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ECONOMIC GROWTH WITH LEARNING BY PRODUCING, LEARNING BY EDUCATION AND LEARNING BY CONSUMING

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SUMMARY

This paper proposes a dynamic economic model with wealth accumulation and human capital accumulation. The economic system consists of one production sector and one education sector. We take account of three ways of improving human capital: learning by producing, learning by education, and learning by consuming. The model describes a dynamic interdependence between wealth accumulation, human capital accumulation, and division of labor under perfect competition. We simulate the model to demonstrate existence of equilibrium points and motion of the dynamic system. We also examine effects of changes in the propensity to receive education, efficiency of learning, and efficiency of education upon dynamic paths of the system.

KEY WORDS

learning by producing, learning by consuming, learning by education, economic growth, education production

CLASSIFICATION

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INTRODUCTION

Dynamic interdependence between economic growth and human capital is currently a main topic in economic theory. This study attempts to provide another contribution to the literature by examining interdependence between savings and research within a new approach to consumers' behavior with endogenous saving). This study attempts to provide another contribution to the literature by examining interdependence between savings and education within a new approach to consumers' behavior with endogenous saving.

Our model is built upon the three main growth models – Solow's one-sector growth model [1, 2], Arrow's learning by doing model [3], and the Uzawa-Lucas's growth model with education [4] – in the growth literature. The main mechanisms of economic growth in these three models are integrated into a single framework. One of the first seminal attempts to render technical progress endogenous in growth models was made by Arrow in 1962. He emphasized one aspect of knowledge accumulation - learning by doing [3]. Uzawa [4] introduced a sector specifying in creating knowledge into growth theory. The knowledge sector utilizes labor and the existing stock of knowledge to produce new knowledge, which enhances productivity of the production sector. In 1981 Schultz emphasized the incentive effects of policy on investment in human capital [5]. There are many other studies on endogenous technical progresses. But on the whole theoretical economists had been relatively silent on the topic from the end of the 70s until the publication of Romer's 1986 paper. The literature on endogenous knowledge and economic growth have increasingly expanded since Romer re-examined issues of endogenous technological change and economic growth in his 1986's paper, e.g., [6 - 9]. Since then various other issues related to innovation, diffusion of technology and behavior of economic agents under various institutions have been discussed in the literature. There are also many other models emphasizing different aspects, such as education, trade, R&D policies, entrepreneurship, division of labor, learning through trading, brain drain, economic geography, of dynamic interactions among economic structure, development and knowledge. This study is to model interaction between physical capital and human capital accumulation by taking account of Arrow's learning by doing, Uzawa-Lucas's learning through education, and Zhang's learning by consuming.

Our purpose is to combine the economic mechanisms in the three key growth models - Solow's growth model, Arrow's learning by doing model, the Uzawa-Lucas education model into a single comprehensive framework. The synthesis of the three growth models within a single framework is still analytically tractable because we propose an alternative approach to consumers' behavior. The paper is organized as follows. Section 2 introduces the basic model with wealth accumulation and human capital accumulation. The model describes a dynamic interdependence between wealth accumulation, human capital accumulation, and division of labor under perfect economic competition. Section 3 examines dynamic properties of the model. We simulate the model to demonstrate effects of changes in some parameters on the economic system. Section 5 concludes the study.

BASIC MODEL

The economy has one production sector and one education sector. Most aspects of the production sector are similar to the standard one-sector growth model, see [10 - 12]. It is assumed that there is only one (durable) good in the economy under consideration. Households own assets of the economy and distribute their incomes to consume and save. Production sectors or firms use inputs such as labor with varied levels of human capital, different kinds of capital, knowledge and natural resources to produce material goods or services. Exchanges take place in perfectly competitive markets. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is undertaken only by households. All earnings of firms are distributed in the form of payments to factors of production, labor, managerial skill and capital ownership. We assume a homogenous and fixed population N_0. The labor force is distributed between the two sectors. We select commodity to serve as numeraire, with all the other prices being measured relative to its price. We assume that wage rates are identical among all professions. We introduce

$F_i(t)$ – output level of the production sector at time t,

$K(t)$ – level of capital stocks of the economy,

$H(t)$ – level of human capital of the population,

$N_i(t)$ and $K_i(t)$ – labor force and capital stocks employed by the production sector, respectivelly,

$N_e(t)$ and $K_e(t)$ – labor force and capital stocks employed by the education sector, respectivelly,

$T(t)$ and $T_e(t)$ – work time and study time, respectivelly,

$p(t)$ – price of education (service) per unit of time, and

$w(t)$ and $r(t)$ – wage rate and rate of interest, respectivelly.

Total capital stock $K(t)$ is allocated between the two sectors. As full employment of labor and capital is assumed, we have

$$K_i(t) + K_e(t) = K(t)\,,\ N_i(t) + N_e(t) = N(t)$$

in which $N(t) \equiv T(t) \cdot N_0$, where $N(t)$ is the total work time of the population. We may rewrite previous relations as follows

$$n_i(t)k_i(t) + n_e(t)k_e(t) = k(t),\,,\ n_i(t) + n_e(t) = 1 \qquad (1)$$

in which

$$k_j(t) \equiv \frac{K_j(t)}{N_j(t)},\quad n_j(t) \equiv \frac{N_j(t)}{N(t)},\quad k(t) \equiv \frac{K(t)}{N(t)},\ \text{j = i, e.} \qquad (1)$$

THE PRODUCTION SECTOR

We assume that production is to combine 'qualified labor force' $H^m(t) \cdot N_i(t)$ and physical capital $K_i(t)$. We use the conventional production function to describe a relationship between inputs and output. The function $F_i(t)$ defines the flow of production at time t. The production process is described by

$$F_i(t) = A_i K_i^{\alpha_i}(t)\left[H^m(t)N_i(t)\right]^{\beta_i},\quad A_i, \alpha_i, \beta_i > 0,\ \ \alpha_i + \beta_i = 1.$$

Markets are competitive; thus labor and capital earn their marginal products, and firms earn zero profits. The rate of interest and wage rate are determined by markets. Hence, for any individual firm $r(t)$ and $w(t)$ are given at each point of time. The production sector chooses the two variables $K_i(t)$ and $N_i(t)$ to maximize its profit. The marginal conditions are given by

$$r(t) + \delta_k = \frac{\alpha_i F_i(t)}{K_i(t)} = \alpha_i A_i H^{m\beta_i} k_i^{-\beta_i},\quad w(t) = \frac{\beta_i F_i(t)}{N_i(t)} = \beta_i A_i H^{m\beta_i} k_i^{\alpha_i}, \qquad (2)$$

where δ_k is depreciation rate of physical capital.

ACCUMULATION OF HUMAN CAPITAL AND THE EDUCATION SECTOR

We assume that there are three sources of improving human capital, through education, "learning by producing", and "learning by leisure". Arrow first introduced learning by doing into growth theory [3]; Uzawa took account of trade offs between investment in education and capital accumulation [4], and Zhang introduced impact of consumption on human capital accumulation (via the so-called creative leisure) into growth theory [13, 14]. We propose that human capital dynamics is given by

$$\dot{H} = \frac{\upsilon_e F_e^{a_e}\left(H^m T_e N_0\right)^{b_e}}{H^{\pi_e} N_0} + \frac{\upsilon_i F_i^{a_i}}{H^{\pi_i} N_0} + \frac{\upsilon_h C^{a_h}}{H^{\pi_h} N_0} - \delta_h H \, , \tag{3}$$

where δ_k (>0) is the depreciation rate of human capital, υ_e, υ_i, υ_h, a_e, b_e, a_i and a_h, are non-negative parameters. The signs of the parameters π_e, π_i and π_h are not specified as they can be either negative or positive.

The above equation is a synthesis and generalization of Arrow's, Uzawa's, and Zhang's ideas about human capital accumulation. The term

$$\frac{\upsilon_e F_e^{a_e}\left(H^m T_e N_0\right)^{b_e}}{H^{\pi_e} N_0} \, ,$$

describes the contribution to human capital improvement through education. Human capital tends to increase with an increase in the level of education service, F_e, and in the (qualified) total study time, $H^m \cdot T_e \cdot N_0$. The population N_0 in the denominator measures the contribution in terms of per capita. The term H^{π_e} indicates that as the level of human capital of the population increases, it may be more difficult (in the case of π_e being large) or easier (in the case of π_e being small) to accumulate more human capital via formal education. The term N_0 in the denominator term measures the contribution in terms of per capita. We take account of learning by producing effects in human capital accumulation by the term $\upsilon_i \cdot F_i^{a_i}/H^{\pi_i}$. This term implies that contribution of the production sector to human capital improvement is positively related to its production scale F_i and is dependent on the level of human capital. The term H^{π_i} takes account of returns to scale effects in human capital accumulation. The case of $\pi_e > (<) \ 0$ implies that as human capital is increased it is more difficult (easier) to further improve the level of human capital. We take account of learning by consuming by the term $\upsilon_h \cdot C^{a_h}/(H^{\pi_h} N_0)$. This term can be interpreted similarly as the term for learning by producing.

