Extension of Intersection Method for Multi-Objective Optimization in Case of Interval Number and its Application

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Abstract: This paper aims to develop the extension of intersection method for multi-objective optimization under condition of interval number. Based on the linear correlation of partial favourable probability and the corresponding performance indicator, and the assumption of uniform distribution of the actual value of performance indicator within the range of its lower and upper limits in case of interval number, it derives that the actual partial favourable probability of a performance indicator is the arithmetic mean value of the partial favourable probabilities of the arithmetic mean value and the variation value of the interval index of the corresponding performance indicator for each candidate, or their desired sum. Furthermore, according to the rule of algorithm for the total favourable probability quantitatively, all candidates are ranked according to their total favourable probabilities to complete the multi- objective optimization in case of interval number. As applications, the quantitative assessments of multi-criteria selections for effective dwelling house walls, project managers and contractor for construction works are given in detail, satisfied results are obtained.

Keywords: arithmetic mean; favourable probability; intersection method; interval number; multi-objective optimization

1 INTRODUCTION

Multi-objective optimization (MOO) is the process of specifying the optimal solution from all feasible alternatives. The category of MOO is to conduct the selection from a limited number of alternatives by assessing the performances of multiple objects in beneficial or non - beneficial manners comparatively. A MOO technique determines how the performance information to be assessed and to get the appropriate selection. Given the proper decision matrix and decision-making procedures, the decision maker could conduct the best alternative and /or rank to complete the selection.

In traditional multi - objective optimizations, the performance indexes and attributes are assumed to be well determined without any uncertainty. However, in some cases, the responses of alternatives to attribute are not always possessing exact values. Especially, decision makers' judgment is often vague, which may not present their preferences with exact numerical values. In order to deal with such uncertain elements of a decision problem, several approaches are proposed. In interval COPRAS-G, complex steps are included involving artificial parameters and personality [1]. Hafezalkotob et al. extended the MULTIMOORA method to include interval number by using fuzzy logic theory and a novel comparison technique [2], by introducing a preferences of $A \geq B$ or $B \geq A,$ and each coordinate of the comparison matrix of P_{qr} being a one – to - one comparison between intervals, as well as specific value of 0.5 for diagonal preferences, i.e., $\{P_{11}, P_{22}, …, P_{vv}\}$. Jahanshahloo et al. proposed an extension of the technique for order preference by similarity to ideal solution (TOPSIS) to deal with decision making problems with interval numbers [3]. Sayadi et al. extended the VIse Kriterijumska Optimizacija i kompromisno Resenje (VIKOR) technique by considering interval value [4]. Pan et al. formulated an interval multi attribute decision making (MADM) approach by using the linear additive utility function and composite utility variance [5]. Chen et al. developed a unique interval MADM method with loss aversion [6]. Cao and Wu extended

the continuous ordered weighted geometric (COWG) operators to solve multiple attributive group decision making (MAGDM) problems with interval numbers [7]. Brauers et al. employed the fuzzy numbers to extend the crisp MULTIMOORA method [8]. Liu et al. utilized fuzzy numbers to determine the risk of failure modes by extending MULTIMOORA method [9]. Mandal and Sarkar used the fuzzy MOORA methodology to perform the selection of an optimal intelligent manufacturing system [10].

Recently, a new "intersection" method for MOO was proposed in the viewpoints of probability theory and set theory [11], which aims to solve the inherent problems of personal and subjective factors in previous multi – object optimizations. In the new MOO method, a novel concept of favourable probability was developed, which reflects the favourable degree of the candidate in the optimization; each performance utility indicator of the candidate contributes to a partial favourable probability quantitatively; the total favourable probability of a candidate is the product of all partial favourable probabilities in the viewpoint of probability theory, which is the overall and unique decisive index in the competitive selection process. The new MOO method was applied in materials selection and multi - objective orthogonal test design (MOOTD) and multi - objective uniform test design (MOUTD) successfully [11].

