# A Distributed Asynchronous Transmission Access Strategy for Optical Single-Hop LANs: An Analytical Performance Study 

Peristera A. Baziana, Member IEEE

Original scientific article


#### Abstract

In this paper, we introduce an optical passive network architecture suitable for wavelength division multiplexing local area networks (LANs) which use a separate control wavelength. The data wavelengths are organized into several sets, while the access rights over them are distributedly determined aiming to totally expunge the packets collisions on the wavelengths and at destination. The proposed access algorithm pertains to the asynchronous transmission schemes. Thus, it is simple enough since it does not require any synchronization among the stations, providing high efficiency especially under high data rates ( 100 Gbps and beyond). The performance is evaluated through exhaustive analysis, whilst closed mathematical formulas provide the performance measures. The comparative study proves that the proposed wavelengths organization into sets strategy along with the access scheme significantly improves the performance. Especially, the throughput improvement is proven to be higher as the number of sets increases, and more than $100 \%$ even by organizing the wavelengths into only two sets, for diverse numbers of data wavelengths, data wavelengths sets and data packets size. Finally, the proposed study could be applied to optical passive single-hop LANs such as intra-rack data center networks or local institutional or enterprise networks.


Index terms-Multiple access algorithm; optical network; performance analysis.

## I. Introduction

Nowadays, an increasing demand for high bandwidth networks to serve the significant traffic growth has been noticed globally. Diverse users network applications and network communication platforms have been utilized and further developed to support the remote communication for various users activities, like work-from-home, education-from-home, entertainment-from-home etc., especially under the pandemic conditions. In this framework, the existing access networks are requested to serve a tremendous traffic growth coming from such users network activities, while large investments in network infrastructures have already been announced. For example, by the end of 2021 Cisco has increased the number of its network infrastructures by $86 \%$ the last five years since it has been expected that the traffic would have quadrupled [1].

Manuscript received November 5, 2021; revised December 27, 2021. Date of publication March 9, 2022. Date of current version March 9, 2022. The associate editor prof. Tianhua Xu has been coordinating the review of this manuscript and approved it for publication.

Peristera A. Baziana is with the University of Thessaly - Computer Science and Telecommunications, Greece (e-mail: baziana@uth.gr).

Digital Object Identifier (DOI): 10.24138/jcomss-2021-0162

Also, Google has already announced its network expansion globally [2] and further investments in digital future [3]. It is remarkable that AT\&T works towards to expand its Fiber-To-The-Home (FTTH) network aiming to more than double its fiber footprint by the end of 2025 [4].
In this context, optical local area networks (LANs) at the access level are of great interest since they can support diverse enterprise or institutional network solutions serving highbandwidth consuming data traffic. Various optical LAN implementations have been proposed to serve several applications and network architectures, like Tactile Internet [5], aerospace applications [6], health-care services [7-9], data center networks [10-14] etc. Their performance strongly depends on the applied medium access control (MAC) algorithm aiming to provide efficiency, access fairness, flexibility and reliability. In literature, many paradigms have been studied. For example, some passive optical health-care LANs along with adaptive resource allocation algorithms have been recently introduced to effectively serve the highbandwidth demanding medical data traffic in a hospital campus [7-8]. Moreover, the optical LANs paradigm has been thoroughly exploited in diverse optical intra-cluster data center network architectures in conjunction with various MAC schemes aiming to improve the network efficiency and satisfactorily serve the end users applications [10-14].

Many of the optical LANs proposals exploit the Wavelength Division Multiplexing (WDM) technique [15] since it is the most suitable one to optimally share the high fiber bandwidth to the stations by providing several distinct wavelengths to them. Since the optical transceivers market currently supports very high data rate implementations (even 400 Gbps per wavelength) [16] and recent research results guarantee even the 400 Gbps WDM LANs deployment [17], the optimum fiber bandwidth exploitation is challenging enough. Diverse WDM access (WDMA) protocols have been investigated to exploit the high data rates, based on either centralized or distributed access control mechanisms. Some of them are based on random access ALOHA-type schemes which do not require any time synchronization among the stations, while their performance has been initially studied in [18]. Nevertheless, they inevitably provide low bandwidth exploitation due to the high packets collision probability as a result of the lack of any pretransmission coordination strategy. On the contrary, recent synchronous transmission WDMA algorithms have been studied to decrease the probability of packets collisions in an
optical LAN environment [10-13]. Despite the theoretical performance improvement compared to the relative asynchronous transmission WDMA protocols, they unavoidably exhibit high complexity and lack of reliability since they can hardly guarantee the compliance with a common clock for transmission at the currently available high data rates of even 400 Gbps . The alternative of the carrier sense multiple access with collisions detection (CSMA/CD) scheme has been proposed for optical LANs implementations [14]. Nonetheless, the CSMA/CD protocol is not suitable for optical LANs since in such high data rates the packet transmission time is not larger than the round trip propagation delay time, which is required by the protocol for the efficient collisions detection.
In this paper, we introduce a single-hop optical passive LAN architecture that uses a separate wavelength for the control channel, along with an effective distributed WDMA protocol for asynchronous transmission over the data wavelengths. The proposed pre-transmission coordination WDMA protocol ensures the total collisions-free transmission over the data wavelengths and at destination. Also we introduce an access strategy according which the data wavelengths are organized into several sets while the access rights over them are distributedly determined, aiming to enhance the overall performance. The proposed WDMA protocol provides flexibility and reliability at high data rates since it does not require any time synchronization for transmission. Also, it reaches high bandwidth utilization and efficiency.

We study the results of an accurate mathematical analysis based on Poisson traffic model. In specific, we derive as closed mathematical formulas the following performance measures: the network throughput, the throughput improvement comparatively to the scenario where the wavelengths organization into sets is not adopted, and the probability to cancel the transmission to expunge the data packets collisions. Also, they are investigated for diverse values of data wavelengths, wavelengths sets and packet size. The comparative analysis results demonstrate that the proposed WDMA protocol accompanied by the wavelengths organization into sets strategy achieves significant throughput improvement, more than $100 \%$ even by organizing the data wavelengths into only two sets, whilst the throughput improvement grows even more as the number of wavelengths sets rises. This is a useful result to build efficient, flexible, and reliable optical LANs of high data rates by just applying the proposed WDMA strategy, without requiring additional cost and complexity.

Finally, the innovative contribution of this study is the proposal of a simple ALOHA-type asynchronous transmission distributed WDMA scheme over a number of sets of data wavelengths, and the proper determination of the wavelengths assignment rules to ensure the totally collisions-free transmission in an optical LAN. This contribution differs from those of our previous studies that propose either synchronous transmission access schemes that require strict time synchronization which cannot be guaranteed especially under such high data rates, and use a multi-channel control architecture [19] or a deterministic time division multiplexing (TDM) control access algorithm [20], or a simplified asynchronous transmission MAC protocol for multi-channel networks [21] that does not take into account the performance
parameters of an optical network like the transceivers tuning time, the propagation delay etc.

The rest of the paper is organized as follows: section II presents the network architecture. Section III studies the performance analysis, whilst it analytically derives the performance measures formulas. Also, section IV comparatively explores the results. Finally, section V concludes our study.

## II. Network Architecture

We consider the network architecture of Fig. 1.


Fig. 1. Network architecture
As Fig 1 depicts, we assume an optical LAN which connects a number $M$ of stations through an optical passive coupler. The network uses $(N+1)$ WDM wavelengths $\left\{\lambda_{0}, \lambda_{1}, \lambda_{2}, \ldots . \lambda_{N}\right\}$ each of which runs in data rate $R$ Gbps. The wavelength $\lambda_{0}$ is referred as control channel whilst it serves the control transmissions. The wavelengths $\left\{\lambda_{1}, \lambda_{2}, \ldots ., \lambda_{N}\right\}$ are referred as data channels serving the data transmissions. The distance between any station and the coupler is equal for all the stations.
Each station has a burst mode tunable transmitter (TT) able to be tuned at all wavelengths, a tunable receiver (TR) able to be tuned at the data channels $\left\{\lambda_{1}, \lambda_{2}, \ldots ., \lambda_{N}\right\}$ and a fixed receiver (FR) that is permanently tuned at the control wavelength $\lambda_{0}$, as Fig. 1 shows. The tunable transceiver normalized tuning time between any two channels is $T$ time units. Each station generates packets independently following the Poisson traffic model. The transmission is assumed to be asynchronous.

