INTEGRATION OF PRICING AND INVENTORY DECISION IN A SUPPLY CHAIN UNDER VENDOR-MANAGED INVENTORY WITH DEFECTIVE ITEMS AND INSPECTION ERRORS: A GAME-THEORETIC APPROACH

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Abstract:

In this paper, the production-inventory-marketing model for a two-stage manufacturer-retailer supply chain under VMI policy with a price-sensitive demand is studied. An imperfect production at the manufacturer and inspection process involving with Type I and II errors at the retailer are considered. We assume that the manufacturer gives products to the retailer in a number of equal-sized shipments. This model is formulated as a Stackelberg game in which the retailer retains a certain degree of autonomy by reserving the right to choose the retail price and the manufacturer determines replenishment frequency, replenishment quantity and wholesale price. The critical supply chain decision factors including the manufacturer's wholesale price, the retailer's price, shipment frequencies and number of shipments are determined maximizing the total profit of each member of the supply chain. A solution procedure is proposed to find the Stackelberg game equilibrium. The performance of the model is assessed by a numerical example. The numerical shows that it is more beneficial for both the manufacturer and the retailer when the demand is less price sensitive.

1 Introduction

Increasing trend of the competition and globalization of the markets has made the managers of manufacturing organization to look at their supply chain for higher coordination. The aim of most researches in this area is to provide mechanism to reduce operational cost in the supply chain and providing high level of service to the costumers. Vendor managed inventory (VMI) is an approach of integrating supply chain, in which supplier is responsible for controlling and replenishing retail inventory [1, 2]. Efficient supply chain management requires accurate information about the chain; in this system, the supplier does not receive market demand only based on the retailer's orders; on the other hand, the supplier tries to collect demand information from the costumers directly [3].

In general, VMI has some advantages for retailers such as an increasing availability of goods and service level and reducing the costs of ordering. VMI has some advantages for suppliers such as reducing the bullwhip effect and better the production capacity use [4, 5]. An early conceptual framework for VMI was described by Magee [6] when discussing who should have authority over the control of inventories.

Razmi et al. [7] compared the supply chain performances of individual firms under the traditional and VMI arrangements. The results have shown that the VMI system works better than traditional mode in reducing the total inventory cost. Mateen and Chatterjee [8] developed an analytical model for various

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approaches through which a supply chain may be coordinated through VMI. Taleizadeh et al. [9] investigated two integrated vendor managed inventory systems under continuous review and periodic review replenishment policies by considering partial backordering and limited storage capacity at the buyer's side.

In competitive markets, prices are one of the most important factors for product selection by the costumers. The first model of this kind was formulated by Whitin [10] who incorporated pricing into the traditional economic order quantity (EOQ) model through a linear price sensitive relation for the end customers. Nachiappan and Jawahar [11] investigated a two-echelon single vendor – multiple buyer supply chain model under VMI. They found that the revenue sharing between the vendor and the buyer plays a vital role in fixing the contract price. Yu et al. [12] developed a VMI model with a price-sensitive demand. They developed a hybrid algorithm combining dynamic programming (DP), genetic algorithm (GA) and analytical methods in order to solve the problem. Taleizadeh and Noori-daryan [13] developed a multi-product economic production quantity (EPQ) model with price- sensitive demand and rework process in a three-level supply chain. They employed the Stackelberg game approach to analyze their model. Bieniek [14] studied retailer managed consignment inventory and vendor managed consignment inventory with additive price–dependent demand.

Product quality, as an important factor in customer purchasing decisions, has strategic role in today's competitive markets. Porteus [15] introduced the concept of quality control for the first time in production systems. Roy et al. [16] investigated EOQ model to minimize the expected total cost by consolidating a number of batches of imperfect quality products of different cycles for a single shipment. Lee and Kim [17] presented an integrated production–distribution model by considering multiple deliveries for items with imperfect quality and deteriorating characteristics. Shah et al. [18] developed an EPQ model under Price-sensitive stock-dependent demand by considering imperfect production. Gautam and Khanna [19] provided a sustainable framework under a supply chain environment by considering the carbon-emissions costs during the transportation process. The manufacturing system is imperfect, thus, the buyer employs the screening process. Khanna et al. [20] developed an integrated inventory model in which the production process is imperfect; thus, the vendor uses preventive maintenance and warranty policy for the efficient operation. Jaggi et al. [21] formulated inventory model for deterioration and imperfect products situation where trade credit and partial backlog scenario were considered.

In the real conditions, inspection process may have an error because of human mistakes or other factors. Yoo et al. [22, 23 and 24] investigated imperfect production and inspection processes incorporating Type I, II errors, investment on production and inspection reliability under one-time and continuous improvement and defective disposal with customers' return. Khan et al. [25] developed an EOQ model by considering imperfect production processes, inspection errors and sales returns. Hsu and Hsu [26] developed two EPQ-based models with imperfect production processes, inspection errors, backorders, and sales returns. Kishore et al. [27] investigated the effect of learning in set-up cost for imperfect production systems by utilizing two-way inspection plan for rework.

The concept of just in time (JIT) was introduced by Toyota Motor Corporation for the first time in the early 1980's, resulted in higher quality and lower cost for the company. Several studies have been done on its implementation and its impact on manufacturing companies of US from different dimensions [28]. Sajadieh and Jokar [29] developed an integrated production–inventory–marketing model under the assumption of price-sensitive demand. Darwish and Odah [30] developed a supply chain model under VMI setting where the production lot is shipped in a number of equal-size shipments. Lin et al. [31] proposed an inventory model in which both the supplier and the retailer have adopted trade credit policies in which the retailer receives an arriving lot with some defective items. Hariga et al. [32] extended the VMI model of Darwish and Odah [30] to include unequal replenishment intervals. Kumar and Uthayakumar [33] introduced five different shipment policies in a supply chain under VMI.

