Calculation of Complex Power Generated by a Transmitting Thin Wire Antenna Radiating above a Lossy Half-space

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Original scientific article

***Abstract*—The paper deals with an efficient approach to determine complex power generated by a thin wire antenna above a lossy ground. Once the current distribution along a thin wire is obtained by numerically solving the Pocklington integro-differential equation the complex power of the antenna can be obtained by solving the integral over the inner product of tangential component of the electric field and current distribution along the wire stemming from Poynting theorem. Numerical calculation procedure uses the vectors and matrices already constructed within the calculation procedure for the current distribution. Some illustrative numerical results for the active power, reactive power and apparent power of the dipole antenna radiating over a lossy half-space are presented in the paper for different values of input parameters.**

***Index terms*—Wire scatterer, Pocklington integro-differential equation, current distribution, complex power, active power, reactive power, apparent power.**

I. Introduction

Modeling of Wireles Power Transfer (WPT) systems includes the analysis of power generated and received by the antennas in the near field, Widely used measure of efficiency of such systems is power transfer efficiency (PTE) defined as a ratio between received and transmitted power.

Though the analysis of WPT systems can be carried out by using simple approaches; circuit theory, or transmission line (TL) models respectively, a rigorous studies use antenna theory (AT) approach is used in some scenarios [1-3]. A trade-off between different approaches is addressed in a number of books, e.g. [4, 5] and papers, e.g. [6-9].

Note that antenna theory (AT), though theoretically and computationally demanding, enables one to account for radiation effects at higher frequencies when analyzing finite length wires.

This is, in principle, not possible by using simplified transmission line (TL) approximation, or oversimplified circuit theory approximation. Particular difficulties with TL approach arise when wires radiate in the presence of a lossy half-space.

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Antenna theory models generally require numerical methods for the solution although there are some examples where the analytical approach has been successfully applied [10-13]. The received complex power along the straight horizontal wire scatterer in the presence of a lossy ground has been recently reported in [14]. The use of generalized telegrapher's equations provide to calculate induced current and the scattered voltage along the scatterer, as derived in [6]. Knowing the current and voltage distribution along the scatterer, the active, reactive and apparent power, respectively, stemming for the standard circuit theory definitions are calculated [14]. which could be useful in the analysis of power transfer efficiency (PTE) for the WPT applications.

This work deals with a novel and rather efficient approach to calculate complex power generated by the transmitting antenna above a lossy ground and could be regarded as a sequel to the research reported in [14]. The proposed approach provides the assessment of the antenna active, reactive and apparent power, respectively, without necessity to evaluate generated electric and magnetic fields and carry out subsequent integration of corresponding Poyinting vector over a closed surface around the wire. The antenna current is obtained by numerically solving Pocklington integro-differential equation via Galerkin-Bubnov Indirect Boundary Element Method (GB-IBEM) [5]. Once the current distribution along dipole antenna is known the complex power of the antenna versus frequency can be computed by solving the integral over the inner product of tangential component of the electric field and current distribution along the wire arising from Poynting integral theorem. It is worth emphasizing that the numerical procedure for the calculation of complex power uses the vectors and matrices already constructed within the numerical procedure for the calculation of current distribution along the wire which is rather attractive advantage of GB-IBEM. Such an approach could be also used to compute input impedance spectrum [5], [15].

The paper is organized, as follows; section II outlines the formulation used to determine the antenna complex and developed corresponding numerical procedures. Some illustrative numerical results for the active power, reactive power and apparent power of the dipole antenna radiating over a lossy ground are presented in the section III for different values of input parameters. Finally, some conclusions are drawn and guidelines for a future work are given.

II. Theoretical Background

A thin wire antenna of an arbitrary shape of finite length Cw and radius a located above a lossy ground and driven by an equivalent voltage generator, Fig 1, is considered.

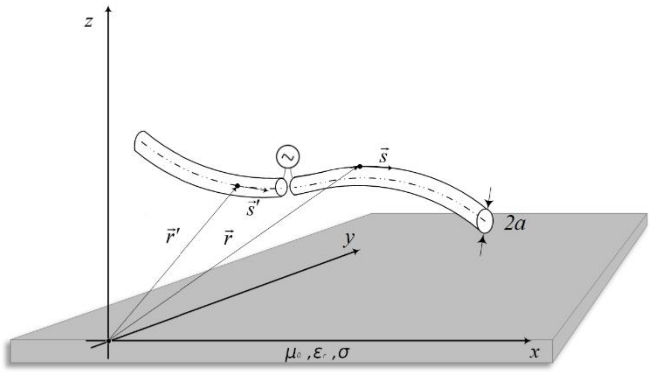


Fig. 1. Center fed arbitrary shaped antenna above a lossy half-space

The apparent complex power P_a generated in the volume of interest V containing sources for time-harmonic fields can be derived from the Poynting integral theorem and is given by

$$P_a = -\frac{1}{2}\int_V \vec{E}\cdot\vec{J}^*\, dV \tag{1}$$

where E stands for the field generated by the source and J denotes the distribution of corresponding current density along the source.