As Fig. 2 illustrates, the set $\left\{\lambda_{1}, \lambda_{2}, \ldots ., \lambda_{N}\right\}$ is divided into $(F+1)$ sets, $A_{1}, A_{2}, \ldots, A_{F}, A_{F+1}$, where $F$ is an integer and $F \geq$ 2. Each set $A_{1}, A_{2}, \ldots, A_{F}$ includes $k$ data channels, where


Fig. 2. Data channels organization into $F$ sets - Asynchronous transmission
$k=N \operatorname{div} F$, while the last set $A_{F+1}$ includes $m$ data channels, where $m=N \bmod F$. In specific, the sets $A_{1}, A_{2}, \ldots A_{F}$ includes the data channels $\left\{\lambda_{1}, \lambda_{2}, \ldots \lambda_{k}\right\}, \quad\left\{\lambda_{k+1}, \lambda_{k+2}, \ldots \lambda_{2 k}\right\}, \ldots$ $\left\{\lambda_{(F-1) k+1}, \lambda_{(F-1) k+2}, \ldots \lambda_{F k}\right\}$ respectively, while the set $A_{F+1}$ includes the remaining data channels $\left\{\lambda_{F k+1}, \lambda_{F k+2}, \ldots ., \lambda_{N}\right\}$ as Fig. 2 shows. Since $k>m$, each channel that belongs to the set $A_{1}$ has an one to one correspondence to the relative channels that belong to the sets $A_{2}, \ldots, A_{F}$. For example the data channel $\lambda_{x}$, $x \in\{1,2, \ldots k\}$, that belongs to the set $A_{1}$ matches with the data wavelengths $\lambda_{k+x}, \lambda_{2 k+x}, \ldots \lambda_{(F-1) k+x}$ that belong to the sets $A_{2}, A_{3}, \ldots, A_{F}$ respectively.

The control packet includes three fields: the source address, the destination address, and the data channel $\lambda_{x}, x \in\{1,2, \ldots k\}$, from the set $A_{1}$ over which the corresponding data packet is selected to be transmitted. In our study, we assume the transmission time of the control packet as time unit. We define as data slot the normalized transmission time of a data packet, which equals to $L$ time units. The round-trip propagation delay between any pair of stations through the passive coupler is $T_{p}$ time units.

## A. Proposed Access Algorithm

Let's assume that a station tends to transmit, for example the time unit $\mathrm{t}_{1}$ as Fig. 2 shows. It runs the following procedure:

1) Data channel selection: At the time unit $t_{1}$, it selects a data channel from the set $A_{1}$, let's say the $\lambda_{j}, j \epsilon\{1,2, \ldots k\}$. Then it has to send over the channel $\lambda_{0}$ a control packet to properly advice the remaining stations about the data packet transmission intension.
2) Tuning of tunable transmitter: At the time unit $t_{1}$ and for the next $T$ time units, it tunes the tunable transmitter to the channel $\lambda_{0}$ to transmit the control packet.
3) Transmission of the control packet: At the time unit $\left(t_{1}+T\right)$ and for the next time unit, it transmits the control packet. Then, for the next $T_{p}$ time units, the control packet propagates to the other stations through the passive optical coupler. A partial or total control packets collision will take place, if any other control packet is transmitted by another station during the interval $\left(t_{1}+T-1, t_{1}+T+1\right)$. In other words, a collision over the control channel will occur only if another station's control packet transmission starts or lasts during the time interval $\left[t_{1}+T, t_{1}+T+1\right]$. Thus, the control packet vulnerable period equals to 2 time units (2 control packets length) [18]: i.e. one time unit for the possible collisions with control packets that have been transmitted in the last time unit will be transmitted in the next time unit.
4) Control information processing: Since the station's fixed receiver always monitors the activity over the channel $\lambda_{0}$, the station recognizes the transmission result of its own control
packet: either the successful transmission or the collision and destruction. Also, the station recognizes the claims for data transmission by all other stations. In case that the data channel $\lambda_{j}$ has been chosen by any other station whose transmission of control packet is collision-free during the appropriate time period, a total or partial data packets collision will take place on the data channel $\lambda_{j}$ and the collided packets will be aborted. At that time instant, the station runs the Bandwidth Exploitation Algorithm (BEA) that follows:

## Bandwidth Exploitation Algorithm (BEA)

Aiming to expunge the collisions on the channels $\left\{\lambda_{1}, \lambda_{2}, \ldots\right.$, , $\left.\lambda_{N}\right\}$, the station processes the received control packets and runs the bandwidth exploitation algorithm (BEA), while the control packet processing lasts for $T_{p r}$ time units, i.e. the control packet processing time is denoted by $T_{p r}$. According to the BEA, if the station has transmitted the control packet without collision, in order to expunge the data channels and the receiver collisions, it examines the remaining control packets which has been successfully transmitted, starting transmission during the time interval $\left(t_{1}+T-L+1, t_{1}+T\right)$. Especially, it examines two fields: the data channel and the destination address. Let' suppose that the same data channel $\lambda_{j}$ has been chosen by $n$, $n \in\{1,2, \ldots M\}$, stations that have transmitted a control packet without collision during the above time interval, while none of them has selected the same destination station. Also let's suppose that the competing stations gain access to a data channel according the priority rule. According to the BEA, an effective data channels assignment strategy is applied to expunge the data packets collisions. Thus, the first competing station gains access to the data channel $i$ of the set $A_{1}$, i.e. the data channel $\lambda_{i}$, the second competing station gains access to the data channel $i$ of the set $A_{2}$, i.e. the data channel $\lambda_{k+i}$, whilst the $n$-th competing station gains access to the data channel $i$ of the set $A_{n}$, i.e. the data channel $\lambda_{(n-1) k+i}$. Thus, the data channel collisions are totally expunged. Moreover, if any of the competing data packets is destined to the same station with another already transmitted data packet during the vulnerable period, the station cancels the transmission in order to expunge the destination conflict.
It is noticeable that if a station that requests the data channel $\lambda_{j}$ for data packet transmission experiences a collision to the corresponding control packet over the control channel $\lambda_{0}$, the requested data channel $\lambda_{j}$ is not assigned to the station, since the collided control packet is destroyed and cannot be processed by the BEA. Thus, the data packet transmission is cancelled. In this case the station should transmit a new control packet in a next time unit, selecting a new data channel for the data packet transmission, and follow the same procedure, hoping to a successful control packet transmission. Finally, the selected data channel $\lambda_{j}$ is allowed to be assigned according to the BEA to another station whose control packet transmission is successful.
On the contrary, if the station's control packet transmission is successful, and a data channel is assigned to it according to the BEA, for example the data channel $\lambda_{z}, z \in\{1,2, \ldots N\}$, then the next procedure is followed by the station:
5) Tuning of tunable transmitter: After the BEA execution, at the time unit $t_{1}+T+1+T_{p}+T_{p r}$ and for the next $T$ time units, the
station tunes the tunable transmitter to the assigned data channel $\lambda_{z}$.
6) Transmission of data packet: At the time unit $t_{1}+T+1+T_{p}+T_{p r}+T$ and for the next $L$ time units, the station transmits the data packet on the data channel $\lambda_{z}$.

Also, the station is aware of the data packets that have chosen itself as a destination, by always monitoring the control channel $\lambda_{0}$. In that case, according to the BEA, the destination station tunes the tunable receiver to the corresponding data channel.

Finally, it is worth mentioning that according to the proposed access scheme, the stations experience only control packets collisions over the control channel $\lambda_{0}$. In this case the collided control packets are destroyed, and the stations must re-run the same procedure, expecting a successful new control packet transmission. On the other hand, the stations do not experience any packets collisions over the data channels. This is grace to the BEA that obliges a station to cancel its data packet transmission in case that a concurrent data packet transmission by another station is inspected over the requested data channel or over the corresponding ones from the other data channels sets. In this case, the data packet remains at the output buffer until the successful control and data packet transmission takes place in the future.

## III. Performance Analysis

Based on the Poisson traffic model, the number of control packets which are on average offered to the channel $\lambda_{0}$ per time unit, is denoted by $G$. Also, we denote as $P_{c}$ a control packet's successful transmission probability [18]:

$$
\begin{equation*}
P_{c}=e^{-2 G} \tag{1}
\end{equation*}
$$

For simplicity, we consider that $A_{F+1}=\{ \}$ and $N \bmod F=0$, i.e. the last set $A_{F+1}$ is empty. For analysis simplicity reasons, we assume that $T_{p}=0$ and $T_{p r}=0$, while the receiver collisions are ignored. In the following, we study the cases $F=2$ and $F=3$.