In recent decades, game theory has been recognized as an applicable tool for supply chain management. Yu et al. [34] presented a supply chain model under VMI with a price-sensitive demand. They utilized Stackelberg approach to formulate the VMI contract. Kim and Park [35] presented a supply chain management model under VMI strategy, where the retailer decides the retail price while the vendor determines its capacity commitment. A system dynamics simulation approach based on game theory is used to solve the problem. Li et al. [36] introduced a two-period supply chain involving price-dependent demands and a linear stochastic cost-learning curve. They employed a Stackelberg game.

Considering the aforementioned researches, despite the development of many theoretical models which have been proposed in two stages supply chain under VMI policy, analysis of decentralized supply chain under VMI policy with defective items and inspection errors have not been studied. The paper discusses a market channel in supply chain under VMI policy assuming that the production process at the manufacturer is imperfect. An imperfect inspection process based on a typical entire lot screening at the retailer is considered. Non-delayed equal-sized shipment policy is used in order to deliver product batches to the retailer's side. This study applies a Stackelberg game approach between the existing members of the given supply chain to find the Stackelberg game equilibrium in order to maximize the total profit of each member.

The remainder of this paper is organized as follows: in Section 2, the assumptions and notations used in the paper are given. The mathematical formulation is presented in Section 3. In Section 4, the solution procedure is proposed in order to find the Stackelberg game equilibrium. In Section 5, a numerical example and the sensitivity analysis is given to analyze the effects of parameters on the decisions variables, retailer and manufacturer's profit. Finally, conclusions is stated in Section 6.

2 Notations and assumptions

2.1 Notation

The notations which are used in this paper are as follows:

δ: Unit retail price (*decision variable*)
Q: Size of a shipment from the manufacturer to the retailer per replenishment cycle (*decision variable*)
n: Number of shipments from manufacturer to the retailer per production cycle (*decision variable*)
C_B: Unit wholesale price at the manufacturer (\$/unit) (*decision variable*)
$D(\delta)$: Demand rate which is a linear function of retailing price
A: Intercept value of demand function
B: Slope of demand function
P: Production rate of the manufacturer
A_S: Setup cost at the manufacturer's site (\$/setup)
A_B: Ordering cost at the retailer (\$/order)
F: Transportation cost per shipment from the manufacturer to the retailer (\$/delivery)
C_S: Unit production cost at the manufacturer (\$/unit)
ω: Unit inventory holding cost per unit time at the retailer
h_S: Unit inventory holding cost per unit time at the manufacturer
h_{B1}: Unit inventory holding cost of good quality items per unit time at the retailer
h_{B2}: Unit inventory hold cost of defective items per unit time at the retailer
y: Percentage of defective items in each lot size of Q, ($0 \le y < 1$)
x: Screening rate per unit time ($x > D$)
s: Unit inspecting cost at the manufacturer
E_1: Type I error (falsely rejecting a non-defective item)
E_2: Type II error (falsely accepting a defective item)
B_1: Number of defective items observed by the manufacturer through screening
B_2: Number of defective items which are returned by customers
v_1: Unit inspection cost of returned items at the manufacturer
v_2: Unit disposal cost at the manufacturer
θ: Unit selling price of poor quality items at the manufacturer
TP_S: Total profit of the manufacturer
TP_B: Total profit of the retailer

2.2 Assumption

We have develop the model considering the following assumptions:

1. The single product supply chain consists of a single manufacture and a single retailer operating under VMI contract. According to the VMI contract, the manufacturer is responsible for managing the inventory at the retailer's site.
2. Customers' demand are satisfied by the retailer; customers' demand is assumed to be a function of retailer price.
3. The manufacturer adopts a continuous review inventory policy; furthermore, it applies "non-delayed equal-sized shipment policy" in order to deliver batches to the retailer's side. The manufacturer produces a batch size of nQ at each production setup and delivers it to the retailer in n batch sizes of Q.
4. The production process is imperfect; the produced items are inspected and both Type I and II errors are existed in the inspection process.
5. The shortage is not authorized at the retailer's side; thus, the numbers of good items have to be at least equal to the sum of the demands at the retailer.
6. The shortage is not authorized at the manufacturer's site; thus, the production time for a lot size of Q has to be less than the replenishment cycle, i.e. $D(\delta)<P(1-y)(1-E_1)$.
7. The returned items by the customer and the items that are classified as defectives are sent back to the manufacturer.
8. The returned items to the manufacturer's side are inspected and the inspection time is assumed to be negligible. Among all returned items, Qy units are useless, thus the manufacturer disposes them with a fixed cost of υ_2. On the other hand, $Q(1-y)E_1$ units which were classified as defectives are sold as poor quality items at unit price of θ at secondary markets.

3 Model formulation

3.1 The retailer's inventory model

Considering the demand as a linear function of retail price, it can be computed as in Eq. (1).

$$D(\delta) = a - b\delta \tag{1}$$

Since D (δ)>0, Constraint (2) gives the maximum retail price.

$$\delta < a / b \tag{2}$$

The retailer's net profit equals to the retailer's revenue minus the sum of retailer's procurement and inventory holding costs paid to the manufacturer to manage its inventory. As a result, the retailer's net profit function can be given as in Eq. (3).

$$\begin{aligned} Max \quad & TP_B(\delta) = (\delta - C_B - \omega)D(\delta) \\ S.t. \quad & D(\delta) = a - b\delta \\ & \delta < a / b \end{aligned} \tag{3}$$

3.2 The manufacturer's inventory model

The manufacturer's net profit equals to the manufacturer's revenue minus the total cost which consists of two parts; the first is the total direct cost of manufacturer which includes the total cost at manufacturer's side and the second is the total indirect cost of manufacturer. According to the VMI, the manufacturer is responsible for supply chain and managing the inventory at the retailer's side. The indirect cost originates from managing the retailer's inventory. The manufacturer's net profit function can be given as in (4).

