

ELEMENTALGEBRAIC STRUCTURE OF KNOWLEDGE TRANSFER MODEL

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ABSTRACT

This paper is an extension of the work originally presented in conference ICERI2019 [0]. We presented the way of understanding the relation of information and knowledge by showing that knowledge is built on previous acquired knowledge, that is just as information in new relation. We find that we have to define mathematical model using algebraic structure to describe this phenomena.

In educational sense we use this as model to give new paradigm in education, we suggest that education should enable students to find relations from knowledge that they acquire and the teacher should use different models (functions) to classify knowledge and enable students to discover (using class of functions) new knowledge. That model will help to develop self-adapting system for autonomous learning (new algorithms) what we have found out that we need in times we live today.

The interpretation of knowledge given in this paper gives us an opportunity to develop different algorithms for acquiring knowledge which is independent from its type, just by defining the function for transforming the set of information. This gives us the opportunity to interpret newly acquired knowledge as information for further interpretation and acquisition of different knowledge. In this paper, we go further in abstracting the knowledge and knowledge representation and show the model that can explain how new knowledge can be acquired from similar but not the same set of information.

Entering into the realm of groups is necessary for the mapping from a set of information to a set of knowledge to be an analytic function, which opens the possibility for us to abstract such a

function by a polynomial, for example, a Taylor polynomial, which will greatly facilitate our knowledge transfer.

In the analysis of knowledge transfer, particular attention should be paid to the domain of function or set of information. A set of information consists of a series of subsets that form a partitive set of domains. One of the characteristics of a partition of a set (the set of all subsets) is its cardinal number, the Bell number.

The property we have shown here may be another approach in the development of artificial intelligence, for which it is necessary to mathematically represent developments in the creation of a set of knowledge.

This model helps us understand the knowledge creation in different disciplines and from different fields of work.

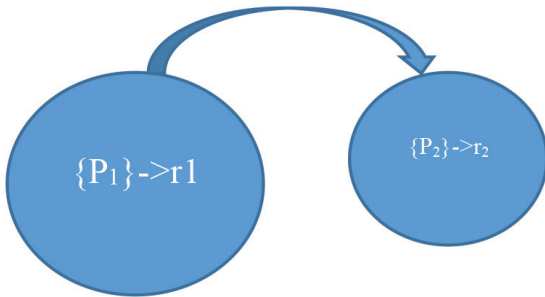
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ALGEBRAIC STRUCTURE

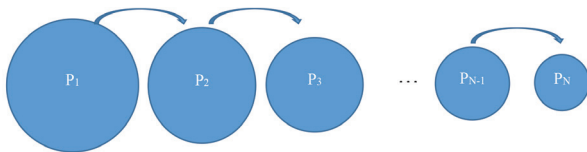
1. INTRODUCTION

In the paper presented at the ICERI 2019 Meeting entitled MATHEMATICAL MODEL OF KNOWLEDGE TRANSFER REPRESENTATION G. Sirovatka, V. Mickovic, P. Cavka, G. Sirovatka (2019) [0] we set up a model of the relationship between an information set and a knowledge set derived by a function that maps an information set into a knowledge set. In doing so, we presented the set of information as a domain of function and the set of knowledge as a codomain.

The relationship between these two sets can be represented by the fact that r_n is the radius of the information set $\{P_n\}$ while r_{n+1} is the radius of the knowledge set $\{f(\{P_n\})\}$.



With term radius we understand the size of set of information or knowledge set. We have found that mapping opens up the possibility, which we have shown, of a series of mappings whereby the knowledge set is for pre-mapping codomains and for subsequent domain mapping. We graphically depicted this series of mappings as follows:



In doing so, we have defined by domain size relative to the codomain that we have three forms of knowledge:

1. Expert knowledge

$$r_k = r_{k+1}, k= 1,2,\dots,n$$

2. Analytical knowledge

$$r_k > r_{k+1}, k= 1,2,\dots,n$$

3. Synthetic knowledge

$$r_k < r_{k+1}, k= 1,2,\dots,n$$

In this work we would assume that in one moment in time we have some set of information and then we organize this information and we got new knowledge, that knowledge we again see as information when we with other knowledge sets build new knowledge. So we do transformation from information to knowledge and then we see that knowledge as information for building new knowledge.

2. MAPPING AS A LIE GROUP

In order to describe a mapping from a set of information to a set of knowledge we use analytical functions from the realm of groups. This opens the possibility to model such a function with a polynomial, for example, a Taylor polynomial, which will greatly facilitate ability to describe the knowledge transfer.