## Case I: $F=2$

In this subsection, we consider that $F=2$ sets, i.e. the data channels set is equally divided in two subsets. Let's consider that at the time unit $t_{x}$, there isn't any station which transmits on the data cannels $\lambda_{i}$ and $\lambda_{k+i}, i \in\{1,2, \ldots k\}$, where $k=N \operatorname{div} 2$. Also, we consider a station that follows the next conditions (denoted as conditions $C$ ): 1) it has finished transmitting a control packet without collision (successful transmission), and 2) the relative transmission of data packet, it has been chosen to be performed over the data channel $\lambda_{i}$. Let's suppose that according to the BEA, the data channel $\lambda_{i}$ has been confided to it, while it starts transmission at the time unit $t_{x}$. The data packet transmission will last till the time unit $t_{x}+L-1$. Also, we consider a $2^{\text {nd }}$ station which follows the conditions $C$. According to the BEA, the data channel $\lambda_{k+i}$ will be confided to the $2^{\text {nd }}$ station to expunge the data packets collision. Moreover, we assume that at the time unit $t_{y}$, where $t_{x} \leq t_{y} \leq t_{x}+L-1$, the $2^{\text {nd }}$ station starts the data packet transmission over the relative data channel $\lambda_{k+i}$, which will last till the $t_{y}+L-1$ time unit. As we understand, if a $3^{\text {rd }}$ station that follows the conditions $C$ claims transmission between the $t_{y}$ and $t_{x}+L-1$ time units, the claim cannot be satisfied since a collision would occur. For this reason according to the BEA, aiming to
expunge the collision, the $3^{\text {rd }}$ station is obliged to go back on transmitting and to cancel its transmission. In other words, if the $3^{\text {rd }}$ station has requested the same data channel $\lambda_{i}$ for data packet transmission as long as the transmissions of the $1^{\text {st }}$ and $2^{\text {nd }}$ stations take place, and its control packet has been successfully transmitted, the $3^{\text {rd }}$ station runs the BEA, and it is informed that neither the requested data channel $\lambda_{i}$ nor the corresponding data channel $\lambda_{k+i}$ from the second set can be assign to it since they carry the $1^{\text {st }}$ and $2^{\text {nd }}$ stations' transmissions respectively. Thus, the $3^{\text {rd }}$ station cancels its corresponding data packet transmission to avoid any collision, while the data packet remains at its output buffer for attempting transmission in the future.

Let's assume the event $E(F=2)$ that happens when a station which follows the conditions $C$ manages to successfully transmit a data packet. It is evident that $E(F=2)$ happens:
1.either if no other station follows the conditions $C$ during the time units $\left[t_{x}, t_{x}+L-1\right]$. (event $E_{1}(F=2)$ ),

2 .or if only one more station follows the conditions $C$, and starts its transmission at time unit $t_{y}$, where: $t_{x} \leq t_{y} \leq t_{x}+L-$ 1 (event $E_{2}(F=2)$ ).
The events $E_{1}(F=2)$ and $E_{2}(F=2)$ are independent. It is:

$$
\begin{equation*}
E(F=2)=E_{1}(F=2) \cup E_{2}(F=2) \tag{2}
\end{equation*}
$$

For the analysis of Case I, we define the parameters $Q_{j}$, $\operatorname{Pr}\left(Q_{j}\right), R_{j}, \operatorname{Pr}\left(R_{j}\right)$ that are described in Table 1. Also, Table 1 summarizes the basic parameters used in the analysis that follows.

It is evident that the probability $\operatorname{Pr}\left(Q_{j}\right)$ is:

$$
\begin{equation*}
\operatorname{Pr}\left(Q_{j}\right)=G P_{c} \tag{3}
\end{equation*}
$$

Also, it is obvious that the probability $\operatorname{Pr}\left(R_{j}\right)$ is:

$$
\begin{equation*}
\operatorname{Pr}\left(R_{j}\right)=\frac{2}{N} \tag{4}
\end{equation*}
$$

Moreover, the events $Q_{j}$ and $R_{j}$ are independent for every $j, t_{x}$ $\leq j \leq t_{x}+L-1$. The event $E_{1}(F=2)$ is:

$$
\begin{equation*}
E_{1}(F=2)=\bigcap_{j=t_{x}}^{t_{x}+L-1} \overline{Q_{j} \cap R_{j}} \tag{5}
\end{equation*}
$$

The probability $\operatorname{Pr}\left(E_{1}(F=2)\right)$ of finding no other station that follows the conditions $C$ during the time interval $\left[t_{x}, t_{x}+L-1\right]$, is [22]:

$$
\begin{equation*}
\operatorname{Pr}\left(E_{1}(F=2)\right)=\left(1-\frac{2 G P_{c}}{N}\right)^{L-1} \tag{6}
\end{equation*}
$$

The event $E_{2}(F=2)$ :

$$
\begin{gather*}
E_{2}(F=2)=\cup_{j=t_{x}+1}^{t_{x}+L-1}\left[\left(Q_{j} \cap R_{j}\right) \cap\left(\cap_{a=t_{x}+1}^{j-1} \overline{Q_{a} \cap R_{a}}\right)\right. \\
\left.\cap\left(\cap_{b=j+1}^{t_{x}+L-1} \overline{Q_{b} \cap R_{b}}\right)\right] \tag{7}
\end{gather*}
$$

The probability $\operatorname{Pr}\left(E_{2}(F=2)\right)$ of finding only one more station which follows the conditions $C$, and which starts transmission at time unit $t_{y}$, where $t_{x} \leq t_{y} \leq t_{x}+L-1$, is:

$$
\begin{align*}
& \operatorname{Pr}\left(E_{2}(F=2)=\sum_{j=t_{x}+1}^{t_{x}+L-1}\right. \operatorname{Pr}\left(Q_{j} \cap R_{j}\right) \prod_{a=t_{x}+1}^{j-1} \operatorname{Pr}\left(\overline{Q_{a} \cap R_{a}}\right) \\
&\left.\prod_{b=j+1}^{t_{x}+L-1} \operatorname{Pr}\left(\overline{Q_{b} \cap R_{b}}\right)\right] \tag{8}
\end{align*}
$$

Since the events $Q_{j}$ and $R_{j}$ are independent for every value of $j, t_{x} \leq j \leq t_{x}+L-1$, it is:

TABLE I
ANALYSIS PARAMETERS - CASE I: $\mathrm{F}=2$

| Symbol | Description |
| :---: | :---: |
| Conditions $C$ | The initial station: <br> Condition 1: has finished transmitting the control packet without collision (successful transmission) <br> AND <br> Condition 2: the data channel $\lambda_{i}$ has been chosen for the relative data packet transmission. |
| $t_{x}$ | The time unit at which the initial station starts transmitting a data packet over the data channel $\lambda_{i}$, according to the BEA. |
| $\lambda_{i}$ | The data channel that has been assigned to the initial station, according to the BEA, to start transmitting a data packet at the time unit $t_{x}$. |
| $Q_{j}$ | The event that an additional station starts transmitting a new data packet at the $j$-th time unit, where: $t_{x} \leq j \leq t_{x}+$ $L-1$, i.e. the additional data packet transmission is started as long as the initial one has been taking place. |
| $\operatorname{Pr}\left(Q_{j}\right)$ | The probability of the event $Q_{j}$ |
| $R_{j}$ | The event that the additional station which starts transmitting a new data packet at the $j$-th time unit, where: $t_{x} \leq j \leq t_{x}+L-1$, has selected the same data channel $\lambda_{i}$ for the transmission. |
| $\operatorname{Pr}\left(R_{j}\right)$ | The probability of the event $R_{j}$ |
| $E(F=2)$ | The event that the initial station which follows the conditions $C$ manages to successfully transmit a data packet. |
| $\operatorname{Pr}(E(F=2))$ | The probability of the event $E(F=2)$ |
| $E_{1}(F=2)$ | The event that no other station follows the conditions $C$ and starts a new data packet transmission during the time units $\left[t_{x}, t_{x}+L-1\right]$, i.e. during the time period the initial station transmits its data packet. (Eq. (5)) |
| $\operatorname{Pr}\left(E_{1}(F=2)\right)$ | The probability of the event $E_{1}(F=2)$ |
| $E_{2}(F=2)$ | The event that only one additional station follows the conditions $C$, and starts its new data packet transmission at time unit $t_{y}$, where: $t_{x} \leq t_{y} \leq t_{x}+L-1$, i.e. during the time period the initial station transmits its data packet. In other words, no other additional station starts a new data packet transmission during the time periods: $\left[t_{x}, t_{y}-1\right]$ and $\left[t_{y}+1, t_{x}+L-1\right]$. (Eq. (7)) |
| $\operatorname{Pr}\left(E_{2}(F=2)\right)$ | The probability of the event $E_{2}(F=2)$ |

$$
\begin{equation*}
\operatorname{Pr}\left(Q_{j} \cap R_{j}\right)=\operatorname{Pr}\left(Q_{j}\right) \operatorname{Pr}\left(\frac{Q_{j}}{R_{j}}\right)=\frac{2 G P_{c}}{N} \tag{9}
\end{equation*}
$$