$$\begin{aligned} &Max \quad TP_S\left(\mathrm{Q}, \mathrm{n}, \mathrm{C_B}\right) = r - C_1 - C_2 \\ &S.t. \quad Q > 0 \\ &\qquad\quad \mathrm{C_B} > 0 \\ &\qquad\quad \mathrm{n} \ \text{integer} \end{aligned} \tag{4}$$

3.2.1 The manufacturer's revenue

According to the given VMI strategy defined in Section 2, the retailer bears his inventory cost only depending on his demand rate by repaying it to the manufacturer. The revenue of the manufacturer consists of three components: the selling of product to retailer at wholesale price, the inventory holding cost which is paid by the retailer to the manufacturer and obtained revenue from selling returned items at secondary market; thus, the revenue of the manufacturer can be obtained from Eq. (5).

$$r = D(\delta)(\mathrm{C_B} + \omega + \frac{\theta\, E_1}{1 - E_1}) \tag{5}$$

3.2.2 The direct inventory cost of the manufacture

The direct inventory cost of the manufacturer is the total cost at manufacturer's side including setup cost, manufacturing cost, inventory holding cost at manufacturer's side, inspection cost of the returned items and disposal cost of defective items. Figure 1 shows the inventory variations at manufacturer's side.

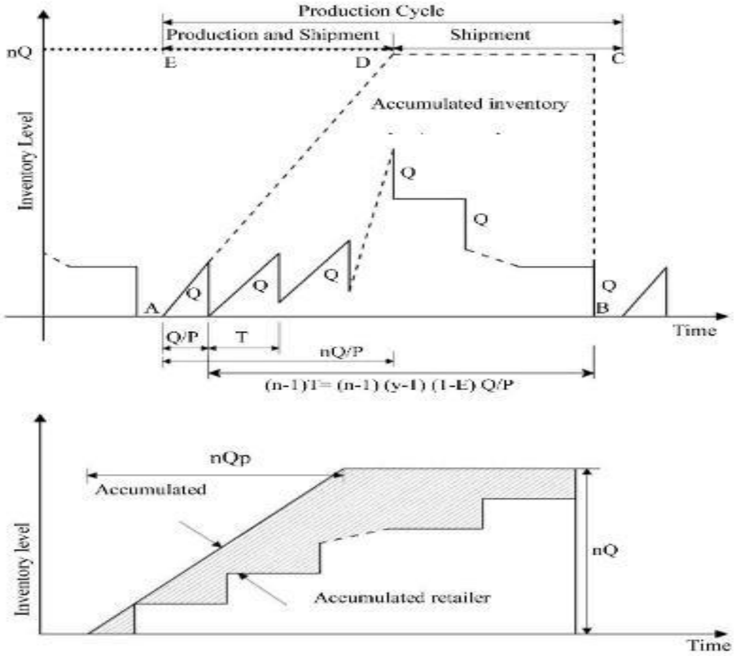

Figure 1. Manufacturer inventory level.

By considering Figure 1, each jump in the section represented by T shows that a shipment with size of *Q* has been delivered to the retailer. In addition, the first shipment from the manufacturer to the retailer takes place as soon as the required shipment quantity, *Q*, is produced. The number of these jumps is equal to *n*. Moreover, during this time, the retailer consumes the delivered products at constant rate.
The accumulation of the manufacturer's inventory during the production cycle, shown by the shaded area in Figure 1, is computed by Eq. (6).

$$S_{ABCE} - S_{ADE} - T\left[Q + 2Q + ... + (n-1)Q\right] \tag{6}$$

The Manufacturer's unit inventory holding cost per unit time and production cycle are h_S and $nT = \frac{Q(1-y)(1-E_1)}{nD(\delta)}$. The manufacturer's inventory holding cost can be given as in (7).

$$\frac{[S_{ABCE} - S_{ADE} - T[Q + 2Q + ... + (n-1)Q]]h_S}{nT} = \frac{\left(\begin{array}{l}\left[nQ(\frac{Q}{P} + (n-1)\frac{(1-y)(1-E_1)Q}{D}) - \frac{n^2Q^2}{2P}\right] \\ -\frac{(1-y)(1-E_1)Q}{D}[Q + 2Q + ... + (n-1)Q]\end{array}\right)h_S}{\frac{n(1-y)(1-E_1)Q}{D}} = \frac{h_S\,\mathrm{Q}}{2}\left[\mathrm{n}(1 - \frac{\mathrm{D}(\delta)}{\mathrm{P}(1-\mathrm{y})(1-\mathrm{E}_1)}) - 1 + \frac{2\mathrm{D}(\delta)}{\mathrm{P}(1-\mathrm{y})(1-\mathrm{E}_1)}\right] \tag{7}$$

Since the shortage is not allowed in the retailer's side, the number of good items has to be at least equal to the sum of the demand and the items that are replaced with the returned items by customer over replenishment cycle.

