The State of the Art in Model Predictive Control Application for Demand Response

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ABSTRACT

Demand response programs have been used to optimize the participation of the demand side. Utilizing the demand response programs maximizes social welfare and reduces energy usage. Model Predictive Control is a suitable control strategy that manages the energy network, and it shows superiority over other predictive controllers. The goal of implementing this controller on the demand side is to minimize energy consumption, carbon footprint, and energy cost and maximize thermal comfort and social welfare. This review paper aims to highlight this control strategy’s excellence in handling the demand response optimization problem. The optimization methods of the controller are compared. Summarization of techniques used in recent publications to solve the Model Predictive Control optimization problem is presented, including demand response programs, renewable energy resources, and thermal comfort. This paper sheds light on the current research challenges and future research directions for applying model-based control techniques to the demand response optimization problem.

KEYWORDS

Stochastic MPC, Robust MPC, Energy management, Demand response, Renewable energy.

INTRODUCTION

Many countries expressed the need to make their power infrastructure more cost-effective, environmentally clean, and sociologically acceptable, thus sustainable. A considerable amount of the generated power is currently being lost due to various technical reasons: a) separated generation from end usage, b) outdated transmission and distribution lines, c) missing demand-responsive technology and policy infrastructures. Also, the load congestion bottlenecks in the existing grid raise barriers to integrating renewable forms of energy. The situation is exacerbated by increasing load demands and historically declining research and development investment by power utilities. Moreover, the dependency on centralized power generation is expected to increase emissions and raise electricity tariff prices [1]. Distributed resources such as renewable energy sources (RES) and demand response (DR) programs reduce transmission congestion, carbon footprint, and electricity price. However, DR’s uncertainties increase the complexity of integrating them into the existing power grid. Therefore, energy management using the DR programs gained a lot of attention in recent years. Energy management can achieve different objectives such as minimizing the cost, reducing greenhouse gases, and minimizing the loss of generation, transmission, and distribution systems [2, 3]. DR programs can improve flexibility in the power system's operation and

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facilitate the low carbon transition in electricity production. The main objective of demand-side management is to mitigate the power supply’s uncertainty and fluctuation, creating electricity demand flexibility. This demand flexibility increases the ability to integrate large penetrations of renewable energy. In other words, demand-side management utilizes distributed generators, including RES, using DR programs. Also, DR has a very high potential to improve power systems’ performance in terms of energy saving, energy cost, emissions, and integrated RES. According to Energy Technology Perspectives Clean Energy Technology Guide [4], utilizing DR increases distributed generators' participation in the electricity market, which encourages installing more RES on the demand side and, as a result reducing transaction costs. The global market for DR has received a lot of attention. The wholesale demand response capacity in the United States grew to 28 GW and 35 GW from the retailer programs. In Italy, a total DR capacity of 280 MW was commissioned, while in Ireland, 415 MW was awarded in a four-year-head-action. In Japan, 1 GW was offered through different DR programs, such as the Interruptible load DR program and incentive-based DR programs.

![Demand Side Management Classification](image)

As shown in Figure 1, the demand-side management can be classified into a demand reduction and DR. The demand reduction achieves using efficient appliances or changing consumer behavior. On the other hand, the demand response program can be an incentive-based program (dispatchable) or a price-based program (non-dispatchable). With a specific contract, the incentive DR programs allow the independent system operator (ISO) to reduce customers’ loads. There are different types of incentive DR programs that can be set up for the customer. Some of these types are [5-10]:

1) Direct load control (DLC): this gives the ISO direct control of the customer processes.
2) Interruptible load: a customer contract with limited sheds.
3) Emergency program: this allows the customers to respond to the emergency signal.
On the other hand, a price-based DR program influences customer consuming behavior by applying different tariffs throughout the day. There are different types of price-based DR program, some of which are:

1) Time-of-use rates: a scheduled fixed price.
2) Real-time pricing (RTP): the end customers have the wholesale price.
3) Critical peak pricing: a less predetermined variant of time or use.

Utilizing the full potential of DR programs needs a control system that manages the energy network. Different control strategies have been applied to manage the demand side, such as classical, soft, and hard control strategies [11], [12-14]. Classical control, such as PID, is integrated with a predictive algorithm to enhance its ability to manage the building energy system. Soft control uses historical data for controlling the system, while the hard controller uses a model to determine future modus operandi. A hard controller’s ability to foresee the upcoming system variability makes it more adaptive to the change. One of the best examples of the hard controller that has been used in building energy management is Model Predictive Control (MPC). MPC technique can adapt and update the model by using a feedback signal. This feature of the MPC allows the system to be rationalized with the new estimation or the measurement. As a result, the ongoing interval will be optimized based on estimating the future time interval. MPC mitigates future uncertainty by predicting the direction of the future and optimize the current decision. Moreover, the fast response of the MPC and the ability to incorporate several control operations makes MPC suitable for energy management optimization problems.

