Power consumption management and simulation of optimized operational conditions of ball mills using the Morrell Power model: A case study

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Abstract
The amount of comminution or fineness of minerals in a mill can be described by various parameters, the most important of which is $d_{80}$ (80% passing size). The purpose of this study is to investigate and simulate the optimal operating conditions of a ball mill in a copper processing plant. The actual operating conditions in the intended mill are performed with a 300 tph tonnage, a 267 second retention time, and a discharge $d_{80} = 53 \, \mu m$. Laboratory studies showed that the optimal economical and metallurgical recovery of copper in this plant is achieved in $65 \, \mu m \leq d_{80} \leq 75 \, \mu m$ with Flotation Recovery (R) = 90.16%, Economical Efficiency (EE) = 93.04% and Separation Efficiency (SE) = 88.64%. In this study, having the optimal $d_{80}$ for the concentration unit, the mill data, and utilizing Excel Software and the Morrell method, first the total power for the optimal set of $d_{80}$ was calculated, which is equal to 7790 to 8005 kW. Then, according to these power values, the corresponding retention times were calculated, which are equal to 236 and 247 seconds respectively. Finally, utilizing the retention time-tonnage relationship and taking into account the specific filling of the mill, the optimal corresponding tonnages to the obtained retention times were calculated, ranging from 324 to 340 tph. The results of these studies showed that by reducing the level of comminution from $d_{80} = 53 \, \mu m$ to $65 \, \mu m \leq d_{80} \leq 75 \, \mu m$, in addition to increasing flotation efficiency to $R = 90.16\%$, EE $= 93.04\%$ and SE $= 88.64\%$, about 4.21% to 7.09% energy savings and an 8.00% to 13.33% tonnage increase will occur.

Keywords:
ball mill; power draw; energy savings; Morrell Power model; simulation; degree of fineness

1. Introduction

Comminution is a process in which the dispersion state of solids changes with the action of mechanical forces (Salopek and Bedeković, 2000). It is one of the most important sections of mineral processing plants, whose task is to liberate valuable minerals from the gangue minerals (for metallic minerals) or to reach a certain fineness size in the final product (for industrial minerals). The grinding and particle size distribution is usually expressed based on a parameter called $d_{80}$ (the size in $\mu m$ which 80% of the material passes). In case this certain size is not reached, the recovery of mineral processing or the quality of the final product will be greatly reduced. On the other hand, over-grinding not only reduces efficiency due to fine particle generation but also dramatically increases comminution costs, such as energy costs, and the costs of grinding media and liners. Therefore, achieving a certain amount of fineness to obtain optimal conditions is one of the important priorities of any mineral processing plant and comminution system (Gharehgheshlagh et al., 2019).

The most significant controllable operating parameters affecting the quality of the grinding product are the retention time of the material inside the mill and the breakage power of the grinding media, which means the amount and size of balls in the case of ball mills. On the other hand, these two parameters are the most important factors affecting the mill power draw. Despite the existence of different methods, types, and technologies in the design of mill drive motors, all of them have common properties, such as motor type, output torque, and motor speed (Gupta and Yan, 2006).

Energy (kWh) and power (kW) are both factors that are used to show the efficiency of comminution equipment in terms of energy or power consumption, as well as their savings. Therefore, the calculation of power (or energy) consumption of ball mills is one of the most important factors in estimating the operating costs of pro-
cessing plants, determining the optimal or problematic operating conditions of the mill, particle size distribution of the mill discharge, mill flowrate, material retention time, breakage mechanism and other parameters. All parameters affecting the efficiency of mills also affect the mill power draw. The diameter and length of the mill, mill charge amount, the amount of grinding media and its density, the rotational speed of the mill, the type and design of the liners, etc. are among the factors that determine the amount of energy consumed in mills.

Energy consumption in different units of a mineral processing plant varies depending on the type and efficiency of equipment, physicochemical and crystalline characteristics of the feed (such as ore hardness, oxidized or sulphide type of ore, degree of liberation and metal/mineral grade), feed size, and final product quality. For example, in terms of equipment type, the hardness of the mineral, which is one of the most important factors in the amount of energy consumed by the milling section, has a minor impact on the amount of energy consumed by jaw crushers (Kujundžić et al., 2008). Therefore, allocating a fixed number to the amount of energy consumed by each of the sections of processing plants is illogical and wrong. Even though various studies have shown (or may show) different numbers for certain sections, the common ground of all the studies and surveys carried out on the mineral processing plants and mine-plant combinations is that they have shown that most of the energy is consumed in the grinding units. For example, the results of the studies in this area are as follows:

- About 60% of investment costs, 40 to 50% of operating costs, and more than 60% of energy costs of processing plants pertain to the comminution section (Wenzheng, 1991);
- Grinding systems (mills) consume about 50% of the total mining-plant energy (Fuerstenau, 1981; Jankovic et al., 2003; Kumar, 2011);
- In some plants, nearly 90% of operating costs are spent on comminution (Wills and Napier-Munn, 2011);
- In a typical mining-processing plant, about 53% of energy is consumed in the comminution section (CEEC, 2013);
- In mining, about 80 to 90 percent of the energy is consumed in grinding (milling) (Wang et al., 2013);
- In mining, about 35 to 50 percent of mining costs pertain to the comminution and the classification systems (crushing - mill - classifiers) (Curry et al., 2014);
- Grinding systems consume about 4% of the total global electrical energy (Fuerstenau, 1981; Jankovic et al., 2003; Kumar, 2011; Jeswiet and Szekeres, 2016).

Therefore, according to the mentioned cases, it can be stated that the milling section is the most sensitive and important part of a mineral processing plant from the optimization and management of operating and maintenance cost standpoints. Therefore, the optimal operation of these machines is one of the most important challenges of all processing plants in the world. Various criteria and parameters can be defined to determine the optimal or poor performance of a comminution system. One of the most important of these criteria is specific energy consumption (SEC) in the comminution section. Comparing similar plants and processes; any plant or process with low specific energy consumption (SEC) has a high energy efficiency. Also, by comparing SEC values, one can understand the energy efficiency potential of a plant (Ali et al., 2011).