If thin wires are of interest the problem becomes one dimensional in nature and it can be written

$$\vec{J}^*\, dV = I^* S\, d\vec{s} \tag{2}$$

where S stands for wire cross-section and I* is the complex conjugate of the antenna current, and (1) simplifies into

$$P_a = -\frac{1}{2}\int_{C_W} \vec{E}\cdot I^*\, d\vec{s} \tag{3}$$

where C_w represents the geometry of a curved wires.

It is worth noting that the real part of (3) represents the active power, while the imaginary part is reactive power.

Pocklington integro-differential equation for the wire of arbitrary shape above a lossy half-space can be written, as follows

$$\begin{aligned} E_S^{exc}(s) = \frac{j}{4\pi\omega\varepsilon_0}\int_{C_W}\Big\{\Big[k^2\vec{e}_s\vec{e}_{s'} - \frac{\partial^2}{\partial s\partial s'}\Big] g_0(s,s') + \\ R_{TM}\Big[k^2\vec{e}_s\vec{e}_{s"} - \frac{\partial^2}{\partial s\partial s"}\Big] g_i(s,s")\Big\} I(s')I^*(s)ds' \end{aligned} \tag{4}$$

where $I(s')$ is the induced current along the wire, s' and s is the source and observation point respectively, $\vec{e}_{s'}$ is the vector tangential to the curved wire and $\vec{e}_s$ is related to the vector tangential to the observation point at the wire surface.

Furthermore, $g_0(s,s`)$ stands for the lossless medium Green function

$$g_0(s,s') = \frac{e^{-jkR}}{R} \tag{5}$$

and R is the distance from the source point to the observation point, respectively, while $g_i(s,s")$ arises from the image theory and is given by

$$g_i(s,s") = \frac{e^{-jkR"}}{R"} \tag{6}$$

and $R"$ is the distance from the image source point to the observation point, respectively.

The propagation constant of the lossless homogeneous medium is

$$k = \omega\sqrt{\mu_0\varepsilon_0} \tag{7}$$

The influence of a lower medium is taken into account via the Fresnel reflection coefficient for transverse magnetic (TM) polarization

$$R'_{TM} = \frac{\underline{n}\cos\theta' - \sqrt{\underline{n} - \sin^2\theta'}}{\underline{n}\cos\theta' + \sqrt{\underline{n} - \sin^2\theta'}} \tag{8}$$

where θ' is the angle of incidence and n is given by:

$$\underline{n} = \frac{\varepsilon_{eff}}{\varepsilon_0}, \quad \varepsilon_{eff} = \varepsilon_r\varepsilon_0 - j\frac{\sigma}{\omega} \tag{9}$$

Once the current distribution is known the scattered field can be obtained by forcing the continuity condition

$$\vec{e}_s\cdot\left(\vec{E}^{exc} + \vec{E}^{sct}\right) = 0 \tag{10}$$

where $\vec{E}^{sct}$ stands for the scattered field.

Inserting (4) into (3) one obtains

$$\begin{aligned} P_a = \frac{j}{8\pi\omega\varepsilon_0}\int_{C_W}\int_{C_W}\Big\{\Big[k^2\vec{e}_s\vec{e}_{s'} - \frac{\partial^2}{\partial s\partial s'}\Big] g_0(s,s') + \\ R_{TM}\Big[k^2\vec{e}_s\vec{e}_{s"} - \frac{\partial^2}{\partial s\partial s"}\Big] g_i(s,s")\Big\} I(s')I^*(s)ds'ds" \end{aligned} \tag{11}$$

