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# Application of Intersection Method for Multi-Objective Optimization in Optimal Test with Desirable Response Variable

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**Abstract:** This paper aims to conduct applications of intersection method for multi-objective optimization in optimal test design with desirable response variable. The partial favourable probability of the desirable response variable of the test is evaluated according to the type of “one side desirability problem” or “one range desirability problem” and the requirements of desirable response first; then the evaluation of total favourable probability $P_t$ of the multi-objective optimization test design is conducted according to the common procedure of the "intersection" method for multi-object optimization of performance indicators of the test. Finally, regression analysis is performed for the total favourable probability and response variables to get maximum total favourable probability and optimal configuration of the optimal test with desirable response variable. As application examples of the intersection method for the test designs of maximizing yield with constraints of viscosity and molecular weight and the maximizing conversion rate with constraints of desirable thermal activity are given in detail, satisfied results are obtained.

**Keywords**: desirable response variable; intersection method; multi-objective optimization; optimal test design; partial favourable probability

## 1 INTRODUCTION

Optimization is an eternal topic in the world, including industrial production, transportation, architecture building, chemical reaction, banking, and social activities. They are likely involving several attributes or performances that must be considered in the analysis. An alternative is recommended to be an optimal one that needs to meet some requirements of response variables (performances), they are even conflicting each other. The proper approach to address this issue is to consider all responses comprehensively and simultaneously. Various techniques have been proposed, including technique for order preference by similarity to ideal solution (TOPSIS), Vlse Kriterijumska Optimizacija Kompromisno Resenje (VIKOR), multi attribute decision making (MADM), Analytical Hierarchy Process (AHP) and Multi-Objective Optimization on the basis of Ratio Analysis (MOORA), etc. [1].

Recently, a new approach named "intersection" method for multi-object optimization was proposed in the viewpoints of set theory and probability theory [1], which attempts to solve the inherent problems of personal and subjective factors in the above multi-object optimizations. The novel concept of favourable probability was developed to reflect the favourable degree of the candidate in the optimization, all performance utility indicators of candidates are divided into beneficial or unbeneficial types to the selection. Each performance utility indicator of the candidate is correlated to a partial favourable probability quantitatively, and the total favourable probability of a candidate is the product of all partial favourable probabilities in the viewpoints of probability theory and "intersection" of set theory, which is the overall and sole decisive index in the competitive selection process. The new multi-object optimization method was also extended in application of multi-objective orthogonal test design method (OTDM) and uniform test design method (UTDM) as well; appropriate achievements have been obtained [1].

In practical engineering fields, besides the beneficial or unbeneficial types of performance utility indicators of candidates, which have the features of the higher the better or the lower the better, there exists third type of performance indicators of candidates, which have the feature of the desired target being the best [2]. In order to address this problem, Derringer and Suich once implemented a multi-response optimization technique with a desirability function [3], which is with the weighting exponent to be assigned instead of favourable probability.

As a further development to the newly proposed "intersection" method for multi-object optimization, here in this paper, applications of intersection method for multi-objective optimization in optimal test design with desirable response are studied.

## 2 EVALUATIONS OF PARTIAL AND TOTAL FAVOURABLE PROBABILITIES OF DESIRABLE RESPONSE IN THE "INTERSECTION" METHOD FOR MULTI-OBJECTIVE OPTIMIZATION

### 2.1 One Range Desirability Problem

Under condition of one range desirability, the response variable $Y_{ij}$ has a desirable response range, i.e., within range of $[a, b]$. In this case, the partial favourable probability $P_{ij}$ of response variable $Y_{ij}$ will have certain value $\alpha_j$ within range of $[a, b]$, and the partial favourable probability $P_{ij}$ of response variable $Y_{ij}$ will be zero outside range of $[a, b]$, i.e.,

$$P_{ij} = \begin{cases} \alpha_j,\ Y_{ij} \in [a, b]; \\ 0,\ Y_{ij} \notin [a, b]; \end{cases} \quad i = 1, 2, \cdots, n;\ j = 1, 2, \cdots, m. \qquad (1)$$

$Y_{ij}$ represents the $j^{th}$ performance indicator of the $i^{th}$ candidate; $P_{ij}$ is the partial favorable probability of the one range desirable response variable $Y_{ij}$; $n$ is the total number of candidates in the candidate group involved; $m$ is the total number of performance indicators of each candidate in the group; $\alpha_j$ is value of the partial favourable probability $P_{ij}$ of the $j^{th}$ response variable $Y_{ij}$.