Simulation is a set of operations by which the necessary changes can be made and possible results can be seen without interfering with the working process of a real system, in a virtual space, such as relevant software or spreadsheets (Gharehgheshlagh et al., 2019), i.e. a simulation is an imitation of a real process or system in a virtual environment (Banks et al., 2001). Using this method, it is possible to view the response or responses of the system without the need to make a real change in one or more parameters of the system under study. Using these observed responses, a final decision can be made about whether or not to change the intended parameter (Gray and Rumpe, 2016).

To perform the simulation, a series of mathematical equations are required, which describe the behaviour of the system components and within the system, which are called mathematical models (Sokolowski and Banks, 2009). These models demonstrate the basic features, behaviour, and functions of the system (Gray and Rumpe, 2016).

The first basic mathematical model in the mineral processing industry was proposed by Rosin and Rammler to model the particle size distribution of materials (Rosin and Rammler, 1933). Later, by studying different components of mineral processing plants, researchers developed mathematical models related to each component or hybrid models of a section of the whole plant. Among the most important models developed, we can refer to the models related to different types of ball mills (Bond, 1952; Lynch, 1977), Autogenous (AG) and Semi autogenous (SAG) mills (Leung, 1987; Leung et al., 1987), etc. Furthermore, models for calculating the mill power (Rose and Sullivan, 1957; Bond, 1961; Austin and Klimpel, 1984; Morrell, 1994; Morrell, 1996a, b; Napier-Munn et al., 1996) or models for calculating the breakage distribution function and the selection function (Epstein, 1947 and 1948; Broadbent and Calcott, 1956; Herbst and Fuerstenau, 1980; Narayanan and Whiten, 1983; Austin et al., 1984; Narayanan, 1985) are among the basic and key mathematical models for calculating unknown parameters used in mineral processing simulation software.

By combining mathematical models of different equipment in a processing plant, in the form of a commercial software package, such as JKSimMet, USIM-
In the operational phase, for optimizing the grinding circuits of mineral processing plants, the design parameters, such as mill length and diameter are immutable and only the operational parameters can be changed. Also, it is very difficult or impossible to change some operating parameters, such as input feed properties. Therefore, the number of changeable operating parameters for optimization will be limited. The most important of these parameters, which can also be called controllable operating parameters, are mill flowrate, solid percentage, filling amount, etc. To evaluate the variability of mill power relative to controllable operating parameters, either the power should be measured by changing these parameters or an estimate of possible outcomes should be predicted utilizing simulation. Changing operating parameters is risky and is not usually done in processing plants. Therefore, to get optimal conditions for milling circuits of a processing plant, the best, cheapest, and most risk-free method is the utilization of simulation (Gharehgheshlagh et al., 2019).

In this study, simulations have been utilized to determine and evaluate the effect of controllable operating parameters on the optimization of a secondary ball mill (SBM) power in the comminution system of a copper processing plant.

2. Calculation of mill power using the Morrell method

In mills utilized for the grinding of minerals, the power applied to the tumbling mills is transferred to the grinding media via rotation and then to the ore, breaking the rock through various breakage mechanisms. During the comminution process, power is one of the most important operational parameters in the mill, and many efforts have been made to estimate this parameter more accurately utilizing laboratory and operational data. The various models and µµs proposed to calculate the power consumption of mills are classified into the following two general groups:

- Calculation of the mill power utilizing mechanical methods (experimental equations);
- Calculation of the mill power utilizing theoretical equations.

Mechanically, the power required in the motor of a mill or the mill power is the force that brings a mill from a static state to the intended rotational speed, i.e. this power in addition to creating the initial torque to move the mill, must be able to rotate the mill and its internal load at the required speeds (Gupta and Yan, 2006). Mechanical methods are developed based on experience from a series of industrial data, and therefore the application of these experimental equations is always accompanied by a certain amount of error. Among the mechanical methods, the Bond method (Bond, 1961) is the most reliable method for designing and optimizing ball and rod mills in which it utilizes a parameter called the Bond work index (Wi) to calculate the energy consumption of mills. The work index is one of the characteristics of an ore and the higher its numerical value, the greater its resistance to comminution, and it therefore requires more energy. For example, studies by Abdelhaffez (2020) on various samples of the Mahd Ad Dahab gold mine showed that the geological, mechanical, and mineralogical properties of these samples have a large effect on the work index. The change in this index is directly reflected in the energy consumption of mills.

The theoretical method in estimating the power draw of mills is based on the fact that in a tumbling mill such as a ball mill, repetitive impact, compression, abrasion and attrition forces work together in a complex situation. Due to the use of these complex forces, energy transfer occurs, which leads to an increase in the concentration of stresses inside the particles. When the stress-generating forces become larger than the bond energy, the particles are broken down into two or more small particles. Particle rupture within a tumbling mill is therefore a complex function of energy transfer from grinding media (balls) and the shells of a mill to particles. The mill rotation and the fall of grinding media (balls) from a certain height cause the transfer of both kinetic and potential energy to the load inside the mill. As a result, heat and sound are generated and the bond energy (chemical bonding) between the particles is broken (Gupta and Yan, 2006). Austin et al. (Austin et al., 1984), Morrell (1994; Morrell, 1996a, b) and Napier-Munn et al. (Napier-Munn et al., 1996) have theoretically evaluated the total energy transferred to the load inside the mill during mill rotation and made it a function of the power required to rotate an empty mill and the rotation of a fully-loaded mill. The experimental model of Napier-Munn (Napier-Munn et al., 1996), which is, in fact, a modified version of Morrell’s method (Morrell, 1996b), is the most important and valid theoretical method for calculating mill power.