Replacing (9) to (8), we get:

$$
\begin{equation*}
\operatorname{Pr}\left(E_{2}(F=2)\right)=(L-2) \frac{2 G P_{c}}{N}\left(1-\frac{2 G P_{c}}{N}\right)^{L-4} \tag{10}
\end{equation*}
$$

We denote as $\operatorname{Pr}(E(F=2))$ the probability of a successful data packet transmission from a station that follows the conditions $C$, at a given time unit. From (2), (6), (10) we get:
$\operatorname{Pr}(E(F=2))=$

$$
\begin{equation*}
\left(1-\frac{2 G P_{c}}{N}\right)^{L-1}+(L-2) \frac{2 G P_{c}}{N}\left(1-\frac{2 G P_{c}}{N}\right)^{L-4} \tag{11}
\end{equation*}
$$

## Case II: $F=3$

In this subsection, we consider that $F=3$ sets, i.e. the data channels set is equally divided in three subsets. Similar to the Case $I$, we assume the event $E(F=3)$ that happens when a station which follows the conditions $C$ manage to successfully transmit a data packet. It is evident that $E(F=3)$ happens:
1.Similar to the previous case 1 . It is referred as the event $E_{1}(F=3)$,
2.Similar to the previous case 2 . It is referred as the event $E_{2}(F=3)$.
3. or if there are only two more stations which follow the conditions $C$ and have started their transmission at time unit $t_{z}$, where $t_{x} \leq t_{y} \leq t_{z} \leq t_{x}+L-1$ (event $E_{3}(F=3)$ ).
The events $E_{1}(F=3), E_{2}(F=3)$ and $E_{3}(F=3)$ are independent. Thus, it is:

$$
\begin{equation*}
E(F=3)=E_{1}(F=3) \cup E_{2}(F=3) \cup E_{3}(F=3) \tag{12}
\end{equation*}
$$

We assume the event $Q_{j}$ as in Case $I$. The probability $\operatorname{Pr}\left(Q_{j}\right)$ is given by (3).
Table 2 summarizes the new parameters used in the analysis of Case II that follows.
It is obvious that the probability $\operatorname{Pr}\left(V_{j}\right)$ is:

$$
\begin{equation*}
\operatorname{Pr}\left(V_{j}\right)=\frac{3}{N} \tag{13}
\end{equation*}
$$

Also, the events $Q_{j}$ and $V_{j}$ are independent for every $j, t_{x} \leq j \leq$ $t_{x}+L-1$. Thus, the event $E_{1}(F=3)$ is:

$$
\begin{equation*}
E_{1}(F=3)=\bigcap_{j=t_{x}}^{t_{x}+L-1} \overline{Q_{j} \cap V_{j}} \tag{14}
\end{equation*}
$$

The probability $\operatorname{Pr}\left(E_{1}(F=3)\right)$ of finding no other station which follows the conditions $C$ in the time interval $\left[t_{x}, t_{x}+L-1\right]$, is [22]:

$$
\begin{equation*}
\operatorname{Pr}\left(E_{1}(F=3)\right)=\left(1-\frac{3 G P_{c}}{N}\right)^{L-1} \tag{15}
\end{equation*}
$$

The event $E_{2}(F=3)$ is:

$$
\begin{gather*}
E_{2}(F=3)=\cup_{j=t_{x}+1}^{t_{x}+L-1}\left[\left(Q_{j} \cap V_{j}\right) \cap\left(\cap_{a=t_{x}+1}^{j-1} \overline{Q_{a} \cap V_{a}}\right)\right. \\
\left.\cap\left(\cap_{b=j+1}^{t_{x}+L-1} \overline{Q_{b} \cap V_{b}}\right)\right] \tag{16}
\end{gather*}
$$

Thus, the probability $\operatorname{Pr}\left(E_{2}(F=3)\right)$ of finding only one more station which follows the conditions $C$ and transmits the time unit $t_{y}, t_{x} \leq j \leq t_{x}+L-1$, is:

$$
\begin{gather*}
\operatorname{Pr}\left(E_{2}(F=3)=\sum_{j=t_{x}+1}^{t_{x}+L-1}[ \right.
\end{gather*} \operatorname{Pr}\left(Q_{j} \cap V_{j}\right) \prod_{a=t_{x}+1}^{j-1} \operatorname{Pr}\left(\overline{Q_{a} \cap V_{a}}\right)
$$

Similar to the Case $I$, since the events $Q_{j}$ and $V_{j}$ are independent for each value of $j, t_{x} \leq j \leq t_{x}+L-1$, it is:

$$
\begin{equation*}
\operatorname{Pr}\left(Q_{j} \cap V_{j}\right)=\operatorname{Pr}\left(Q_{j}\right) \operatorname{Pr}\left(\frac{Q_{j}}{V_{j}}\right)=\frac{3 G P_{c}}{N} \tag{18}
\end{equation*}
$$

Replacing (18) to (17), we get:

$$
\begin{equation*}
\operatorname{Pr}\left(E_{2}(F=3)\right)=(L-2) \frac{3 G P_{c}}{N}\left(1-\frac{3 G P_{c}}{N}\right)^{L-4} \tag{19}
\end{equation*}
$$

Finally, the event $E_{3}(F=3)$ is:

$$
\begin{align*}
E_{3}(F= & 3)=\cup_{j=t_{x}+1}^{t_{x}+L-1}\left[\left(Q_{j} \cap V_{j}\right)\right. \\
& \cap\left[\cup _ { u = t _ { x } + 1 } ^ { j - 1 } \left[\left(Q_{u} \cap V_{u}\right) \cap\left(\cap_{a=t_{x}+1}^{u-1}\left(\overline{Q_{a} \cap V_{a}}\right)\right)\right.\right. \\
& \left.\cap\left(\cap_{b=u+1}^{j-1}\left(\overline{Q_{b} \cap V_{b}}\right)\right) \cap\left(\cap_{c=j+1}^{t_{x}+L-1}\left(\overline{Q_{c} \cap V_{c}}\right)\right)\right] \\
& \cup_{u=j+1}^{t_{x}+L-1}\left[\left(Q_{u} \cap V_{u}\right) \cap\left(\cap_{a=t_{x}+1}^{j-1}\left(\overline{Q_{a} \cap V_{a}}\right)\right)\right. \\
& \left.\left.\cap\left(\cap_{b=u+1}^{u-1}\left(\overline{Q_{b} \cap V_{b}}\right)\right) \cap\left(\cap_{c=u+1}^{t_{x}+L-1}\left(\overline{Q_{c} \cap V_{c}}\right)\right)\right]\right] \tag{20}
\end{align*}
$$