Thus:

$$\begin{aligned} &Q - Q(1-y)E_1 - Qy(1-E_2) \geq D(\delta)T + QyE_2 \\ &Q(1-y) - Q(1-y)E_1 \geq D(\delta)T \\ &Q(1-y)(1-E_1) \geq D(\delta)T \end{aligned} \tag{8}$$

Therefore, the replenishment and manufacturing cycle length can be obtained from (9) and (10).

$$T = \frac{Q(1-y)(1-E_1)}{D(\delta)} \tag{9}$$

$$nT = \frac{Q(1-y)(1-E_1)}{nD(\delta)} \tag{10}$$

Then, the direct inventory cost of the manufacturer per unit time can be calculated as (11).

$$\begin{aligned} C_1 = &\frac{\mathrm{D}(\delta)A_S}{\mathrm{nQ}(1-\mathrm{y})(1-\mathrm{E}_1)} + \frac{C_S\,\mathrm{D}(\delta)}{(1-\mathrm{y})(1-\mathrm{E}_1)} \\ &+ \frac{h_S\,\mathrm{Q}}{2}\left[\mathrm{n}(1 - \frac{\mathrm{D}(\delta)}{\mathrm{P}(1-\mathrm{y})(1-\mathrm{E}_1)}) - 1 + \frac{2\mathrm{D}(\delta)}{\mathrm{P}(1-\mathrm{y})(1-\mathrm{E}_1)}\right] \\ &+ \frac{\nu_1(B_1 + B_2)D(\delta)}{Q(1-y)(1-E_1)} + \frac{\nu_2\,\mathrm{y}\,\mathrm{D}(\delta)}{(1-\mathrm{y})(1-\mathrm{E}_1)} \end{aligned} \tag{11}$$

Considering inspection error of type I, the number of good items which are incorrectly recognized as defectives are equal to $Q(1\text{-}y)E_1$. Considering inspection error of type II, the number of defective items which are recognized as defectives are equal to $Qy(1\text{-}E_2)$. Hence, the number of items which are recognized as defectives can be calculated as in Eq. (12):

$$B_1 = Q(1-y)E_1 + Qy(1-E_2) \tag{12}$$

Considering inspection error of type II, the number of defective items which are incorrectly recognized as good items are equal to QyE_2, and therefore what is returned by the customers can be given as in Eq. (13).

$$B_2 = QyE_2 \tag{13}$$

Substituting Eq. (12) and (13) into Eq. (11), the direct inventory cost of manufacturer per unit time can be given by (14).

$$
\begin{aligned}
C_1 = {} & \frac{\mathrm{D}(\delta)A_S}{\mathrm{nQ}(1-\mathrm{y})(1-\mathrm{E}_1)} + \frac{C_S\,\mathrm{D}(\delta)}{(1-\mathrm{y})(1-\mathrm{E}_1)} \\
& + \frac{h_S\,\mathrm{Q}}{2}\left[\mathrm{n}\left(1-\frac{\mathrm{D}(\delta)}{\mathrm{P}(1-\mathrm{y})(1-\mathrm{E}_1)}\right)-1+\frac{2\mathrm{D}(\delta)}{\mathrm{P}(1-\mathrm{y})(1-\mathrm{E}_1)}\right] \\
& + \frac{v_1E_1D(\delta)}{(1-E_1)} + \frac{v_1\mathrm{y}\,\mathrm{D}(\delta)}{(1-\mathrm{y})(1-\mathrm{E}_1)} + \frac{v_2\mathrm{y}\,\mathrm{D}(\delta)}{(1-\mathrm{y})(1-\mathrm{E}_1)}
\end{aligned}
\tag{14}
$$

3.2.3 *The manufacture's indirect cost*

The manufacturer's indirect cost is the cost of retailer's inventory management which includes the ordering cost, the inspection cost, the transportation cost, and defective and good items holding costs. The behaviour of on-hand inventory level over time is depicted in Figure 2.

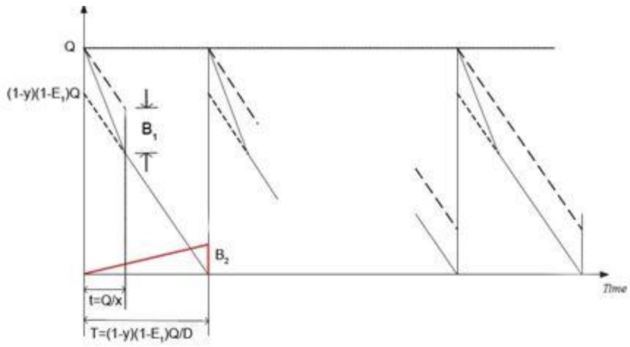

Figure 2. Manufacturer inventory level.

The retailer receives an arriving lot size including defective items with defective rate of *y*; all the items in the addressed lot are inspected with screening rate of *x* during the inspection time. The screening process is also imperfect because an inspector may incorrectly recognize non-defective item as defective one, or a defective item as non-defective one.

From Figure 2, the inventory holding cost at retailer's side can be divided into two components:

- The inventory holding cost of items which are classified as good items until the end of the replenishment cycle and the holding cost of items which are classified as defective items over screening time can be obtained from Eq. (15).

$$
\begin{aligned}
& \frac{nh_{B1}}{nT}\left(\frac{1}{2}(\mathrm{Q}-\mathrm{B}_1)T+\frac{1}{2}B_1\left(\frac{Q}{X}\right)\right)= \\
& h_{B1}\left(\frac{\mathrm{Q}-\mathrm{B}_1}{2}+\frac{\mathrm{D}(\delta)\mathrm{B}_1}{2\mathrm{x}(1-\mathrm{y})(1-\mathrm{E}_1)}\right)
\end{aligned}
\tag{15}
$$

- The inventory holding cost of items that are classified as defective items after screening time until the end of the replenishment cycle and the holding cost of defective items that are returned by the customer can be obtained from Eq. (16).

$$
\begin{aligned}
& \frac{nh_{B2}}{nT}\left(\mathrm{B}_1T-\frac{1}{2}B_1\left(\frac{Q}{X}\right)+\frac{1}{2}\mathrm{B}_2T\right)= \\
& h_{B2}\left(\mathrm{B}_1-\frac{\mathrm{D}(\delta)\mathrm{B}_1}{2\mathrm{x}(1-\mathrm{y})(1-\mathrm{E}_1)}+\frac{\mathrm{B}_2}{2}\right)
\end{aligned}
\tag{16}
$$