The developed theoretical methods due to consideration of all of the design and operational conditions, very high manoeuvrability, considering all mechanisms that cause breakage, total potential and kinetic energies of particles, etc. have a relative advantage over mechanical methods (experimental) and therefore are widely used in new mill design methods that are model-based.

According to Morrell’s method (Morrell, 1996b), the total power input to a mill motor ($P_{total}$) is equal to the
total power input to the mill motor to rotate an empty mill \((P_{(No-Load)})\), the power needed to move the load inside the mill \((P_{net})\) and the power to overcome losses. The power required to rotate the load inside the mill \((P_{net})\), also known as the net power, is equal to the sum of the power required to rotate the load inside the cylindrical section \((P_{CYL})\) and the power required in the two conical sections at the beginning and end of the mill \((P_{CON})\) (Morrel, 1996 b; Napier-Munn et al., 1996; Gupta and Yan, 2016). On the other hand, due to the partial loss of the input power to the mill motor in the path of the mill motor to the load inside, a power calibration factor \((K)\) is required to calculate the total rotating load power \((P_{GC})\) (Morrell, 1996 b; Napier­Munn et al., 1996). Therefore, the total power input to a mill motor \((P_{Total})\) is equal to Equation 1.

\[
P_{Total} = P_{(No-Load)} + P_{GC} = P_{(No-Load)} + KP_{net} = P_{(No-Load)} + K(P_{CYL} + P_{CON}) \tag{1}
\]

It should be noted that \(K\) is a composite factor that includes a set of heat loss due to internal friction, various mechanisms of breakage and rotation of grinding media (balls), and defects due to assumptions and measurements of the shape and motion of the load. The results of studies by researchers at the JKMRC (Julius Kruttschnitt Mineral Research Centre) on 63 different types of mills showed that the value of \(K\) is 1.26 (Napier-Munn et al., 1996).

To calculate each of these components, it is necessary to define the design and operational parameters in a ball mill. Figures 1 to 3 show a schematic diagram of a tumbling mill and the operating parameters of the load inside the mill, and Table 1 shows the parameters required to calculate the mill power draw.

### 2.1. No-load mill power

Morrell (Morrell, 1996 b) have presented the following equation for the estimation of no-load mill power:

\[
P_{(No-Load)} = 1.68[D^{2.5} \phi(0.667L_{CONE} + L_{CYL})]^{0.82} \tag{2}
\]

### 2.2. Power required for the cylindrical section of grate-type ball mills

Figure 1 shows a simplified schematic diagram of the possible slurry position (solid + water), bulk solids and grinding media (balls) in a static wet mill. Figures 2 and 3 also show a cross-section of a wet ball mill in which the positions of the slurry, toe, and shoulder for a given load are presented. As a mill rotates, the material in the toe and shoulder shifts in the direction of the shell rotation. This overall displacement of the material is given by the travelled distance of the toe and shoulder position of the load. This distance can also be estimated using the changes in the toe (\(\alpha_t\)) and shoulder (\(\alpha_s\)) angles of the load. These angles are measured relative to the horizon line and the 3 o’clock position is considered as a reference point of the mentioned measuring system (see Figures 2 and 3). The increase in angle will also be in the direction of mill rotation, which in this study is assumed to be counter clockwise (see Figures 2 and 3). With these assumptions, the power required for the cylindrical section of grate-type mills is given by (Morrell, 1996 b; Napier-Munn et al., 1996):
Table 1: The required parameters for power draw calculations of ball mills (Morrell, 1996 b; Napier-Munn et al., 1996)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>unit</th>
<th>symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>length of cylindrical section: belly length (inside liners)</td>
<td>m</td>
<td>L\textsubscript{CYL}</td>
</tr>
<tr>
<td>length of conical section (sum of two cone-end)</td>
<td>m</td>
<td>L\textsubscript{CONE}</td>
</tr>
<tr>
<td>mean cone length (length of each end)</td>
<td>m</td>
<td>L\textsubscript{CONE}</td>
</tr>
<tr>
<td>mill diameter (inside liners)</td>
<td>m</td>
<td>D</td>
</tr>
<tr>
<td>mill radius (inside liners)</td>
<td>m</td>
<td>R</td>
</tr>
<tr>
<td>trunnion inner radius</td>
<td>m</td>
<td>R\textsubscript{T}</td>
</tr>
<tr>
<td>cylindrical section power</td>
<td>kW</td>
<td>P\textsubscript{CYL}</td>
</tr>
<tr>
<td>conical section power</td>
<td>kW</td>
<td>P\textsubscript{CONE}</td>
</tr>
<tr>
<td>no-load power</td>
<td>kW</td>
<td>P\textsubscript{(No-Load)}</td>
</tr>
<tr>
<td>net power or charge motion power</td>
<td>kW</td>
<td>P\textsubscript{net}</td>
</tr>
<tr>
<td>power calibration factor</td>
<td></td>
<td>K</td>
</tr>
<tr>
<td>total power for charge rotation</td>
<td>kW</td>
<td>P\textsubscript{GC}</td>
</tr>
<tr>
<td>total power</td>
<td>kW</td>
<td>P\textsubscript{Total}</td>
</tr>
<tr>
<td>ore (rock) density</td>
<td>t / m\textsuperscript{3}</td>
<td>\rho\textsubscript{R}</td>
</tr>
<tr>
<td>water density</td>
<td>t / m\textsuperscript{3}</td>
<td>\rho\textsubscript{w}</td>
</tr>
<tr>
<td>density of ball (grinding media)</td>
<td>t / m\textsuperscript{3}</td>
<td>\rho\textsubscript{b}</td>
</tr>
<tr>
<td>density of slurry (pulp)</td>
<td>t / m\textsuperscript{3}</td>
<td>\rho\textsubscript{P}</td>
</tr>
<tr>
<td>density of the total charge (ore + water + ball)</td>
<td>t / m\textsuperscript{3}</td>
<td>\rho\textsubscript{C}</td>
</tr>
<tr>
<td>volumetric solid percent of mill load</td>
<td>%</td>
<td>S</td>
</tr>
<tr>
<td>weighted solid percent of mill load</td>
<td>%</td>
<td>X\textsubscript{s}</td>
</tr>
<tr>
<td>ball fractional mill filling</td>
<td>fraction</td>
<td>J\textsubscript{B}</td>
</tr>
<tr>
<td>charge (slurry + ball) fractional mill filling (for overflow mills J\textsubscript{c} = J\textsubscript{S})</td>
<td>fraction</td>
<td>J\textsubscript{c}</td>
</tr>
<tr>
<td>theoretical mill critical speed</td>
<td>rps</td>
<td>C\textsubscript{S}</td>
</tr>
<tr>
<td>mill rotational speed</td>
<td>rps</td>
<td>\omega</td>
</tr>
<tr>
<td>mean rotational speed</td>
<td>rps</td>
<td>\bar{\omega}</td>
</tr>
<tr>
<td>fraction of theoretical critical speed</td>
<td>fraction</td>
<td>\phi</td>
</tr>
<tr>
<td>ratio of experimental critical speed to the theoretical critical speed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>angular displacement of charge toe</td>
<td>radians</td>
<td>\alpha\textsubscript{T}</td>
</tr>
<tr>
<td>angular displacement of charge shoulder</td>
<td>radians</td>
<td>\alpha\textsubscript{S}</td>
</tr>
<tr>
<td>angular displacement of surface of slurry pool at toe or slurry toe angle (for overflow mills \alpha\textsubscript{TS} = 3.395 radians and for grate mills \alpha\textsubscript{TS} = \alpha\textsubscript{C})</td>
<td>radians</td>
<td>\alpha\textsubscript{TS}</td>
</tr>
<tr>
<td>porosity of the charge (porosity between crushed rock and ball load)</td>
<td>fraction</td>
<td>\phi</td>
</tr>
<tr>
<td>fraction of ball voidage at rest that is occupied by the slurry</td>
<td>fraction</td>
<td>\U</td>
</tr>
<tr>
<td>function of the volumetric filling of the mill</td>
<td></td>
<td>\zeta</td>
</tr>
<tr>
<td>mean time taken for the active part to travel from the toe to the shoulder</td>
<td>sec</td>
<td>t\textsubscript{A}</td>
</tr>
<tr>
<td>mean time for free fall from the shoulder to the toe</td>
<td>sec</td>
<td>t\textsubscript{g}</td>
</tr>
<tr>
<td>mean radial position of the active part of the charge</td>
<td>m</td>
<td>R\textsubscript{c}</td>
</tr>
<tr>
<td>radial location of inner surface of charge</td>
<td>m</td>
<td>R\textsubscript{i}</td>
</tr>
<tr>
<td>volume fraction of the active part of the charge to the total charge</td>
<td>fraction</td>
<td>\gamma</td>
</tr>
</tbody>
</table>