Definition 1.

The finite groups have an associated binary operation satisfying four axioms:

1. Closure
2. Associativity
3. Invertibility
4. Identity
5. Neutral element

[Saunders MacLane](#) (2012) [2]

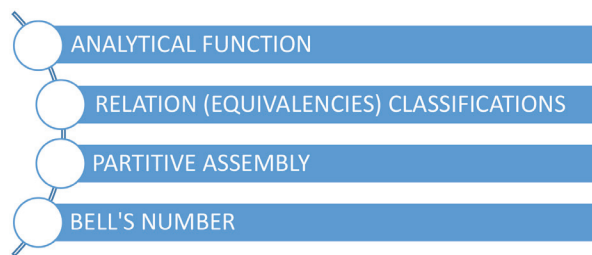
Mappings from a set of information to a set of knowledge satisfy the four axioms of a finite group:

1. Closure: The influence of a function on two information transforms this information into a third information, which under certain circumstances becomes an element of the knowledge set, which is an element of the information set (domain) of further mapping. So the information set is a closed set.
2. Associativity: If we influence the information of two information with a function, we have obtained an element of a set of knowledge that we assume will be information for further mapping. Therefore, it follows from the axiom of closure of a set of information that the addition of new information results in a new element of the set of information for some further mapping.
3. Invertibility: Each element of a set of information has its negation, which in this sense represents an inverse element.
4. Identity: For each element of a set of information, there is at least one element of a set of information that is irrelevant to any information in the set of information elements.
5. Neutral element: is set of no information.

3. EQUIVALENCY RELATION AS A SUBSCRIPTION OF A GROUP OF ANALYTICAL FUNCTIONS

The reason we insisted on the analytic function will be shown in further considerations. Mapping from a set of information to a set of knowledge that is an analytic function that we have proven by applying Lie groups, and we can tighten the criterion on the analytic function and obtain an equivalence relation. An equivalence relation (or a synonym of a classification relation) will serve to create equivalence classes that, by the set conditions, have as many members as the members have a pertinent domain set in our case of the information set, and in further mapping any further set of knowledge that becomes the set of information for the second third and n-th mapping. Now let's set the sequence as follows:

Definition 3.



We define a relation $R \subseteq S \times S$ as an **equivalence relation (or classification relation)** if it is reflexive, symmetric, and transitive.

1. A relation $R \subseteq S \times S$ is reflexive if and only if it is aRa .
2. A relation $R \subseteq S \times S$ is symmetric if and only if it has a property: if it is aRb , then it is also bRa .
3. A relation $R \subseteq S \times S$ is transitive if and only if it has a property: if it is aRb and bRc , then it is also aRc .

Definition 4.

If R is the equivalence relation on the set S , and a some element from S , then the set of all elements x from S , for which it is true that xRa , are denoted by C_a and termed the equivalence class, i.e.

$$C_a = \{x \in S \mid xRa\}.$$

Theorem 1.

Each equivalence relation defined in the set S determines the dividing of the set S into disjunctive subsets that are equivalence classes with respect to the given equivalence relation.

Theorem 2.

Let R be the equivalence relation on the finite set S . Then equivalence classes form a partition of the set S . Conversely, for a given partition of a set S , there is a unique equivalence relation whose classes are partition elements.

Definition 5.

Let S be a given set. We call each subset \mathcal{P} of $P(S) \setminus \{\emptyset\}$ a **partition of a set S** if these two conditions are met:

1. the union of all elements in \mathcal{P} , is S .
2. if A and B are different elements from \mathcal{P} , then $A \cap B = \emptyset$.

Therefore, the number of partitions of a set equals the number of equivalence relations on that set. We call that number **Bell's number**. Bell's number form an sequence (B_n) and n-th Bell's number B_n gives the number of partitions of a finite set of n elements or the number of equivalence relations on that set.

So, we have a sequence:

$$B_0, B_1, B_2, \dots, B_n$$

which satisfies recursion

$$(**) \quad B_{n+1} = \sum_{k=0}^n \binom{n}{k} B_k, \quad n \in \mathbb{N}_0$$

with the initial condition defined $B_0 = 1$.

We have a table of values of the first ten Bell's numbers, obtained by recursion (**). Zrno Ž. [11]

n	1	2	3	4	5	6	7	8	9	10
B_n	1	2	5	15	52	203	877	4140	21147	115975

4. EXAMPLE

For an example of a mapping from a set of information to a set of knowledge, we will use the current CORONA-19 disease.