Considering the uncertainties of the power systems in energy management optimization problems will increase the solution’s optimality. The difficulty of solving a real-time optimization problem that considers the uncertainty of the energy management problem can be tackled by different optimization techniques. Different review articles have been published on control strategies of demand-side energy management problems. In [5], the authors focused on the intelligent control system that achieved a building’s comfort level using different control strategies such as a fuzzy logic controller and a neural network controller. On the other hand, the authors in [15] focused their review on agent-based control and model-based predictive control. In [16], the authors reviewed the supervisory and optimal control of the Heating, Ventilation, and Air Conditioning (HVAC) system in a building. The review paper in [17] focused on MPC’s HVAC system theory and applications. Reference [18] provided different building energy management strategies such as MPC, fault detection, stochastic optimization, and robust optimization for residential and non-residential buildings. Unlike the aforementioned reviews, this present paper mainly focused on the recent journal publications that showed the MPC approach’s capability to handle demand response optimization problems considering a high uncertainty level. In other words, this paper tries to summarize algorithms and techniques that have been used in recent publications to solve MPC optimization problem that includes RES, thermal comfort and different type of DR. Based on the uncertainty level, the optimization problems can be formulated as deterministic MPC, stochastic MPC, scenario approach MPC, or robust MPC. A comparison between different type of MPC formulations and MPC optimization methods are conducted. Also, this paper classifies the recent publications in demand-side management based on MPC formulations.

This paper is organized as follows: Section 2 provides an overview of different MPC formulations: deterministic MPC, stochastic MPC, scenario approach MPC, and robust MPC. Section 3 reviews the existing literature on the application of MPC in managing the demand side, focusing on the demand response. Section 4 concludes the paper and presents future research directions.
MODEL PREDICTIVE CONTROL

MPC is an optimization-based control technique that aims to drive the closed-loop system to an optimal operation set-point while meeting state, input, and output constraints. Using the MPC feedback mechanism, the optimization problem inside the moving horizon window is solved at each time step. Only the first control action is implemented, and the rest is discarded. Therefore, MPC can predict the evolution of the states over the prediction horizon. However, modeling a building for MPC is time-consuming since each building has a specific model [19]. To prepare a building model for demand response using MPC, white-box, black-box, or gray-box model structures have been used in the literature. The white-box modeling is developed based on the system's physical process, while the black-box model is developed based on measuring the inputs and outputs of the system [20]. Gray-box modeling is a mix of white-box and black-box approaches. For example, reference [21] applied the white-box approach for modeling, while reference [22] utilized the block-box approach. Gray-box modeling is used in [23, 24]. In terms of the simulation tool, researchers use different software environments for modeling buildings, such as DYMOLA [25], TRNSYS [26], EnergyPlus [27], and ESP-r [28]. A review paper in building modeling techniques can be found in [29]. After modeling the building, the optimal control strategy, such as MPC, can be applied.

Depending on data uncertainty, different optimization methods, such as deterministic, stochastic, and robust optimization, have been used to the formulated energy management problem. In the deterministic MPC formulation, the uncertainty parameters are assumed to be time-independent parameters (perfect prediction). Therefore, the deterministic formulation is less complicated, which lowers the computational time of solving the optimization problem. However, the perfect prediction assumption of the uncertainties in the deterministic approach is not realistic, which may lead to a sub-optimal solution. On the other hand, the stochastic approach is more realistic since it considers the uncertainties in the decision-making process. Nevertheless, the stochastic approach's computational time is very high due to the complexity of the formulation. Since stochastic MPC requires prior knowledge of the underlying probability distribution function for the uncertainties, which is hard to find for complex processes, robust MPC is an alternative MPC paradigm that can deal with uncertainty without knowing the probability distribution. This paradigm can be achieved by deriving a robust invariant set of the error system, which is the difference between the real and nominal systems. To construct the invariant set, a feedback control law is used (e.g., LQR based control law and feedback linearization). The robust MPC can then optimize the process performance online while maintaining the close loop state within the stability region. However, deriving a robust invariant set can be challenging.

Several papers have compared deterministic and stochastic optimization methods for demand-side energy management problems [30-32]. References [30, 31] applied deterministic and stochastic MPC on a single room with an HVAC system, while reference [30] applied MPC and weather prediction in integrated room automation by controlling the HVAC system. The result of these studies showed that deterministic and stochastic MPC had similar performance in terms of energy use. The authors in [33] compare the deterministic and robust MPC method in a single room. The result showed the robust MPC outperforms the deterministic in case of high uncertainty consideration. Comparison of deterministic, stochastic, and robust MPC optimization methods are shown in Table 1 [34-44]. In the following subsection, the deterministic MPC schemes will be given. In the second and third subsections, stochastic MPC formulations that utilize the uncertainties' probabilistic measures will be presented. Finally, another form of MPC, famously known as robust MPC, will be provided.
Table 1. Comparison of MPC optimization methods

<table>
<thead>
<tr>
<th>Optimization Methods</th>
<th>Reference</th>
<th>Complexity</th>
<th>Computational Speed</th>
<th>Accuracy</th>
<th>Robustness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic MPC</td>
<td>[34, 35]</td>
<td>Simple</td>
<td>High</td>
<td>Low</td>
<td>Low</td>
</tr>
<tr>
<td>Stochastic MPC</td>
<td>[36-38]</td>
<td>More complexity in joint chance-constrained compared to recourse problems</td>
<td>It leads to a considerable size expansion of the problem and eventually increases the computational burden</td>
<td>Very good</td>
<td>Good</td>
</tr>
<tr>
<td>Scenario Approach MPC</td>
<td>[39,40]</td>
<td>Choosing approximations and models is difficult</td>
<td>It is inversely proportional to the number of samples</td>
<td>It depends upon the size of the sampling</td>
<td>It is very good for a large number of samples</td>
</tr>
<tr>
<td>Robust MPC</td>
<td>[41-44]</td>
<td>Very complex</td>
<td>Slow</td>
<td>High</td>
<td>High</td>
</tr>
</tbody>
</table>

