ISSN 1330-3651 (Print), ISSN 1848-6339 (Online) https://doi.org/10.17559/TV-20220406014211
Original scientific paper

A Lévy Option Pricing model of FFT-Based High-order Multinomial Tree

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Abstract: This paper studies the method of constructing high order recombined multinomial tree based on fast Fourier transform (FFT), and applies multinomial tree option pricing under the Lévy process. First, the Lévy option pricing model and Fourier transform are introduced. Then, the network model based on FFT (Markov chain) is presented. After that, a method of constructing a recombined multinomial tree based on FFT is given. It is proved that the discrete random variables corresponding to the multinomial tree converge to the Lévy distributed continuous random variable. Next, we obtain the European option pricing formula of FFT multinomial tree pricing, and apply the reverse iteration method to the American option pricing. Finally, under the Jump-diffuse process, the difference between the computational accuracy and computational efficiency of the Semi-analytical solution of European Option and Merton European Call Option which are priced under FFT is compared. The results show that the method of constructing a high-order recombined multinomial tree based on FFT has very high calculation precision and calculation speed, which can solve the problem of traditional risk-neutral multinomial tree construction and it is a promising pricing method for derivative products.

Keywords: FFT; Lévy process; multinomial tree; option pricing

1 INTRODUCTION

In recent years, the Fourier transform has been widely used in derivatives pricing. On one hand, because of the use of fast Fourier transform, the computational complexity can be reduced from N^2 to $N\log_2^N$, which greatly improves the computational efficiency. On the other hand, the Lévy process is better able to describe the underlying asset price movement, while the analytical form of the characteristic function of the Lévy process does exist. The majority of the Lévy process does not exist in an analytic form of a density function. Fang and sterlee [1] based on Fourier Cosine Series, DIA [2]used regular Fourier transform（Regularized Fourier transform）, Kwok, Leung and Wong [3], Zhang and Wang [4] based on Fast Fourier Transform, Ortiz-Gracia and Oosterlee [5] based on Shannon wavelet inverse Fourier transform studies the European option pricing process with stochastic dividends, stochastic volatility and jumping risk. Eberlein [6] systematically introduced various methods of Fourier transform pricing European options. Dempster and Hong [7], Pellegrino and Sabino [8], Andersson [9] discussed the Spread options pricing problem based on Fourier transform. Ramponi [10] under the condition of jump-diffuse of the underlying asset price obedience mechanism the Fourier transform method of pricing the forward start option is given. Zhang and Oosterlee [11] introduced the pricing of the American Asian option under the Lévy model by using Fourier cosine expansion. Ibrahim, O'Hara and Constantinou [12] gave the Fourier transform method pricing extendable options (extendible options) method, Fusai [13], Shu [14] and Huang, O'Hara and Mataramvura [15] used the Fourier method to study pricing problem of different forms of Asian options. Fang and Oosterlee [16], discussed the pricing problem of barrier options and exotic options under Fourier transform. Mordecki [18], Sheu and Tsai [19], Yamazaki [20], Boyarchenko and Levendorskii [21] showed the optimal stochastic and long-term American option pricing problem by taking Fourier transform. Based on the Fourier transform, Zhylyevskyy [22], Gyulov and Valkov [23] generalized the pricing of American options on Stochastic Volatility and finite interval Pellegrino and Sabino [8], Ruijter and Oosterlee [24], Chan [25] took multi-dimensional Fourier transform to derive option pricing. In view of FFT, a Markov chain is used for option pricing by Wong and Guan [26]. In Madan and Yor [27], the expression theorem of time-varying Brownian for a class of Lévy processes was given. The risk pricing under the Lévy process is given in Asiimwe, Mahera and Menoukeu-Pamen [28]. Kulczycki and Ryznar [29] focused on the Lévy process transfer probability estimation problem. Neufeld and Nutz [30] studied the characteristic functions of nonlinear Lévy processes. And the characteristic function of time-inhomogeneous Lévy driven O-U process is given in Vrins [31]. Zeng and Kwok [32] developed the pricing and approximation of arithmetic average Asian options under time-varying Lévy process. The Lévy model in Jovan and Ahčan [33] is applied to the problem of default prediction using structured methods. Gong and Zhuang [34], Lian, Zhu, Elliott and Cui [35] studied the pricing methods of American options and discrete barrier options under Lévy process respectively.

