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Analysis of Ship Propulsion Shafting Vibration Using Coupled Torsional-Bending Model

Abstract

The paper describes a coupled torsional-bending model of ship propulsion system vibrations. The developed model is based on the modified model of the Jeffcott rotor. In order to test the model and determine the coupled torsional-bending vibrations, several cases were analyzed. First, a reference case corresponding to a fully axisymmetric ship propulsion system is set up. Then, the influence of the constant radial force in the vertical direction at the position of the stern bearing was analyzed, such as the conditions of navigation of the ship at calm sea under partial or fully loaded hull. Finally, the case of sailing on rough sea is analyzed, when the propeller racing occurs due to the stern lifting out of the sea. For simplicity, the harmonic law of amplitude changing of the vertical radial force on the stern tube bearing as well as of the propeller load was adopted. Based on the results of numerical analysis, it was found that the proposed model well describes the case of coupling of torsional and bending vibrations of the propulsion system.

Keywords: ship propulsion system, coupled torsional-bending vibration model, numerical analysis, propeller racing

1. Introduction

The ship's propulsion system suffers various complex loads during operation. The shaft line is usually loaded externally due to combination of radial forces that represent reactions in the bearings and the torque variations caused by the torsional vibration applicable for continuous operation, basically due to engine firing pulses [1, 2].

A proper consideration of the single and coupled vibration modes of the propeller shaft, including torsional, longitudinal, and transverse vibrations is important to ensure safe ship propulsion and navigation at sea, [3, 4].

The weather conditions under which the ship sails can also have a impact on the shaft line loadings since the rough sea can cause periodical emerging and immersion from the sea, causing a propeller racing phenomenon, [5, 6].

Due to mentioned loads the torsional and transverse vibrations of a shaft line are expected, having the influence on fatigue [7-9] as well as fractural [10,11] behavior of shaft line system.

Additionally, due to always present shaft unbalance and bow [12], the conditions for appearance of coupled torsional – transverse vibrations, are met.

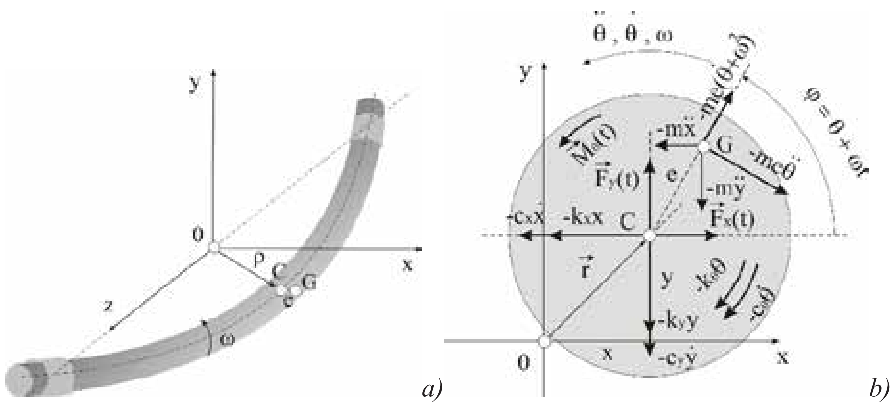
This paper presents a contribution in terms of explaining the coupling between torsional and bending vibrations of the shaft line when the ship is navigating on calm and rough seas.

2. Mathematical and numerical model for coupled torsional-bending vibrations of ship propulsion shafting

In this paper mathematical model based on the modified model of the Jeffcott rotor is used, Figure 1. The propulsion shaft, subjected to both torsional as well as bending vibrations is modelled as rigid disc characterized with inertial parameters such as mass *m* and mass moment of inertia *J*. Propulsion shaft is also subjected to cross – section eccentricity due to unbalance *e*. The equations of motion for the disc due to planar motion are

$$x = x(t),\ \ y = y(t),\ \ \varphi = \varphi(t) = \theta(t) + \omega t \tag{1}$$