Hence, the manufacturer's indirect inventory cost per unit time can be formulated as in Eq. (17).

$$C_2 = \frac{D(\delta)A_B}{Q(1\text{-}y)(1\text{-}E_1)} + \frac{S\ D(\delta)}{(1\text{-}y)(1\text{-}E_1)} + \frac{D(\delta)F}{Q(1\text{-}y)(1\text{-}E_1)} + h_{B1}\left(\frac{Q\text{-}B_1}{2} + \frac{D(\delta)B_1}{2x(1\text{-}y)(1\text{-}E_1)}\right) + h_{B2}\left(B_1 \text{-} \frac{D(\delta)B_1}{2x(1\text{-}y)(1\text{-}E_1)} + \frac{B_2}{2}\right) \tag{17}$$

Substituting Eq. (12) and (13) into Eq. (17), the manufacturer's indirect inventory cost per unit time is given by Eq. (18).

$$C_2 = \frac{D(\delta)A_B}{Q(1\text{-}y)(1\text{-}E_1)} + \frac{S\ D(\delta)}{(1\text{-}y)(1\text{-}E_1)} + \frac{D(\delta)F}{Q(1\text{-}y)(1\text{-}E_1)} + h_{B1}\left(\frac{Q(1\text{-}y)(1\text{-}E_1) + QyE_2}{2} + \frac{D(\delta)QE_1}{2x(1\text{-}E_1)} + \frac{D(\delta)Qy(1\text{-}E_2)}{2x(1\text{-}y)(1\text{-}E_1)}\right) + h_{B2}\left(Q(1-y)E_1 + Qy(1-E_2) \text{-} \frac{D(\delta)QE_1}{2x(1\text{-}E_1)} - \frac{D(\delta)Qy(1\text{-}E_2)}{2x(1\text{-}y)(1\text{-}E_1)} + \frac{QyE_2}{2}\right) \tag{18}$$

4 Solution Procedure

4.1 Stackelberg game

The Stackelberg game is a dynamic game with perfect information. In this game, the leader (manufacturer) will take into account the follower's best-response when choosing his decision and the follower (retailer) makes her decision subject to the leader's choice.

4.2 The best reaction of the retailer

Substituting D (δ) =a-bδ into the Eq. (3), the total profit of the retailer is obtained from Eq. (19).

$$Max \quad TP_B(\delta) = (\delta - C_B - \omega)(a - b\delta) \tag{19}$$

Taking the first and second partial derivatives of Eq. (19) with respect to δ, we have:

$$\frac{\partial^2 TP_B(\delta)}{\partial \delta^2} = -2b < 0 \tag{20}$$

The retailer's profit function is strictly concave in the retail price; letting the first derivative equal to zero as in Eq. (21), we can obtain optimal retail price from Eq. (22).

$$\frac{\partial TP_B(\delta)}{\partial \delta} = 0 \tag{21}$$

$$\delta^* = \frac{a}{2b} + \frac{(C_B + \omega)}{2} \tag{22}$$

4.3 The manufacturer's decisions

The manufacturer makes his decision by considering the best reaction function of the retailer. Therefore the manufacturer's model can be reformulated as in (23).

$$
\begin{aligned}
Max\ TP_S(\mathrm{Q},\mathrm{n},\mathrm{C_B}) = {} & (\mathrm{C_B} + \omega + \frac{\theta E_1}{1-E_1})\mathrm{D}(\delta^*(\mathrm{C_B})) \\
& - \frac{\mathrm{D}(\delta^*(\mathrm{C_B}))A_S}{\mathrm{nQ(1-y)(1-E_1)}} - \frac{h_S\,\mathrm{Q}}{2}\begin{bmatrix} \mathrm{n}(1 - \frac{\mathrm{D}(\delta^*(\mathrm{C_B}))}{\mathrm{P(1-y)(1-E_1)}}) - 1 \\ + \frac{2\mathrm{D}(\delta^*(\mathrm{C_B}))}{\mathrm{P(1-y)(1-E_1)}} \end{bmatrix} \\
& - \frac{C_S\,\mathrm{D}(\delta^*(\mathrm{C_B}))}{\mathrm{(1-y)(1-E_1)}} - \frac{v_1 E_1 D(\delta^*(\mathrm{C_B}))}{(1-E_1)} \\
& - \frac{(v_1+v_2)\mathrm{y\,D}(\delta^*(\mathrm{C_B}))}{\mathrm{(1-y)(1-E_1)}} - \frac{S\,\mathrm{D}(\delta^*(\mathrm{C_B}))}{\mathrm{(1-y)(1-E_1)}} \\
& - h_{B1}\begin{bmatrix} \frac{\mathrm{Q(1-y)(1-E_1)} + QyE_2}{2} \\ + \frac{\mathrm{D}(\delta^*(\mathrm{C_B}))\mathrm{QE_1}}{\mathrm{2x(1-E_1)}} + \frac{\mathrm{D}(\delta^*(\mathrm{C_B}))\mathrm{Qy(1-E_2)}}{\mathrm{2x(1-y)(1-E_1)}} \end{bmatrix} \\
& - h_{B2}\begin{bmatrix} Q(1-y)E_1 + Qy(1-E_2) - \frac{\mathrm{D}(\delta^*(\mathrm{C_B}))\mathrm{QE_1}}{\mathrm{2x(1-E_1)}} \\ - \frac{\mathrm{D}(\delta^*(\mathrm{C_B}))\mathrm{Qy(1-E_2)}}{\mathrm{2x(1-y)(1-E_1)}} + \frac{QyE_2}{2} \end{bmatrix} \\
& - \frac{\mathrm{D}(\delta^*(\mathrm{C_B}))A_B}{\mathrm{Q(1-y)(1-E_1)}} - \frac{\mathrm{D}(\delta^*(\mathrm{C_B}))F}{\mathrm{Q(1-y)(1-E_1)}} \\
S.t. \quad & Q > 0 \\
& \mathrm{C_B} > 0 \\
& \mathrm{n\ \ integer}
\end{aligned}
\tag{23}
$$