Fourier transform itself has high computational efficiency, however, the existing Fourier transform method for calculating derivatives prices requires massive unnecessary calculations, such as the need to calculate the price of options under different strike prices, whether the strike prices are required or not, and the market is whether existent, which resulted in a waste of computing resources and low computational efficiency. Markov chain method has too many nodes in the early stage of calculations, and it has fixed nodes at the later stage of the calculation. Therefore, the same calculation efficiency is not high. Based on FFT, this paper proposes the option pricing for Lévy process with a high-order recombined multinomial tree. The node grows linearly with the number of multinomial tree periods, which has high computational efficiency and simple program implementation.

The rest of this paper is organized as follows: Section 2 introduces the Lévy model option pricing and Fourier transform. Section 3 describes the FFT-based network. Section 4 gives a method to construct recombined multinomial tree visa FFT. In section 5, we introduce the options pricing formula and reverse iteration method for pricing path-dependent options. Section 6 uses the jump-diffuse procedure with semi-analytical European option pricing formula to demonstrate the accuracy and efficiency of the FFT tree. Section 7 gives conclusions.

2 LÉVY OPTION PRICING AND FOURIER TRANSFORM

The Lévy process is an independent and steady increment process, each Lévy process corresponds to an infinitely separable distribution. Denote X_t as a Lévy process. According to Lévy-Khintchine formula, the characteristic exponent of X_1 is:

$$\Psi(u) = i\mu u - \frac{1}{2}\sigma^2 u^2 + \int_{\mathbb{R}} \left(e^{iux} - 1 - iux\mathbf{1}_{(-1,1)}\right)\nu(dx) \quad (1)$$

the characteristic function of X_t is:

$$\Phi(u) = E_{X_t}\left[e^{iux}\right] = e^{t\Psi(u)} \quad (2)$$

Wong and Guan [26] showed several characteristic functions of the Lévy process which are commonly used in the pricing of financial derivatives. In addition to the Wiener process, most of the commonly used Lévy process only has an analytical form of the characteristic function, and there is no analytical form of probability density function (PDF) $f(x)$ or cumulative distribution function(CDF) $F(x)$.

The characteristic function is the Fourier transform of the probability density function, as follows: $\Phi(u) = \int_{-\infty}^{+\infty} \mathrm{e}^{iux} f(x)\,\mathrm{d}x$

By using the inverse Fourier transform, the probability density function and the cumulative distribution function can be obtained from the characteristic function:

$$\begin{aligned} f(x) &= \frac{1}{\pi}\int_0^{+\infty} \Re\left(e^{-ixu}\Phi(u)\right)\mathrm{d}u \\ F(x) &= \frac{1}{2} - \frac{1}{\pi}\int_0^{+\infty} \Im\left(\frac{e^{-ixu}\Phi(u)}{u}\right)\mathrm{d}u \end{aligned} \quad (3)$$

$\Re(.)$ is the real part of the complex number and $\Im(.)$ is the imaginary part of the complex number. Under the risk-neutral probability measure, the price of the European option with the maturity T and the payment function $g(.)$ in $t = 0$ is:

$$C = \mathrm{e}^{-rT} E^{Q}\left[g(x)\right] = \mathrm{e}^{-rT}\int_{-\infty}^{+\infty} g(x) f(x)\,\mathrm{d}x \quad (4)$$

Theoretically, the probability density function can be obtained from Eq. (3), and the value of the option can be calculated by Eq. (4). But we cannot obtain analytical form of the probability density function for most Lévy processes unless the numerical method is used, which is extremely inefficient. Carr and Madan [36], Lewis [37] changed strike price of European call option and introduced an exponential damping factor into the payoff function. After that, option pricing was transformed to the inner product of Fourier space by Plancherel-Parseval's Theorem. Based on the discounted cumulative distribution function, the analytic pricing formula of the Fourier transform space is obtained in Bates [38].

Even if the European option pricing formula is obtained in the Fourier space, it is necessary to use the inverse Fourier to get the true European option price. There is no analytic form pricing formula in Fourier space for path-dependent exotic options and American options can be executed ahead of time. When we compute the Fourier transform or the inverse Fourier transform by numerical method, FFT (IFFT) uses the symmetry and periodicity of a complex number to simplify the calculation of N^2 times which is necessary for calculating discrete Fourier transform or inverse Fourier transform to $N\log_2^N$ times. In derivatives pricing, if we can get the analytic pricing formula of Fourier space, it is also known as the analytic pricing formula.