In the previous equation, the total rotational angle φ of disc is a sum of rotational angle due to shaft rotation with constant rotational speed ω and torsional vibrations angle θ. The unknown displacement vector, as function of time *t*, consists of two translational displacements and one rotational angular displacement:



a) b)

Figure 1. Shaft with the eccentricity of cross section

$$\{u(t)\} = \{x(t), y(t), \theta(t)\}^T \tag{2}$$

Having in mind the elastic and dissipative forces acting in shaft geometrical centre C in both vibrating directions x and y, as well as inertial forces in this directions and inertial forces due to centripetal and tangential rotational accelerations, acting on center of mass G, the equations of motion obtained due D'Alambert principle take the following form (Figure 1b):

$$m\ddot{x} = \sum F_x \rightarrow m\ddot{x} - me\ddot{\theta}\sin(\omega t + \theta) + c_x\dot{x} + k_x x == me(\omega + \dot{\theta})^2 \cos(\omega t + \theta)$$

$$m\ddot{y} = \sum F_y \rightarrow m\ddot{y} + me\ddot{\theta}\cos(\omega t + \theta) + c_y\dot{y} + k_y y == me(\omega + \dot{\theta})^2 \sin(\omega t + \theta) \tag{3}$$

$$J\ddot{\theta} = \sum M_\theta \rightarrow (J + me^2)\ddot{\theta} - me\ddot{x}\sin(\omega t + \theta) + +me\ddot{y}\cos(\omega t + \theta) + c_\theta\dot{\theta} + k_\theta\theta = 0$$

The nonlinear equations of motion (3) are coupled through the presence of torsional angle θ and its time derivations in bending equations as well through the presence of horizontal and vertical accelerations in the equation for motion in torsional direction. Equations (3) can be further rearranged by means of angles addition theorem as well as torsional angle approximation as small quantities ($\sin\theta \approx \theta$, $\cos\theta \approx 1$):

$$m\ddot{x} - me\ddot{\theta}sin(\omega t)\sin(\omega t) + c_x\dot{x} + k_x x - 2me\omega\dot{\theta}\cos(\omega t) + me\omega^2\theta\sin(\omega t) =$$

$$= me\omega^2\cos(\omega t) + F_x(t) m\ddot{y} + me\ddot{\theta}\cos(\omega t) + c_y\dot{y} + k_y y -$$

$$-2me\omega\ddot{\theta}\sin(\omega t) - me\omega^2\theta\cos(\omega t) = me\omega^2\sin(\omega t) + F_y(t)(J + me^2)\ddot{\theta} - \tag{4}$$

$$-me\ddot{x}\sin(\omega t) + me\ddot{y}\cos(\omega t) + c_\theta\dot{\theta} + k_\theta\theta = M_\theta(t)$$

Finally, equations (4) should be written in the matrix form as follows:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F_c\} + \{F_{ext}\} \tag{5}$$

where inertia matrix $[M]$, matrix of dissipative forces $[C]$, stiffness matrix $[K]$ and vectors of time dependent centrifugal $\{F_c\}$ and external forces $\{F_{ext}\}$, are defined as:

$$[M]=\begin{bmatrix} m & 0 & -me\sin(\omega t) \\ 0 & m & me\cos(\omega t) \\ -me\sin(\omega t) & me\cos(\omega t) & J+me^2 \end{bmatrix},$$

$$[C]=\begin{bmatrix} c_x & 0 & -2me\omega\cos(\omega t) \\ 0 & c_y & -2me\omega\sin(\omega t) \\ 0 & 0 & c_\theta \end{bmatrix}, \tag{6}$$

$$[K]=\begin{bmatrix} k_x & 0 & me\omega^2\sin(\omega t) \\ 0 & k_y & -me\omega^2\cos(\omega t) \\ 0 & 0 & k_\theta \end{bmatrix},$$

$$\{F_c\}=\begin{Bmatrix} me\omega^2\cos(\omega t) \\ me\omega^2\sin(\omega t) \\ 0 \end{Bmatrix},\{F_{ext}\}=\begin{Bmatrix} F_x(t) \\ F_y(t) \\ M_\theta(t) \end{Bmatrix}$$

In the matrix equation (6) parameters c_x, c_y, c_θ, k_x, k_y and k_θ represent dampings and stiffnesses of the system in directions of coordinate axis x and y while c_θ and k_θ are torsional damping and stiffness.