\[ P_{\text{CYL-GR}} = \left( \frac{\pi g L \rho \omega R}{3(1 - \zeta R)} \right) \left[ 2R^3 - 3\zeta R^2 R_1 + R_1^3 (3\zeta - 2) \right] (\sin \sin \alpha_s - \sin \sin \alpha_T) + \left[ L \rho_c \left( \frac{\omega \pi R}{R - \zeta R_1} \right)^{\frac{3}{4}} \left[ (R - \zeta R_1)^{\frac{3}{4}} - R_1^{\frac{3}{4}} (\zeta - 1)^{\frac{3}{4}} \right] \right] \] (3)

\[ \rho_c = \frac{J_c \rho_b (1 - \phi + \phi U/S) + J_s (\rho_b - \rho_s) (1 - \phi) + J_c \phi U (1 - S)}{J_c} \] (4)

To calculate the mill power, it is necessary to calculate the total density of the load (solid + water + grinding media). To calculate the total density of the load, it is necessary to know the amount of porosity of the load, the pulp density and the amount of space of balls that are filled by slurry. In general, the total load density is equal to:
The amount of porosity of mill charge ($\phi$) is usually considered to be between 0.3 and 0.4. On the other hand, the fraction of space of balls filled by slurry ($U$) is usually considered to be 1 (Napier-Munn et al., 1996).

The parameter $\zeta$ is a function of the volumetric filling of the mill and is defined as follows:

$$\log\zeta = 0.4532\log(1 - J_c)$$

On the other hand, according to Figures 2 and 3 and performing the necessary analyses, the necessary equations for calculating the toe and shoulder angles will be obtained as follows:

$$\alpha_T = 2.5307(1.2796 - J_c)(1 - e^{-0.42(\alpha_T - \alpha_S)}) + \frac{\pi}{2}$$

$$\alpha_S = \frac{\pi}{2} - \left(\left[\alpha_T - \frac{\pi}{2}\right] \left(0.3386 + 0.1041\phi\right) + \left(1.54 - (2.5673\phi)\right)J_c\right)$$

If $\alpha_{TS}$ is the angle or angular displacement of slurry pool surface at the toe, then in grate type ball mills $\alpha_{TS}$ will be equal to the $\alpha_T$ ($\alpha_{TS} = \alpha_T$) and for overflow-type ball mills $\alpha_{TS}$ will be equal to the 3.395 radians ($\alpha_{TS} = 3.395$ rad) (Napier-Munn et al., 1996).