Deterministic model predictive control

The mathematical formulation of the deterministic MPC for the class of nonlinear continuous system is as follows [34]:

\[
\min_{u(t) \in S(\lambda)} \int_{t_k}^{t_{k+N}} [\ddot{x}(\tau)^T Q \ddot{x}(\tau) + u(\tau)^T R u(\tau)] d\tau \tag{1a}
\]

\[s.t \quad \dot{x}(\tau) = f(\dot{x}(\tau), u(\tau)) \tag{1b}\]

\[u(\tau) \in U, \forall \tau \in [t_k, t_{k+N}] \tag{1c}\]

\[\ddot{x}(t_k) = x(t_k) \tag{1d}\]

\[x(t) \in X, \forall t \in [t_k, t_{k+N}] \tag{1e}\]

where \(u(t)\) is the decision variable defined over the prediction horizon length \(N\). The control objective is to minimize a quadratic function that penalizes the deviations of the predicted states and inputs from their corresponding set-points (eq. (1a)). The nominal model of eq. (1b) is applied to predict the process of state evolution over the prediction horizon. To mitigate a feedback control scheme, the predicted model is initiated at each sampling time \(t_k\) by the measured state from the real system \(x(t_k)\). The input constraints \(U\) and state constraints \(X\) are enforced over the entire prediction horizon. The above MPC formulation can be applied to nonlinear discrete systems by replacing the nominal model of eq. (1b) by the nominal discrete system (i.e., \(x(k+1) = f_d(x(k), u(k), 0)\)) [35]. Up to this point, addressing the external disturbances and model uncertainties of the controlled process is not considered within the above deterministic MPC formulation. Such uncertainties will be taken into account via stochastic MPC paradigms, presented in the following subsection.

Stochastic model predictive control

A stochastic MPC algorithm can be developed using stochastic programming that can be reformulated as an optimal control problem considering the system's uncertainties. To
understand how stochastic programming optimization problem is structured as an optimal control problem, the following stochastic discrete-time system is considered [36]:

\[
\begin{align*}
x_{t+1} &= f(x_t, u_t, w_t) \\
y_t &= h(x_t, u_t, v_t)
\end{align*}
\]

where \( t \in \mathbb{N}, x, \) and \( u \) are the state and input vectors, respectively. The disturbance vector \( w \) and \( v \) can represent a wide range of uncertainties with known probability distributions. The term \( f \) is the function that describes the system dynamics, while \( h \) is the function that describes the outputs. For full state-feedback control, the \( N \)-stage feedback control policy for stochastic MPC can be defined as follows:

\[
\pi: = [\pi_0(.), \pi_1(.), ... \pi_{N-1}(.)]
\]

where \( \pi(.) \) is the Borel-measurable function for all \( i=0...N-1 \). The stochastic discrete-time system can be formulated as a finite moving-horizon optimal control problem. By applying the MPC feedback mechanism, the value function of the resulting stochastic optimal control is commonly defined as follows:

\[
V_N: = E_x \left[ \sum_{i=0}^{N-1} J_c(\tilde{x}_i, u_i) + J_f(\tilde{x}_N) \right]
\]

where \( J_c \) and \( J_f \) are the stage cost function and the final cost function, respectively. Given the initial states, the term \( \tilde{x}_i \) represents the predicted states at the time \( i \). The objective function (eq. (5)) is usually subjected to chance constraints. Using the conditional probability \( Pr_{xt} \), the joint chance constraint over the prediction horizon formulation as follows [45, 46]:

\[
Pr_{xt}[g_i(\tilde{y}_i) \leq 0, \forall j = 1, ..., s] \leq \beta, \forall i = 1, ..., N
\]

where \( g_i \) is the Borel-measurable function, \( \tilde{y}_i \) is the predicted outputs at time \( i \). \( s \) represents the number of inequality constraints, and the probability lower bound is represented by \( \beta \). By using value function (eq. (5)) with joint chance constraints (eq. (6)), the stochastic optimal control problem for the stochastic discrete-time system (2-3) can be formulated as follows [39, 40]:

\[
V^0_N: = \min_{\pi} V_N(x_t, \pi)
\]

Subject to:

\[
\begin{align*}
x_{i+1} &= f(x_i, u_i, w_i) \\
y_i &= h(x_i, u_i, v_i) \\
Pr_{xt}[g_i(\tilde{y}_i) \leq 0, \forall j = 1, ..., s] \geq \beta \\
w_i &\sim P_i \\
\tilde{x}_0 &= x_t
\end{align*}
\]

where \( V^0_N \) is the function that represents the optimal value under the feedback control policy \( \pi(0) \). The optimal control sequence is implemented in a receding-horizon fashion (i.e., the first element of the optimal sequence \( \pi^* \) is only applied between two consecutive time instants). Stochastic MPC is an optimal control scheme that aims to balance the trade-offs between fulfilling the overall control objectives and ensuring the satisfaction of the probabilistic constraints resulted from the uncertainty [36-38].
Scenario approach model predictive control