In terms of getting results of simulation i.e. vector $\{u\}$, system (6) is solved by using of Newmark's time stepping method. Between it's parameters, the combination containing $\gamma = ½$ and $\beta = ¼$ (average acceleration method) is selected [14].

2.1. External forces and torsional torque

External loadings on the shaft line can be simply defined as a combination of radial forces that represent reactions in the bearings and the torque caused by the propulsion torque load. Two cases are considered in the paper. The first case represents the shaft line response to loads when the ship is navigating on calm seas and the other one represents the shaft line response to loads when the ship is navigating on rough seas.

In the first case, different hull structure deflections and their impact on propulsion shaft line are assumed regarding different ship service load conditions, Figure 2. It is assumed that ship hull structure has constant deflections when navigating at the same service load condition. Therefore, three subcases are considered, $F_x = 0; 0; 0$, $F_y = 0; 300; 600$ N and $M_\theta = M_{\theta 0}\sin(\omega_s t)$, where F_x and F_y are external radial forces, M_θ is external torque on propulsion shaft and $\omega_s = 2\pi/T$ ($f_s = 1/T$) is shaft rotational frequency.

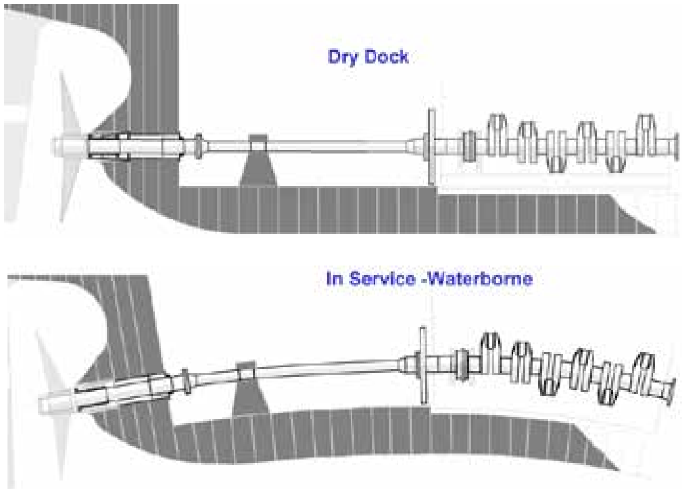


Figure 2. Hull structure deflections and their impact on propulsion shaft line, [13]

In the second case it is assumed that rough sea causes a propeller racing, Figure 3. This phenomenon happens when a ship pitches and heaves heavily. Due to these motions, the stern lifts out of the sea periodically exposing part of the propeller and causing instant increase of propeller speed as well as instant drop of propeller load, Figure 4. It is assumed that wave period T which causes propeller racing is equal to 6 s. To simulate rough sea beside the parameters used for calm sea the following conditions are additionally set: $F_x = 0$, $F_{yr} = F_{yr0}\,(0.5 + 0.5\sin(\omega_r t))$ and $M_{\theta r} = M_{\theta r0}\,(0.6 + 0.4\,\sin(\omega_r t))$, where $\omega_r = 2\pi/T$ ($f_r = 1/T$) is wave rotational frequency and $M_{\theta r0}$ is torque amplitude due to static propeller torque drop caused by propeller racing. Table 1 presents torque and transverse force loads used in numerical analysis.