It should be noted that in the literature on education and economic growth, it is assumed that human capital evolves according to the following equation (see [12])

$$\dot{H}(t) = H(t)^{\eta} G[T_e(t)] \, ,$$

where the function $G(\cdot)$ is increasing as the effort rises with $G(0) = 0$. In the case of $\eta < 1$, there is diminishing return to the human capital accumulation. This formation is due to Lucas [7]. As $\dot{H}/H < H^{\eta-1}G(1)$, we conclude that the growth rate of human capital must eventually tend to zero no matter how much effort is devoted to accumulating human capital. Uzawa's model may be considered a special case of the Lucas model with $\gamma = 0$, $U(c) = c$, and the assumption that the right-hand side of the above equation is linear in the effort. It seems reasonable to consider diminishing returns in human capital accumulation: people accumulate it rapidly early in life, then less rapidly, then not at all – as though each additional percentage increment were harder to gain than the preceding one. Solow adapts the Uzawa formation to the following form

$$\dot{H}(t) = H(t) \cdot \kappa \cdot T_e(t) \, .$$

This is a special case of the previous equation. The new formation implies that if no effort is devoted to human capital accumulation, then $\dot{H}(0) = 0$ (human capital does not vary as time passes. This results from depreciation of human capital being ignored); if all effort is devoted to human capital accumulation, then $G_H(t) = \kappa$ (human capital grows at its maximum rate as results from the assumption of potentially unlimited growth of human capital). Between the two extremes, there is no diminishing return to the stock $H(t)$. To achieve a given percentage increase in $H(t)$ requires the same effort. As remarked by Solow, the above formulation is very far from a plausible relationship. If we consider the above equation as a production for new human capital (i.e., $\dot{H}(t)$), and if the inputs are already accumulated human capital and study time, then this production function is homogenous of degree two. It has strong increasing returns to scale and constant returns to $H(t)$ itself. It can be seen that our approach is more general to the traditional formation with regard to education. Moreover, we treat teaching also as a significant factor in human capital accumulation. Efforts in teaching are neglected in Uzawa-Lucas model.

We assume that the education sector is also characterized of perfect competition. Here, we neglect any government's financial support for education. Indeed, it is important to introduce government's intervention in education. Students are supposed to pay the education fee $p(t)$ per unit of time. The education sector pays teachers and capital with the market rates. The cost of the education sector is given by $w(t) \cdot N_e(t) + r(t) \cdot K_e(t)$. The total education service is measured by the total education time received by the population, $T_e \cdot N_0$. The production function of the education sector is assumed to be a function of $K_e(t)$ and $N_e(t)$. We specify the production function of the education sector as follows

$$F_e(t) = A_e K_e^{\alpha_e} \left(H^m N_e\right)^{\beta_e}, \ \ \alpha_e, \beta_e > 0, \ \ \alpha_e + \beta_e = 1, \tag{4}$$

where A_e, α_e and β_e are positive parameters. The education sector maximizes the following profit

$$\pi(t) = p(t) A_e K_e^{\alpha_e} \left(H^m N_e\right)^{\beta_e} - \left(r(t) + \delta_k\right) K_e(t) - w(t) N_e(t).$$

For given $p(t)$, $H(t)$, $r(t)$ and $w(t)$ the education sector chooses $K_e(t)$ and $N_e(t)$ to maximize the profit. The optimal solution is given by

$$r + \delta_k = \frac{\alpha_e p F_e}{K_e} = \alpha_e A_e p H^{m\beta_e} k_e^{-\beta_e}, \ \ w(t) = \frac{\beta_e p F_e}{N_e} = \beta_e A_e p H^{m\beta_e} k_e^{\alpha_e}. \tag{5}$$

The demand for labor force for given price of education, wage rate and level of human capital is given by

$$N_e = K_e \left(\frac{\beta_e A_e p H^{m\beta_e}}{w} \right)^{1/\alpha_e}.$$

We see that the demand for labor force from the education sector increases in the price and level of human capital and decreases in the wage rate.

CONSUMER BEHAVIORS

Consumers make decisions on choice of consumption levels of services and commodities as well as on how much to save. Different from the optimal growth theory in which utility defined over future consumption streams is used, we assume that we can find preference structure of consumers over consumption and saving at the current state. The preference over current and future consumption is reflected in the consumer's preference structure over current consumption and saving. We denote *per capita* wealth by $\bar{k}(t)$, where $\bar{k}(t) \equiv k(t)/N_0$. By the definitions, we have $\bar{k}(t) = T(t) \cdot k(t)$. *Per capita* current income from the interest payment $r(t) \cdot \bar{k}(t)$ and the wage payment $T(t) \cdot w(t)$ is given by

$$y(t) = r(t)\bar{k}(t) + T(t)w(t).$$

We call $y(t)$ the current income in the sense that it comes from consumers' daily toils (payment for human capital) and consumers' current earnings from ownership of wealth. The current income is equal to the total output. The sum of money that consumers are using for consuming, saving, and education are not necessarily equal to the temporary income because consumers can sell wealth to pay, for instance, the current consumption if the temporary income is not sufficient for buying food and touring the country. Retired people may live not only on the interest payment but also have to spend some of their wealth. The total value of wealth that consumers can sell to purchase goods and to save is equal to $\bar{k}(t)$. Here, we assume that selling and buying wealth can be conducted instantaneously without any transaction cost. The per capita disposable income is given by

$$\hat{y}(t) = y(t) + \bar{k}(t) = [1 + r(t)]\bar{k}(t) + T(t)w(t). \tag{6}$$

The disposable income is used for saving, consumption, and education. At each point of time, a consumer would distribute the total available budget among saving $s(t)$, consumption of goods $c(t)$, and education $p(t) \cdot T_e(t)$. The budget constraint is given by

$$c(t) + s(t) + p(t)T_e(t) = \hat{y}(t) = (1 + r(t))\bar{k}(t) + T(t)w(t). \tag{7}$$

The consumer is faced with the following time constraint

$$T(t) + T_e(t) = T_0,$$

where T_0 is the total available time for work and study. Substituting this function into the budget constraint (7) yields

$$c(t) + s(t) + (p(t) + w(t))T_e(t) = \bar{y}(t) \equiv (1 + r(t))\bar{k}(t) + T_0 w(t). \tag{8}$$

In our model, at each point of time, consumers have three variables, the level of consumption, the level of saving, and the education time, to decide. We assume that consumers' utility function is a function of level of goods $c(t)$ and level of saving $s(t)$ and education service $T_e(t)$ as follows

$$U(t) = U(c(t), s(t), T_e(t)).$$

The utility function can be considered as a function of $c(t)$, $s(t)$ and $T_e(t)$. For simplicity of analysis, we specify the utility function as follows

$$U(t) = c^{\xi}(t)s^{\lambda}(t)T_e^{\eta}(t), \quad \xi, \lambda, \eta > 0; \quad \xi + \lambda + \eta = 1, \tag{9}$$

where ξ is called the propensity to consume, λ the propensity to own wealth, and η the propensity to obtain education. This utility function is applied to different economic problems [13, 15]. A detailed explanation of the approach and its applications to different problems of economic dynamics are provided in [16].

For the representative consumer, wage rate $w(t)$ and rate of interest $r(t)$ are given in markets and wealth $\bar{k}(t)$ is predetermined before decision. Maximizing $U(t)$ in (9) subject to the budget constraint (8) yields

$$c = \xi\bar{y}, \quad s = \lambda\bar{y}, \quad (p + w)T_e = \eta\bar{y}. \tag{10}$$

The demand for education is given by $T_e = \eta\bar{y}/(p + w)$. The demand for education decreases in the price of education and the wage rate and increases in $\bar{y}$. An increase in the propensity to get educated increases the education time when the other conditions are fixed. In this dynamic system, as any factor is related to all the other factors over time, it is difficult to see how one factor affects any other variable over time in the dynamic system. We will demonstrate complicated interactions by simulation.