TABLE II
ANALYSIS NEW PARAMETERS - CASE II: $\mathrm{F}=3$

| Symbol | Description |
| :---: | :---: |
| $V_{j}$ | The event that the additional station which starts transmitting a new data packet at the $j$-th time unit, where: $t_{x} \leq j \leq t_{x}+L-1$, has selected the same data channel $\lambda_{i}$ for the transmission (when $F=3$ ). |
| $\operatorname{Pr}\left(V_{j}\right)$ | The probability of the event $V_{j}$ |
| $E(F=3)$ | The event that the initial station which follows the conditions $C$ manages to successfully transmit a data packet. |
| $\operatorname{Pr}(E(F=3))$ | The probability of the event $E(F=3)$ |
| $E_{1}(F=3)$ | The event that no other station follows the conditions $C$ and starts its transmission during the time units $\left[t_{x}, t_{x}+\right.$ $L-1$ ], i.e. during the time period the initial station transmits its data packet. (Eq. (14)) |
| $\operatorname{Pr}\left(E_{1}(F=3)\right)$ | The probability of the event $E_{1}(F=3)$ |
| $E_{2}(F=3)$ | The event that only one additional station follows the conditions $C$, and starts its new data packet transmission at time unit $t_{y}$, where: $t_{x} \leq t_{y} \leq t_{x}+L-1$, i.e. during the time period the initial station transmits its data packet. In other words, no other additional station starts a new data packet transmission during the time periods: $\left[t_{x}, t_{y}-1\right]$ and $\left[t_{y}+1, t_{x}+L-1\right]$. (Eq. (16)) |
| $\operatorname{Pr}\left(E_{2}(F=3)\right)$ | The probability of the event $E_{2}(F=3)$ |
| $E_{3}(F=3)$ | The event that only two additional stations follow the conditions $C$, and start their new data packet transmissions at the time units $t_{y}$ and $t_{z}$ respectively, where: $t_{x} \leq t_{y} \leq t_{z} \leq t_{x}+L-1$, i.e. during the time period the initial station transmits its data packet. In other words, no other additional station starts a new data packet transmission during the time periods: $\left[t_{x}, t_{y}-\right.$ $1],\left[t_{y}+1, t_{z}-1\right],\left[t_{z}+1, t_{x}+L-1\right]$. (Eq. (20)) |
| $\operatorname{Pr}\left(E_{3}(F=3)\right)$ | The probability of the event $E_{3}(F=3)$ |

The probability $\operatorname{Pr}\left(E_{3}(F=3)\right)$ of finding only two more stations that follow the conditions C, while their transmission starts at time unit $t_{z}, t_{x} \leq t_{y} \leq t_{z} \leq t_{x}+L-1$, is:
$\operatorname{Pr}\left(E_{3}(F=3)\right)=\sum_{j=t_{x}+1}^{t_{x}+L-1}\left[\operatorname{Pr}\left(Q_{j} \cap V_{j}\right) \times\right.$

$$
\begin{align*}
& {\left[\sum _ { u = t _ { x } + 1 } ^ { j - 1 } \left[\operatorname{Pr}\left(Q_{u} \cap V_{u}\right) \prod_{a=t_{x}+1}^{u-1} \operatorname{Pr}\left(\overline{Q_{a} \cap V_{a}}\right)\right.\right.} \\
& \left.\prod_{b=u+1}^{j-1} \operatorname{Pr}\left(\overline{Q_{b} \cap V_{b}}\right) \prod_{c=j+1}^{t_{x}+L-1} \operatorname{Pr}\left(\overline{Q_{c} \cap V_{c}}\right)\right]+ \\
& \sum_{u=j+1}^{t_{x}+L-1}\left[\operatorname{Pr}\left(Q_{u} \cap V_{u}\right) \prod_{a=t_{x}+1}^{j-1} \operatorname{Pr}\left(\overline{Q_{a} \cap V_{a}}\right)\right. \\
& \left.\left.\prod_{b=j+1}^{u-1} \operatorname{Pr}\left(\overline{Q_{b} \cap V_{b}}\right) \prod_{c=u+1}^{t_{x}+L-1} \operatorname{Pr}\left(\overline{Q_{c} \cap V_{c}}\right)\right]\right] \tag{21}
\end{align*}
$$

Replacing (18) to (21), we get:
$\operatorname{Pr}\left(E_{3}(F=3)\right)=(L-2)(L-4)\left(\frac{3 G P_{c}}{N}\right)^{2}\left(1-\frac{3 G P_{c}}{N}\right)^{L-6}$
Thus, the probability $\operatorname{Pr}(E(F=3))$ of a successfully transmitted packet by a station which follows the conditions $C$, at a given time unit, is:

$$
\begin{align*}
& \operatorname{Pr}(E(F=3))= \\
& \quad \operatorname{Pr}\left(E_{1}(F=3)\right)+\operatorname{Pr}\left(E_{2}(F=3)\right)+\operatorname{Pr}\left(E_{3}(F=3)\right) \tag{23}
\end{align*}
$$

Replacing (15), (19) and (22) to (23), we get:

$$
\begin{align*}
\operatorname{Pr}(E(F=3))= & \left(1-\frac{3 G P_{c}}{N}\right)^{L-1}+ \\
& (L-2) \frac{3 G P_{c}}{N}\left(1-\frac{3 G P_{c}}{N}\right)^{L-4}  \tag{24}\\
& (L-2)(L-4)\left(\frac{3 G P_{c}}{N}\right)^{2}\left(1-\frac{3 G P_{c}}{N}\right)^{L-6}
\end{align*}
$$

## Average throughput

The average throughput $S_{c}$ is defined as the average output rate from the channel $\lambda_{0}$ per time unit [18]:

$$
\begin{equation*}
S_{c}=G L P_{c} \tag{25}
\end{equation*}
$$

Thus, we define the average throughput $S_{d}(F)$ per data slot as:

$$
\begin{equation*}
S_{d}(F)=S_{c} \operatorname{Pr}(E(F)) \tag{26}
\end{equation*}
$$

## Average transmission cancelation probability

The average probability $P_{t c}(F)$ of a packet transmission cancelation according to the BEA in steady state, to expunge collisions over the data channels, is defined as:

$$
\begin{equation*}
P_{t c}(F)=\frac{S_{c}-S_{d}(F)}{S_{c}} \tag{27}
\end{equation*}
$$

## Average probability of throughput improvement

The average probability $P_{s i}(F)$ of throughput improvement grace to the data channels organization into $F$ sets strategy and the BEA assumption, in steady state, is defined as:

$$
\begin{equation*}
P_{s i}(F)=\frac{s_{d}(F)-S_{A}}{S_{A}} \tag{28}
\end{equation*}
$$

where: $S_{A}$ is the average throughput per data slot if the data channels organization into $F$ sets strategy is not assumed, i.e. each station selects randomly 1 of the $N$ data channels and is transmitting according to the ALOHA scheme. $S_{A}$ is [18]:

$$
\begin{equation*}
S_{A}=G L P_{c} e^{-2 \frac{G}{N}(L-1)} \tag{29}
\end{equation*}
$$

## Average delay

Finally, the average delay $D(F)$ is defined as the ratio of the average offered traffic to the average throughput [18], i.e.:

$$
\begin{equation*}
D(F)=(\mathrm{L}+1) \frac{G L}{S_{d}(F)} \tag{30}
\end{equation*}
$$

while the average delay $D_{A}$ if the data channels organization into $F$ sets strategy is not assumed is [18]:

$$
\begin{equation*}
D_{A}=(\mathrm{L}+1) \frac{G L}{S_{A}} \tag{31}
\end{equation*}
$$

It is noticeable that (30) gives the system delay taking into account the possible collisions over the control channel, either they are partial or total, and considering the data packet transmission cancelations in order to avoid any data packet collisions over the data channels, since the throughput $S_{d}(F)$ considers both of them according to (26). In case of a collided control packet, it is destroyed, and a new control packet should be transmitted in a next time unit for the relative data packet transmission attempt. On the other hand, in case of a data packet transmission cancelation, the data packet remains at the output buffer until the successful control and data packet transmission takes place in the future.

## IV. Numerical Results

In this section, we extensively investigate the numerical results of the proposed data channels organization strategy into $F$ sets, considering various values for $F, N$, and $L$, i.e. the numbers of sets, of channels, and of packet size respectively.