According to the general principle of probability theory [4], the summation of each $P_{ij}$ for the index $i$ in $j^{th}$ performance factor is normalized and equal to 1, i.e., $\sum_{i=1}^{n} P_{ij} = 1$, thus, it obtains naturally

$$\sum_{i=1}^{n} \alpha_j = 1,\ \alpha_j = \frac{1}{l}, \tag{2}$$

$l$ is the number of the value of the response variable $Y_{ij}$ falling within range of the $[a, b]$.

### 2.2 One Side Desirability Problem

In condition of one side desirability, the response variable $Y_{ij}$ has a desirable response limit, e.g., within range of $[a, \propto]$, which can be taken as a special case of one side desirable condition by setting $b = \propto$. In this case, the partial favourable probability $P_{ij}$ of response variable $Y_{ij}$ will have certain value $\beta_j$ within range of $[a, \propto]$, and the partial favourable probability $P_{ij}$ of response variable $Y_{ij}$ will be zero outside range of $[a, \propto]$, i.e.,

$$P_{ij} = \begin{cases} \beta_j,\ Y_{ij} \in [a, \infty]; \\ 0,\ Y_{ij} \notin [a, \infty]; \end{cases} \quad i = 1, 2, \cdots, n;\ j = 1, 2, \cdots, m. \tag{3}$$

Similarly, according to the general principle of probability theory [4], the summation of each $P_{ij}$ for the index $i$ in $j^{th}$ performance factor is normalized and equal to 1, i.e., $\sum_{i=1}^{n} P_{ij} = 1$, thus, it obtains naturally

$$\sum_{i=1}^{n} \beta_j = 1,\ \beta_j = \frac{1}{k} \tag{4}$$

$k$ is the number of the value of the response variable $Y_{ij}$ falling within range of the $[a, \propto]$.

The treatment for the situation for one side desirability within range of $[0, b]$ is similar to above procedure.

As the partial favourable probability $P_{ij}$ of response variable $Y_{ij}$ is obtained, the evaluations of total probability $P_t$ of candidate and the ranking of the multi-objective optimization can be conducted according to the common procedure of the "intersection" method for multi-object optimization [1].

## 3 APPLICATIONS OF THE "INTERSECTION" METHOD FOR MULTI-OBJECTIVE OPTIMIZATION IN CASES OF DESIRABLE RESPONSE

### 3.1 Maximizing Yield with Constraints of Viscosity and Molecular Weight

Montgomery et al showed a maximizing yield optimization with constraints of viscosity and molecular weight problem two input variables reaction time $x_1$ and temperature $x_2$ [5], and three responses variables, i.e., the yield $y_1$ (%), the viscosity $y_2$ (cSt) and the molecular weight $y_3$ (Mr.) of the product, which is cited and displayed in Tab. 1. The optimization of this problem is to get maximum yield $y_1$ with the constraints of viscosity $y_2$ and molecular weight $y_3$ by $62 \le y_2 \le 68$ cSt and $y_3 \le 3400$ Mr. In this second phase of the study, two additional responses were of special interest: the viscosity $y_2$ and the molecular weight $y_3$ of the product in addition to yield $y_1$, which are responses with desirable values. The experimenter called it as a central composite design (or CCD).

In this problem, it involves complex optimization for yield $y_1$ as a beneficial performance index, viscosity $y_2$ and molecular weight $y_3$ as desirable response indexes. Therefore, the partial favourable probability for yield $y_1$ could be evaluated by the assessment for beneficial type variable proposed in [1], and partial favourable probabilities for the viscosity $y_2$ and molecular weight $y_3$ should be evaluated by the assessment methods developed in the last section for both one side desirability and one range desirability problems, respectively. The evaluated results of partial and total favourable probabilities $P_{y1}$, $P_{y2}$, $P_{y3}$ and $P_t$ of this product experiment are shown in Tab. 2. Tab. 2 shows that the test No. 1 exhibits the maximum total favourable probability at first glance, so the optimal configuration could be around test No. 1.