The scenario approach can be used to reformulate the stochastic optimization programming problem into a deterministic equivalent problem. To illustrate, a two-stage stochastic linear programming is used as an example. In two-stage stochastic programming with recourse, the decision-maker can take corrective actions (recourse decisions) after realizing the uncertainty over sequence stages. A general formulation for a two-stage stochastic linear programming with linear constraints is given by [47]:

\[ \min f(x) + E[Q(x, \omega)] \]  

Subject to

\[ Ax = b, \quad x \in X \]  

where \( Q(x, \omega) \) is the second stage optimal objective value

\[ Q(x, \omega) = \min g(x, y, \omega) \]  

\[ W_\omega y_\omega + T_\omega x = h_\omega, \quad y_\omega \in Y \]  

where \( \omega \) is the probability distribution of the uncertain data for the second-stage, \( x \) represents the first-stage decision variable, and \( y \) represents the decision variable of the second-stage with the recourse action cost. \( E_\omega \) is the optimal objective value expectation of the second-stage decision variable. \( Q(x, \omega) \) represents the recourse action cost. \( W_\omega \) represents the compensation of the system's variation of the \( T_\omega \leq h_\omega \). To overcome the difficulty of obtaining the random variable's probability distribution function, the continuous probability distribution can be approximated using a finite scenario set \( \{s\} \) with their probabilities \( \{\pi_s\} \). As a result, the two-stage stochastic programming problem can be reformulated as a deterministic equivalent problem as follows:

\[ \min f(x) + \sum_{s \in S} \pi_s q_s^T y_s \]  

Subject to

\[ Ax = b \]  

\[ W_\omega y_\omega + T_\omega x = h_\omega, \quad y_\omega \in Y \]

To apply the MPC algorithm, the deterministic optimization problem can be formulated as an optimal control problem. To have an accurate solution to the optimization problem, a large number of scenarios have to be generated to represent the system's uncertainties. However, including a high number of scenarios raises the computational time or causes an intractable problem. Different approaches have been used to overcome this issue [48]. One way to reduce the computational time is to apply scenario reduction techniques [49, 50], which reduces the number of scenarios. As a result, the computational time is reduced, whereas the solution accuracy is compromised. Another way to circumvent the issue of a high number of scenarios is by using an online MPC algorithm [51-53].

Robust model predictive control

The decision in the robust optimization can be a one-stage decision that has to be taken before the uncertainty is realized, and no corrective action can be taken after the realization of the uncertainty. The robust optimization can also be formulated as multiple stages, where the
decision can be taken depending on the flow of the uncertainty realization. It is worth noting that it is challenging to incorporate the dynamic of the uncertainty in the robust optimization [54]. The general robust optimization formulation is [55]:

\[
\min f_0(\bar{x}), \quad \text{s.t.} \quad f_i(\bar{x}, \bar{u}_i) \leq 0, \quad i = 1, 2, \ldots m
\]  

(15)

where \(f_0(\bar{x})\) is the objective to be optimized, and \(f_i(\bar{x}, \bar{u}_i)\) is the system constraints. \(f_0\) and \(f_i\) are \(\mathbb{R}^n \rightarrow \mathbb{R}\) functions. \(x\) is a vector of decision variables, and \(\bar{u}_i\) is the parameter uncertainties of the uncertainty set \((U_i)\). \(m\) is the number of uncertain parameters. A comprehensive survey of robust optimization can be found in [56]. The robust optimization problem can be formulated as a robust MPC optimization problem, iteratively over a finite-moving horizon window. In other words, given the initial state, the state-feedback control law is used to minimize the worst-case scenario subjected to control input and output constraints [57]. The summary of the formulations of MPC optimization methods is shown in Table 2.

Table 2. Formulations of MPC optimization methods

<table>
<thead>
<tr>
<th>Optimization Methods</th>
<th>Objective Function</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic MPC</td>
<td>(\min_{u(t) \in U(t)} \int_{t_k}^{t_{k+N}} [x(t)^T Q x(t) + u(t)^T R u(t)] , dt)</td>
<td>(\ddot{x}(t) = f(\dot{x}(t), u(t))) (u(t) \in U, \forall t \in [t_k, t_{k+N}]) (\ddot{x}(t_k) = x(t_k)) (x(t) \in X, \forall t \in [t_k, t_{k+N}])</td>
</tr>
<tr>
<td>Stochastic MPC</td>
<td>(V_N^* = \min_{\pi} V_N(x_N, \pi))</td>
<td>(x_{i+1} = f(x_i, u_i, w_i)) (y_j = h(x_i, u_i, v_i)) (P_{r_{g_i}}(g_i(x_i)) \leq 0, \forall j = 1, \ldots, s) (w_i \sim P_i) (x_0 = x_t) (\forall i \in Z_{[0, N-1]})</td>
</tr>
<tr>
<td>Robust MPC</td>
<td>(\min f_0(\bar{x}))</td>
<td>(f_i(\bar{x}, \bar{u}_i) \leq 0, \quad i = 1, 2, \ldots m)</td>
</tr>
</tbody>
</table>
techniques can incorporate energy conservation strategies and disturbance rejection in its algorithm [17, 66]. However, there are several challenges when MPC is implemented, such as control design and building modeling. The decision to use the MPC strategy for a building mainly depends on its cost and performance [67].