## A. Dependence on the number $F$

In Fig. 3 and 4 we extensively study the performance enhancement given by the proposed wavelengths organization into $F$ sets strategy. Specifically, Fig. 3 presents the throughput as a function of the load for $M=100$ stations, $N=60$ channels and $L=100$ time units, when the data channels are organized in: i) $F=2$ sets each of which includes 30 data channels: curve $S_{d}(2)$, and ii) $F=3$ sets each of which includes 20 data channels: curve $S_{d}(3)$. The $S_{d}(2)$ and $S_{d}(3)$ are comparatively presented with the throughput $S_{A}$ from the data channels in case that the data channels organization into sets strategy is not assumed [18]. In Fig. 3, the horizontal axis (x-axis) represents the offered load to the multichannel system. For the $S_{d}(2)$ and $S_{d}(3)$ cases, the offered load refers to the number of control packets arrived at the system per time unit (i.e. control packets/time unit). Similar, for the $S_{A}$ case, the offered load refers to the number of data packets arrived per data slot. Since, the data packet size and the data slot size are both $L$ times larger than the control packet size and the control slot size respectively, the ratios (data packets/data slot) and (control packets/time unit) are equal. Thus, the x -axis numbering is equivalent regarding the offered load of the $S_{d}(2), S_{d}(3)$ and $S_{A}$ cases, while the comparison of the throughput values is completely meaningful and fair. As Fig. 3 depicts, for $G<0.1$ (control packets/time unit) the $S_{d}(2)$, $S_{d}(3)$ and $S_{A}$ values are similar since the offered control traffic is low enough. As a result, there are few collisions on the data channels and the system performance does not depend on the data channels organization into $F$ sets strategy. Thus, no throughput improvement is provided.

In opposition to the low loads, for $G>0.1$ the throughput keenly depends on the data channels collisions, whilst the data channels organization into $F$ sets strategy along with the BEA grants essential throughput improvement. Specifically, if we


Fig. 3. Throughput $S_{d}(2), S_{d}(3), S_{A}$ vs load, $M=100, N=60, L=100$.
study the $S_{A}$ curve we understand that as $G$ increases from $G>0.1$ up to $G \leq 0.2$, the collided packets over the control channel $\lambda_{0}$ are more, although the congestion condition has not been reached yet and for this reason the throughput $S_{A}$ increases, obtaining the maximum for $G=0.2$. On the contrary, the control channel congestion conditions are reached at much higher loads, i.e. for $G=0.5$, for both $F=2$ and $F=3$ cases. This is because the use of the $F$ sets along with the BEA, provides lower probability of channels collisions, enhancing the throughput. Thus as the value $G$ increases from $G>0.1$ up to $G \leq 0.5$, both the $S_{d}(2)$ and $S_{d}(3)$ rise, while they get their maximum for $G=0.5$. In specific, $S_{A}$ gets the maximum value for $G=0.2$ and it is 6.92 data packets/data slot, while $S_{d}(2)$ and $S_{d}(3)$ reach their maximum value for $G=0.5$ and they are 16.12 and 20.33 data packets/data slot respectively. In other words, the $S_{d}(2)$ and $S_{d}(3)$ are increasing functions of $G$, since the $F$ data channels sets accompanied by the BEA manage to provide high probability of successful transmission on the channels.
For higher loads $G>0.5$, the system behavior changes for both $F=2$ and $F=3$ cases, as Fig. 3 depicts. Since the probability of a collision of control packets gradually rises, lower probability of data packet transmission success is provided. Thus, both the $S_{d}(2)$ and $S_{d}(3)$ values decrease. For example, for $G=1$ it is: $S_{d}(2)=12.51$ and $S_{d}(3)=14.65$ data packets/data slot, while the $S_{A}$ has been reduced to 0.49 data packets/time unit. Finally for $G>3$, the use of the $F$ data channels sets strategy in conjunction with BEA seems to have no actual effect on the system throughput. This is understood since at these high loads, the collisions over the control channel are many, thus the offered data packets are less. Concluding, the throughput improvement grace to the proposed strategy is more noticeable for loads $0.5<G<1$ and exceeds $100 \%$.
These results are also confirmed in Fig. 4, which illustrates the probability $P_{s i}(F)$ of throughput improvement grace to the use of the data channels organization into $F$ sets strategy in conjunction with the BEA, as compared to the $S_{A}$ case, for $M=100, N=60$ and $L=100$, for the cases that the data channels are organized into: i) $F=2$ sets: curve $P s i(2)$, and ii) $F=3$ sets: curve $\operatorname{Psi}(3)$. It is depicted that the throughput improvement is very high. For example, for $G=0.5$ we achieve improvement


Fig. 4. Throughput improvement $P_{\text {si }}(2)$ and $P_{\text {si }}(3)$ vs load, $M=100, N=60$, $\mathrm{L}=100$.
equal to: $\operatorname{Psi}(2)=358 \%$ for $F=2$ and $\operatorname{Psi}(3)=477 \%$ for $F=3$, while for $G=1$ it increases to: $\operatorname{Psi}(2)=2418 \%$ for $F=2$ and $\operatorname{Psi}(3)=2847 \%$ for $F=3$. This perfect behavior is explained by the combined study of Fig. 4 and Fig. 5. The latest figure presents the average probability $P_{t c}(F)$ that a station cancels its transmission according to the BEA to expunge data packets collisions, for the cases that the data channels are organized into: i) $F=2$ sets: curve $P_{t c}(2)$, and ii) $F=3$ sets: curve $P_{t c}(3)$. For a given $G$ value, $P_{s i}(F)$ grows as the number $F$ increases. This is explained since as $F$ is getting higher, the data packet transmission cancelation probability according to the BEA is getting lower. Consequently, the throughput $S_{d}(F)$ rises. Thus, for $G=0.5$ is: $P_{t c}(2)=95 \%$ and $P_{t c}(3)=93 \%$, while for $G=1$ it decreases to: $P_{t c}(2)=93 \%$ and $P_{t c}(3)=91 \%$. In other words, the $P_{t c}(F)$ decreases as $F$ rises. This useful result demonstrates that we can reach high performance improvement, if we increase the data channels sets number, without any additional cost or hardware complexity.
As Fig. 5 depicts for all $F$ values, $P_{s i}(F)$ is an increasing function of $G$, for $G<0.5$. This behavior is explained by the fact that for these low offered load conditions, the probability of a control packet successful transmission (without collision) is high enough. Consequently, the number of stations that compete for a data packet transmission and run the BEA in order for a data channel to be assigned to them is high too. As a result, the probability of a data packet transmission cancelation in order to avoid a data channel collision is also high, giving rise to the throughput as Fig. 3 presents. On the contrary, as the offered load increases for $G>0.5$, the number of control packets collisions over the control channel $\lambda_{0}$ are getting higher. This means that the number of stations that manage to successfully transmit a control packet and compete for data packet transmission according to the BEA is getting lower, providing lower values of $P_{s i}(F)$. This behavior is more noticeable for $G>2$ where the system throughput $S_{d}(F)$ is low enough, as Fig. 3 presents.
The performance improvement that the proposed data channels organization into $F$ sets strategy along with the BEA provides is representatively depicted in Fig. 6 that shows the delay $D(F)$ as a function of load when $M=100, N=60$ and


Fig. 5. Transmission cancelation probability $P_{t c}(2)$ and $P_{t c}(3)$ vs load, $M=100$, $\mathrm{N}=60, \mathrm{~L}=100$.
$L=100$, for the cases that the data channels are organized into: i) $F=2$ sets: curve $D(2)$, and ii) $F=3$ sets: curve $D(3)$. These curves are compared with the delay $D_{A}$ where the data channels organization into sets strategy is not assumed. The comparison regarding the offered load conditions is meaningful, as explained in Fig. 3. As it is presented, the delay $D(2)$ and $D(3)$ achieved is lower than $D_{A}$ even under low loads. Especially, the delay $D(2)$ and $D(3)$ are respectively: 162 and 139 time units for $G=0.2,255$ and 203 time units for $G=0.4$, and 381 and 304 for $G=0.6$. It is proved that the delay improvement is an increasing function of $F$. For example it is: $D(2)=44 \%$ and $D(3)=53 \%$ for $G=0.2, D(2)=70 \%$ and $D(3)=76 \%$ for $G=0.4$, and $D(2)=84 \%$ and $D(3)=88 \%$ for $G=0.6$. This behavior is an immediate result of the above discussion. In other words, as $F$ increases, the throughput increases too as Fig. 3 shows, while the transmission cancelation probability is getting lower as Fig. 5 presents. Thus, the time interval that a data packet has to wait until its successful transmission decreases, providing lower delay, as Fig. 6 presents. It is worth mentioning that in Fig. 6 the delay axis unit equals to the control packet transmission time (time unit), which provides results independent from the channels data rate $R$ Gbps. It is evident that one time unit on the delay axis corresponds to $(L / R)$ s.
Finally, it is noticeable that for certain traffic types, like timesensitive traffic of control messages that require immediate service, the strict preconditions regarding the delay pose challenges. In this case, the network design should provide appropriate solutions considering two main performance parameters that affect the end-to-end (e2e) delay achieved: the data rate $R$ and the size of buffer in each server. Especially, by upgrading the $R$ assumed, the time unit duration decreases providing lower delay values. On the other hand, by reducing the buffer size, the queuing delay and by extension the total delay are restricted. Thus, by appropriately determining these performance parameters values, a desired e2e delay level is expected to be reached.
As it is proved by the above comments, the system performance is improved as the number $F$ of data channels sets increases. A question may arise: which is the optimum number $F$ of data channels sets in order to obtain a desired level of throughput? Since the response cannot be obtained as a closed