Taking the first and second partial derivatives with respect to Q, we have:

$$
\begin{aligned}
\frac{\partial^2 TP_S(Q,n,C_B)}{\partial Q^2} = {} & -\frac{2\mathrm{D}(\delta^*(\mathrm{C_B}))A_S}{\mathrm{nQ^3(1-y)(1-E_1)}} - \\
& \frac{2\mathrm{D}(\delta^*(\mathrm{C_B}))F}{\mathrm{Q^3(1-y)(1-E_1)}} - \frac{2\mathrm{D}(\delta^*(\mathrm{C_B}))A_\mathrm{B}}{\mathrm{Q^2(1-y)(1-E_1)}} < 0
\end{aligned}
\tag{24}
$$

The profit function of manufacturer is strictly concave with the size of shipment; consequently, the optimal values can be obtained considering (25) and (26).

$$
\frac{\partial TP_S(Q,n,C_B)}{\partial Q} = 0 \tag{25}
$$

$$
Q^* = \sqrt{\frac{\frac{2\mathrm{D}(\delta^*(\mathrm{C_B}))}{\mathrm{(1-y)(1-E_1)}}[\frac{A_S}{n} + A_B + F]}{G}} \tag{26}
$$

Where:

$$G = h_S \begin{bmatrix} \mathrm{n}(1 - \dfrac{\mathrm{D}(\delta^*(\mathrm{C_B}))}{\mathrm{P}(1-\mathrm{y})(1-\mathrm{E}_1)}) - 1 \\ + \dfrac{2\mathrm{D}(\delta^*(\mathrm{C_B}))}{\mathrm{P}(1-\mathrm{y})(1-\mathrm{E}_1)} \end{bmatrix} + h_{B1} \begin{bmatrix} (1-\mathrm{y})(1-\mathrm{E}_1) + yE_2 + \dfrac{\mathrm{D}(\delta^*(\mathrm{C_B}))\mathrm{E}_1}{\mathrm{x}(1-\mathrm{E}_1)} \\ + \dfrac{\mathrm{D}(\delta^*(\mathrm{C_B}))\mathrm{y}(1-\mathrm{E}_2)}{\mathrm{x}(1-\mathrm{y})(1-\mathrm{E}_1)} \end{bmatrix} - h_{B2} \begin{bmatrix} 2(1-y)E_1 + Qy(1-E_2) - \dfrac{\mathrm{D}(\delta^*(\mathrm{C_B}))\mathrm{E}_1}{\mathrm{x}(1-\mathrm{E}_1)} \\ - \dfrac{\mathrm{D}(\delta^*(\mathrm{C_B}))\mathrm{y}(1-\mathrm{E}_2)}{\mathrm{x}(1-\mathrm{y})(1-\mathrm{E}_1)} + yE_2 \end{bmatrix} \tag{27}$$

By substituting the optimal values of *Q* into Eq. (23), the total profit of the manufacturer can be obtained from (28).

$$\begin{aligned} Max\, TP_S(\mathrm{n}, \mathrm{C_B}) = {} & (\mathrm{C_B} + \omega + \frac{\theta E_1}{1-E_1})\mathrm{D}(\delta^*(\mathrm{C_B})) \\ & - \frac{C_S\, \mathrm{D}(\delta^*(\mathrm{C_B}))}{(1-\mathrm{y})(1-\mathrm{E}_1)} - \frac{\nu_1 E_1 D(\delta^*(\mathrm{C_B}))}{(1-E_1)} \\ & - \frac{(\nu_1+\nu_2)\mathrm{y}\, \mathrm{D}(\delta^*(\mathrm{C_B}))}{(1-\mathrm{y})(1-\mathrm{E}_1)} - \frac{S\, \mathrm{D}(\delta^*(\mathrm{C_B}))}{(1-\mathrm{y})(1-\mathrm{E}_1)} \\ & - \sqrt{\frac{2\mathrm{D}(\delta^*(\mathrm{C_B}))}{(1-\mathrm{y})(1-\mathrm{E}_1)}} \sqrt{[\frac{A_S}{n} + A_B + F]G} \end{aligned} \tag{28}$$

From Eq. (28), it is clear that only wholesale price and the number of shipment decision variables remain to be calculated; the following procedure is proposed in order to find them.
Under this condition, for any fixed C_B, the optimal value of *n* can be uniquely determined by the following condition. Maximizing the profit of manufacturer is equivalent to minimizing the following expression.

$$\begin{aligned} & Min[\frac{A_S}{n} + A_B + F]G = \\ & Min[A_B + F][h_S \begin{bmatrix} \text{-}1 + \dfrac{2\mathrm{D}(\delta^*(\mathrm{C_B}))}{\mathrm{P}(1\text{-y})(1\text{-E}_1)} \end{bmatrix} \\ & + h_{B1} \begin{bmatrix} (1\text{-y})(1\text{-E}_1) + \mathrm{yE}_2 + \dfrac{\mathrm{D}(\delta^*(\mathrm{C_B}))\mathrm{E}_1}{\mathrm{x}(1\text{-E}_1)}) \\ + \dfrac{\mathrm{D}(\delta^*(\mathrm{C_B}))\mathrm{y}(1\text{-E}_2)}{\mathrm{x}(1\text{-y})(1\text{-E}_1)} \end{bmatrix} \\ & + h_{B2} \begin{bmatrix} 2(1-y)E_1 + 2y(1-E_2)\text{-}\dfrac{\mathrm{D}(\delta^*(\mathrm{C_B}))\mathrm{QE}_1}{\mathrm{x}(1\text{-E}_1)} \\ - \dfrac{\mathrm{D}(\delta^*(\mathrm{C_B}))\mathrm{y}(1\text{-E}_2)}{\mathrm{x}(1\text{-y})(1\text{-E}_1)} + yE_2 \end{bmatrix}] \\ & + A_S h_S (1\text{-}\frac{\mathrm{D}(\delta^*(\mathrm{C_B}))}{\mathrm{P}(1\text{-y})(1\text{-E}_1)}) + Z(n) \end{aligned} \tag{29}$$