In general, the controller's goal in a building energy management system can be categorized as follows. 1) minimize the operational cost, 2) maximize the utilization of RES, 3) achieve thermal comfort level by using a minimal amount of energy. 4) minimize the peak load or reschedule it. To achieve the control goal using MPC strategies, an optimal control optimization problem has to be formulated to minimize an objective function considering several constraints. The objective is usually to minimize the energy cost, but multi-objectives can minimize the cost and guarantee thermal comfort. Also, time-dependent constraints can be used for different comfort levels. Constraints can also be constructed to limit some of the parameters. Various Constraints can be considered in the MPC, such as equipment, energy match, economic, environmental, and political constraints.

![Figure 2. Smart home energy management](image)

Figure 2 illustrates the basic methodology of MPC for a building. The design parameters and predicted disturbances are the inputs to the MPC. Considering these inputs, the MPC optimizer minimizes the objective function is subjected to the constraints and the dynamic of the building model. Using the feed mechanism, the MPC controller applied only the first step of the solution and discarded the rest. In each time step, the MPC is updated with the current states. Based on the DR program, the MPC controller can manage the different types of loads to minimize energy costs. For example, the storage loads and the shiftable loads can be feed during the off-peak hours, where the electricity price is low.

As previously discussed, the MPC formulation can be categorized into deterministic MPC, stochastic MPC, scenario approach MPC, and robust MPC. The main difference in these formulations is how the system's uncertainties are considered on the optimization problem. Researchers used all these formulations to develop an MPC algorithm that can manage the building's energy system. Regardless of the MPC formulation, the MPC algorithm's implementation can be centralized, decentralized, distributed [68, 69]. Table 3 summarizes the techniques and evaluation of recent publications' contributions to utilizing RES, thermal comfort, cost reductions, shaving, shifting, or shaping load peaks for buildings' energy consumption, and characteristics of the demand response [30, 32, 70-107].
### Table 3. MPC formulations for demand-side energy management

<table>
<thead>
<tr>
<th>Formulation</th>
<th>Ref</th>
<th>Techniques</th>
<th>Utilized RES</th>
<th>Thermal Comfort</th>
<th>Reduce Cost</th>
<th>Load Peak</th>
<th>DR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic MPC</td>
<td>[70]</td>
<td>EMPC</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Shaving 26%</td>
<td>TOU</td>
</tr>
<tr>
<td></td>
<td>[71]</td>
<td>Mixed-Integer Nonlinear Programming, EMPC</td>
<td>PV</td>
<td>Yes</td>
<td>Yes</td>
<td>Shifting</td>
<td>Price-Based</td>
</tr>
<tr>
<td></td>
<td>[72]</td>
<td>Artificial Neural Networks Dynamic Programming</td>
<td>Wind</td>
<td>Yes</td>
<td>Yes</td>
<td>Shaping</td>
<td>Control Load</td>
</tr>
<tr>
<td></td>
<td>[73]</td>
<td>MINLP Branch &amp; Bound Algorithm</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Shaving</td>
<td>Auxiliary Services</td>
</tr>
<tr>
<td></td>
<td>[74]</td>
<td>Cluster Analysis</td>
<td>PV</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>DLC</td>
</tr>
<tr>
<td></td>
<td>[75]</td>
<td>Proposed Algorithm</td>
<td>PV</td>
<td>Yes</td>
<td>Yes</td>
<td>Shaving %23</td>
<td>Ancillary Services</td>
</tr>
<tr>
<td></td>
<td>[76]</td>
<td>Mixed-Integer Linear Programming</td>
<td>PV</td>
<td>No</td>
<td>7%</td>
<td>Shaving</td>
<td>TOU</td>
</tr>
<tr>
<td></td>
<td>[77]</td>
<td>Linear State-Space Model, Discretized</td>
<td>Wind</td>
<td>No</td>
<td>Yes</td>
<td>Shaving</td>
<td>Dispatchable</td>
</tr>
<tr>
<td></td>
<td>[78]</td>
<td>Quadratic Program</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Shaving</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>[79]</td>
<td>Exponentially Weighted Moving Average Algorithm</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Shifting</td>
<td>DCL</td>
</tr>
<tr>
<td></td>
<td>[80]</td>
<td>Linear Quadratic Method Monte Carlo</td>
<td>No</td>
<td>Yes</td>
<td>Energy Saving 43%</td>
<td>Shaving</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>[81]</td>
<td>Linear Quadratic Method Monte Carlo</td>
<td>No</td>
<td>Yes</td>
<td>Energy Saving 43%</td>
<td>Shaving</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>[82]</td>
<td>Mixed-Integer Programming</td>
<td>Solar, Thermal</td>
<td>Yes</td>
<td>Yes</td>
<td>Shifting</td>
<td>Price-Based</td>
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<tr>
<td></td>
<td>[83]</td>
<td>Cooperative Optimization, EMPC</td>
<td>No</td>
<td>Yes</td>
<td>15%</td>
<td>Shifting</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>[84]</td>
<td>Discrete Quadratic Programming</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Shifting</td>
<td>Incentive-Based</td>
</tr>
<tr>
<td></td>
<td>[85]</td>
<td>Adaptive Approach</td>
<td>Wind, PV</td>
<td>Yes</td>
<td>46%</td>
<td>Shaving</td>
<td>TOU</td>
</tr>
<tr>
<td>Stochastic MPC</td>
<td>[30]</td>
<td>Probabilistic Constraints Factorial Simulation</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>[32]</td>
<td>Discrete Algorithm Sampling Algorithm</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Shifting</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>[86]</td>
<td>Monte Carlo Probabilistic Constraints</td>
<td>PV</td>
<td>Yes</td>
<td>Yes</td>
<td>Shaving</td>
<td>DCL</td>
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<td></td>
<td>[87]</td>
<td>Monte Carlo Probabilistic Constraint</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
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<tr>
<td></td>
<td>[88]</td>
<td>Probabilistic Time-Varying Constraint</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Shaving</td>
<td>Incentive-Based</td>
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<td></td>
<td>[89]</td>
<td>Two-Stage Optimization Discrete</td>
<td>No</td>
<td>No</td>
<td>7.5%</td>
<td>Shifting</td>
<td>Price-Based</td>
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<td></td>
<td>[90]</td>
<td>Quadratic Programming</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
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<td>Scenario Approach MPC</td>
<td>[91]</td>
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<td>Wind</td>
<td>No</td>
<td>12%</td>
<td>Shifting</td>
<td>DCL</td>
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<td></td>
<td>[92]</td>
<td>Probabilistic Monte Carlo</td>
<td>Solar</td>
<td>Yes</td>
<td>20%</td>
<td>Ramp Shaving 50%</td>
<td>RTP</td>
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<td></td>
<td>[93]</td>
<td>Probabilistic Gaussian Distribution</td>
<td>No</td>
<td>Yes</td>
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<td>Shifting</td>
<td>RTP</td>
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<tr>
<td></td>
<td>[94]</td>
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<td>Solar</td>
<td>Yes</td>
<td>25%</td>
<td>No</td>
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<td></td>
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<td>Yes</td>
<td>Yes</td>
<td>Shifting</td>
<td>TOU</td>
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<td></td>
<td>[96]</td>
<td>EMPC Probabilistic Scenario</td>
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<td>9-32%</td>
<td>Shaving</td>
<td>RTP</td>
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<td>Wind, PV</td>
<td>Yes</td>
<td>21%-75%</td>
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<td>Price-Based</td>
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<tr>
<td></td>
<td>[98]</td>
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<td>Yes</td>
<td>Energy Savings 35%</td>
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<tr>
<td></td>
<td>[99]</td>
<td>Neural Network Predictive Control</td>
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<td>No</td>
<td>Yes</td>
<td>Shaving</td>
<td>Reliability</td>
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<td>Robust</td>
<td>[100]</td>
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<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
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</table>
Deterministic model predictive control in demand response