Utilizing the integration by parts it is possible to avoid quasingularity problems with the kernel, i.e. (11) becomes

$$\begin{aligned} P_a = \frac{j}{8\pi\omega\varepsilon_0}\Big[-\int_{C_W}\int_{C_W}\frac{\partial I(s')}{\partial s'}\cdot\frac{\partial I^*(s)}{\partial s} g_0(s,s')ds'ds + \\ k^2\int_{C_W}\int_{C_W}\vec{e}_s\vec{e}_{s'} I(s')I^*(s)ds'ds g_0(s,s')ds'ds - \\ R_{TM}\int_{C_W}\int_{C_W}\frac{\partial I(s")}{\partial s"}\cdot\frac{\partial I^*(s)}{\partial s} g_i(s,s")ds"ds + \\ R_{TM}\int_{C_W}\int_{C_W}\vec{e}_s\vec{e}_{s"} I(s")I^*(s) g_i(s,s")ds"ds\Big] \end{aligned} \tag{12}$$

Applying the GB-IBEM featuring isoparametric elements integral expression (12) is transformed into matrix equation

$$P_a = \frac{j}{8\pi\omega\varepsilon_0}\{I\}^T[Z]\{I^*\} \tag{13}$$

where $\{I\}$ and $\{I\}^T$ is the vector of current and its transponse vector, respectively, while $[Z]$ is impedance matrix stemming from the numerical procedure for the current distribution determination. Global impedance matrix is assembled from the generalized mutual impedance matrices for the *j-th* and *i-th* isoparametric elements, given by

$$\begin{aligned}[Z]_{ji}^{e} &= \int_{-1}^{1}\int_{-1}^{1}[-\{D\}_j \cdot \{D'\}_i^T g_0(s,s') + \\ &+k^2\vec{e}_s\vec{e}_{s'}\{f\}_j \cdot \{f'\}_i^T ds' ds\, g_0(s,s')]\frac{ds'}{d\xi'}d\xi'\frac{ds}{d\xi}d\xi + \\ &R_{TM}\int_{-1}^{1}\int_{-1}^{1}[-\{D\}_j \cdot \{D'\}_i^T g_i(s,s") + \\ &+k^2\vec{e}_s\vec{e}_{s"}\{f\}_j \cdot \{f'\}_i^T g_i(s,s")]\frac{ds'}{d\xi'}d\xi'\frac{ds}{d\xi}d\xi \end{aligned} \qquad (14)$$

where vectors $\{f\}$ and $\{f'\}$ contain shape functions while vectors $\{D\}_i^n$ and $\{D\}_j^m$ contain shape functions derivatives.

Note that vectors $\{I\}^T$ and vector of complex conjugate values of current $\{I\}^*$ and mutual impedance matrix $[Z]$ are already available from the calculation procedure for the current distribution along the wire.

Input impedance of the antenna can be defined, as follows [5]

$$Z_{in} = \frac{2P_a}{|I_0|^2} = -\frac{1}{|I_0|^2}\int_C \vec{E}\cdot\vec{I}^*\, ds \qquad (15)$$

where I_0 stands for the input current.

Taking into account matrix equation (13) the expression for computation of input impedance becomes

$$Z_{in} = -\frac{1}{j4\pi\omega\varepsilon_0|I_0|^2}\{I\}^T[Z]\{I^*\} \qquad (16)$$

More details on the calculation of the input impedance of a dipole antenna can be found in [5].

It is worth stressing out that the proposed procedure is highly efficient as it practically requires no additional computation, only instantaneous manipulation with already existing matrices and vectors from the numerical modeling of Pocklington equation which is, in a sense of computational cost, a matter of few seconds.

III. NUMERICAL RESULTS

A straight thin wire antenna of a finite length L and radius a located at height h above a lossy ground and driven by an equivalent voltage generator, Fig 2, is considered.

The dipole is excited by the unity voltage generator. Length of the wire L has been varied between 0.25 m and 1m, while height above ground has been changed from 0.1m up to 0.5m. Wire radius is fixed at a = 2 mm and frequency range up to 1 GHz is considered.

First example deals with the free space scenario. Figures 3 to 5 show frequency spectra of the active, reactive and apparent power, respectively, for the wire lengths L=0.25m, L=0.5m and L=1m.

As it can be observed at Fig 6, peaks of the active power coincide with minima of the input impedance and vice-versa. Figs 7 to 9 show the results for the active, reactive and apparent power, respectively, of the wire placed at height h=0.25m above lossy ground with conductivity σ = 0.001 S/m and relative permittivity εr=10.

As could be seen from the numerical results, the power amplitude is slightly changed due to the presence of the lossy ground when compared to the case of free space. Furthermore, influence of the ground is more obvious at the first maximum which is expected as the wire is electrically closer to the ground at lower frequencies.