As a further study on the newly proposed "intersection" method for MOO, here in this paper, an extension of the intersection method for MOO under condition of interval indexes is developed so as to include this case in the evaluations.

2 EXTENSION OF THE "INTERSECTION" METHOD FOR MOO IN CASE OF INTERVAL NUMBER

In traditional MOO, the performance indexes and attributes are assumed to be well determined without any uncertainty. However, in some cases, the performance indexes and attributes are often vague, which results in un-exact numerical values instead of well determined data. In

order to deal with such decision problem with uncertain elements, proper approach is in need.

In general, the decision-making matrix X is with the form of Eq. (1) in case of interval numbers,

$$X = \begin{bmatrix} [w_{11};b_{11}][w_{12};b_{12}]\cdots[w_{1m};b_{1m}] \\ [w_{21};b_{21}][w_{22};b_{22}]\cdots[w_{2m};b_{2m}] \\ \cdots[w_{ij};b_{ij}]\cdots \\ [w_{n1};bn1][w_{n2};b_{n2}]\cdots[w_{nm};b_{nm}] \end{bmatrix} \quad (1)$$

$i = 1, 2, ..., n; \; j = 1, 2, ..., m.$

In Eq. (1), w_{ij} represents the lower limit, and b_{ij} is the upper limit of the performance index.

The main feature of the new "intersection" method for MOO is that there is no necessary to make normalization for the decision-making matrix [11], the partial favourable probability can be obtained from the decision-making matrix directly for the performance index with exact value, and the total favourable probability is the overall representative in the viewpoint of probability theory, which is the unique decisive index in the competitive selection process.

However, in some cases there gives the interval value instead of exact value for the performance index, i.e., the lower limit and upper limit of the performance indicator.

In cases of interval values, it is no doubt that the value of the performance index is within the range of the lower and upper limits of the performance indicator. Thus the lower and upper limits of the performance indicator may lead to uncertainty of the partial favorable probability of the performance index.

In [11], it assumed that the partial favourable probability is correlative to the corresponding performance index linearly from the principle of simplicity, if we further assume that the actual value of the performance indicator under condition of interval number is uniformly distributed within the range of its lower and upper limits, then the actual partial favourable probability of a performance index is the arithmetic mean value of the partial favourable probabilities of the arithmetic mean value $m_{ij} = (w_{ij} + b_{ij})/2$ and the variation value $v_{ij} = (b_{ij} - w_{ij})/2$ of the interval index, i.e. or their desired sum,

$$P_{ij} = \frac{\left[P(m_{ij}) + P(v_{ij}) \right]}{2}, \text{ or}$$
$$P_{ij} = a_j * P(m_{ij}) + (1 - a_j) * P(v_{ij}). \quad (2)$$

In Eq. (2), $P(m_{ij})$ and $P(v_{ij})$ represent the partial favorable probabilities of the arithmetic mean value and variation value of the interval index, respectively. The assessments of $P(m_{ij})$ and $P(v_{ij})$ are in accordance with Ref. [11], individually; in general, $P(v_{ij})$ is attributed to non-beneficial type; a_j is the adjust parameter for desired sum.

In this operation, the interval index is divided into two parts, i.e., m_{ij} and v_{ij}, which represent the various properties of interval index in different aspects. This operation opens a new treatment to interval index, which breaks away from the

persecution of comparison of interval indexes, while the comparison of interval indexes is continuously an unsolvable problem.

Afterwards, the assessment of the intersection method for MOO under condition of interval number will be as usual as that in [11].

Till now, the extension of intersection method for multi-objective optimization under condition of interval number is completed.

3 APPLICATIONS OF THE EXTENSION OF INTERSECTION METHOD FOR MULTI-OBJECTIVE OPTIMIZATION

In this section, the extended intersection method for MOO in case of interval number is applied to deal with the following multi-criteria decision-making problems.