The uncertainty parameters usually come from solar irradiation, occupancy, RES, and weather forecast in the energy management optimization problem. These uncertainty parameters are assumed to be time-independent parameters in the deterministic MPC approach. A vast body of literature applied deterministic MPC to manage energy networks in a building [70-79]. Table 3 shows the techniques and controller objectives for each reference that applied the deterministic MPC approach.

A group of publications [70, 73, 77] used the hot water system in a building as an energy storage system. Based on the DR, the authors in [70] used an EMPC controller that optimized hot water system consumption by determining the optimal set-point of water temperature. Reference [77] proposed an MPC controller scheme that aggregated electric water heaters and provided the ISO with ancillary services. Also, the authors in [73] proposed an MPC controller scheme that aggregated thermostatically controlled appliances and provided them to the ISO as ancillary services.

A few publications [71, 72, 74, 75, 79] have directly dealt with the issue of the generation intermittency of renewable energy by using DR programs. References [79] and [74] used the DLC program to balance the renewable generation fluctuation by applying distributed and centralized MPC algorithms. The authors in [79] control the HVAC and the water level to shape the load while [74] controls the thermostat set-points for air conditioners. Reference [75] developed an MPC framework that optimizes the interaction between renewable generation and the battery storage system while maintaining the comfort level and reducing peak load. The authors in [72] proposed an MPC strategy to reduce the fluctuation of wind energy by regulating grid consumption and on-site energy generation and controlling the elastic loads. By modeling the building’s behavior and weather forecasting, [71] proposed an EMPC controller that can match demand with fluctuations in supply.

In [76], MPC based on a deep reinforcement learning method was used to utilize dispatchable loads and storage resources in a DR program. A prototype was installed to demonstrate the performance of their control method using the internet of things devices. In [78], a machine learning technique for MPC was applied to minimize energy usage and guarantee the end-user comfort level. Using machine learning reduced the hardware and software complexity of the controller and, as a result, the implementation cost. Experiments were conducted in [80, 81] to show the MPC performance's superiority to minimize energy consumption while maintaining comfort. Reference [82] proposed a nonlinear model predictive controller that optimizes the energy usage and comfort level based on a linear thermal model, which reduces the problem complexity, resulting in reducing the computational time.