We now find dynamics of capital accumulation. According to the definition of $s(t)$, the change in the household's wealth is given by

$$\dot{\bar{k}}(t) = s(t) - \bar{k}(t) = \lambda\bar{y}(t) - \bar{k}(t). \tag{11}$$

For the education sector, the demand and supply balances at any point of time

$$T_e N_0 = F_e(t). \tag{12}$$

As output of the production sector is equal to the sum of the level of consumption, the depreciation of capital stock and the net savings, we have

$$C(t) + S(t) - K(t) + \delta_k K(t) = F_i(t) \tag{13}$$

where $C(t)$ is the total consumption, $S(t) - K(t) + \delta_k K(t)$ is the sum of the net saving and depreciation. We have

$$C(t) = c(t)N_0, \quad S(t) = s(t)N_0.$$

It is straightforward to show that this equation can be derived from the other equations in the system. We have thus built the dynamic model. We now examine dynamics of the model.

DYNAMICS AND ITS PROPERTIES

This section examines dynamics of the model. First, we show that the dynamics can be expressed by the two-dimensional differential equations system with $k_i(t)$ and $H(t)$ as the variables.

LEMMA

The dynamics of the economic system is governed by the two-dimensional differential equations

$$\dot{k}_i(t) = \tilde{\Omega}_i(k_i, H),$$
$$\dot{H}(t) = \tilde{\Omega}_h(k_i, H), \tag{14}$$

where the functions $\tilde{\Omega}_i(k_i, H)$ and $\tilde{\Omega}_h(k_i, H)$ are functions of $k_i(t)$ and $H(t)$ defined in (A10) and (A13) in the Appendix. Moreover, all the other variables can be determined as functions of $k_i(t)$ and $H(t)$ at any point of time by the following procedure: $\bar{k}(t) = \varphi_0(k_i, H)\varphi(k_i, H)$ (where φ_0 and φ are defined respectively in (22) and (21)) → $k_h(t) = \alpha k_i(t)$ → $T(t)$ and $\bar{y}(t)$ by (A9) → $k(t)$ by (A8) → $p(t)$ by (A2) → $n_i(t)$ and $n_h(t)$ by (A3) → $r(t)$ and $w(t)$ by (2) → $c(t)$, $T_e(t)$, and $s(t)$ by (10) → $N(t) = N_0T(t)$ → $N_j(t) = n_j(t)N(t)$ (j = i, e), → $K(t) = k(t)N(t)$ → $K_j(t) = k_j(t)N_j(t)$ → $F_j(K_j(t), N_j(t))$.

The differential equations system (14) contains two variables $k_i(t)$ and $H(t)$. Although we can analyze its dynamic properties as we have explicitly expressed the dynamics, we omit analyzing the model as the expressions are too complicated. Instead, we simulate the model to illustrate behavior of the system. In the remainder of this study, we specify the depreciation rates by δ_k = 0,05; δ_h = 0,04 and let T_0 = 1. The requirement T_0 = 1 will not affect our analysis. We specify the other parameters as follows

$$\alpha_i = 0{,}3;\ \alpha_e = 0{,}34;\ \lambda = 0{,}8;\ \eta = 0{,}008;\ N_0 = 50\,000;\ m = 0{,}6;$$
$$v_e = 0{,}8;\ v_i = 2{,}5;\ v_h = 0{,}7;\ \pi_e = -0{,}2;\ \pi_i = 0{,}7;\ \pi_h = 0{,}1; \tag{15}$$
$$a_e = 0{,}3;\ b_e = 0{,}5;\ a_i = 0{,}4;\ a_h = 0{,}1;\ A_i = A_e = 0{,}9.$$

The propensity to save λ is 0,8 and the propensity to consume education is 0,008. The propensity to consume goods ξ = 1 – 0,8 – 0,008 = 0,192. The technological parameters of the two sectors are specified at $A_i = A_e$ = 0,9. The specification m = 0,6 implies that there is a diminishing effect in turning human capital to labor force. The conditions π_e = – 0,2; π_i = 0,7 and π_h = 0,1 mean respectively that the learning by education exhibits increasing effects in human capital; the learning by producing exhibits (strong) decreasing effects in human capital; and the learning by consuming exhibits (weak) increasing effects in human capital.

By (14), an equilibrium point of the dynamic system is given by

$$\tilde{\Omega}_i(k_i, H) = 0,$$

$$\tilde{\Omega}_h(k_i, H) = 0. \tag{16}$$

Simulation demonstrates that the above equations have the following unique equilibrium solution

$$k_i = 7{,}9643, \; H = 1{,}1716.$$

The equilibrium values of the other variables are given by the procedure in Lemma. We list them as follows

$$r = 0{,}0175; \quad w = 1{,}255; \quad p = 0{,}920; \quad H = 1{,}172; \quad N = 32138{,}7$$

$$\begin{pmatrix} f_i \\ f_e \end{pmatrix} = \begin{pmatrix} 1{,}793 \\ 1{,}790 \end{pmatrix}, \begin{pmatrix} n_i \\ n_e \end{pmatrix} = \begin{pmatrix} 0{,}731 \\ 0{,}269 \end{pmatrix}, \begin{pmatrix} k_i \\ k_e \end{pmatrix} = \begin{pmatrix} 7{,}964 \\ 9{,}573 \end{pmatrix}, \begin{pmatrix} F_i \\ F_e \end{pmatrix} = \begin{pmatrix} 42109{,}7 \\ 16778{,}1 \end{pmatrix}, \begin{pmatrix} K_i \\ K_e \end{pmatrix} = \begin{pmatrix} 187\,092 \\ 82\,781{,}4 \end{pmatrix},$$

$$k = 8{,}397; \quad T = 0{,}643; \quad T_e = 0{,}357; \quad c = 1{,}295.$$

The consumer spends about 35,7 % of the total available time for study. The relative importance of the education sector is given by the following variables

$$\frac{pF_e}{F_i + pF_e} = 0{,}268; \quad \frac{K_e}{K_i + K_e} = 0{,}307; \quad \frac{N_e}{N_i + N_e} = 0{,}269.$$

The relative share of the education sector is about 26,8 % percent of the national product. The share seems to be large if one considers any real economy. As we are mainly concerned with effects of changes in some parameters, it seems that these unrealistic shares will not affect our main conclusions about comparative dynamic analyses.

We are now concerned with motion of the system. We specify initial conditions for the differential equations (14) as follows

$$k_i(0) = 7{,}32 \text{ and } H(0) = 0{,}7.$$

As shown in Figure 1, only one variable monotonously changes – the level of the human capital monotonously increases from the initial state to the equilibrium value. The economic development experiences a kind of *J*-curve process. It first experiences declination in per capita levels of consumption and wealth. After a few years these variables start to increase. The wage slightly declines and soon begins to increase. During the simulation period, the price of education increases and then starts to decline. It should be remarked that the price of education changes only slightly during the whole period. The education time also experiences a *J*-curve change during the study period. It first declines as the price increases and the real wage rate declines. The rate of interest increases and then starts to decline.