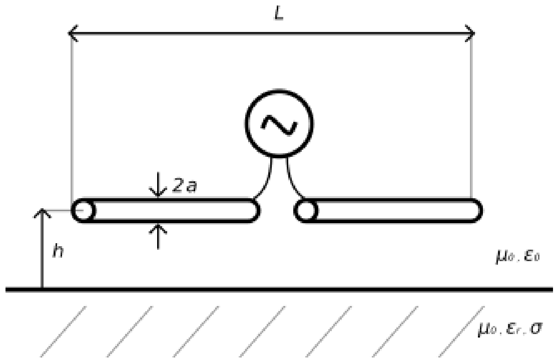


Fig. 2. Center fed dipole antenna above a lossy half-space

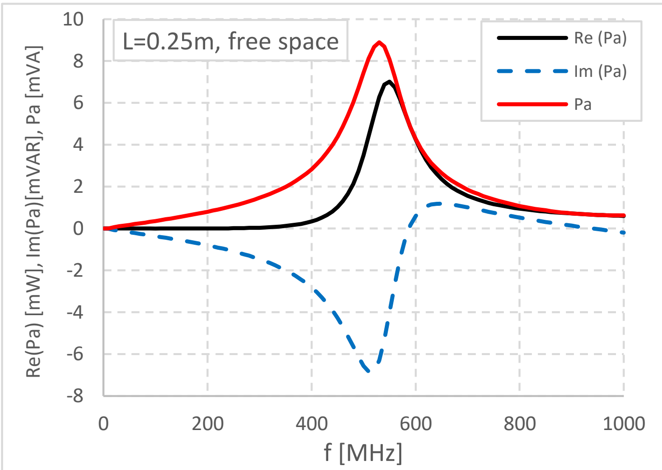


Fig. 3. Power vs frequency for the 0.25m long wire in free space

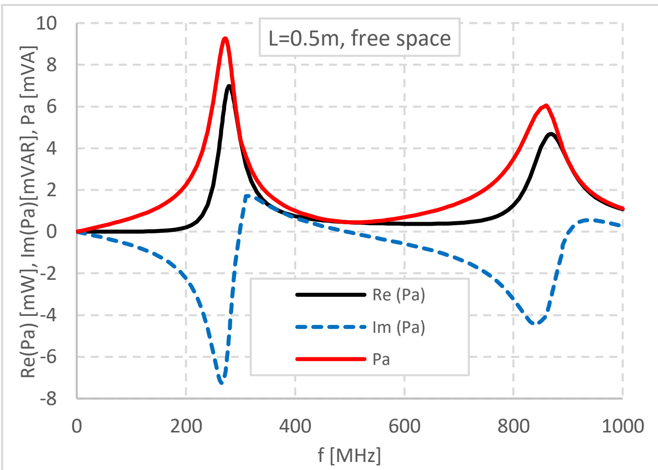


Fig 4. Power vs frequency for the 0.5m long wire in free space

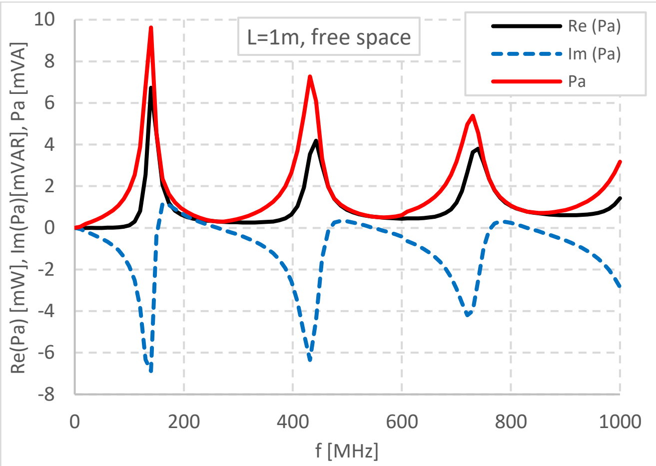


Fig. 5. Power vs frequency for the 1m long wire in free space

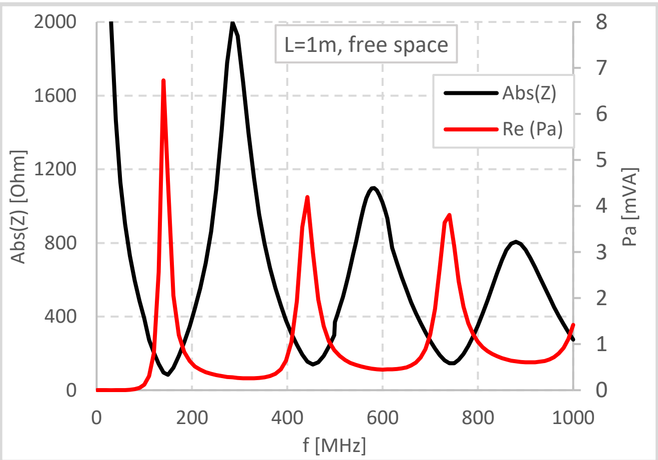


Fig. 6. Input impedance and active power for the 1m long wire in free space

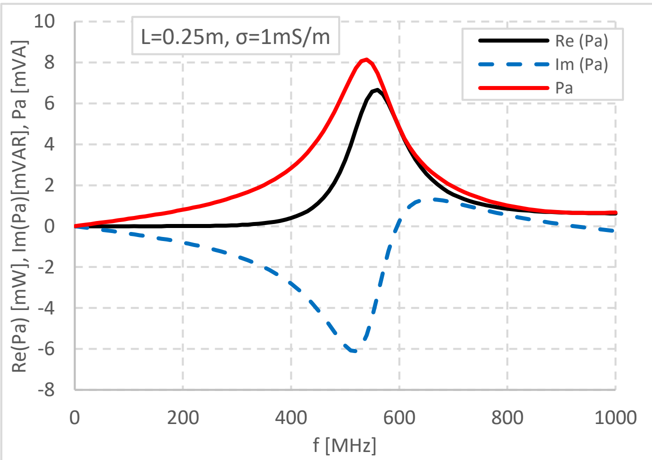


Fig. 7. Power vs frequency for the 0.25m long wire placed at height h=0.25m above a lossy ground (σ = 0.001 S/m, ε_r =10)