In which:

$$Z(n)=[A_B+F]h_S n(1-\frac{D(\delta^*(C_B))}{P(1-y)(1-E_1)})+\frac{A_S}{n}V \tag{30}$$

And:

$$\begin{aligned} V = {} & h_S\left[-1+\frac{2D(\delta^*(C_B))}{P(1-y)(1-E_1)}\right] \\ & + h_{B1}\left[\begin{array}{l}(1-y)(1-E_1)+yE_2+\frac{D(\delta^*(C_B))E_1}{x(1-E_1)} \\ +\frac{D(\delta^*(C_B))y(1-E_2)}{x(1-y)(1-E_1)}\end{array}\right] \\ & + h_{B2}\left[\begin{array}{l}2(1-y)E_1+2y(1-E_2)-\frac{D(\delta^*(C_B))QE_1}{x(1-E_1)} \\ -\frac{D(\delta^*(C_B))y(1-E_2)}{x(1-y)(1-E_1)}+yE_2\end{array}\right] \end{aligned} \tag{31}$$

Since *Z(n)* only depends on *n*, Maximizing the manufacturer's profit function in Eq. (28) can be equivalent to minimizing *Z(n)* in Eq. (30). Since *n* is a discrete positive integer, we use the derivative in order to calculate the optimal value of *n*. In this case two scenarios may be come across:

1. If

$$V\le 0 \quad \Rightarrow \quad Z(n+1)-z(n)<0 \tag{32}$$

Where:

$$Z(n+1)-z(n)=[A_B+F]h_S n(1-\frac{D(\delta^*(C_B))}{P(1-y)(1-E_1)})-\frac{A_S V}{n(n+1)} \tag{33}$$

No shortage is authorized in the manufacturer's side and also from condition $V\le 0$, *Z(n)* is an increasing function of *n* and the optimal solution of the shipment number is $n^*=1$.

2. If

$$V>0 \;\Rightarrow\; [Z(n+1)-z(n)]-[Z(n)-Z(n-1)]>0 \tag{34}$$

Where:

$$[Z(n+1)-z(n)]-[Z(n)-Z(n-1)]=\frac{2A_S V}{n(n-1)} \tag{35}$$

From the condition $V>0$, it is known that *Z(n)* is convex with respect to *n* and n^* is the minimum shipment number enough for *Z(n)*.

For any given n, the optimal *n* can be determined by the condition given in (36).

$$\begin{cases} n^*(n^*-1)<\dfrac{A_S V}{[A_B+F]h_S n(1-\frac{D(\delta^*(C_B))}{P(1-y)(1-E_1)})}\le n^*(n^*+1) & V>0 \\ n^*=1 & V\le 0 \end{cases} \tag{36}$$

For any given *n*, Eq. (28) is continuous function of   and the maximum value of Eq. (28) can be obtained; for a fixed value of *n*, the optimal wholesale price can be obtained from (37).

$$\frac{\partial TP_S(C_B)}{\partial C_B} = 0 \tag{37}$$

The optimal value of C_B must satisfy the following conditions:

$$\delta^* < a/b \quad \Rightarrow \quad \frac{a}{2b} + \frac{C_B^* + \omega}{2} < \frac{a}{b} \quad \Rightarrow \quad C_B^* < (\frac{a}{b} - \omega) \tag{38}$$

According to Eq. (19), TP_B is a decreasing function of C_B; thus, if there are two critical points for C_B represented by $C_{B1}^* > C_{B2}^*$, the manufacturer, as the leader knows $TP_B^*(C_{B1}^*) < TP_B^*(C_{B2}^*)$, and C_{B2}^* is selected as the final optimal solution. Therefore, the unique Stackelberg equilibrium is obtained as in [34].

4.4 Algorithm procedure

The following solution procedure is applied to find the optimal values for the model developed (See Yu et al. [34] for a similar solution methodology):

- Step 1: Initialize $n=1$, obtain the wholesale price by solving Eq. (37) and check whether (38) is satisfied. By substituting C_B into Eq. (32), if it is satisfied and $Z(n+1)-Z(n) < 0$, then n^* *and* C_B^* are obtained and go to Step 4; otherwise, go to Step 2.
- Step 2: Compute C_B for $n=2$ using Eq. (37), and check whether (38) is satisfied. By substituting C_B into Eq. (34), if it is satisfied, then n^* *and* C_B^* are obtained and go to Step 4; otherwise, go to Step 3.
- Step 3: Set $n = n + 1$, and repeat Steps 2.
- Step 4: Utilize Eq. (28) to calculate $TP_B^*(n, C_B)$, Eq. (27) to calculate Q^* and Eq. (22) to calculate δ^*.

5 Numerical example and sensitivity analysis

To illustrate the results obtained from the model developed in this paper, we applied the following numerical example. Referring to existing literature, we consider an example with the following data that is given in Table 1.

Table 1. Values of input parameters.

a	b	P	A_S	A_B
50000	1000	60000	400	100
F	C_S	θ	ω	S
25	14	16	8	0.5
x	**y**	**h_S**	**h_{B1}**	**h_{B2}**
87600	0.01	3	5	2
v_1	**v_2**	**E_1**	**E_2**	
3	2	0.01	0.01	

Result of the solution procedure for the illustrative example is given in Table 2.