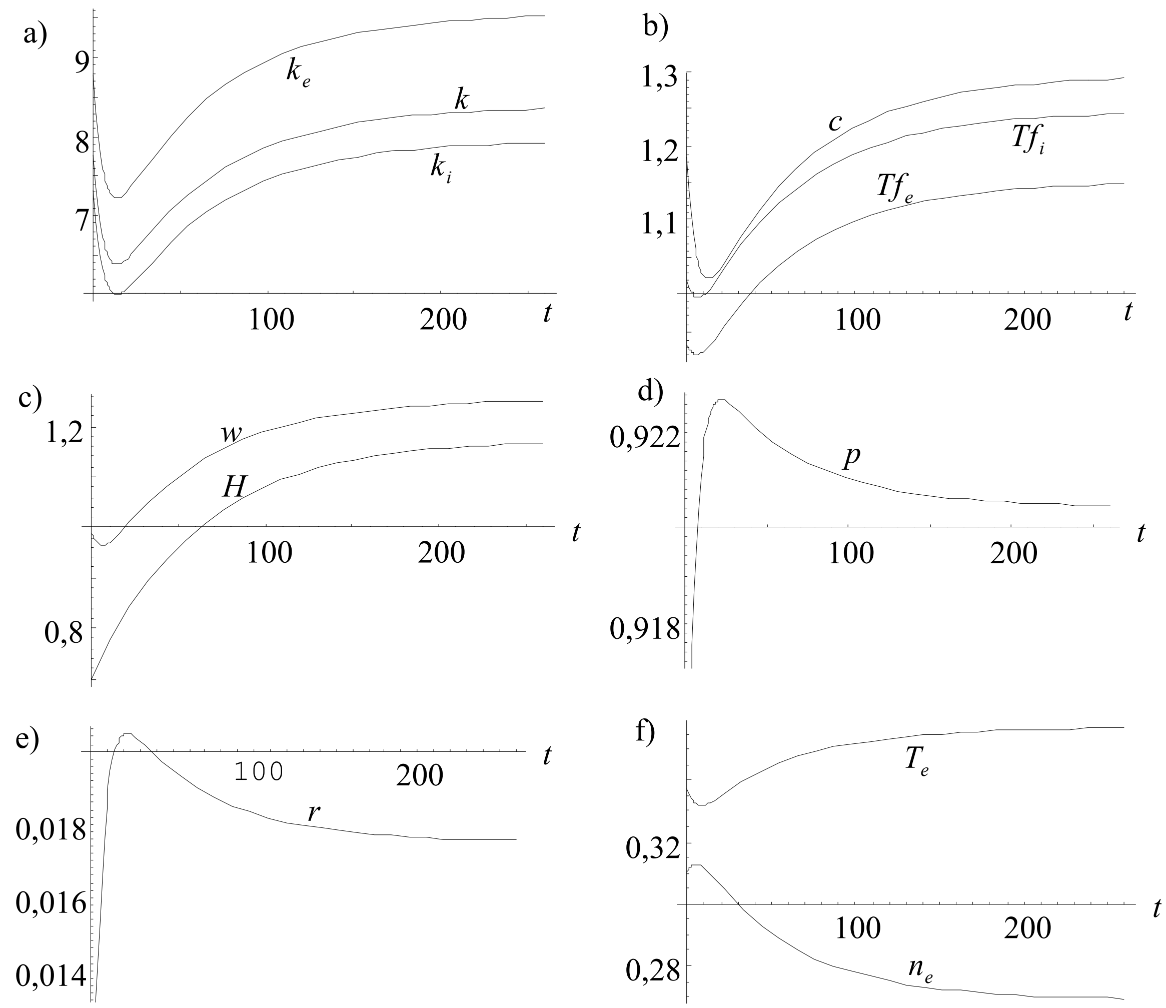

Figure 1. The motion of the economic system. Graph a) shows the capital intensities for production (k_i) and education (k_e) and per capita wealth ($\bar{k}$), b) the per capita consumption (c) and production (Tf_i), c) the wage rate (w) and the level of human capital (H), d) the price of education p, e) the rate of interest r; and f) the study time T_e and the sectorial share of labor force n_e. Values of parameters are as in (15).

COMPARATIVE DYNAMIC ANALYSIS IN SOME PARAMETERS BY SIMULATION

We now examine impact of changes on dynamic processes of the system. First, we examine the case that all the parameters, except the education efficiency parameter, A_e, are the same as in (15). We increase the education efficiency parameter from $A_e = 0,9$ to $A_e = 1,2$. The simulation results are demonstrated in Figure 2. The solid lines in Figure 2 are the same as in Figure 1, representing the values of the corresponding variables when $A_e = 0,9$; the dashed lines in Figure 2 represent the new values of the variables when $A_e = 1,2$. We see that as the education sector improves its productivity, the price of education will be reduced and the study time is increased. The level of human capital increases and the wage rate is increased. The per capita levels of consumption and wealth are increased. The share of the labor force of the education sector in the total labor force declines as the productivity of the education sector is improved. The per capita level of the education sector's output is increased and the per capita level of the production sector's output is reduced.

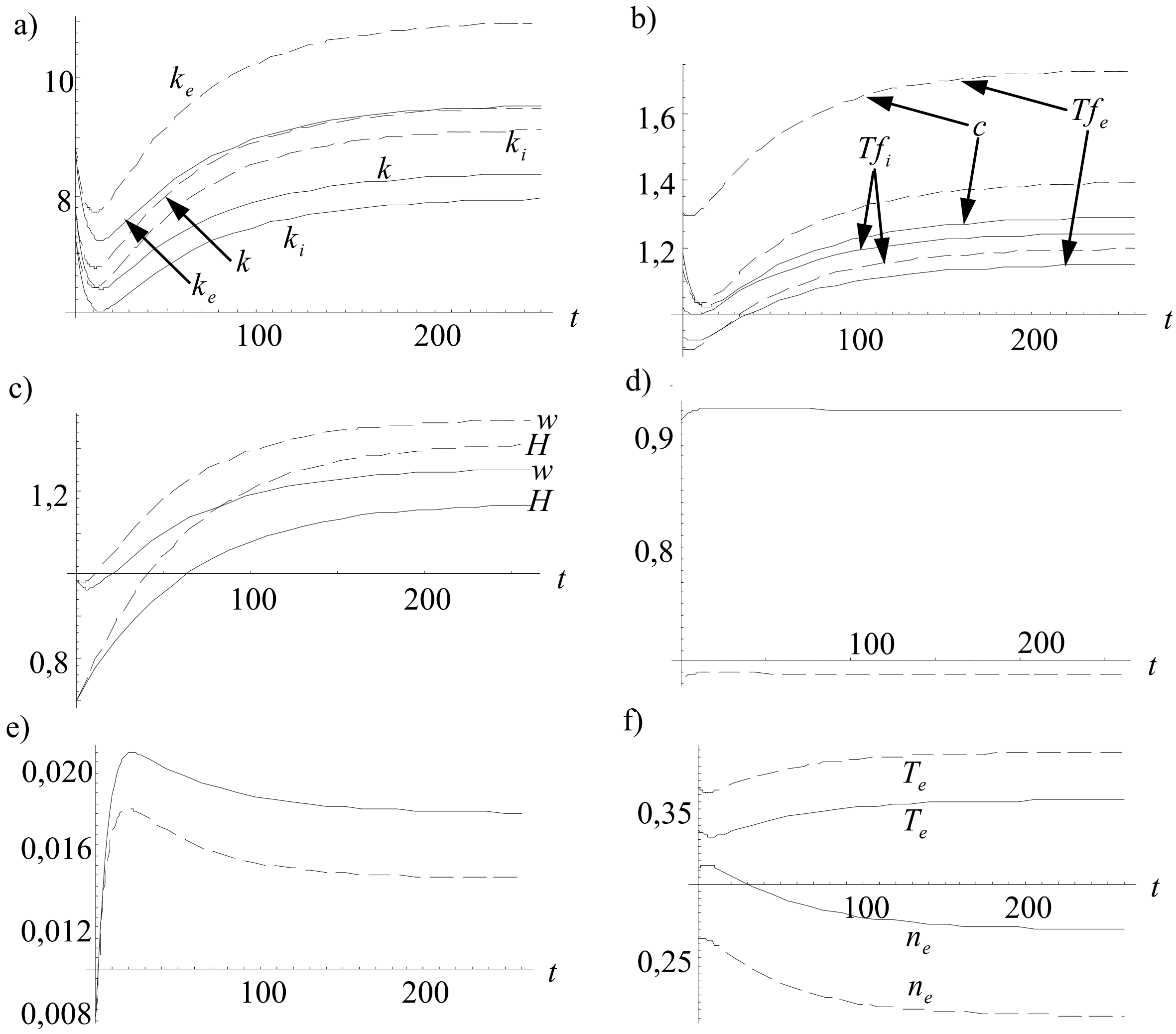

Figure 2. For A_e equal 0,9 (solid lines) and 1,2 (dashed lines) the graphs show a) the capital intensities and wealth for production (k_i) and education (k_e) and per capita wealth ($\bar{k}$), b) the per capita consumption (c) and production (Tf_i), c) the wage rate (w) and the level of human capital (H), d) the price of education, e) the rate of interest and f) the study time T_e and the sectorial share of labor force n_e.

We now increase the production sector's productivity from $A_i = 0{,}9$ to $A_i = 1{,}2$. The simulation results are demonstrated in Figure 3. The solid lines in Figure 3 are the same as in Figure 1, representing the values of the corresponding variables when $A_i = 0{,}9$; the dashed lines in Figure 3 represent the new values of the variables when $A_i = 1{,}2$. We see that as the production sector improves its productivity, both the price of education is increased and the study time is increased. The effects are different from the effects due to increases in the education sector's productivity. The level of human capital increases and the wage rate is increased. The per capita levels of consumption and wealth are increased. The share of the labor force of the education sector in the total labor force declines as the productivity of the production sector is improved. The per capita levels of the education sector's output and the production sector's output are increased.