Fig. 6. Delay $\mathrm{D}(2), \mathrm{D}(3), \mathrm{D}_{\mathrm{A}}$ vs load, $\mathrm{M}=100, \mathrm{~N}=60, \mathrm{~L}=100$.
analytical formula (i.e. the analysis differs for each $F$ value, like Case I and II in Section III), we could evaluate the number $F$ limits with respect to the number of data channels $N$ (it is $1 \leq \underline{F} \leq N$ ), while the number $F$ is not related to the number of stations $M$. Thus, if $F=1$ the system does not organize the data channels into sets and its throughput and delay are depicted by the $S_{A}$ and $D_{A}$ curves in Fig. 3 and 6 respectively, which are significantly worse as compared to the $F=2$ and $F=3$ cases. On the other hand, if $F=N$ each set has a single data channel, while all the stations that attempt a data packet transmission select the specific one from the first set. In this case, the BEA assigns the data channels from the other channels sets to the competing for the same single channel stations, giving them more opportunities for a successful data packet transmission and maximizing the throughput. Concluding, the $F$ value is determined between the marginal values $F=2$ (since the $F=1$ case does not assume the proposed data channels organization into sets strategy) and $F=N$, taking into account the desired level of throughput improvement $P_{s i}(F)$ according to (28). Since the transmission cancellation probability is a decreasing function of $F$ as Fig. 5 shows, it is obvious that it gets its minimum value for $F=N$. This is explained by the fact that as $F$ increases up to $N$, the probability of assigning a channel from another set in case that the first set's channel is busy, is getting higher while it is maximized for $F=N$, reaching the minimum $P_{t c}(F)$ value.

## B. Dependence on the number $N$

Fig. 7 depicts the throughput $S_{d}(2)$ as a function of load when $M=100, F=2, L=100$ and $N=30,60,90$. It is worth mentioning that $S_{d}(2)$ is an increasing function of the number of data channels $N$ for $G>0.1$ up to almost $G<1.9$. This is because as $N$ is getting higher, the number of successfully transmitted data packets according to the BEA increases, giving rise to $S_{d}(2)$. Thus, when $G=0.5, S_{d}(2)$ in data packets/data slot is: 12.18 for $N=30,16.12$ for $N=60$, and 17.23 for $N=90$.

Thus, the probability that a station cancels its transmission is getting lower as $N$ increases. This is studied in Fig. 8 that shows the probability $P_{t c}(2)$ as a function of load, when $M=100, F=2$, $L=100$ and $N=30,60,90$. Thus, when $G=0.5, P_{t c}(2)$ equals to: $96 \%$ if $N=30,95 \%$ if $N=60$, and $94 \%$ if $N=90$. This is because


Fig. 7. Throughput $S_{d}(2)$ vs load, $M=100, F=2, L=100, N=30,60,90$.
as $N$ increases, the probability that a station selects a data channel from the set $A_{1}$ that no other station has already selected, is getting higher. So, the probability of a collision on a data channel and consequently the BEA effect decreases.
Finally, Fig. 9 presents the average probability $P_{s i}(2)$ of throughput improvement as a function of load when $M=100, F$ $=2, L=100$ and $N=30,60,90$. As it is shown in Fig. 9, the $P_{s i}(2)$ is a decreasing function of $N$, for all load values up to the those we can have comparable results. For example, for $G=0.5$ (where the maximum $S_{d}(2)$ value is achieved for all $N$ values as Fig. 7 depicts), the $P_{s i}(2)$ is: $1707 \%$ for $N=30,358 \%$ for $N=60$, and $182 \%$ for $N=90$. This is understood since the $S_{A}$ value is getting higher with the $N$ increase, as (29) proves. As a result, the $P_{s i}(2)$ values as getting lower according to (28). Thus, for very low offered load conditions ( $G<0.1$ ) we could conclude that there is no need to increase the number $N$ (and consequently the number of data channels per set) in order to achieve higher throughput since the competed stations are few. This result is similar to that of Fig. 3. On the other hand, for $G>0.1$ the applied data channels organization into $F=2$ sets strategy with the BEA, provides


Fig. 8. Transmission cancelation probability $\mathrm{P}_{\mathrm{tc}}(2)$ vs load, $\mathrm{M}=100, \mathrm{~F}=2$, $\mathrm{L}=100$, $\mathrm{N}=30,60,90$.


Fig. 9. Throughput improvement $P_{\text {si }}(2)$ vs load, $\mathrm{M}=100, \mathrm{~F}=2, \mathrm{~L}=100$, $\mathrm{N}=30,60,90$.
throughput improvement with the $N$ increase as Fig. 7 depicts, while the improvement is getting lower with $N$ for the same $F$ value. Consequently, a desired level of throughput improvement is achieved at higher load as $N$ increases. For example, the throughput improvement $P_{s i}(2)=1000 \%$ is reached: at $G=0.4$ for $N=30$, at $G=0.72$ for $N=60$, and at $G=1.17$ for $N=90$.

## C. Dependence on the number $L$

Finally, the effect of the data packet size $L$ differentiation on the throughput is examined in Fig. 10. This figure presents the $S_{d}(2)$ curves as a function of the load, when $M=100, N=60, F$ $=2$ and $L=50,100,150$. It is noticeable that $S_{d}(2)$ is an increasing function of $L$. For example for $G=0.5, S_{d}(2)$ in data packets/data slot is: 8.84 for $L=50,16.12$ for $L=100$, and 21.23 for $L=150$. This fact totally conforms with the mathematical analysis results of (25) and (26) which prove that the $S_{d}(F)$ values are analogous to $L$. Indeed, as the size $L$ increases, the total successfully transmitted bits on the data channels are getting more, providing higher throughput.
The throughput improvement dependance on $L$ is shown in Fig. 11 where the $\operatorname{Psi}(2)$ curves as a function of load are shown, when $M=100, N=60, F=2$ and $L=50,100,150$. Thus, for a given $G$, we notice that the throughput improvement as compared with the $S_{A}$, is an increasing function of $L$. So, when $G=0.5, \operatorname{Psi}(2)$ is: $118 \%$ for $L=50,358 \%$ for $L=100$, and $827 \%$ for $L=150$. This is understood since if no data channels organization strategy is followed, the $S_{A}$ decreases as the data packet size $L$ increases. Also, it is proven by (28) and explains that the probability of a data channels collision increases as $L$ increases too. In other words, the proposed access protocol grants higher throughput improvement as $L$ increases. As a result, the delay experienced is getting higher as $L$ increases. This fact is shown in Fig. 12 that presents the delay $D(2)$ as a function of load, when $M=100, N=60, F=2$ and $L=50,100$, 150. For example for $G=0.4$, the $D(2)$ is: 118 time units for $L=50,255$ time units for $L=100$, and 433 time units for $L=150$. Similar, for $G=0.6$, the $D(2)$ is: 176 time units for $L=50,381$ time units for $L=100$, and 647 time units for $L=150$. This is


Fig. 10. Throughput $\mathrm{S}_{\mathrm{d}}(2)$ vs load, $\mathrm{M}=100, \mathrm{~N}=60, \mathrm{~F}=2, \mathrm{~L}=50,100,150$.


Fig. 11. Throughput improvement $\mathrm{P}_{\mathrm{si}}(2)$ vs load, $\mathrm{M}=100, \mathrm{~N}=60, \mathrm{~F}=2$, $\mathrm{L}=50,100,150$.


Fig. 12. Delay $\mathrm{D}(2)$ vs load, $\mathrm{M}=100, \mathrm{~N}=60, \mathrm{~F}=2, \mathrm{~L}=50,100,150$.
understood since as $L$ increases the probability of a data packet collisions is getting higher giving rise to the total system delay.

