can be conducted on how to better derive a closed-form expression for link travel time estimation in order to improve the estimation accuracy and even guarantee the FIFO property. Such improved link travel approximation will help better model and solve the ideal DUE problems with queue spillbacks.

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基于双队列概念的交通流模型中的路段运行时间估测

摘要

由于双队列概念（*Double queue concept*）能有效地模拟实际交通流动态特征包括排队溢出现象等，双队列概念已经开始应用于理想/预测型动态用户均衡建模。由于基于双队列概念的动态用户均衡问题直接求解非常困难，最近一研究提出了一种一阶近似方案以简化动态用户均衡问题中的路段运行时间估测，但该研究并没有研究该一阶近似方案的特性和性能。本文通过设计并模拟单线路网动态加载问题而非复杂网络的动态配流问题，以直接研究该一阶近似方案在路段运行时间估测中的先进先出（*FIFO*）特性及估测性能。建模后我们首先分析了该一阶近似方案的先进先出特性。接着我们进行了一系列算例分析以展示该一阶近似方案的先进先出特性，并对比一阶近似方案，二阶近似方案，点队列模型和累积流入和流出流量曲线的路段运行时间估测性能。研究结果显示一阶近似方案不能确保先进先出特性，并且尤其当路段的流出流量增长时应

推荐使用二阶近似方案。该研究对将来更进一步研究新的路段运行时间估测方法提供了参考。

关键词

双队列概念；动态用户均衡；动态网络加载；路段运行时间估测；先进先出

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