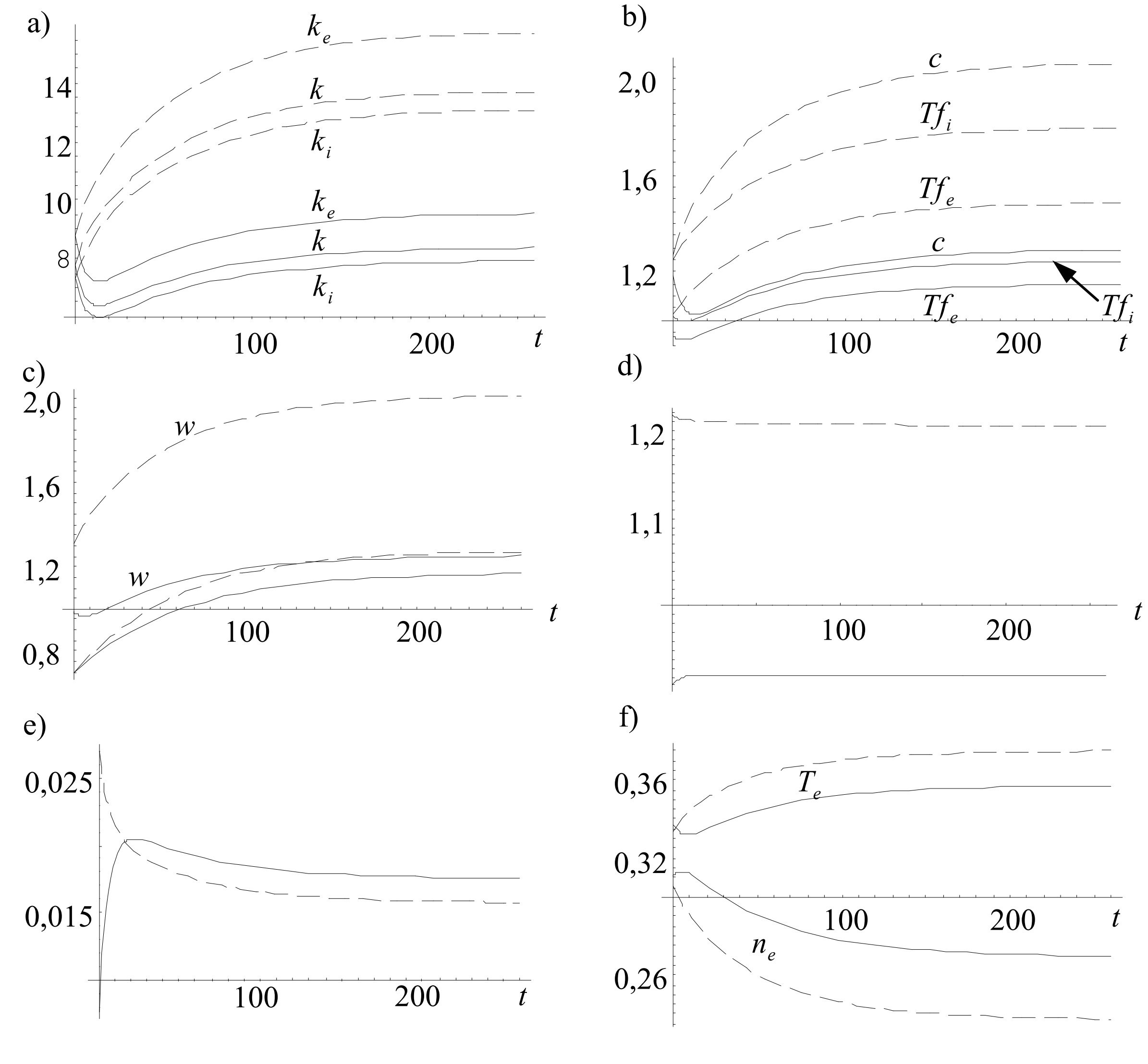

Figure 3. For A_i equal 0,9 (solid lines) and 1,2 (dashed lines) the graphs show a) the capital intensities and wealth for production (k_i) and education (k_e) and per capita wealth ($\bar{k}$), b) the per capita consumption (c) and production (Tf_i), c) the wage rate (w) and the level of human capital (curves without letters), d) the price of education, e) the rate of interest and f) the study time T_e and the sectorial share of labor force n_e.

It is important to examine effects of changes in the household's preference for education. We allow the propensity to receive education to increase from η = 0,008 to η = 0,014. The simulation results are demonstrated in Figure 4. The solid lines in Figure 4 are the same as in Figure 1, representing the values of the corresponding variables when η = 0,008; the dashed lines in Figure 4 represent the new values of the variables when η = 0,014. We see that as the household's propensity to receive education increases, the per capita level of consumption declines first and then increases. The level is only slightly increased. The per capita level of the education sector increases and that of the production sector declines as the household's preference for education is increased. The per capita level of wealth increases. The wage rate and level of human capital are increased. As the propensity to receive education is increased, the study time increases and the price level falls down.

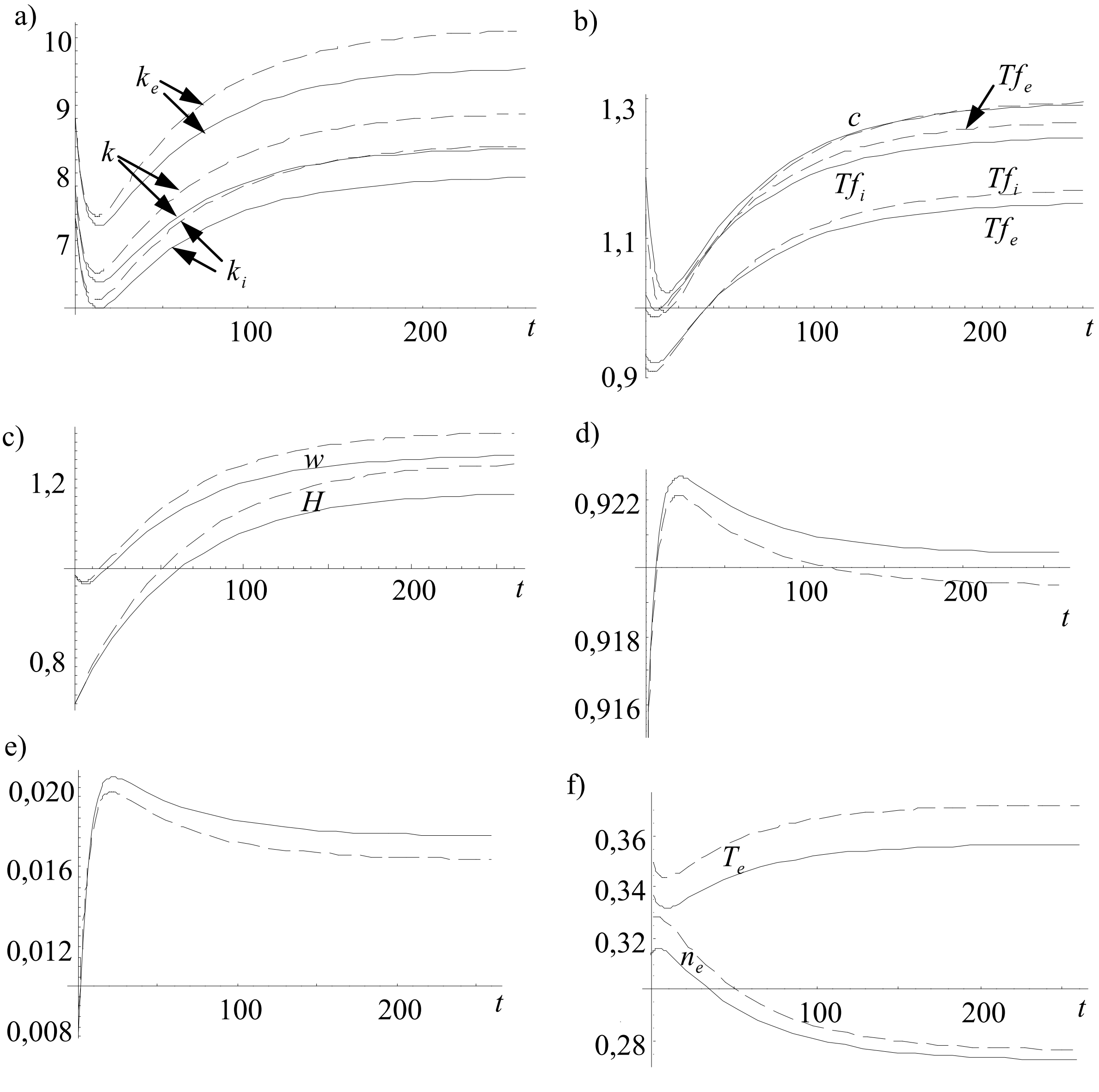

Figure 4. For η equal 0,008 (solid lines) and 0,014 (dashed lines) the graphs show a) the capital intensities and wealth for production (k_i) and education (k_e) and per capita wealth ($\bar{k}$), b) the per capita consumption (c, with solid and dashed curve almost overlapped) and production (Tf_i), c) the wage rate (w) and the level of human capital, d) the price of education, e) the rate of interest and f) the study time T_e and the sectorial share of labor force n_e.