**Table 1** Experimental results of maximizing yield with constraints of viscosity and molecular weight

| No. | Reaction time, $x_1$ / min | Temperature, $x_2$ / °C | Yield, $y_1$ / % | Viscosity, $y_2$ / cSt | Molecular weight, $y_3$ / Mr. |
|---|---|---|---|---|---|
| 1 | 80 | 76.67 | 76.5 | 62 | 2940 |
| 2 | 80 | 82.22 | 77 | 60 | 3470 |
| 3 | 90 | 76.67 | 78 | 66 | 3680 |
| 4 | 90 | 82.22 | 79.5 | 59 | 3890 |
| 5 | 85 | 79.44 | 79.9 | 72 | 3480 |
| 6 | 85 | 79.44 | 80.3 | 69 | 3200 |
| 7 | 85 | 79.44 | 80 | 68 | 3410 |
| 8 | 85 | 79.44 | 79.7 | 70 | 3290 |
| 9 | 85 | 79.44 | 79.8 | 71 | 3500 |
| 10 | 92.07 | 79.44 | 78.4 | 68 | 3360 |
| 11 | 77.93 | 79.44 | 75.6 | 71 | 3020 |
| 12 | 85 | 83.37 | 78.5 | 58 | 3630 |
| 13 | 85 | 75.52 | 77 | 57 | 3150 |

**Table 2** The evaluated results of partial and total favourable probabilities for the chemical experiment

| | Response Variables | | | Favourable Probability | | | |
|---|---|---|---|---|---|---|---|
| No. | $y_1$/% | $y_2$/cSt | $y_3$/Mr | $P_{y1}$ | $P_{y2}$ | $P_{y3}$ | $P_t \times 10^3$ |
| 1 | 76.5 | 62 | 2940 | 0.0750 | 0.1997 | 0.0869 | 2.2437 |
| 2 | 77 | 60 | 3470 | 0.0755 | 1.46E-05 | 0.0751 | 0 |
| 3 | 78 | 66 | 3680 | 0.0765 | 0.3995 | 0.0704 | 0.1163 |
| 4 | 79.5 | 59 | 3890 | 0.0779 | 3.81E-07 | 0.0657 | 0 |
| 5 | 79.9 | 72 | 3480 | 0.0783 | 1.75E-08 | 0.0748 | 0 |
| 6 | 80.3 | 69 | 3200 | 0.0787 | 0.0013 | 0.0811 | 0.0142 |
| 7 | 80 | 68 | 3410 | 0.0784 | 0.1997 | 0.0764 | 1.0902 |
| 8 | 79.7 | 70 | 3290 | 0.0781 | 1.46E-05 | 0.0791 | 0.0001 |
| 9 | 79.8 | 71 | 3500 | 0.0782 | 3.81E-07 | 0.0744 | 0 |
| 10 | 78.4 | 68 | 3360 | 0.0768 | 0.1997 | 0.0775 | 1.4744 |
| 11 | 75.6 | 71 | 3020 | 0.0741 | 3.81E-07 | 0.0851 | 0 |
| 12 | 78.5 | 58 | 3630 | 0.0769 | 1.75E-08 | 0.0715 | 0 |
| 13 | 77 | 57 | 3150 | 0.0755 | 1.21E-09 | 0.0822 | 0 |

In order to get an accurate optimization, the data in Tab. 2 is regressed. The fitted result for the total favourable probability is

$$P_t \times 10^3 = -210149.5010 - 2424.3807x_1 + 520.1057x_2 + \\ +0.0383x_1x_2 + 13.1983x_1^2 - 6.5788x_2^2 + \\ +73485.15 \cdot \ln(x_1) - 0.0317x_1^3 + 0.0275x_2^3, \\ R^2 = 0.8554. \quad (5)$$

$P_t$ gets its maximum value $P_{tmax} \times 10^3 = 2.2750$ at $x_1 = 80.39$ minutes, and $x_2 = 76.91$°C.

While, the fitted result for the yield $y_1$ is

$$y_1 = -326843.3710 - 4410.0970x_1 + 48.0746x_2 + \\ +0.0180x_1x_2 + 25.9987x_1^2 - 0.4803x_2^2 + \\ +124747.8522 \cdot \ln(x_1) + 0.0682x_1^3 + 0.0014x_2^3, \\ R^2 = 0.9926. \quad (6)$$

The yield $y_1$ gets its optimal value $y_{1opt} = 76.979$ % at $x_1 = 80.39$ minutes, and $x_2 = 76.91$°C.

The fitted result for viscosity $y_2$ is

$$y_2 = 1454310.2050 + 20242.9592x_1 + 2436.0490x_2 - \\ -0.0900x_1x_2 + 117.4347x_1^2 - 29.7790x_2^2 - \\ -580304.3480 \cdot \ln(x_1) + 0.3025x_1^3 + 0.1215x_2^3, \\ R^2 = 0.9723. \quad (7)$$

The desirable variable viscosity $y_2$ gets it optimal value $y_{2opt} = 63.351$ cSt at $x_1 = 80.39$ minutes, and $x_2 = 76.91$°C.