Figs 10 and 11 show frequency span of the active and reactive power respectively, for the case of the L=0.25m and h=0.25 m for the different values of the ground conductivity. Furthermore, Fig 12 shows detail from the curves shown in Fig 10 around the first maximum.

It could be noticed that presence of the ground causes a slight shift of the maximum towards lower frequencies (about 6 MHz) while in the case of the perfect ground this shift is even more appreciable (around 12 MHz). The same behavior could be observed for the minima of the input impedance (Fig 13).

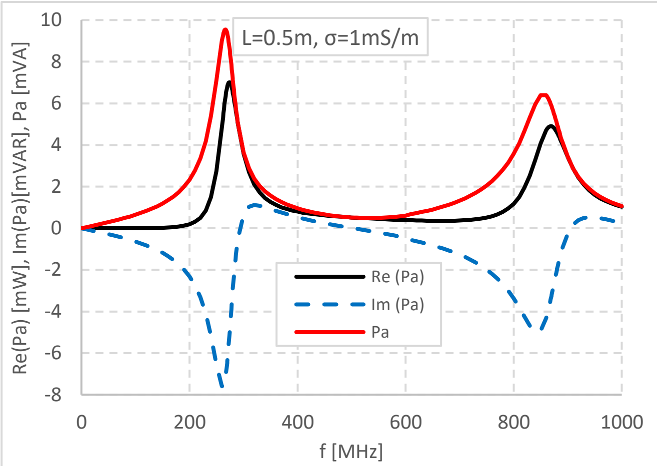


Fig. 8. Power vs frequency for the 0.5m long wire placed at height h=0.25m above a lossy ground (σ = 0.001 S/m, ε_r =10)

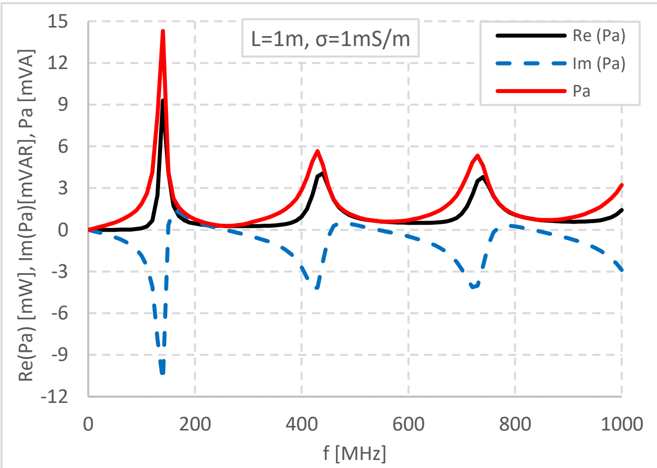


Fig. 9. Power vs frequency for the 1m long wire placed at height h=0.25m above a lossy ground (σ = 0.001 S/m, ε_r =10)