Table 1. Torque and transverse force loads used in numerical analysis

Case No.	f_s, Hz	F_y , N	$M_{\theta 0}$, Nm	f_r, Hz	F_{yr0} , N	$M_{\theta r0}$, Nm
1	1.67	0	0.06	0	0	0
2	1.67	300	0.06	0	0	0
3	1.67	600	0.06	0	0	0
4	1.67	0	0.06	0.167	600	1

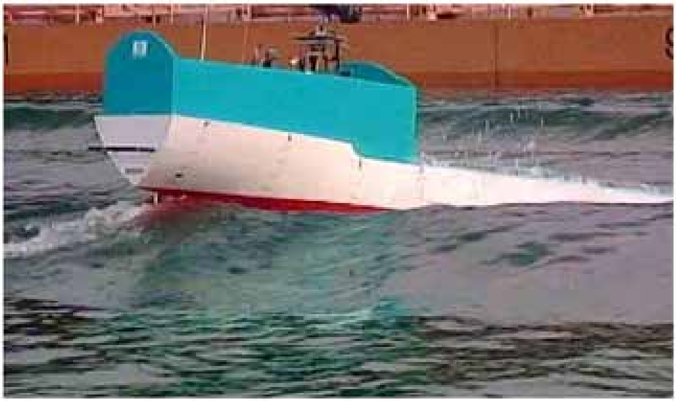


Figure 3. Propeller racing [6]

Figure 4. The emergence of the stern at the rough sea [2]

3. Numerical simulation

To investigate the coupling effect between torsional and bending vibration, propulsion shaft with following parameters was used: shaft density ρ = 7800 kg/m^3, Poisson's ratio ν = 0.3, Young's modulus, E = 206 GPa, shear modulus, G = 77 GPa, shaft length, L = 2.665 m, shaft diameter, D = 0.086 m, rotational speed n = 100 rpm, stiffness of system $k_x = k_y = 7\times10^5$ N/m, $k_y = 1.7\times10^5$ N·m/rad, damping of system, $k_x = k_y = 7\times10^5$ N·s/m and $k_y = 1.7\times10^5$ N·m·s/rad.

Equation (6) was solved by using of Newmark's time stepping method. Time step in each simulation case was 1×10^{-3} s whereas the initial conditions of displacements x_0, y_0, θ_0 were set to zero.

The first three cases of simulation, according to Table 1, correspond to the conditions of navigation on calm seas while the fourth simulation corresponds to the conditions of navigation on rough seas. The total simulation time was equal to 30 s. Results are presented as displacements in time domain, frequency domain as well as in the form of Short time Fourier transform – STFT spectrograms. Figure 5 presents the response of the propulsion shaft displacements in horizontal, vertical direction and torsional angles for case 1 in time and frequency domain. Likewise, Figures 6 and 7 show the frequency responses of shaft displacements in horizontal, vertical direction and torsional angles for cases 2, 3 and 4. In order to gain a more detailed insight into what happens to the vibrations of the shaft line when navigating rough seas, the spectrograms for case 4 are additionally presented in Fig. 8.