The value of $R_i$ can be estimated by considering the active mass of the charge, the time required to move the load from the toe to the shoulder, and the time required to move the load from the shoulder to the moment of falling to the toe (see Figure 2).

$$R_i = R \left(1 - \left(\frac{2\pi J_c}{\pi + \alpha_S - \alpha_T}\right)^{0.5}\right)$$

Where $\gamma$ represents the volume fraction of the active charge relative to the total load.

$$\gamma = \frac{T_r}{T_r + t_s}$$

$$T_r = \frac{2\pi + \alpha_S - \alpha_T}{2\pi \omega} \left(\frac{2\pi + \alpha_S - \alpha_T}{\pi \omega}\right)$$

$$T_s = \frac{2\pi \sin \alpha_S - \sin \alpha_T}{g} \left(\frac{2\pi \sin \alpha_S - \sin \alpha_T}{g}\right)^{0.5}$$

$\bar{R}$ is the average radial position of the active part of the charge, which depends on the volumetric filling of the mill and is calculated as Equation 14.

By placing Equations 4 to 14 in Equation 3, the power of the cylindrical section of the grate-type mills can be calculated.

2.3. Power required for the cylindrical section of the overflow-type ball mills

In the overflow-type mills, slurries reduce the frictional forces between the solids and the inner shell of the mill. Therefore, the power required for an overflow-type mill will be less than that of grate-type ones. On the other hand, due to the collision of falling particles with the lower slurry and their transfer downwards, the slurry also provides a floating action. Taking these factors into account, Morrell provided the power required for the cylindrical section of the overflow-type ball mills as follows:

$$P_{\text{cyl-over}} = \left(\frac{\pi g \rho_w \omega R^3}{3(R - \zeta R_s)}\right)\left[2R^3 - 3\bar{R}^2 R_s + R_s^3(3\zeta - 2)\right] + \left[\rho_w \left(\sin \alpha_S - \sin \alpha_T\right) + \rho_r \left(\sin \alpha_T - \sin \alpha_{TS}\right)\right] + \left(\left[\left(\frac{\omega R}{R - \zeta R_s}\right)^4 - \left(\frac{\omega R}{R - \zeta R_s}\right)^4(3\zeta - 2)\right]\right)$$

2.4. Power required for conical sections of the mill

Large-sized mills are designed to have a considerable volume in their two conical sections. These two end portions hold a significant amount of slurry and solids and are therefore effective in the total mill power. Morrell (1996b) presented the needed equation to estimate the power required for the two conical end sections like the cylindrical shape, the final form of which is as follows:

$$P_{\text{cone}} = \left(\frac{\pi g \omega \rho_w R_s^3}{3(R - R_s)}\right)\left[R^4 - 4RR_s^3 + 3R_s^4\right] + \left[\rho_w \left(\sin \alpha_S - \sin \alpha_T\right) + \rho_r \left(\sin \alpha_T - \sin \alpha_{TS}\right)\right] + \left(\left[\left(\frac{2\pi \omega}{5(R - R_s)}\right)^4 - \left(\frac{2\pi \omega}{5(R - R_s)}\right)^4\right]\right)$$

Finally, by calculating all the components of Equation 1, the total power required for the mill is obtained.

3. Case study: copper processing plant

In this study, simulations have been utilized to determine and evaluate the effect of controllable operating parameters on the optimization of a secondary ball mill (SBM) power in the comminution system of a copper processing plant, and subsequently, the optimal operating conditions are predicted. This mill operates in real
operating conditions with a capacity of 300 tons per hour (tph) and a discharge $d_{80}$ of 53 μm. The general schematic of the comminution system of this processing plant is given in Figure 4 and the specifications of the SBM and its design and operational parameters are given in Table 2.

The aforementioned mill is a regrinding mill with a ball filling ratio of about 33% (33% of mill volume which is equal to $0.33 \times 137.10 = 45.24$ m$^3$), which can be increased by as much as 45%. It is assumed that the porosity of the ball charge in this mill is 0.4 and this means 40% of all the ball charge’s volume, i.e. $13.2 \% (0.4 \times 0.33 = 0.132)$ of the total mill volume, which is equal to 18.10 m$^3$, can be filled with the feed slurry. In addition to, by considering the $X_s$ (solid weight% of mill feed) and the specific gravity of the ore, water and balls, the weight of solid ore in the mill can be calculated for each amount of filling ratio (i.e. ball filling). The maximum ball size in this mill is 40 mm and to achieve a more effective and efficient grinding, ball size distribution is used instead of a single ball size.

In this study, the effect of degree of fineness ($d_{80}$), retention time ($h$) and solid flow rate ($T$) on the power draw of the ball mill ($P$) are investigated. The data in Table 2, the formulas mentioned above and the Excel spreadsheet are utilized to simulate the data.

3.1. Laboratory scale flotation tests and evaluation of the effect of feed size on flotation efficiency

The particle size of the feed entering the flotation system is one of the key parameters of mineral processing plants because on the one hand, the efficiency of the concentration system is completely dependent on it, and on the other hand, the cost of mineral comminution, which is the largest cost parameter of any processing plant, is directly dependent on the amount of comminution, i.e. fineness or coarseness of the mill discharge (flotation feed). Therefore, it can be stated that particle size plays a critical role in froth flotation, especially when the goal is to separate more than two minerals by flotation (Drzymala, 1999; Kowalczyk et al., 2011). Studying the effect of particle size of input feed on flotation efficiency without considering other parameters can produce misleading results. The size of the feed alone cannot directly determine the flotation efficiency, but rather

![Figure 4: Schematic of the grinding system of the examined copper processing plant](image)