3.1 Selection for Effective Dwelling House Walls

Zavadskas et al. once developed a "specific method of multiple criteria decision-making combining with grey relations" to deal with the problem of attribute values within intervals [1]. Tab. 1 shows the initial decision-making index for effective dwelling house walls with values expressed in intervals, which is cited from [1].

Table 1 Initial decision-making matrix with values expressed in intervals

Alternate No.	Durability		Thermal transmit.		Cost		Weight of wall		Human expenditure	
Weight q	0.21		0.33		0.26		0.09		0.11	
Index	X_1		X_2		X_3		X_4		X_5	
Optimum	max		min		min		min		min	
Alternat.	w_1	b_1	w_2	b_2	w_3	b_3	w_4	b_4	w_5	b_5
A_1	75	100	0.22	0.25	72.08	94.71	590	652	4.60	4.60
A_2	75	100	0.22	0.25	89.01	100.93	596	625	4.60	4.60
A_3	75	100	0.21	0.25	80.32	96.42	581	604	4.60	4.60
A_4	25	25	0.24	0.27	67.76	98.10	455	479	4.55	5.01

Table 2 Evaluated results for actual partial favourable probability, total favourable probability and ranking

Probability	P_{X1}	P_{X2}	P_{X3}	P_{X4}	P_{X5}	P_t	Rank
1	0.1522	0.2602	0.2420	0.1702	0.2929	0.2225	4
2	0.1522	0.2602	0.2869	0.2545	0.2929	0.2411	2
3	0.1522	0.2294	0.2722	0.2737	0.2929	0.2230	3
4	0.5435	0.2501	0.1989	0.3017	0.1214	0.2605	1

In Tab. 1, b_{ij} means the upper limit of the j performance indicator in the i alternative of a solution, w_{ij} means the lower limit of the j performance indicator in the i alternative of a solution; the word "max" in Tab. 1 means "the higher the better", i.e., beneficial performance index, and the word "min" means "the lower the better", i.e., unbeneficial performance index in the multi-objective optimization evaluation for the arithmetic mean value of the interval index.

The partial favourable probabilities of $P(m_{ij})$ and $P(r_{ij})$ are assessed from the performance data of the interval indexes shown in Tab. 1. According to Eq. (2), the arithmetic mean value of the partial favourable probabilities of

performance indexes of the candidates of mean value and variation represents the actual partial favourable probabilities of performance indicators, which is shown in Tab. 2.

The evaluated results for total favourable probability together the ranking results are given in Tab. 2 as well.

The sequence in Tab. 2 is "$A_4 > A_2 > A_3 > A_1$", which is not the same as the ranking of Zavadskas [1], however one may use the formula of desired sum with adjust parameter α_j to improve the consequence for one's aim; the expression "$A_4 > A_2$" here means that A_4 is prior to A_2.

3.2 Multi-Criteria Selection of Project Managers

In addition, in Ref. [12], Zavadskas et al. applied the "specific method of multiple criteria decision-making combining with grey relations" to deal with a project manager problem.

Tab. 3 shows the initial decision-making index for project manager problem, which is cited from [12].

Table 3 Initial decision-making index with the criterion values described in intervals and normalized weighted matrix

Index	X_1		X_2		X_3		X_4		X_5		X_6	
Optimum	max		max		max		max		max		min	
Weight q	0.25		0.15		0.12		0.20		0.13		0.15	
Manager	$\underline{x_1}$	$\overline{x_1}$	$\underline{x_2}$	$\overline{x_2}$	$\underline{x_3}$	$\overline{x_3}$	$\underline{x_4}$	$\overline{x_4}$	$\underline{x_5}$	$\overline{x_5}$	$\underline{x_6}$	$\overline{x_6}$
1	50	60	40	55	10	20	50	70	45	50	30	40
2	70	80	60	70	40	45	60	75	70	80	60	70
3	60	70	55	70	30	40	70	80	55	65	40	50

In Tab. 3, $\overline{x_{ij}}$ means the upper limit of the j attribute in the i alternative of a solution, $\underline{x_{ij}}$ means the lower limit of the j attribute in the i alternative of a solution.