Stochastic model predictive control in demand response

In building energy management, researchers have used stochastic MPC formulation to include uncertainties such as occupancy, ambient temperature, solar radiation, and renewable...
energy generation. Table 3 shows the techniques and controller objectives for each reference that applied the stochastic MPC approach. In contrast with the deterministic approach, the stochastic MPC considers the uncertainty in the decision-making process. To include uncertainties in the DR optimization problem, chance-constrained is usually used in a stochastic MPC algorithm. For instance, reference [86] proposed a chance-constrained MPC to take into account the uncertainties of ambient temperature and PV generation. The developed model optimizes the scheduling of the controllable appliances based on energy cost, thermal comfort, and PV system. Chance constraints can be transformed into deterministic using a sample-based method and discrete convolution integrals, as shown in [32]. This reference considers the uncertainties of occupancy loads and weather and used stochastic MPC to control small-scale HVAC systems while guaranteeing the occupancy's comfort level. However, using chance constraints on a large system sometimes leads to computational intractability issues. To overcome this problem, the authors in [87] developed a closed-loop disturbance feedback formulation to reduce the conservatism of the problem. This reference used Monte-Carlo simulations to validate the chance-constrained solution. The stochastic algorithm was capable of considering the weather forecast and ensure temperature preferences and DR requests. The authors in [30] and [88] focused on including weather prediction uncertainty in the stochastic MPC to increase energy efficiency and maintain the thermal comfort level. The authors compared the predictive controller and rule-based controller with a stochastic MPC controller, which outperforms both controllers. A recent publication [89] compared deterministic and stochastic MPC controllers, which outperformed both controllers. A recent publication [89] compared deterministic and stochastic MPC controllers, which outperformed both controllers. A recent publication [89] compared deterministic and stochastic MPC controllers, which outperformed both controllers. A recent publication [89] compared deterministic and stochastic MPC controllers, which outperformed both controllers. A recent publication [89] compared deterministic and stochastic MPC controllers, which outperformed both controllers.

**Scenario approach model predictive control in demand response**

Stochastic MPC based on chance constraints is difficult to be solved. Therefore, some researchers use a scenario approach to reformulate the stochastic MPC optimal problem to a deterministic equivalent problem. Table 3 shows the techniques and controller objectives for each reference that applied the scenario approach MPC. The system's uncertainty can be captured using a sampling method based on the probability distribution function [108].

The Monte-Carlo technique is commonly used to sample a probability distribution randomly. Applying the Monte-Carlo technique, reference [91] used energy storage as a DR to shave the load and reduce wind generation fluctuation. In this reference, the wind generation and customer behavior uncertainties were considered in the scenario-based MPC to maximize social welfare.

Another approach to include the uncertainty in MPC algorithms is using the Markov chain modeling framework. For instance, the wind power uncertainty was modeled in [91] using the Markov chain Monte-Carlo method.

The high penetration of renewable resources increases the probabilistic variations of power generation. These probabilistic variations can be handled using energy storage systems and DR programs. However, some of the DR programs may increase the system's uncertainty due to the customer's behavior. Therefore, the online MPC approach can be more adaptable to the probabilistic variations of the model and enhance the solution's accuracy [91, 92, 99].

A real-time optimization framework MPC can utilize thermal mass storage and energy storage systems to control power flow between the grid, a PV system, and a commercial building [92]. Reference [99] developed a real-time MPC algorithm based on a neural network technique that manages the energy system in a zero-energy building.

Considering the uncertainties of the model, such as solar irradiation, occupancy, renewable energy resources, and weather forecast, the energy management stochastic optimization problems are formulated to minimize the operational cost of integrating renewable energy resources, DR, and controllable, and storage devices Figure 2 [93-98]. The authors in [93] took advantage of a commercial building's flexible operation and proposed an MPC strategy.
that considered real-time pricing and thermal comfort level. To increase the model's accuracy, the authors consider the uncertainty of cooling demands in their stochastic optimization problem. In [92], the authors applied MPC to optimize the HVAC system and the storage devices considering thermal comfort constraints and external temperature uncertainty. Reference [95] investigated the EMPC strategy's ability to utilize a high penetration of renewable energy in the system to reduce the operational cost and maintain the system's reliability. On the other hand, reference [96] applied EMPC on supermarket refrigeration systems, which enable it to be used as ancillary services. In [95], the authors used sequential linear programming to achieve an EMPC strategy that reduces computational time and minimizes the energy cost significantly. In contrast, the authors in [98] used cloud parallel computation to consider the full complexity simulation in the proposed MPC algorithm.

Robust model predictive control in demand response

The robust optimization deals with the range or region of a deterministic uncertainty while taking into account the worst-case scenario over the predetermined deterministic uncertainty set. Since robust optimization does not need probability distribution, it is preferable when the probability distribution is difficult to obtain from uncertain data. The robust optimization approach uses an uncertainty set that covers all the possible outcomes of the uncertain parameters. Thus, the optimality and feasibility of a solution are guaranteed within any realizations of the uncertainty set. Therefore, the uncertainty set must be carefully constructed to guarantee computational tractability. The objective is to find the optimal solution considering the worst-case scenario; hence, there is no need to include a large number of scenarios, like in the case of stochastic programming. Considering the worst-case scenario increases the reliability of solutions but leads to very costly (conservative) solutions. Therefore, by adding a constraint to the uncertainty set, a trade-off between the cost and reliability can be optimized [109, 110].

Due to the conservative solution and the implementation complexity of robust MPC optimization [111], a few researchers have applied robust MPC to the DSM optimization problem. Table 3 shows the techniques and controller objectives for each reference that applied the robust MPC. The authors in [101] formulated a min-max robust optimization problem taking into an account comfort level, controllable load, and electricity price. Considering the uncertainties of load predictions and ambient temperature, authors in [100] applied a fuzzy interval model to define the uncertainty bounds in the robust MPC formulation. The authors in [102] formulate a robust MPC optimization problem that optimizes multiple energy forms considering source-network-load flexibilities. In [103, 104], the authors propose a robust MPC that guarantees an optimal energy dispatch in a smart micro-grid considering bounded demand uncertainty. An adaptive robust MPC is presented in [105] to perform online estimation of uncertain parameters of the building, while the adaptive robust MPC proposed in [106] relies on recursive set membership identification to updated the close-loop operation in each time step. Robustness analysis to state estimation for a hybrid ground coupled heat pump system is applied in [107] using robust MPC. The result shows that robust method did not improve the state estimation for the investigated system.