### Table 2: Ball mill specifications

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Symbol</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>mill diameter (inside liners)</td>
<td>m</td>
<td>D</td>
<td>4.88</td>
</tr>
<tr>
<td>trunnion inner radius</td>
<td>m</td>
<td>R_T</td>
<td>0.60</td>
</tr>
<tr>
<td>length of cylindrical section:</td>
<td>m</td>
<td>L_CYL</td>
<td>7.33</td>
</tr>
<tr>
<td>belly length (inside liners)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>length of conical section:</td>
<td>m</td>
<td>L_CONE</td>
<td>0</td>
</tr>
<tr>
<td>(sum of two cone-end):</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>there are no conical sections at</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>both ends of this mill.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mill rotational speed</td>
<td>rps</td>
<td>$\omega$</td>
<td>0.25</td>
</tr>
<tr>
<td>fraction of theoretical</td>
<td>%</td>
<td>$\varphi$</td>
<td>78.34</td>
</tr>
<tr>
<td>critical speed</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>solid flowrate</td>
<td>t/h</td>
<td>$T$</td>
<td>300</td>
</tr>
<tr>
<td>density of ball (grinding media)</td>
<td>t / m$^3$</td>
<td>$\rho_b$</td>
<td>7.8</td>
</tr>
<tr>
<td>ore (rock) density</td>
<td>t / m$^3$</td>
<td>$\rho_R$</td>
<td>2.68</td>
</tr>
<tr>
<td>water density</td>
<td>t / m$^3$</td>
<td>$\rho_w$</td>
<td>1</td>
</tr>
<tr>
<td>weighted solid percent of mill load</td>
<td>%</td>
<td>$X_s$</td>
<td>68</td>
</tr>
<tr>
<td>porosity of the charge</td>
<td>fraction</td>
<td>$\phi$</td>
<td>0.40</td>
</tr>
<tr>
<td>fraction of ball voidage at rest</td>
<td>fraction</td>
<td>$U$</td>
<td>1</td>
</tr>
<tr>
<td>that is occupied by the slurry</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mill discharge 80% passing size</td>
<td>µm</td>
<td>$d_{80}$</td>
<td>53</td>
</tr>
</tbody>
</table>
the chemical composition of the input ore (valuable and gangue minerals), hydrophobic properties of minerals, bubble size, and some other operational parameters and natural characteristics of the ore create a direct or inverse relationship between feed size and flotation efficiency. Therefore, a case study approach should be considered for this issue. In general, various studies have shown that intermediate size has better flotation performance than coarse and fine size fractions (Zhang et al., 2017; Farrokhpay and Fornasiero, 2017), i.e. it is more difficult to float very coarse and very fine minerals, while intermediate-sized minerals float relatively easy (Yin et al., 2011; Ng et al., 2015; Xu et al., 2016). This is because of particle-bubble collision efficiency and collector surface coverage in intermediate-sized particles or optimum flotation particle sizes are higher than in other particles (Jameson, 2013).

In this study, the chemicals used in the flotation section of the studied plant were utilized to establish comparative conditions and also to create a suitable correlation between the results of laboratory-scale studies and industrial-scale sampling. To investigate the effect of narrow size fractions, the feed prepared for flotation tests was classified into different size fractions of -150+106, -106+75, -75+53, -53+38 and -38 µm utilizing a standard sieve series. Despite the differences in the range of variation of these size fractions, the reason for using these size ranges is the use of a standard screening system, which makes the preparation of feed using these screens easier and more convenient. At the same time, it should be noted that the average feed grade of -150 µm is 1.07% copper. Nevertheless, the results of feed grade analysis of different size fractions show a very small amount of difference in the amount of copper content. Therefore, in this study, to accurately calculate the recovery of each sample, the corresponding feed grade was used separately, although, in the flotation plants the average grade of feed \(F_{100}\) (the size in µm which 100% of the material passes) is used.

After selecting the desired sizes, the preparation conditions were provided as per the industrial conditions. After providing the operating conditions in the laboratory, flotation tests were performed on each size fraction. Finally, the concentrate and tailing of each test were collected separately, dried and the weight of dry samples was measured and then, their elemental analysis was performed (see Table 3).

Having valuable mineral characteristics (CuFeS\(_2\) and maximum grade of mineral \((m)\)), feed, tailing and concentrate grades, the weight of concentrate \((C)\), as well as up-to-date economic parameters (copper price \((P = 9700\ S/\text{t})\), smelting \((S = 50\ S/\text{t})\) and transfer \((T = 35\ S/\text{t})\) costs, the values of flotation recovery \((R)\), economic efficiency \((EE)\), and separation efficiency \((SE)\) for all samples were calculated by using Equations 17-19. The summarized results of these tests are presented in Figure 5.

### Table 3: Feed, tailing and concentrate grade results of samples of flotation tests

<table>
<thead>
<tr>
<th>Size fraction, µm</th>
<th>Cu grade of stream, %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Feed (f)</td>
</tr>
<tr>
<td>-150 + 106</td>
<td>1.07</td>
</tr>
<tr>
<td>-106 + 75</td>
<td>1.06</td>
</tr>
<tr>
<td>-75 + 53</td>
<td>1.08</td>
</tr>
<tr>
<td>-53 + 38</td>
<td>1.06</td>
</tr>
<tr>
<td>-38</td>
<td>1.07</td>
</tr>
</tbody>
</table>

\[
R = \frac{C}{F} \times \frac{c}{f} \times 100 = \left(\frac{f-t}{c-t}\right) \times \frac{c}{f} \times 100 \quad (17)
\]

\[
EE = \frac{NSR_{\text{ideal}}}{NSR_{\text{actual}}} = \frac{(P \times \frac{C}{100} \times C_{\text{ideal}}) - (S + T)_{\text{ideal}}}{(P \times \frac{m}{100} \times C_{\text{ideal}}) - (S + T)_{\text{ideal}}} \times 100 \quad (18)
\]

\[
SE = \left(\frac{f-t}{c-t}\right) \times \frac{m}{f} \times 100 \quad (19)
\]

Where:

- \(C\) and \(F\) – the feed and concentrate weights (t), respectively;
- \(f, c \) and \( t\) – the feed, concentrate and tailing grades (%), respectively;
- \(m\) – maximum grade of valuable metal in the mineral (%);
- \(R\) – flotation recovery (%);
- \(EE\) – economic efficiency (%);
- \(SE\) – separation efficiency (%);
- \(NSR\) – Net Smelter Return ($);
- \(P\) – copper price ($/t);
- \(S\) – smelting cost ($/t);
- \(T\) – transfer cost ($/t).