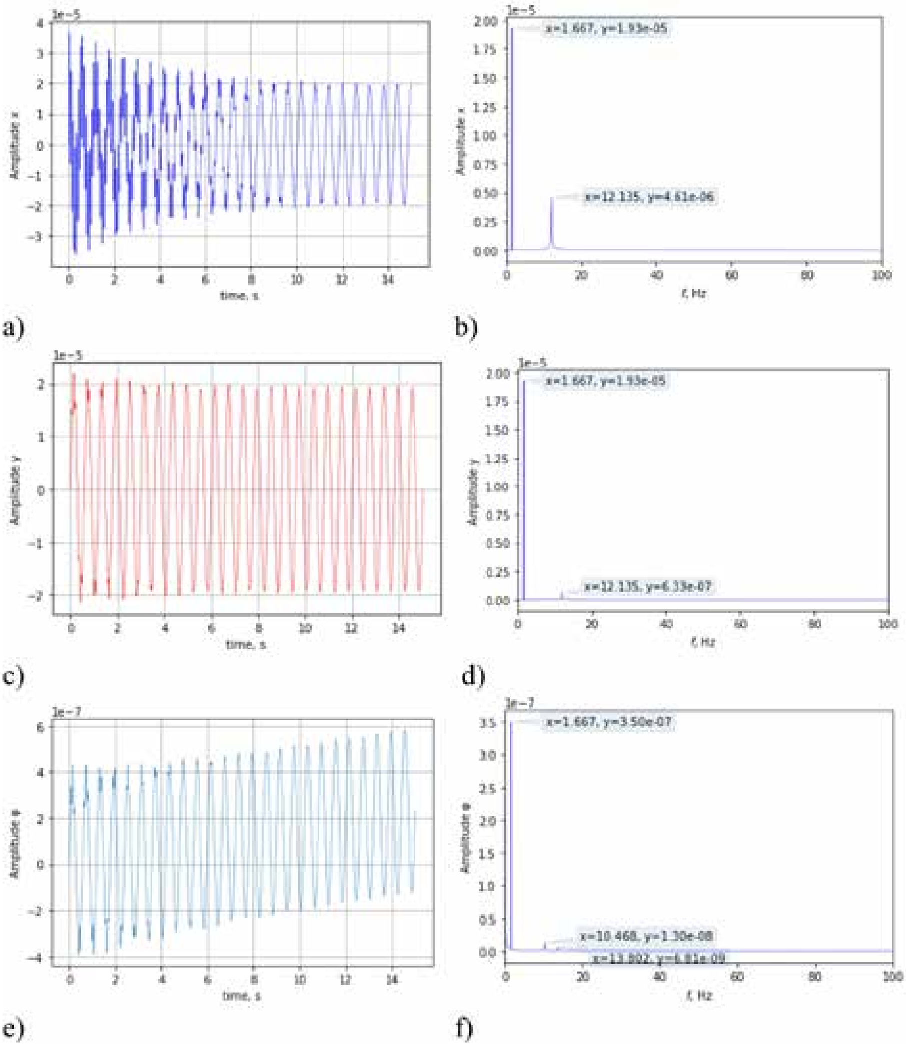


Figure 5. Response of the propulsion shaft displacements in horizontal, vertical and torsional angles for case 1, a) $x(t)$, b) frequency spectrum of $x(t)$, c) $y(t)$, d) frequency spectrum of $y(t)$, e) $\theta(t)$, f) frequency spectrum of $\theta(t)$