CONCLUSION AND FUTURE DIRECTIONS

A smart grid advances two-way communication between the generation and end-users. This advancement of smart grid communication technologies allows consumers to participate in the electricity market through DR programs. Various DR programs have been used to optimize the participation of the demand-side. Utilizing the full potential of DR programs needs a control system that manages the energy network. Different methods of controlling techniques have been applied to manage the demand response in the literature. This paper provides a review of different MPC formulations, which are deterministic MPC, stochastic MPC, scenario approach MPC, and robust MPC. The deterministic MPC approach has the
lowest computational time and most straightforward formulation comparing to other approaches. However, the perfect prediction assumption for the uncertainties may lead to a sub-optimal solution. The stochastic MPC approach considers the uncertainties in the decision-making process, resulting in a more realistic solution. The significant challenges of the stochastic approach are computing time and obtaining the probability distribution of the random variables. On the other hand, robust MPC deals with uncertainty without knowing the probability distribution by constructing an uncertainty set, which leads to a robust solution. However, the robust MPC solution is very costly since it considers the worst-case scenario.

The demand-side management optimization problems are subject to various uncertainties, including weather forecast, solar irradiation, occupant thermal comfort level, and electricity price. The high penetration of renewable distributed generation, such as wind and solar, has added additional uncertainties in the demand side due to the renewable energy sources fluctuations. However, these fluctuations can be mitigated by using an energy storage system. Smart grid capabilities also facilitate the utilization of the demand response programs to handle the demand side's uncertainties. Utilizing DR programs can increase power system reliability, reduce energy consumption, and minimize operational costs. To this end, an MPC strategy considers design parameters and predicted disturbances to come up with optimal control actions that maximize social welfare. Since the computing power has been improving, recent publications focus on including the uncertainties of the system in the MPC formulation. The main research challenge is how to optimize the energy flow and cost, considering the variability of the renewable energy, weather forecast, solar irradiation, thermal comfort, DR programs, and emission constraints. Most of the researchers applied the stochastic approach and considered some of these constraints. A few researchers used robust MPC since it generates a conservative solution.

The objective of implementing MPC on the demand side is to minimize energy consumption, carbon footprint, and energy cost; and maximize thermal comfort and social well-fare. However, several challenges can face researchers when considering the MPC in the demand response optimization problem. These challenges can be summarized as follows:

- Modeling a building for MPC implementation.
- Considering all kinds of uncertainties, such as weather prediction, RES, DR programs, and occupancy, in one model.
- Reducing the computational time to solve the optimization problem.
- Affordability and availability of the communication infrastructure to collect system measurements.
- Handling big data that is collected from the system.

These challenges cause an observable discrepancy in the simulation results in the publications. A comprehensive model that considers all these challenges is needed. Taking advantage of smart grid technologies and cloud computing, artificial intelligence, and machine learning combined with MPC strategy has the potential to overcome these challenges and provide a cost-effective solution and ensure the security and reliability of the power system.

NOMENCLATURE

Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCL</td>
<td>Direct Control Load</td>
</tr>
<tr>
<td>DR</td>
<td>Demand Response</td>
</tr>
<tr>
<td>DSM</td>
<td>Demand Side Management</td>
</tr>
<tr>
<td>EMPC</td>
<td>Economic Model Predictive Control</td>
</tr>
<tr>
<td>ESS</td>
<td>Energy Storage System</td>
</tr>
<tr>
<td>HVAC</td>
<td>Heating, Ventilation, and Air Conditioning</td>
</tr>
<tr>
<td>ISO</td>
<td>Independent System Operator</td>
</tr>
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</table>
MILP  Mixed-Integer Linear Programming
MINLP  Mixed-Integer Nonlinear Programming
MPC  Model Predictive Control
R&D  Research and Development
RES  Renewable Energy Sources
RTP  Real-Time Pricing
TOU  Time of Use

**Variables**

- \( u(t) \): Decision variable for the class of nonlinear continuous system
- \( \tilde{x}(t) \): State variable for the class of nonlinear continuous system
- \( x_t \): State vectors for the stochastic discrete-time system
- \( u_t \): Input vectors for the stochastic discrete-time system
- \( w \): Disturbance vector can represent a wide range of uncertainties with known probability distributions for the stochastic discrete-time system
- \( v \): Disturbance vector can represent a wide range of uncertainties with known probability distributions for the stochastic discrete-time system
- \( x \): First-stage decision for the two-stage stochastic linear programming
- \( y \): Second-stage decision for the two-stage stochastic linear programming
- \( \omega \): Probability distribution of the uncertain data for the second-stage
- \( \tilde{x} \): Vector of decision variables for the robust optimization formulation
- \( \bar{u}_i \): Parameter uncertainties of the uncertainty set for the robust optimization formulation

**REFERENCES**

34. Magni, L., D.M. Raimondo, and F. Allgöwer, Nonlinear model predictive control. Lecture Notes in Control and Information Sciences, 2009(384), https://doi.org/10.1007/978-3-642-01094-1