Table 2. Optimal results.

Q	n	C_B	δ	TP_S	TP_B
860.55	2	24.164	41.082	148745.29	79530.72

In order to find the effectiveness of model's relevant parameters (E_2, E_1, y, b), sensitivity analysis is done and the results are given in Table 3.

5.1 Sensitivity analysis of inspection type I, II error parameters

We understand from Table 3 that the chain members' profit is sensitive to the change in inspection type I error so that the by increasing *E1*, the manufacturer's and retailer's profits are decreased. As the optimal values increase, the optimal number of shipments does not vary. Inspection type II error has little influence on the chain members' profit and optimal values so that the total profit doesn't dramatically change.

Table 3. Sensitivity analysis.

	Value	Q	N	C_B	δ	TP_S	TP_B
b	500	951.91	2	49.18	78.59	447066.59	229183.34
	750	902.53	2	32.51	53.59	245816.23	128316.44
	1250	816.55	2	19.15	33.58	93355.84	51568.66
	1000	860.55	2	24.164	41.082	148745.29	79530.72
	1500	770.27	2	15.81	28.57	58815.84	34024.49
y	0.01	860.55	2	24.164	41.082	148745.29	79530.72
	0.02	860.19	2	24.264	41.132	146946.4	78641.42
	0.05	869.656	2	24.577	41.288	141390.01	75890.23
	0.1	882.167	2	25.145	41.571	131577.07	71022.76
E_1	0.01	860.55	2	24.164	41.082	148745.29	79530.72
	0.02	865.09	2	24.172	41.086	148560.36	79459.39
	0.05	879.15	2	24.196	41.098	147983.21	79245.6
	0.1	903.92	2	24.24	41.12	146942.09	78854.4
E_2	0.01	860.55	2	24.164	41.082	148745.29	79530.72
	0.02	860.53	2	24.164	41.082	148745.17	79530.72
	0.05	860.49	2	24.164	41.082	148744.82	79530.72
	0.1	860.42	2	24.164	41.082	148744.13	79530.72

5.2 Sensitivity analysis of demand Slope parameter

According to Table 3 and Figure 3, Slope of demand parameter has significant influence on the manufacturer's and his retailer's profit so that the decrease of demand Slope by 25% (retailer's demands becomes less sensitive to his prices) the manufacturer's and retailer's profits increase by 60%. Furthermore, it is observed that increasing *b*, decreases the optimal value of decision variables.

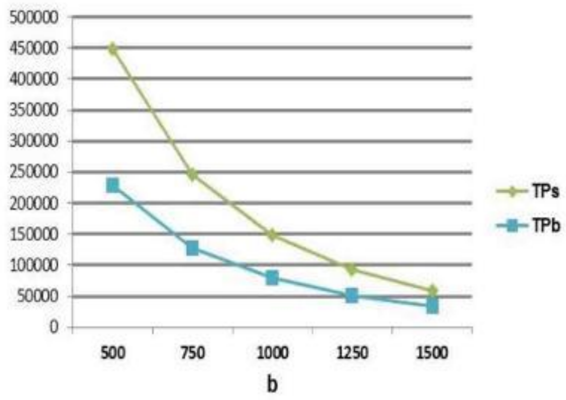

Figure 3. The variations of demand Slope.

5.3 Sensitivity analysis of Percentage of defective items parameter

Based on Table 3, increasing the percentage of defective items leads to increase in the optimal values of size of a shipment, wholesale price and retailer price and does not have any effect on the optimal number of shipments. Figure 4 represents the relationship between percentages of defective items and the chain members' profit. It is clear from the figure that the percentage of defective items has an inverse relationship with inventory.

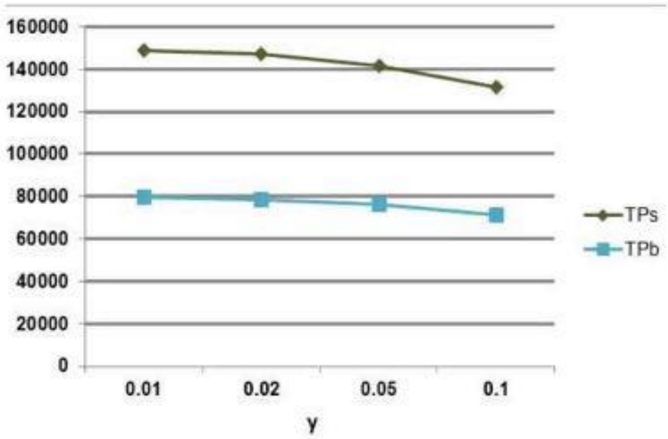

Figure 4. The influence of percentage of defective items.

6 Conclusion

In this article, a two-stage manufacturer-retailer supply chain under VMI policy was studied. We assumed that the demand of the retailer is price sensitive and the manufacturer sends lot sizes to the retailer in several smaller lots. The contribution of this article was on considering a decentralized supply chain using the Stackelberg game with the objective of optimizing the total system costs while the manufacturer produces defective items and there is type I, II errors of inspection process at retailer side. A solution procedure was developed to find the Stackelberg game equilibrium and was proved to be unique. A numerical study was conducted to understand the proposed models.

The numerical results showed that the increase in the percentage of defective items and type I, II inspection errors leads to decrease in the supply chain members' profits. Furthermore, it was shown that when demand is less sensitive to the price variations, the manufacturer and retailer profits are more increased.

As future research, the given model could be further extended to more practical situations such as considering multi-item case, multiple periods, and capacity constraints. Another possible research idea can be considering demand uncertainty under VMI policy.

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