The fitted result for molecular weight $y_3$ is

$$y_3 = -176217474.0000 - 2438309.5500x_1 - \\ -13078.9964x_2 - 5.7600x_1x_2 + 14549.0282x_1^2 + \\ +170.7936x_2^2 + 68063495.1500 \cdot \ln(x_1) - 38.5337 - \\ -0.7128x_2^3, \\ R^2 = 0.9238. \quad (8)$$

The desirable variable molecular weight $y_2$ gets it optimal value $y_{3Opt} = 2972.375$ Mr. at $x_1 = 80.39$ minutes, and $x_2 = 76.91$°C.

Above optimal results meet the requirements of the original idea of the problem, which shows that all the optimized responses are better than those of test No. 1 of Tab. 1 in overall view and the optimal configuration is close to test No. 1.

### 3.2 Maximizing Conversion Rate with Constraints of Desirable Thermal Activity

Myers raised a problem of maximizing conversion rate with constraints of desirable thermal activity [6]. The experiment considers three input variables, i.e., reaction time $x_1$, temperature $x_2$ and percentage of catalyst $x_3$, and two response variables, conversion rate $y_1$ (%) and thermal activity $y_2$ (W·s$^{0.5}$/(m$^2$·K)) using a central composite design with six central runs. The data are cited and shown in Tab. 3.

**Table 3** Experimental results of maximizing conversion rate with constraints of desirable thermal activity

| No. | Input Variables | | | Response Variables | |
|---|---|---|---|---|---|
| | Reaction time $x_1$ / min. | Temperature $x_2$ / °C | Catalyst $x_3$ / % | Conversion rate $y_1$ / % | Thermal activity $y_2$ / W·s$^{0.5}$/(m$^2$·K) |
| 1 | 45 | 48 | 0.682 | 74 | 53.2 |
| 2 | 55 | 48 | 0.682 | 51 | 62.9 |
| 3 | 45 | 58 | 0.682 | 88 | 53.4 |
| 4 | 55 | 58 | 0.682 | 70 | 62.6 |
| 5 | 45 | 48 | 2.682 | 71 | 57.3 |
| 6 | 55 | 48 | 2.682 | 90 | 67.9 |
| 7 | 45 | 58 | 2.682 | 66 | 59.8 |
| 8 | 55 | 58 | 2.682 | 97 | 67.8 |
| 9 | 41.59 | 53 | 1.682 | 76 | 59.1 |
| 10 | 58.41 | 53 | 1.682 | 79 | 65.9 |
| 11 | 50 | 44.59 | 1.682 | 85 | 60 |
| 12 | 50 | 61.41 | 1.682 | 97 | 60.7 |
| 13 | 50 | 53 | 0 | 55 | 57.4 |
| 14 | 50 | 53 | 3.364 | 81 | 63.2 |
| 15 | 50 | 53 | 1.682 | 81 | 59.2 |
| 16 | 50 | 53 | 1.682 | 75 | 60.4 |
| 17 | 50 | 53 | 1.682 | 76 | 59.1 |
| 18 | 50 | 53 | 1.682 | 83 | 60.6 |
| 19 | 50 | 53 | 1.682 | 80 | 60.8 |
| 20 | 50 | 53 | 1.682 | 91 | 58.9 |

**Table 4** The evaluated results of partial and total favourable probabilities for the maximizing conversion rate with constraints of desirable thermal activity

| | Response variables | | Favourable probability | | |
|---|---|---|---|---|---|
| No. | $y_1$ | $y_2$ | $P_{y1}$ | $P_{y2}$ | $P_t \times 10^3$ |
| 1 | 74 | 53.2 | 0.0473 | 0.0585 | 2.7648 |
| 2 | 51 | 62.9 | 0.0326 | 0.0584 | 1.9028 |
| 3 | 88 | 53.4 | 0.0562 | 0.0585 | 3.2879 |
| 4 | 70 | 62.6 | 0.0447 | 0.0585 | 2.6142 |
| 5 | 71 | 57.3 | 0.0453 | 0.0585 | 2.6527 |
| 6 | 90 | 67.9 | 0.0575 | 8.4559E-05 | 0.0049 |
| 7 | 66 | 59.8 | 0.0421 | 0.0585 | 2.4659 |
| 8 | 97 | 67.8 | 0.0619 | 0.0001 | 0.0064 |
| 9 | 76 | 59.1 | 0.0485 | 0.0585 | 2.8396 |
| 10 | 79 | 65.9 | 0.0504 | 0.0055 | 0.2772 |
| 11 | 85 | 60 | 0.0543 | 0.0585 | 3.1758 |
| 12 | 97 | 60.7 | 0.0619 | 0.0585 | 3.6242 |
| 13 | 55 | 57.4 | 0.0351 | 0.0585 | 2.0549 |
| 14 | 81 | 63.2 | 0.0517 | 0.0583 | 3.0139 |
| 15 | 81 | 59.2 | 0.0517 | 0.0585 | 3.0264 |
| 16 | 75 | 60.4 | 0.0479 | 0.0585 | 2.8022 |
| 17 | 76 | 59.1 | 0.0486 | 0.0585 | 2.8396 |
| 18 | 83 | 60.6 | 0.0531 | 0.0585 | 3.1011 |
| 19 | 80 | 60.8 | 0.0511 | 0.0585 | 2.9890 |
| 20 | 91 | 58.9 | 0.0581 | 0.0585 | 3.4000 |