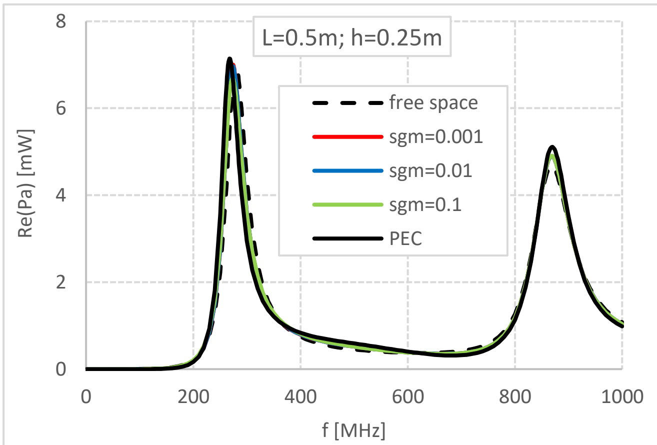


Fig. 10. Active power vs frequency for the 0.5m long wire placed at height h=0.25m above a lossy ground for various values of the ground conductivity (ε_r =10)

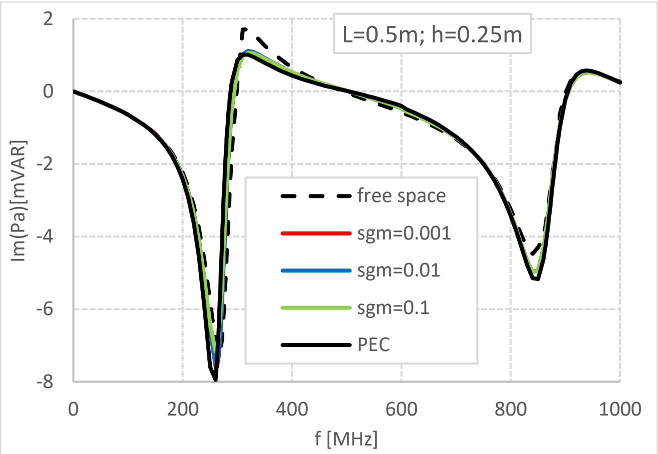


Fig. 11. Reactive power vs frequency for the 0.5m long wire placed at height h=0.25m above a lossy ground for various values of the ground conductivity (ε_r =10)

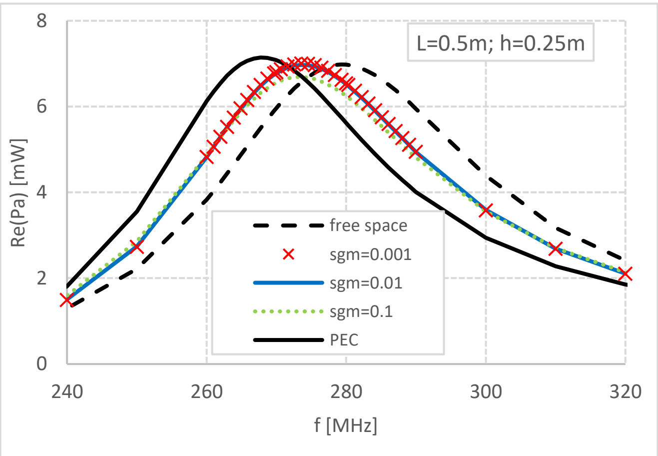


Fig. 12. First maximum of the active power vs frequency for the 0.5m long wire placed at height h=0.25m above a lossy ground for various values of the ground conductivity (ε_r=10)

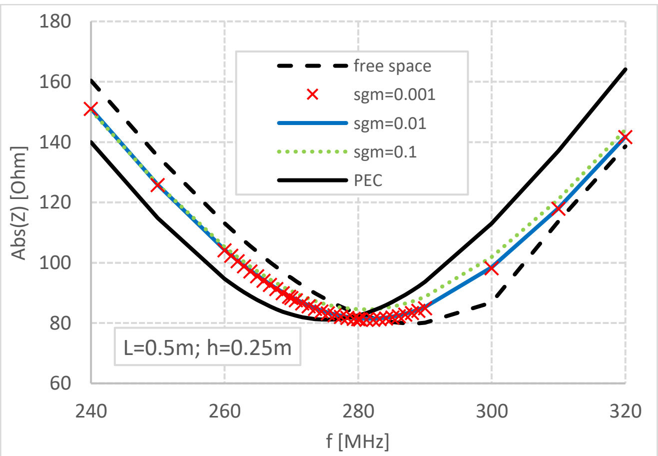


Fig. 13. First minimum of the imput impedance vs frequency for the 0.5m long wire placed at height h=0.25m above a lossy ground for various values of the ground conductivity (ε_r=10)