The effects of change in the population have negative effects on the living conditions, as demonstrated in Figure 5. In Figure 5, we increase the population from N_0 = 50 000 to N_0 = 60 000. We see that as the population is increased, the per capita level of consumption declines. The per capita levels of the two sectors are reduced. The per capita level of wealth declines. The wage rate and level of human capital are reduced. The study time falls and the price level rises.

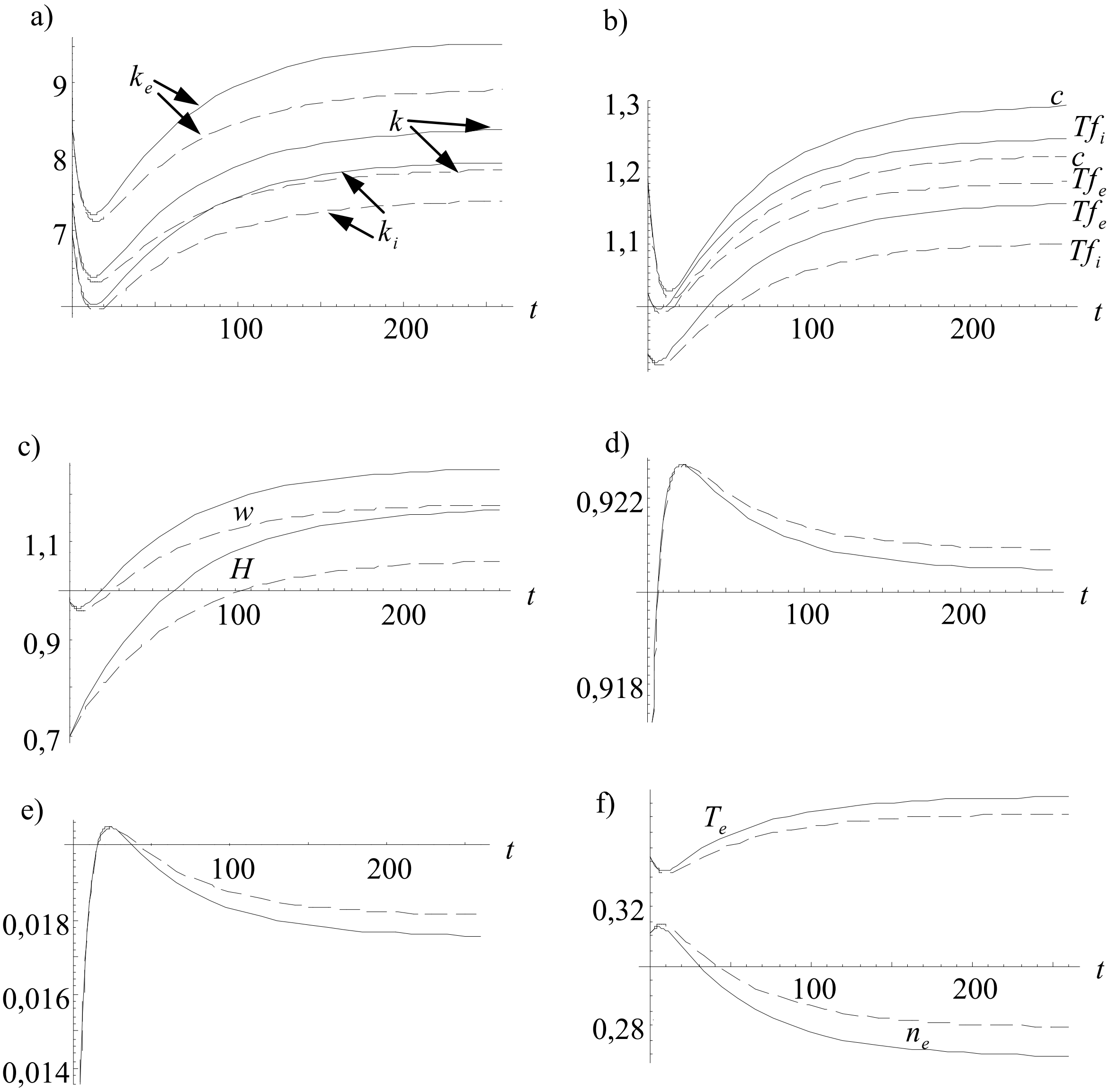

Figure 5. For N_0 equal 50 000 (solid lines) and 60 000 (dashed lines) the graphs show a) the capital intensities and wealth for production (k_i) and education (k_e) and per capita wealth ($\bar{k}$), b) the per capita consumption (c, with solid and dashed curve almost overlapped) and production (Tf_i), c) the wage rate (w) and the level of human capital, d) the price of education, e) the rate of interest and f) the study time T_e and the sectorial share of labor force n_e.

In Figure 6, we show the effects of change in the efficiency of learning by consuming. We increase the efficiency parameter from v_h = 0,7 to v_h = 10. We see that as the efficiency of learning by consuming, the economic conditions are improved and the level of human capital is improved.