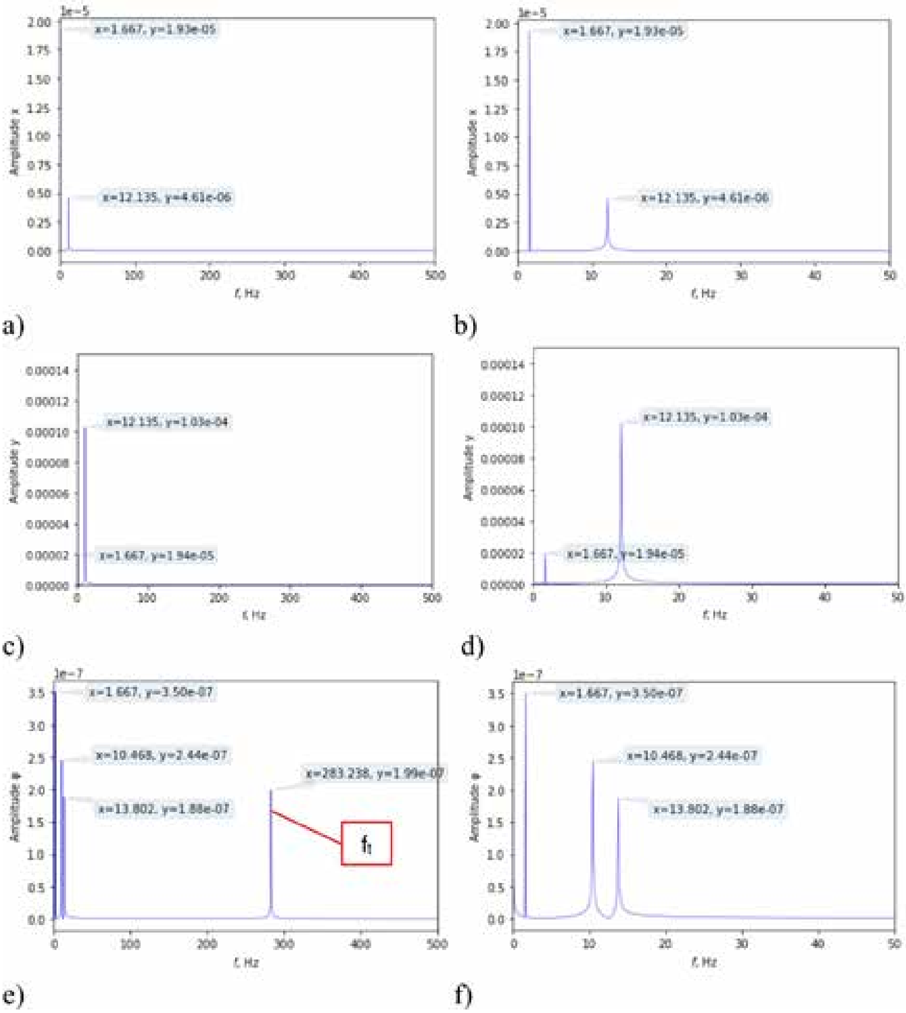


Figure 6. Response of the propulsion shaft displacements in horizontal, vertical and torsional angles for case 2, a) FFT of $x(t)$, b) FFT of $x(t)$ 0-50 Hz, c) FFT of $y(t)$, d) FFT of $y(t)$ 0-50 Hz, e) FFT of $\theta(t)$, f) FFT of $\theta(t)$ 0-50 Hz

Figure 6e shows the frequency responses of the torsional angles of shaft in frequency range 0-500 Hz for case 2. It is important to note that 1 natural torsional frequency has a value of f_t = 283.2 Hz, i.e. the frequency components that are visible on the zoom display (Figure 6f) do not actually have anything to do with the torsional free vibrations of the propulsion shaft.