The first set of information will include the symptoms of CORONA-19 disease: fever, cough, shortness of breath, sore throat, muscle soreness and fatigue, and many others - using a system to collect a large number of different data and their connections, and by creating information (systems for collecting and analyzing large amounts of data - Big Data analysis).

The function will map this set to a knowledge set containing the following elements: CORONA-19, Flu, Cold, and more. The domain and function for this mapping in this known cases is already a known set of symptoms - it forms a model of description of known knowledge, of course the domain may also contain an unknown element that will only be discovered by some new mapping function from the original set or by mapping from the original set and the set of knowledge created by other mappings.

We will now show that for these sets, we consider that the function is analytic, i.e. that it is closed, associative, inverse, and neutral.

For each element of the information set (domain), the mapping will map into one element of the knowledge set (codomain), and only the mapping composition has a unique value in the codomain. If we take the cough symptom, it will be mapped with "Map_Flu" function to the codomain element of Flu. So the other elements have their own mappings. The Map Flu function will pick up other domain elements and map them to the codomain element "Flu". In this sense, this mapping has the property of being closed. Associativity is a characteristic that shows that it does not matter if we first take the element "cough" and "fatigue" and add "muscle pain" or do it in another order, the result is always the same we get a combination of symptoms that characterize the "Flu".

The inverse element exists in the sense that if the cough element is a symptom of the codomain element "Flu", then the reverse codomain element "Flu" also has its symptom in the elements of the cough domain. And there is a neutral element that does not describe any disease, which in our case is "throat pain".

We have derived that mapping from a set of information to a set of knowledge by Lie groups and proved that this function is defined as an analytic function.

We will now show that this mapping is an equivalence relation, that is, a classification relation. The conditions are reflexivity, symmetry and transitivity. Reflexivity is obvious because the element of the cough domain information set is in relation to itself. Also, symmetry is obvious when "Cough" is in relation to "Fatigue" and they make symptoms of "Flu" then the reverse is also true, so "Fatigue" is in relation to "Cough" and again they make symptoms of "Flu". Transitivity in this example can be shown as follows: "Cough" is in relation to "Fatigue" and furthermore "Muscle pain" and all of them are symptoms of "Flu".

Therefore, we can conclude that it is an equivalence relation, that is, a classification relation that classifies elements of a set of information and maps it into an element of a set of knowledge.

How many of these classes are there for us is the number of partition of the information set, or Bell's number. Of course, not every subset of a partisan set is usable, but this discernment requires future research and reflection.

5. EDUCATIONAL CONSEQUENCES

This abstract model describes the ways in which knowledge is created from existing knowledge, the continuous recognition of data that is transformed into information by some procedure (mapping function).

According to our model, this new information can be interpreted, both as knowledge by the function of equivalence as knowledge and interpreted with existing knowledge as a new domain of knowledge by which known teaching models (functions with all characteristics described in 2.) acquire new knowledge which in different condoms which are defined by the class of functions we used.

Accepting this model enables us to develop automated teaching that is adaptive to the learner and the learning goal, and enables a fully autonomous variable system of autonomous learning that is achieved through the preparation of knowledge sources that make up the "subject matter expert", "transfer method", "educator-prepared", and connecting with the entity of coded knowledge and information in the "Internet cloud" with methods (functions) to grasp what IT and Big Data professionals create.

6. CONCLUSION

The interpretation of knowledge given in this paper gives us an opportunity to develop different algorithms for acquiring knowledge which is independent from its type, just by defining the function for transforming the set of information. This gives us opportunity to interpret newly acquired knowledge as information for further interpretation and acquisition of different knowledge. In this paper we go further in abstracting the knowledge and knowledge representation, and show a model that can explain how new knowledge can be acquired from similar but not the same set of information.

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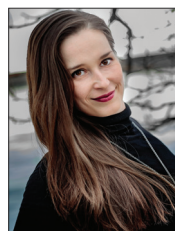
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Prvo zaposlenje bilo je u Partner banci d.d., Sektor riznice i trgovanja. Ubrzo prelazi na Tehničko veleučilište u Zagrebu, gdje i danas radi. Tijekom svog rada na TVZ-u, izvodi nastavu iz raznih matematičkih predmeta (Matematika 1, Matematika 2, Vjerojatnost i statistika, Primijenjena matematika). Za potrebe navedenih predmeta za neke izrađuje nastavne materijale.

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