The results of flotation tests showed that the highest values of \(R\), \(EE\), and \(SE\) parameters belong to the size fraction of -75 + 53 µm, whose values are equal to
88.47%, 90.46%, and 87.00%, respectively (see Figure 5). Since the purpose of this study is to investigate and obtain the optimal size fraction to achieve the highest efficiency, the size range of -75 + 53 µm is divided into two fine ranges of approximately -75 + 65 and -65 + 53 µm and the same laboratory scale tests were performed on these two size fractions. The results showed that the size fraction of -75 + 65 µm with R = 90.16%, EE = 93.04% and SE = 88.64% provides the best economical and metallurgical conditions for the laboratory scale tests (see Figure 6).

![Figure 6: Laboratory flotation tests results for -75 + 53 µm](image)

The results obtained from the grade analysis of different size fractions (see Table 3) show that the tailings grade of -150 +106 µm fraction is very high and this indicates high locking of the valuable mineral and its lack of the appropriate degree of freedom. On the other hand, the results show that the lowest tailing grade belongs to the -75 + 53 µm fraction. Therefore, due to the lack of a suitable degree of freedom, decomposition of other size fractions into narrow sizes is not necessary to perform additional flotation tests.

3.2. Investigation and simulation of operational parameters of mill power

Material retention time in the mill (h) is simply equal to the ratio of slurry volume inside the mill to the mill volumetric flow (h = V_s / Q_v). By altering any of these values, the retention time of the material inside the mill can be changed. Since retention time and comminution rate are directly related to each other, in the industry if a finer-grained product is needed, the retention time will be increased and vice versa. There are several ways to change the retention time of materials, one of the most important of which is to change the speed of the conveyor belt of the mill feeder (for primary mills) or to change the speed of the mill feeder pumps (secondary or regrind mills). For example, as the speed of the conveyor of the mill feeder increases or the speed of the pumps increases, the amount of material entering the mill increases per unit time, i.e. flow rate to the mill and the speed of product discharge from it increases. Therefore, the retention time of materials and subsequently the comminution time of these materials is reduced and a larger grained product is discharged.

To determine the effect of retention time on d_80 of the studied mill, different feed flowrates to the mill have been utilized. For this purpose, samples are obtained from the feed and discharge of the mill by changing the pump speed. The mill feed sample is provided by sampling from the underflow of hydrocyclone. Samples are also taken from the hydrocyclone overflow and discharge of both mills to mass balance and obtain accurate data. Having the wet and dry weight of these materials and the required calculations, the feed flowrate of the desired mill, the solid weight percentage as well as the particle size distribution of the mill product are determined. Finally, using the mill product size distribution, the value of mill discharge d_80 is determined. After taking a series of samples, the feed pump speed is changed and the sampling process is repeated. After each change, the mill works with the same flow for about an hour to reach steady-state conditions. Simultaneously with sampling, mill power is also recorded through the control room. This sampling operation is performed on four different discharges. On the other hand, the theoretical power of the mill is calculated via the Morrell relation (Morrell, 1996b) and the volume of slurry materials within the mill via the corresponding mathematical relations. According to Equation 2, the no-load power of this mill is 181.553 kW and the values of calculated total power and operating power (control room) are presented in Figure 7.

![Figure 7: Comparison of the Morrell theoretical model and industrial results for mill total power](image)

By comparing operational and theoretical power (see Figure 7), it is observed that the total operational power values are greater than the theoretical computational power values obtained via the Morrell method. This difference in power values can be the consequence of many reasons, and one of the most important is the calibration coefficient considered for the calculation of theoretical
power. The results show that the power calibration coefficient of this mill is equal to 1.276, which is about 1.7% more than the Morrell coefficient, and this also indicates a higher energy loss.

By correcting the studied mill calibration coefficient and using the Morrell model, the mathematical relations between $P_{\text{total}}$ and $d_{80}$ values are created and presented in Figures 8 and 9.

Given the $d_{80}$’s and $P_{\text{total}}$’s of the four samplings performed, a mathematical relationship between the experimental data is formed as follows (see Figure 8):

$$P_{\text{total}} = 17693d_{80}^{-0.19}$$ (20)

Now, for all operating conditions in which the goal is to reach a certain fineness size ($d_{80}$), it is possible to obtain the total mill power through the simulated model (Equation 20). On the other hand, with the availability of the total simulated power, the corresponding flowrates, filling ratios, or the retention times can be obtained through simulation. For the studied mill, Figure 9 and Equation 21 indicates the relationship between retention time and power consumption.

$$h = \left(3.84 \times 10^{-5}\right)\frac{P_{\text{total}}^{1.7444}}{P_{\text{total}}}$$ (21)

By performing this simulation process, for each of the desired values of retention time ($h$), solid flow rate ($T$), the values of degree of fineness ($d_{80}$) and the ball mill power ($P_{\text{total}}$), the unknown parameter’s value can be obtained. In other words, since this study aims to simulate and obtain the retention time ($h$), solid flow rate ($T$) and ball mill power ($P_{\text{total}}$) for the optimal degree of fineness ($d_{80}$), first for the optimal $d_{80}$, the total power of mill ($P_{\text{total}}$) is calculated via Equation 20. Then, for each simulated ($P_{\text{total}}$) the retention time or the mill discharge flowrate (which can be converted to each other via the equation $h = \frac{V}{Q}$) is calculated via Equation 21. Namely, the optimal values of $h$ and $P_{\text{total}}$ can be calculated for the intended $d_{80}$, and finally, the necessary decision can be made to change or not to change the intended parameters.