As to this problem, it involves complex optimization for conversion rate $y_1$ (%) as a beneficial performance index, and the thermal activity $y_2$ as desirable response index by $50 \le y_2 \le 65$ W·s$^{0.5}$/(m$^2$·K) and as close to 57.5 W·s$^{0.5}$/(m$^2$·K) as possible. Therefore, the partial favourable probability for conversion rate $y_1$ could be evaluated by the assessment for beneficial type variable proposed in [1], and partial favourable probability for the thermal activity $y_2$ should be evaluated by the assessment methods developed in the last section for one range desirability problem. The evaluated results of partial and total favourable probabilities $P_{y1}$, $P_{y2}$

and $P_t$ of this product experiment are shown in Tab. 4. Tab. 4 shows that the test No. 12 exhibits the maximum total favourable probability at first glance, so the optimal configuration might be around test No. 12.

The data in Tab. 4 is regressed, the fitted result for the total favourable probability is

$$\begin{aligned}&P_t \times 10^3 = -55.9337 + 2.3901x_1 - 0.1179x_2 + 7.0079x_3 - \\&-0.0250x_1^2 + 0.0010x_2^2 - 0.2800x_3^2 + 0.0019x_1x_2 - \\&-0.0355x_2x_3 - 0.0893x_3x_1, \\&R^2 = 0.8166.\end{aligned} \tag{9}$$

$P_t$ gets its maximum value $P_{tmax} \times 10^3$ = 3.657 at $x_1$ = 48.525 min., and $x_2$ = 61.41 °C, and $x_3$ = 2.473%.

While, the fitted result for the conversion rate $y_1$ is

$$\begin{aligned}&y_1 = 497.1314 - 0.7916x_1 - 14.5965x_2 - 49.0071x_3 - \\&0.0733x_1^2 + 0.1175x_2^2 - 5.1915x_3^2 + 0.0850x_1x_2 - \\&-0.7750x_2x_3 + 2.2750x_3x_1, \\&R^2 = 0.9199.\end{aligned} \tag{10}$$

The conversion rate $y_1$ gets its optimal value $y_{1opt}$ = 94.337 % at $x_1$ = 48.525 min., and $x_2$ = 61.41 °C, and $x_3$ = 2.473%.

The fitted result for the desirable thermal activity $y_2$ is

$$\begin{aligned}&y_2 = 73.4856 - 1.7884x_1 + 0.4035x_2 - 0.9000x_3 + \\&+0.0334x_1^2 + 0.0030x_2^2 + 0.05716x_3^2 - 0.0155x_1x_2 + \\&+0.0625x_2x_3 - 0.0075x_3x_1, \\&R^2 = 0.8918.\end{aligned} \tag{11}$$

The desirable thermal activity $y_2$ gets it optimal value $y_{2Opt}$ = 57.545 $W \cdot s^{0.5}/(m^2 \cdot K)$ at $x_1$ = 48.525 min., and $x_2$ = 61.41 °C, and $x_3$ = 2.473%.

Above optimal results meet the requirements of the original intention of the problem, which shows that all the optimized responses are more proper than those of test No. 12 of Tab. 3 comprehensively and the optimal configuration is not far from test No. 12.

## 4 CONCLUSION

From above discussion, the partial favourable probability of the desirable response variable of the test is evaluated, according to the type of "one side desirability problem" or "one range desirability problem" and the requirements of desirable response properly. The regression analysis for the total favourable probability and response variables provide the optimal configuration of the optimal test with desirable response variable, which corresponds to the maximum of the total favourable probability. The application examples of the intersection method for multi-objective optimization in optimal test designs of maximizing yield with constraints of viscosity and molecular weight and the maximizing conversion rate with constraints of desirable thermal activity present satisfied results, which indicate the validity of the assessment.

## Conflict Statement

There is no conflict of interest.

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