Again, let's using Eq. (2) to conduct the assessment of the actual partial favourable probabilities for the performance indicators, the results are shown in Tab. 4.

Tab. 4 shows evaluated results for total favourable probability and ranking as well.

Table 4 Evaluated results for partial favourable probability, total favourable probability and ranking for project manager selection

Probability	P_{X1}	P_{X2}	P_{X3}	P_{X4}	P_{X5}	P_{X6}	P_t	Rank
1	0.3077	0.2784	0.2061	0.2593	0.3801	0.3763	0.2958	3
2	0.3590	0.4000	0.4797	0.3333	0.3305	0.2796	0.3547	1
3	0.3333	0.3214	0.3142	0.4074	0.2894	0.3441	0.3380	2

The sequence in Tab. 4 is "project manager 2 > project manager 3 > project manager 1", which is the same as the ranking in [12] fortuitously.

3.3 Multi-Criteria Selection of Contractor for Construction Works

Beside, in Ref. [13], Zavadskas et al. studied contractor selection problem for construction works by applying saw-g and TOPSIS grey techniques.

Tab. 5 shows the initial decision-making index values of contractor selection for construction works problem, which is cited from [13].

Table 5 Initial decision-making index values of contractor selection for construction works problem

Index	X_1		X_2		X_3		X_4		X_5		X_6	
Optimum	max		max		max		min		max		max	
Alternative	w_1	b_1	w_2	b_2	w_3	b_3	w_4	b_4	w_5	b_5	w_6	b_6
A_1	11	15	10	15	3.30	4.5	35	48	0.152	0.203	1	2
A_2	10	14	7	13	2.54	3.68	40	58	0.111	0.162	1	2
A_3	14	18	5	9	1.95	2.46	42	53	0.079	0.121	1	3
A_4	12	16	1	4	0.42	1.73	15	63	0.01	0.054	1	2
A_5	6	10	2	9	0.62	2.67	10	46	0.012	0.122	1	2

Once more, Eq. (2) is used to conduct the assessment of the actual partial favourable probabilities for the performance indicators, the results are shown in Tab. 6.

Tab. 6 shows evaluated results for total favourable probability and ranking of each candidate as well.

Table 6 Evaluated results for partial favorable probability, total favorable probability and ranking of contractor selection for construction works problem

Probability	P_{X1}	P_{X2}	P_{X3}	P_{X4}	P_{X5}	P_{X6}	$P_t \times 10^5$	Rank
1	0.2032	0.2667	0.2666	0.2347	0.2823	0.2049	19.6047	1
2	0.1952	0.2133	0.2380	0.1991	0.2424	0.2049	9.7989	3
3	0.2270	0.2133	0.2479	0.2240	0.2165	0.1806	10.5103	2
4	0.2111	0.1733	0.1399	0.1381	0.1481	0.2049	2.1442	4
5	0.1635	0.1333	0.1076	0.2042	0.1108	0.2049	1.0866	5

The sequence in Tab. 6 is "$A_1 > A_3 > A_2 > A_4 > A_5$", which is not the same as the ranking in [13].

The applications of the extension of new method for MOO under condition of interval number by above cases indicate its validity.

4 CONCLUSION

From above discussion, the extension of the new method for multi-objective optimization in case of interval number is an appropriate approach. The arithmetic mean value of the partial favourable probabilities of arithmetic mean value and variation value of the interval index or their desired sum gives the actual value of the partial favourable probabilities of the corresponding performance indexes. The total favourable probability is the overall and unique indicator to decide the final result of the assessment comprehensively. The approach is full probability theory based, which is categorical and simple without any artificial and personal factors.

Conflict Statement

There is no conflict of interest.

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