The above result is very useful and could be exploited by optical LANs engineers working, for example, on intra-rack data centers networks ( DCNs ) where the packets size varies depending on the users applications supported. Specifically in an intra-rack DCN environment, the use of the proposed wavelength organization into sets strategy could essentially improve the throughput of the long packets traffic that corresponds to the users applications messages, as compared to the short or medium packets traffic that corresponds to control messages communication among the servers [14].

## V. Concluding Remarks

This paper focuses on the bandwidth exploitation criteria of passive optical WDM LANs. In the control plane, the proposed network configuration employs a separate control wavelength, while the rest of the wavelengths are used in parallel for the data communication. We adopt an effective pre-transmission coordination WDMA protocol of asynchronous transmission
providing a reliable and flexible access algorithm especially under high data rates, since it does not require any time synchronization for transmission. The proposed WDMA protocol suggests the data wavelengths organization into a number of sets, while the access rights over them are distributedly determined pursuing the total packet collisions avoidance on the data wavelengths and at destination.
By an accurate analysis, the throughput and the performance improvement are derived as closed mathematical formulas. The exhaustive numerical solutions demonstrate that the throughput improvement grace to the data channels organization strategy along with the BEA rises as the number of sets increases. Specifically, it reaches more than $100 \%$ by simply assuming two data channels sets, providing significant enhancement without requiring any additional expense and complexity. Also, the analytical results prove that the throughput improvement increases as the data packets size rises, providing a useful criterion for performance enhancement in optical intra-rack data centers networks which handle variable size packets traffic. Finally, the results could be a significant tool to optical LANs engineers to improve the network performance and to optimally serve modern bandwidth-demanding users applications.

## References

[1] Cisco Global Cloud Index: Forecast and Methodology, 2016-2021: Cisco Global Cloud Index: Forecast and Methodology, 2016-2021 White Paper.
[2] Google Cloud: Cloud Locations: Global Locations - Regions \& Zones | Google Cloud.
[3] D. Holz, P. Justus. (2021). Google invests 1 billion euros in Germany's digital future. Google Cloud. [Online].
Available: https://cloud.google.com/blog/products/infrastructure/google-invests-1-billion-euros-in-germanys-digital-future
[4] J. Baumgartner. (2021). AT\&T's fiber expansion could reach beyond 30 million locations. [Online]. Available:
https://www.lightreading.com/opticalip/atandts-fiber-expansion-could-reach-beyond-30-million-locations/d/d-id/772225
[5] E. Wong, M. Pubudini, I. Dias, L. Ruan, Predictive Resource Allocation for Tactile Internet Capable Passive Optical LANs, J. Lightwave Technol., Vol. 35, pp. 2629-2641, 2017, DOI: 10.1109/JLT.2017. 2654365.
[6] S. F. Habiby, Advances in WDM LAN Standards Development for Aerospace Applications, in Proc. of IEEE Conf. Avionics Fiber-Optics and Photonics, 2006, DOI: 10.1109/AVFOP.2006.1707480.
[7] A. Valkanis, P. Nicopolitidis, G. Papadimitriou, D. Kallergis, C. Douligeris, P. Bamidis, Efficient Resource Allocation in Tactile-Capable Ethernet Passive Optical Healthcare LANs, IEEE Access, Vol. 8, pp. 52981 - 52995, 2020, DOI: 10.1109/ACCESS.2020. 2980995.
[8] K. Kantelis, A. Valkanis, P. Nikopolitidis, G. Papadimitriou, D. Kallergis, C. Douligeris, P. Bamidis, Adaptive Protocol with Weights for Fair Distribution of Resources in Healthcare-Oriented PONs, in Proc. of 2020 Int. Conf. on Comp. Information and Telecom. Systems (CITS), 2020, DOI: 10.1109/CITS49457. 2020.9232638.
[9] G. Tziroglou, A. Valkanis, C. Kyriakopoulos, G. Papadimitriou, P. Nicopolitidis, On the use of prediction in Passive Optical LANs for healthcare latency-stringent applications, in Proc. of IEEE EUROCON 18th Int. Conf. on Smart Technol., 2019, DOI: 10.1109/EUROCON.2019. 8861580.
[10] W. Ni, C. Huang, Y. L. Liu, W. Li, K.W. Leong, and J. Wu, POXN: A new passive optical cross-connection network for low-cost powerefficient datacenters, J. Lightwave Technol., Vol. 32, No 8, pp. 14821500, 2014, DOI: 10.1109/JLT.2013.2295599.
[11] Y. Cheng, M. Fiorani, R. Lin, L. Wosinska, and J. Chen, POTORI: A passive optical top-of-rack interconnect architecture for data centers, J. Opt. Commun. Netw., Vol. 9, pp. 401-411, 2017, DOI: 10.1364/JOCN.9. 000401.
[12] Y. P. Cai, Z. Yao, T. Li, S. Luo, and L. Zhou, SD-MAC: Design and evaluation of a software-defined passive optical intrarack network in data centers, Trans. Emerging Telecomm. Technol., e3764, 2019, DOI: 10.1002/ett. 3764.
[13] Y. Cai, S. Luo, L. Zhou, Software defined passive optical intra-rack networks in data centers, in Proc. IEEE Globecom, 1 - 6, 2016, DOI: 10.1109/GLOCOMW.2016.7849048.
[14] Y. Zheng and X. Sun, Dual MAC Based Hierarchical Optical Access Network for Hyperscale Data Centers, IEEE J. Lightw. Techn., Vol. 38, No 7, pp. 1608-1617, 2020, DOI: 10.1109/JLT. 2019.2959882.
[15] I. Kaminow, T. Koch, Optical Fiber Telecommunications IIIA, New York: Academic, 1997.
[16] FS Europe, https://www.fs.com/de-en/c/400g-qsfp-dd-3652, accessed: 2022.
[17] Y. Kawamoto, N. Ohata, T. Murao, H. Sano, M. Binkai, Y. Imai, T. Hatta, K. Hasegawa, An integrated LAN-WDM 400-Gb/s (53Gbaud-PAM4) EML TOSA, in Proc. of 2019 24th OptoElectr. and Comm.. Conf. and 2019 Int. Conf. on Photon. in Switching and Computing, 2019, DOI: 10.23919/PS.2019.8817691.
[18] M. I. Habbab, M. Kavehrad, C.E.W. Sundberg, Protocols for very highspeed optical fiber local area networks using a passive star topology, J. Lightw. Technol., LT-5, pp. 1782-1794, 1987, DOI: 10.1109/JLT.1987. 1075467.
[19] P. A. Baziana: Exploring the optimum number of receivers per station in WDM networks, Computer Networks J., Vol. 143, pp. 153-165, 2018, DOI: 10.1016/j.comnet.2018.07.015.
[20] P. A. Baziana: Resource Allocation Mechanism for Efficiency Improvement in WDM Networks: An Analytical Comparison Study, Optical Switching and Networking J., Vol. 33, pp. 15-24, 2019, DOI: 10.1016/j.osn.2019.02.002.
[21] P. A. Baziana: Channel Grouping Architecture for Multi-Channel Networks, in Proc. of IEEE ICECCT 2017, 2017, DOI: 10.1109/ICECCT. 2017.8117936.
[22] I. E. Pountourakis, Performance Evaluation with Receiver Collisions Analysis in Very High-Speed Optical Fiber Local Area Networks Using Passive Star Topology, IEEE J. Lightw. Techn., Vol. 16, pp. 2303-2310, 1998, DOI: 10.1109/50.736596.


Peristera Baziana received her Dr.-Ing. degree in Electrical and Computer Engineering from the National Technical University of Athens (NTUA), Greece. She is currently serving as an Assistant Professor at the Department of Computer Science and Telecommunications, University of Thessaly, Greece. Her research interests include optical communications, optical networks architectures and transmission protocols, analytical modeling and optimization, and telecommunication networks simulation. She has several publications in international journals and proceedings of conferences with reviewers, related to these fields. During the period 20002002, she was a research fellow of the University of Patras, Greece, while from 2002 to 2015 she was a researcher of the NTUA. During the period 2015-2019 she was an adjunct lecturer at the Department of Engineering Informatics and Telecommunications, University of Western Macedonia, Greece and at the Department of Computer Science and Telecommunications, University of Thessaly, Greece.