Since the ideal conditions for optimal recovery of copper in the processing plant are provided in a $d_{80}$ range of between 65 and 75 μm, the total optimal power for these conditions is 7790 to 8005 kW. Also, depending on the total optimum power obtained, the optimal retention time will be in the 236 to 247 second range. The optimum tonnage for this retention time will be 324 to 340 tons of dry solids per hour.

The results of this study show that by reducing the comminution rate from $d_{80} = 53 \mu m$ to $65 \mu m \leq d_{80} \leq 75 \mu m$, in addition to increasing flotation efficiency to $R = 90.16\%$, $EE = 93.04\%$ and $SE = 88.64\%$, about 4.21% to 7.09% energy savings and an 8.00% to 13.33% tonnage increase will occur. These results indicate that a simulation’s role in developing decision-making criteria is imperative.

4. Conclusions

Actual operating conditions take place in the ball mill of the studied plant with a tonnage of 300 tons per hour and discharge $d_{80}$ equal to 53 microns. The corresponding retention time to the mentioned tonnage and degree of fineness is about 267 seconds.

Laboratory studies showed that the ideal conditions for optimal economical and metallurgical recovery of copper in this plant are at $d_{80}$ of between 65 to 75 μm with $R = 90.16\%$, $EE = 93.04\%$ and $SE = 88.64\%$. Therefore, over-grinding ($d_{80}$ less than 65 μm) not only reduces the efficiency of copper recovery but also reduces discharge and increases comminution costs, such as energy costs, and wear of grinding media and liners.

In this study, with the information of the intended mill, the optimal degree of fineness for the concentration and separation unit, and utilizing the Excel Software and Morrell method, first, the total power for the optimal $d_{80}$ was calculated, which is equal to 7790 to 8005 kW. Subsequently, the range of the corresponding retention time was obtained, which is equivalent to 236 to 247 seconds. Finally, using the retention time-tonnage relationship...
and taking into account the specific filling ratio of the mill, the optimal corresponding tonnages were calculated, which amounted to between 324 and 340 tons of ore per hour.

The results of this study show that by reducing the comminution rate from $d_{80}$ of 53 μm to 65 μm ≤ $d_{80}$ ≤ 75 μm in addition to increasing flotation efficiencies to $R = 90.16\%$, EE = 93.04% and SE = 88.64%, about 4.21% to 7.09% energy savings and 8.00% to 13.33% tonnage increase will occur. These results indicate that a simulation is imperative in developing decision-making criteria. The results also indicate that the use of simulation in obtaining the optimal conditions for a ball mill or the whole mineral processing process is very useful and logical, and through the use of it, possible results can be predicted without making real and risky changes. Although these results have some errors, the scope of these errors is small and provides a near-realistic understanding of the new conditions (changes), and provides a very strong decision-making criterion for system management.

5. References


SAŽETAK

Upravljanje potrošnjom energije i simulacija optimiziranih radnih uvjeta kugličnih mlinova korištenjem Morrellova modela snage: studija slučaja

Stupanj sitnjenja ili finoča mljevenih minerala u mlinu može se opisati raznim parametrima, od kojih je najvažniji \( d_{80} \) (80 % prolazi kroz otvor sita). Svrha je ovoga rada istražiti i simulirati optimalne uvjete rada kugličnoga mlina u postrojenju za preradu bakra. Stvarni radni uvjeti u predviđenome mlinu izvode se s kapacitetom od 300 t/h, vremenom zadržavanja od 267 sekundi i izlaznom veličinom \( d_{80} = 53 \, \mu m \). Laboratorijske studije pokazale su da se optimalna ekonomičnost i metalurško iskorištenje bakra u ovome postrojenju postiže u rasponu 65 μm ≤ \( d_{80} \) ≤ 75 μm s flotacijskim iskorištenjem (R) = 90,16 %, ekonomskom učinkovitošću (EE) = 93,04 % i učinkovitošću odvajanja (SE) = 88,64 %. U ovoj studiji, poznavajući optimalni \( d_{80} \) za jedinicu koncentracije i podatke iz mlina te korištenjem Excel softvera i Morrell metode, najprije je izračunana ukupna snaga za optimalni skup \( d_{80} \), koja je jednaka 7790 do 8005 kW. Zatim su, prema ovim vrijednostima snage, izračunana odgovarajuća vremena zadržavanja, koja su jednaka 236 odnosno 247 sekundi. Zaključno, korištenjem odnosa vremena zadržavanja i kapaciteta te uzimajući u obzir specifično punjenje mlina, izračunani su optimalni odgovarajući kapaciteti sukladno dobivenim vremenima zadržavanja u rasponu od 324 do 340 t/h. Rezultati ovih istraživanja pokazali su da smanjenjem stupnja usitnjavanja s \( d_{80} = 53 \, \mu m \) na 65 μm ≤ \( d_{80} \) ≤ 75 μm, osim povećanja učinkovitosti flotacije na R = 90,16 %, EE = 93,04 % i SE = 88,64 %, doći će do uštede energije od oko 4,21 % do 7,09 % i povećanja kapaciteta od 8,00 % do 13,33 %.

Ključne riječi:

kuglični mlin, potrebna snaga, ušteda energije, Morrellov model snage, simulacija, stupanj sitnjenja

Author’s contribution

Sajjad Chehreghani (Assistant Professor of Mineral Processing Engineering) collaborated on the literature review, data collection and analysis procedure of this manuscript. Hojjat Hosseinzadeh Gharehgheshlagh (Assistant Professor of Mineral Processing Engineering) collaborated on the literature review, data collection, analysis procedure and managed the whole process for this manuscript. Sahand Haghikia (Expert of Mineral Processing Engineering) collaborated on the data collection and writing process of this manuscript.