DETERMINING THE CULMINATION OF A CELESTIAL BODY

Prilog problemu određivanja kulminacije nebeskoga tijela

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Summary

The meridian passage time does not always coincide with the culmination time of the celestial body. The difference is in the local hour angle. At the culminate the celestial body is crossing the observer's celestial meridian and is said to be in the upper transit. Often, culmination is used to mean upper culmination. A few celestial body altitudes measured around local apparent noon can give a more accurate position fix than using single measurement. In the following article the problem solving equation based on the interpolation polynomial in the Lagrange's form will be shown. This method can easily be solved using a programmable hand-held calculator.

Key words: Nautical astronomy

Sažetak

Meridijanska visina neće uvijek odgovarati kulminacijskoj visini nebeskoga tijela. Razliku čini promjena satnog kuta. Nebesko tijelo kulminira prolazeći nebeskim meridijanom opažača, za što se često koristi izrazom „gornji prolaz". Mjerenje nekoliko visina nebeskog tijela u blizini nebeskog meridijana podloga je za točniji račun opažačeve pozicije (geografske širine). U rješavanju problema u članku se koristi interpolacijskim polinomom u Lagrangeovu formatu. Pretpostavlja se uporaba računala.

Ključna riječi: Nautička astronomija

INTRODUCTION / *Uvod*

The procedure of obtaining ship's position at sea through observation of the highest altitude in the meridian is based on the assumption that the observed altitude is a symmetrical function of time. Nothing the time of maximum sextant altitude is not sufficient for determining the time of meridian passage because the celestial body appears to 'hang' for a finite time at its local maximum altitude. A few celestial body altitudes measured around local apparent noon give a more accurate position fix than using single measurement. Since 1 min of time corresponds to 15 min of arc, the longitude estimate will be less accurate than the latitude one. Time measurements should be exact and the sextant arc is to be returned to zero between sights. A high-quality micrometer sextant would improve accuracy. The problem solving equation based on interpolation polynomial in the Lagrange form will be evaluated.

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CULMINATION OF A CELESTIAL BODY / *Kulminacija nebeskoga tijela*

The meridian passage time does not always coincide with the culmination time of the celestial body. The equation of a circle of equal altitude is:

$$H_o = f(\delta, LHA) = \arcsin(\sin\varphi\sin\delta + \cos\varphi\cos\delta\cos LHA) \quad (1)$$

Ho – observed height or altitude of a celestial body;
LHA – Local Hour Angle, δ – declination;
φ and λ are the coordinates of a dead reckoning (assumed) position.

The differentiation of equation (1) with respect to time (t) gives:

$$dH_0 = d\varphi\cos Z_n + d\delta\cos\pi - dLHA\sin Z_n\cos\varphi \quad (2)$$

where Z_n stands for the azimuth and π is the parallactic angle of a celestial body.

In the culmination a celestial body attains its highest or lowest observed altitude i.e. the function $H_0=H_0(t)$ has its extremum. The mathematical condition for it is the vanishing of the derivative dH_0/dt. For that reason the values $d\varphi$, $d\delta$ and dLHA have to be calculated accurately. As the several observed heights include all these rates of changes, this forms the basic concept of the following method.

APPLICATION OF INTERPOLATION POLYNOMIAL IN THE LAGRANGE FORM / *Primjena interpolacijskog polinoma u Lagrangeovu formatu*

Interpolation polynomial in the Lagrange form is particularly applicable for a non-equidistant nodes i.e. nonequal time differences between consecutive sights. For a three observed off culmination heights the polynomial reads:

$$L_2(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2) \quad (3)$$

where xi - time of observation; f(xi) – observed height.

The expression (3) is the equation of parabola:

$$L_2(x) \equiv f(x) = ax^2 + bx + c \quad (4)$$

with vertex at $V\left(-\frac{b}{2a}, -\frac{b^2-4ac}{4a}\right)$ (5)

Solving for latitude requires the declination and the zenith distance. As the vertex time and height are deduced from (5), well known nautical astronomy rules are simply applied.

AN EXAMPLE / *Primjer*

The following example illustrates this process. On 8th April 2006 a vessel at the assumed position:

Lat = 30°06′*N; Long* = 039°*W*, proceeds at course 41° with speed 15 kts. A fixed star *Betelgeuse was observed as per table below:*

x_i = time (UT)	19^h18^m	19^h27^m	19^h30^m
$f(x_i)$ = height (observed)	67°11.8'	67°10.8'	67°07.8'

Applying expressions (3), (4) and (5) we get culmination time and height (vertex):

H_{culm} = 67°12.8′;*UT* = 19^h22^m. Consequently, latitude equals *Lat* = 30°11.7′*N*.

CONCLUSION / *Zaključak*

The shown parabola fits the rate of change of altitude and its peak shows culmination height and culmination time respectively. Albeit, in a given example, a fixed star and slow steaming vessel have been used, resulting in consecutive heights' differences not been substantial, the shown mathematical model gives good result in latitude. Obtaining a vessel's longitude is less accurate as the difficult part lies in determining the precise moment of culmination. Since 1 min of time corresponds to 15 min of arc, the longitude estimate will be less accurate than the latitude one. This method can easily be solved using a programmable hand-held calculator.

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