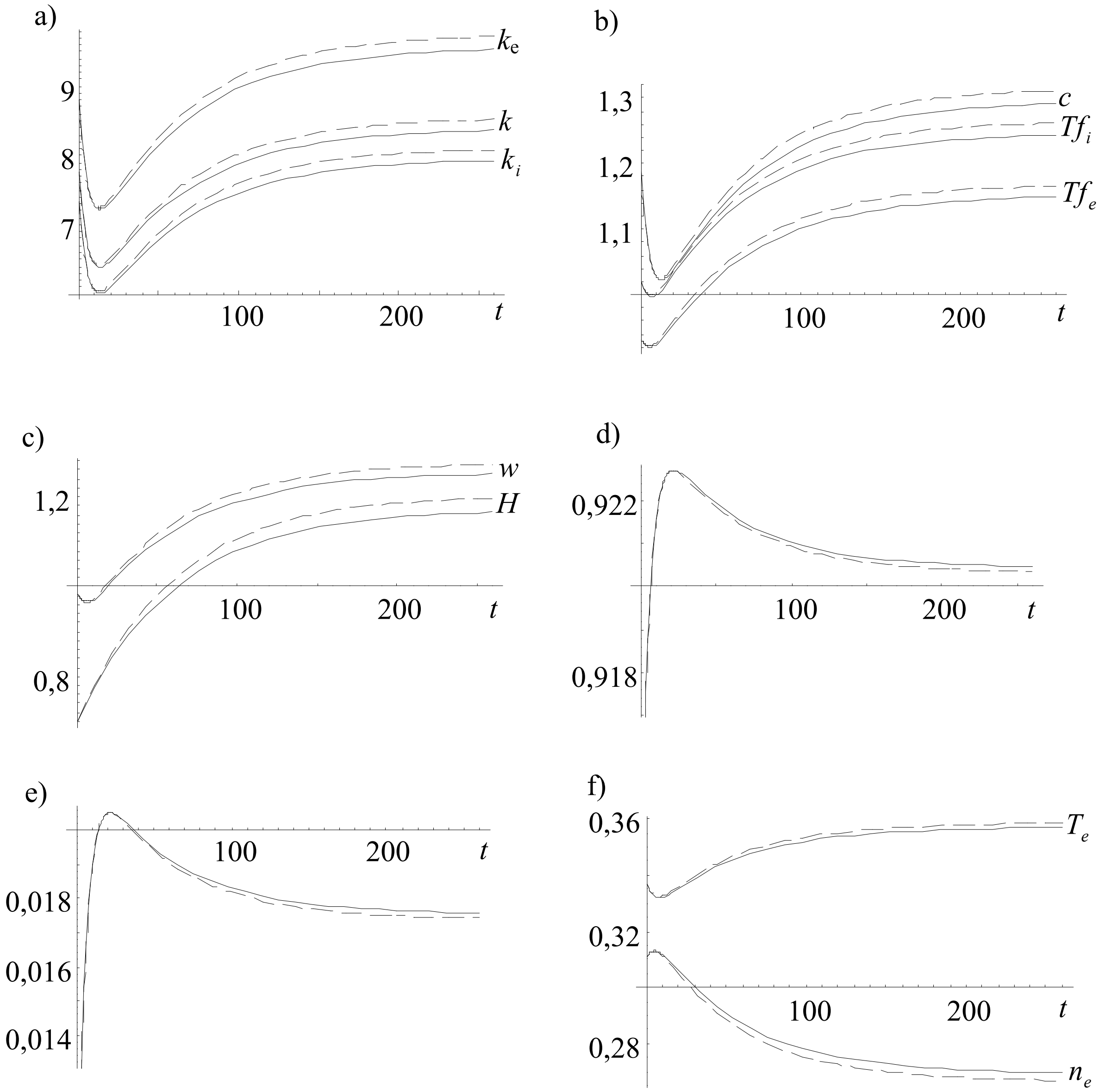

Figure 6. For v_h equal to 0,7 (solid lines) and 10 (dashed lines) the graphs show a) the capital intensities and wealth for production (k_i) and education (k_e) and per capita wealth ($\bar{k}$), b) the per capita consumption (c) and production (Tf_i), c) the wage rate (w) and the level of human capital, d) the price of education, e) the rate of interest and f) the study time T_e and the sectorial share of labor force n_e.

In Figure 7, we show the effects of change in the efficiency of education in improving human capital. We increase the efficiency parameter from $\nu_e = 0{,}8$ to $\nu_e = 1$. We see that as the efficiency of learning by consuming, the economic conditions are improved and the level of human capital is improved.

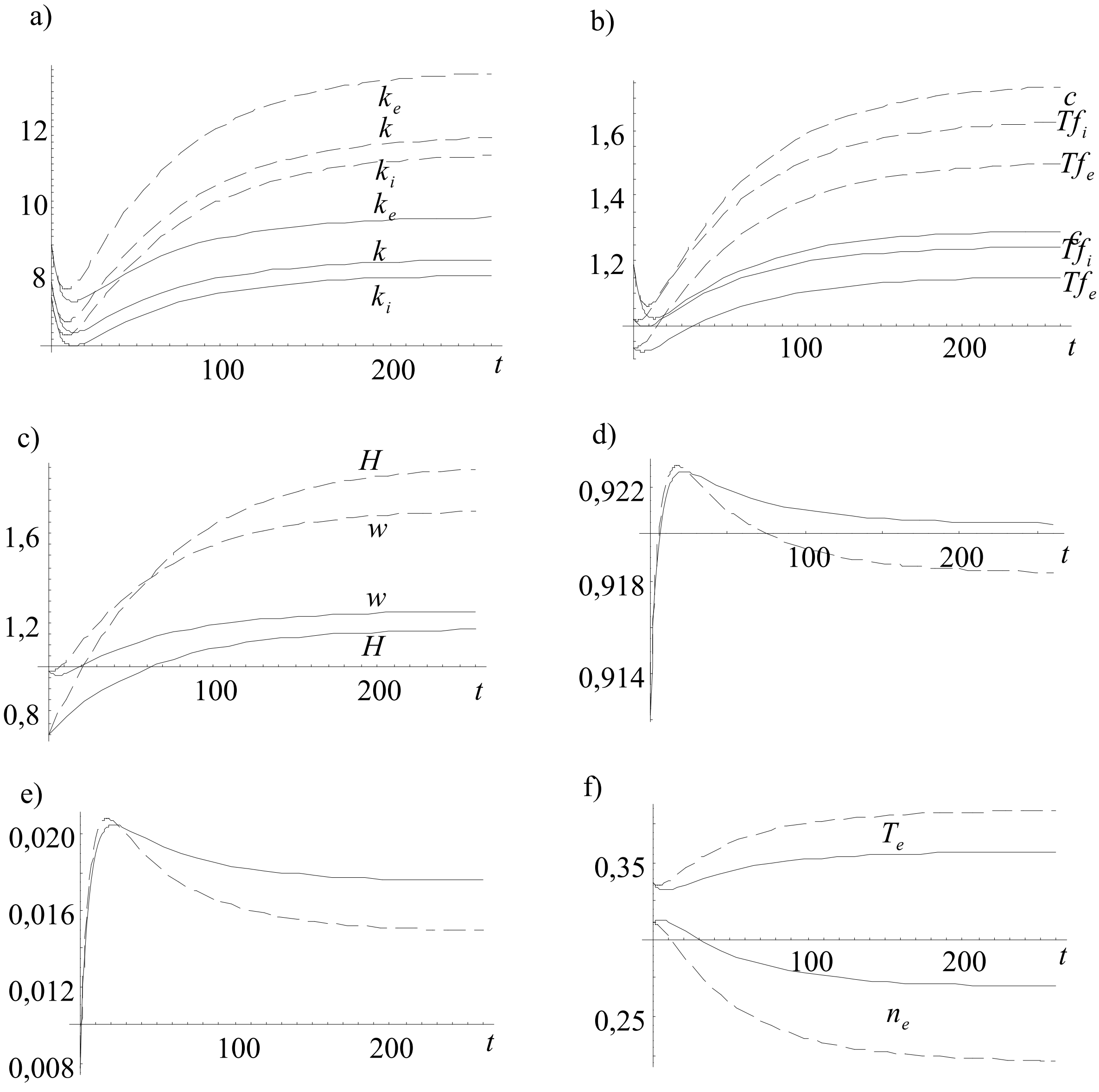

Figure 7. For ν_e equal to 0,8 (solid lines) and 1 (dashed lines) the graphs show a) the capital intensities and wealth for production (k_i) and education (k_e) and per capita wealth ($\bar{k}$), b) the per capita consumption (c) and production (Tf_i), c) the wage rate (w) and the level of human capital, d) the price of education, e) the rate of interest and f) the study time T_e and the sectorial share of labor force n_e.

CONCLUDING REMARKS

This paper proposes a dynamic economic model with wealth accumulation and human capital accumulation. The economic system consists of one production sector and one education sector. We took account of three ways of improving human capital: learning by producing, learning by education, and learning by consuming. The model describes a dynamic interdependence between wealth accumulation, human capital accumulation, and division of labor under perfect competition. We simulated the model to demonstrate existence of equilibrium points and motion of the dynamic system. We also examined effects of changes in the propensity to receive education, efficiency of learning, and efficiency of education upon dynamic paths of the system. We may extend the model in some directions. For instance, we may introduce some kind of government intervention in education into the model. It is also desirable to treat leisure time as an endogenous variable.

APPENDIX: THE TWO-DIMENSIONAL DIFFERENTIAL EQUATIONS

This section examines dynamics of the model. First, we show that the dynamics can be expressed by a two-dimensional differential equations system. From (2) and (3), we obtain

$$\frac{K_e(t)}{N_e(t)} = \alpha \frac{K_i(t)}{N_i(t)}, \quad \text{i.e.} \quad k_e(t) = \alpha k_i(t), \tag{A1}$$

where $\alpha \equiv \alpha_e \beta_i / \alpha_i \beta_e$ ($\neq 1$, assumed). From (A1), (2) and (4), we obtain

$$p(t) = \frac{\alpha_i A_i}{\alpha_e A_e} \alpha^{\beta_e} H^{-m\beta} k_i^{\beta}, \tag{A2}$$

where $\beta \equiv \beta_e - \beta_i$. From (A1) and (1), we solve the labor distribution as functions of $k_i(t)$ and $k(t)$

$$n_i = \frac{\alpha k_i - k}{(\alpha - 1) k_i}, \quad n_e = \frac{k - k_i}{(\alpha - 1) k_i}. \tag{A3}$$