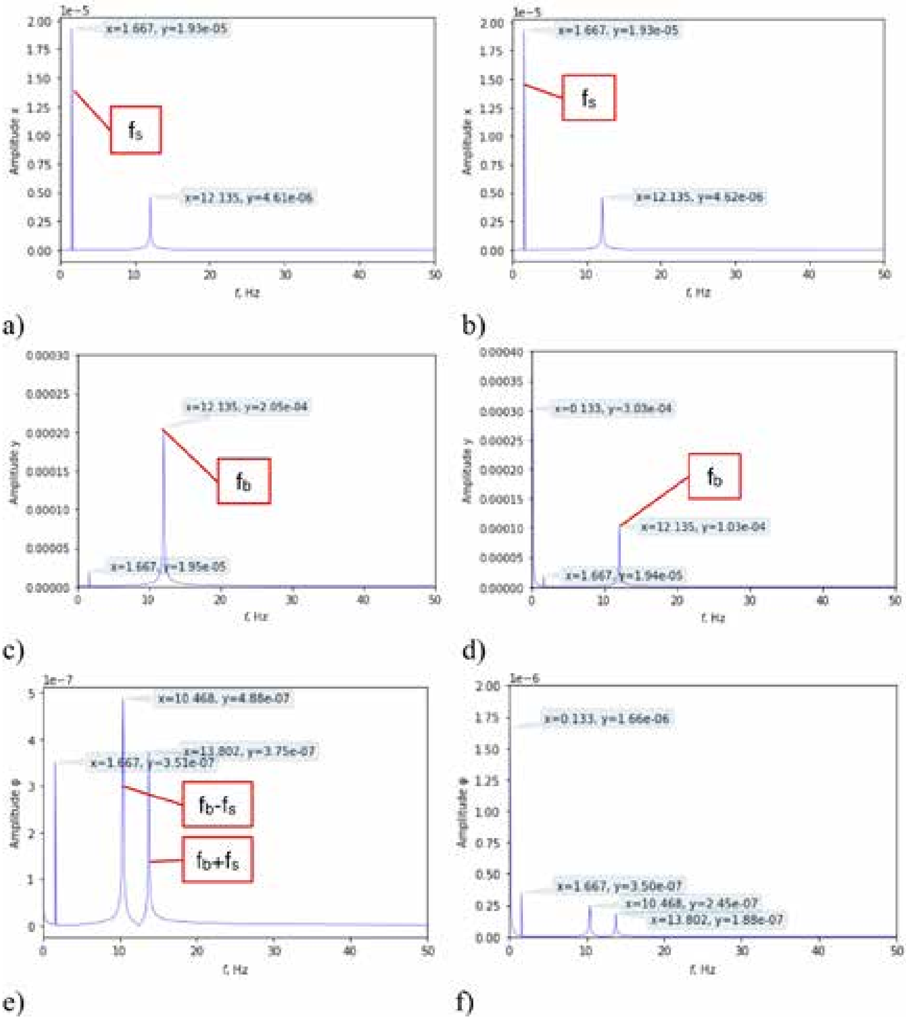


Figure 7. Response of the propulsion shaft displacements in horizontal, vertical and torsional angles for cases 3 and 4, a) c3 FFT of $x(t)$, b) c4 FFT of $x(t)$, c) c3 FFT of $y(t)$, d) c4 FFT of $y(t)$, e) c3 FFT of $\theta(t)$, f) c4 FFT of $\theta(t)$

As the first natural bending frequency of the shaft is at $f_b = 12.1$ Hz, the frequency components, visible in Figures 6f, 7e, 7f and 8c, actually represent the modulated frequencies $f_b \pm f_s = 10.5$; 13.8 Hz and thus prove the coupling between the bending and torsional vibrations of the propulsion shaft.

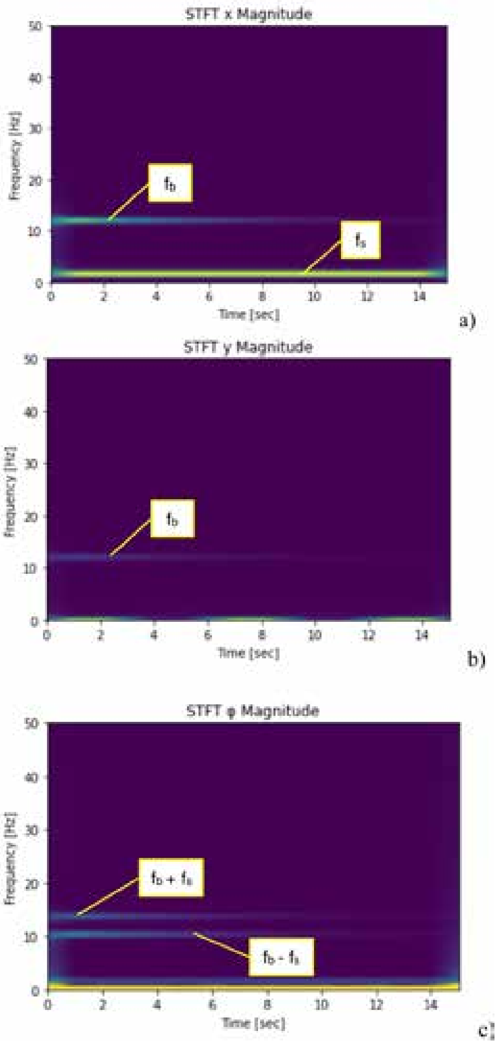


Figure 8. Response of the propulsion shaft displacements in horizontal, vertical and torsional angles for case 4, a) STFT of $x(t)$, b) STFT of $y(t)$, c) STFT of $\theta(t)$,

5. Conclusion

A coupled torsional-bending vibration model of ship propulsion system is considered in this paper. This model is based on the modified Jeffcott rotor model. Two cases are considered. First, when the ship is navigating on calm seas and second, when navigating on rough seas. Coupling between bending and torsional vibration was observed in each of analyzed cases. It is manifested in the appearance of modulated frequency components ($f_b \pm f_s$) in the torsional domain even though modulation occurs around a central frequency which is actually first bending natural frequency.

Coupling is minimal in the first case when the lateral force is equal to zero but it increases with increasing amplitude of the lateral force. In the STFT spectrogram of the 4th case the wave beating in vertical direction at 0.1333 Hz is more intense than speed frequency harmonic at 1.667 Hz.

Acknowledgements

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