The same analysis has been undertaken for the wire located closer to the ground at the height of h=0.1m. Fig 14 and 15 show the results for the apparent and active power, respectively. In this case the frequency shift is also visible for the second maximum. The amplitude of the apparent power at the first maximum is significantly higher for the perfect ground case. It is worth nothing that for the cases of the real ground, curves remain almost the same regardless of the conductivity values.

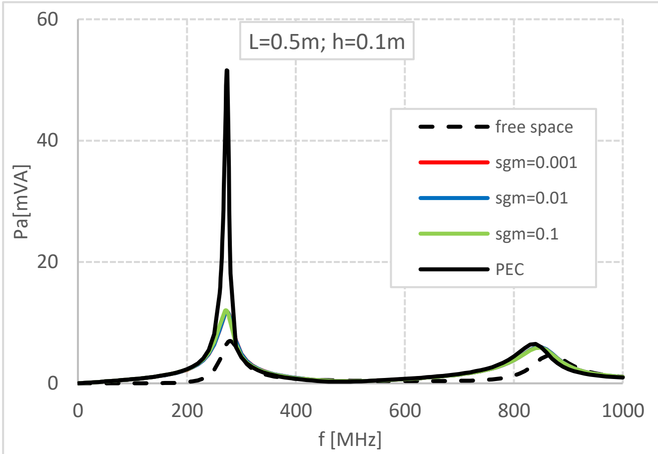


Fig. 14. Apparent power vs frequency for the 0.5m long wire placed at height h=0.1m above a lossy ground for various values of the ground conductivity (ε_r =10)

Fig 16 shows the apparent power for the same wire located 0.1m above the ground but for different values of the ground permittivity. The frequency shift of the maximum is minimal and the amplitude change due to height variation is comparable to the influence of the ground conductivity.

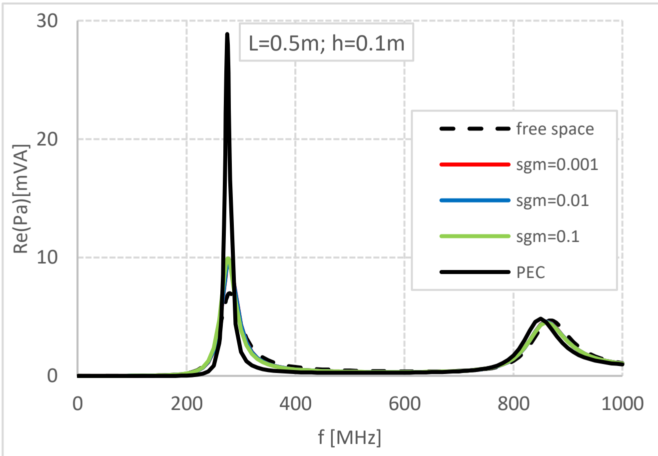
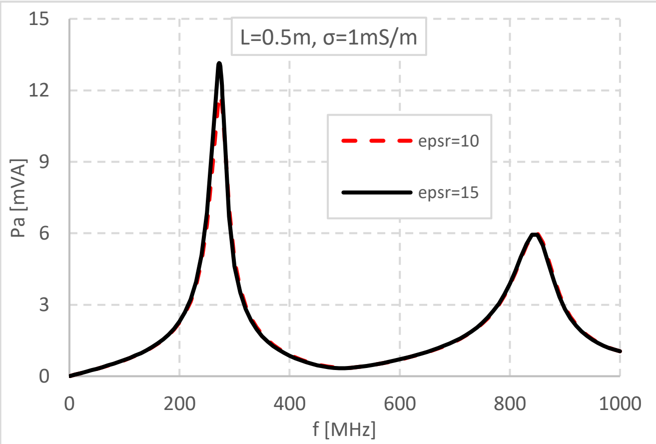