Dividing (13) by N_0, we have

$$c + s - \delta \bar{k} = A_i T n_i H^{\beta_i m} k_i^{\alpha_i},$$

where $\delta \equiv 1 - \delta_k$. Substituting $c = \xi \bar{y}$ and $s = \lambda \bar{y}$ into the above equation yields

$$\bar{y} = \left\{ \frac{\alpha A_i H^{\beta_i m} k_i^{\alpha_i}}{\alpha - 1} + \delta k - \frac{A_i H^{\beta_i m} k}{(\alpha - 1) k_i^{\beta_i}} \right\} \frac{T}{\xi + \lambda}. \tag{A4}$$

where we use the equation for n_i in (A3) and $\bar{k} = Tk$. Insert (2) and $\bar{k} = Tk$ into the definition of $\bar{y}$ in (8)

$$\bar{y} = \left(\delta + \alpha_i A_i H^{m\beta_i} k_i^{-\beta_i} \right) k T + T_0 \beta_i A_i H^{m\beta_i} k_i^{\alpha_i}. \tag{A5}$$

From (A4) and (A5), we solve

$$\left(\frac{\alpha A_i H^{\beta_i m} k_i}{\alpha - 1} + \delta \eta k k_i^{\beta_i} - \frac{A_i H^{\beta_i m} k}{(\alpha - 1)} + (\xi + \lambda) \alpha_i A_i H^{m\beta_i} k \right) T = (\xi + \lambda) T_0 \beta_i A_i H^{m\beta_i} k_i. \tag{A6}$$

From (12) and (4), we have

$$T_e = A_e T n_e H^{\beta_e m} k_e^{\alpha_e}.$$

Insert $T + T_e = T_0$ and n_e in (A3) into the above equation

$$T = \left(1 + \frac{\alpha^{\alpha_e} A_e H^{\beta_e m} (k - k_i)}{(\alpha - 1) k_i^{\beta_e}} \right)^{-1} T_0. \tag{A7}$$

Substituting (A7) into (A6) yields

$$k = \varphi(k_i, H) \equiv \left(\frac{\alpha_0 \beta_i - \alpha - A k_i^{\alpha_e} H^{\beta_e m}}{(\alpha - 1) \delta \eta k_i^{\beta_i} / A_i H^{\beta_i m} - A k_i^{\alpha_e} H^{\beta_e m} - 1 + \alpha_0 \alpha_i} \right) k_i, \tag{A8}$$

where $\alpha_0 \equiv (\alpha - 1)(\xi + \lambda)$ and $A \equiv \alpha^{\alpha_e} \beta_i A_e (\xi + \lambda)$. By (A8), we can express $k(t)$ as functions of $k_i(t)$ and $H(t)$ at any point of time. By (A7) and (A5), we can also express $T(t)$ and $\bar{y}(t)$ as functions of $k_i(t)$ and $H(t)$ as follows

$$T = \varphi_0(k_i, H) \equiv \left(1 + \frac{\alpha^{\alpha_e} A_e H^{\beta_e m}(\varphi(k_i, H) - k_i)}{(\alpha - 1)k_i^{\beta_e}}\right)^{-1} T_0,$$

$$\bar{y} = \Lambda(k_i, H) \equiv \left(\delta + \alpha_i A_i H^{m\beta_i} k_i^{-\beta_i}\right)\varphi(k_i, H)\varphi_0(k_i, H) + T_0 \beta_i A_i H^{m\beta_i} k_i^{\alpha_i}. \quad \text{(A9)}$$

These functions show that $T(t)$, $\bar{y}(t)$, $N(t)$ (with $N(t) = T(t)N_0$), and $K(t)$ (with $K(t) = k(t)N(t)$) can be treated as functions of $k_i(t)$ and $H(t)$ at any point of time. By (A3) and $N_j(t) = n_j(t)\, N(t)$, we see that the labor distribution, $n_j(t)$ and $N_j(t)$ (j = i, s), are functions of $k_j(t)$ and $H(t)$. It is straightforward to see that $F_j(t)$ and $C(t)$ can be expressed as functions of $k_j(t)$ and $H(t)$ at any point of time.

We now express dynamics of the system in terms of $k_j(t)$ and $H(t)$. First, substituting the functions $T = T_0 - T_e$, $F_j(t)$ and $C(t) = \xi N_0\, \bar{y}\,(t)$ into (3), we obtain

$$\dot{H}(t) = \tilde{\Omega}_h(k_i, H) \equiv \Omega_e(k_i, H) + \Omega_i(k_i, H) + \Omega_h(k_i, H) - \delta_h H, \quad \text{(A10)}$$

where

$$\Omega_e(k_i, H) \equiv \frac{\upsilon_e N_0^{a_e + b_e - 1} A_e^{a_e} \alpha^{a_e \alpha_e}}{(\alpha - 1)^{a_e}} (\varphi(k_i, H) - k_i)^{a_e} (T_0 - \varphi_0(k_i, H))^{b_e} \varphi_0^{a_e}(k_i, H) k_i^{-a_e \beta_e} H^{m b_e + a_e \beta_e m - \pi_e},$$

$$\Omega_i(k_i, H) \equiv \frac{\upsilon_i A_i^{a_i} N_0^{a_i - 1}}{(\alpha - 1)^{a_i}} (\alpha k_i - \varphi(k_i, H))^{a_i} \varphi_0^{a_i}(k_i, H) k_i^{-a_i \beta_i} H^{m a_i \beta_i - \pi_i},$$

$$\Omega_h(k_i, H) \equiv \upsilon_h \xi^{a_h} N_0^{a_h - 1} \Lambda^{a_h}(k_i, H) H^{-\pi_h}.$$

The one-dimensional differential equation expresses change in $H(t)$ as a function of $k_i(t)$ and $H(t)$.

We now show that change in $k_i(t)$ can also be expressed as a differential equation in terms of $k_i(t)$ and $H(t)$. First, substitute $\bar{y} = \Lambda(k_i, H)$ and $\bar{k} = Tk = \varphi_0(k_i, H)\varphi(k_i, H)$ into (11)

$$\dot{\bar{k}}(t) = \lambda\Lambda(k_i, H) - \varphi_0(k_i, H)\varphi(k_i, H). \quad \text{(A11)}$$

Taking derivatives of $\bar{k} = Tk = \varphi_0(k_i, H)\varphi(k_i, H)$ with respect to time, we have

$$\dot{\bar{k}} = \left(\frac{\partial \varphi_0}{\partial k_i}\varphi + \frac{\partial \varphi}{\partial k_i}\varphi_0\right)\dot{k}_i + \left(\frac{\partial \varphi_0}{\partial H}\varphi + \frac{\partial \varphi}{\partial H}\varphi_0\right)\tilde{\Omega}_h. \quad \text{(A12)}$$

where we use (A10). Substituting (A12) into (A11) yields

$$\dot{k}_i = \tilde{\Omega}_i(k_i, H) \equiv \left[\lambda\Lambda - \varphi_0\varphi - \left(\frac{\partial \varphi_0}{\partial H}\varphi + \frac{\partial \varphi}{\partial H}\varphi_0\right)\tilde{\Omega}_h\right]\left(\frac{\partial \varphi_0}{\partial k_i}\varphi + \frac{\partial \varphi}{\partial k_i}\varphi_0\right)^{-1}. \quad \text{(A13)}$$

The one-dimensional differential equation (A13) expresses change in $k(t)$ as a function of $k_i(t)$ and $H(t)$. The two differential equations (A10) and (A13) contain two variables $k_i(t)$ and $H(t)$. We thus proved Lemma.

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EKONOMSKI RAST S UČENJEM IZ PROIZVODNJE, UČENJEM IZ OBRAZOVANJA I UČENJEM IZ POTROŠNJE

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SAŽETAK

U radu je predložen ekonomski model s akumulacijom bogatstva i ljudskih resursa. Ekonomski sustav sastoji se od jednog proizvodnog i jednog obrazovnog sektora. Ujedno se uzimaju tri načina unaprijeđenja ljudskih resursa: učenje iz proizvodnje, učenje iz obrazovanja i učenje iz potrošnje. Model opisuje dinamičku povezanost između akumulacije bogatstva, ljudskih resursa i podjele posla u slučaju idealne kompeticije. Simulacijom modela demonstrirana je egzistencija ravnotežnih točaka i gibanje dinamičkog sustava. Također je ispitan učinak promjene mogućnosti obrazovanja, učinkovitosti učenja i učinkovitosti obrazovanja na dinamiku sustava.

KLJUČNE RIJEČI

učenje iz proizvodnje, učenje iz potrošnje, učenje iz obrazovanja, ekonomski rast, produkcija u obrazovanju