Fig. 15. Active power vs frequency for the 0.5m long wire placed at height h=0.1m above a lossy ground for various values of the ground conductivity (ε_r =10)

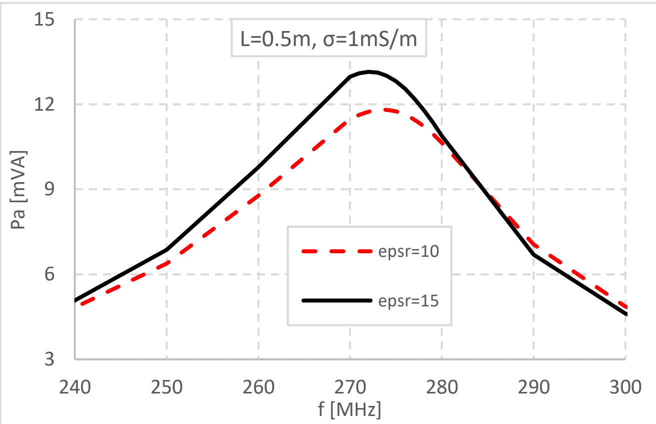
Finally, Fig 17 and 18 show the results obtained for the apparent and active power in the case of the wire above a perfectly conducting (PEC) ground for the different heights. As it is expected, the closer is the wire positioned to the ground the higher is the impact to the peak of the apparent power.

IV. Concluding Remarks

The paper deals with an efficient procedure to assess the complex power generated by thin wires radiating above a lossy half-space without need to calculate generated electric and magnetic fields and corresponding Poynting vector, respectively. Once the current distribution along dipole antenna is obtained by numerically solving the Pocklington integro-differential equation the complex power of the antenna can be determined by solving the integral over the inner product of tangential component of the electric field and current distribution along the wire arising from Poynting theorem.



a)



b)

Fig 16. a) Apparent power vs frequency for the 0.5m long wire placed at height h=0.1m above a lossy ground for various values of the ground permittivity (σ = 0.001 S/m) b) detailed view

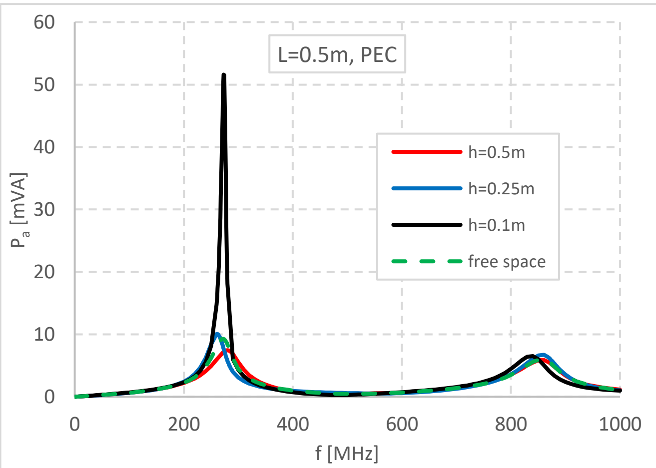


Fig. 17. Apparent power vs frequency for the 0.5m long wire placed at different heights above a perfect ground

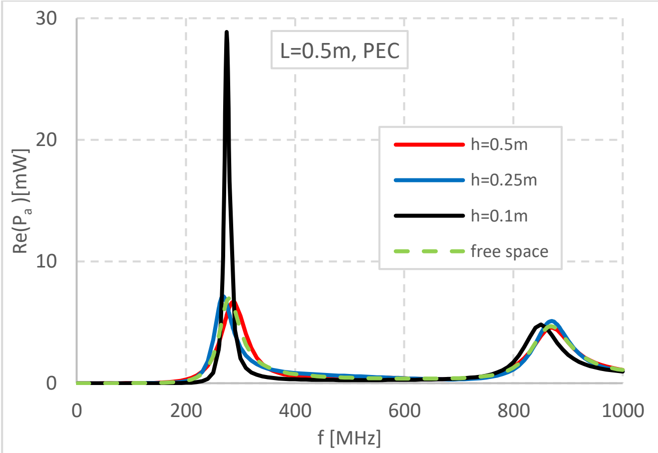


Fig. 18. Active power vs frequency for the 0.5m long wire placed at different heights above a perfect ground

Numerical calculation procedure uses the vectors and matrices already constructed within the current distribution calculation procedure. Some illustrative numerical results for the active power, reactive power and apparent power are presented in the paper for different values of input parameters. Future work is likely to deal with more demanding radiating configurations.

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