3 FFT-BASED NETWORKS

For the path-dependent option under the Lévy model, Wong and Guan [26] presented a method of building a network model using FFT, which is essentially a discrete-time finite-state Markov chain. In the network model of Wong and Guan [26], the time is evenly discretized. In each period, the number of nodes in the network is fixed. Each node is connected to all the nodes in the next period. The network structure is shown in Fig. 1.

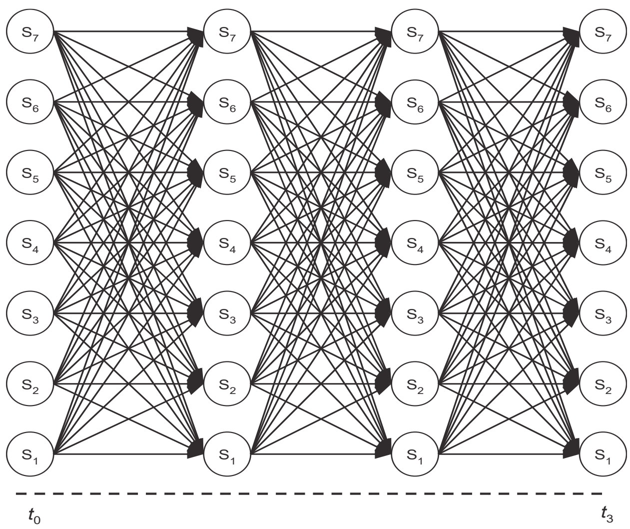

Figure 1 FFT-network models

When the probability transfer matrix is calculated, the conditional density function $f(x_j)$ at the N - equidistant node is calculated by FFT using the conditional feature $\Phi(u \mid x_i)$ of x_i at the i - th node, and the transition probability is calculated as follows:

$$P_{ij}(t) = \frac{f(x_j \mid x_i)}{\sum_{k=1}^{N} f(x_k \mid x_i)} \quad (5)$$

In the above formula, the denominator is an approximate probability density function in a finite interval, and there exists a truncation error. The denominator can be seen as a solution to ensure that the transition probability matrix satisfies the sum of the row elements is 1. Based on the network model, Wong and Guan [26]. Wong and Guan [26] described the pricing method for European option and American option. Combining forward shot grids (FSGs), we also give pricing methods for Asian option and Lookback option.

The network model is applied to the pricing of derivatives. In the whole calculation process, the nodes remain fixed and the computation is controllable. But in the initial stage, the price movement of the underlying asset is concentrated in the vicinity of the initial price. The excessively large calculation interval and the number of computing nodes cannot significantly affect the calculation accuracy, resulting in the waste of computing resources. Near maturity, the underlying asset price fluctuates greatly. This region has a significant influence on the derivative price (especially for the European option). It requires a larger calculation interval and more computing nodes. In addition, when calculating the transition probability matrix, It is necessary to use the conditional characteristic function of the Lévy process. These factors lead to the inefficiency of the network method when calculating the derivative price.

4 FFT-BASED RECOMBINED MULTINOMIAL TREES

The recombined multinomial tree uses the node recombination technology to realize that the node number grows linearly with the number of periods, and overcomes the shortcomings of the general multi-tree nodes growing exponentially with the number of periods. Compared with the Markov chain, a network of nodes that remain constant during computation, the recombined multinomial tree has fewer nodes at the point of relatively small price fluctuation and more nodes at the point of relatively big price fluctuation relatively. It is more efficient in the calculation of derivatives prices. In the existing references of constructing the recombined multinomial tree, the moment matching technique is used to make the moment of the single period multinomial tree as approximate as possible to the moments of continuous distribution. It is very difficult to determine parameters meeting conditions in the multi-tree. It is often necessary to add some extra constraints, such as Yamada and Primbs [39] established Penta-nomial tree parameter estimation formulation and required that the kurtosis and skewness of the continuous distribution must satisfy certain conditional relations, which limits the use scope of the recombined multinomial tree. In addition, it is because of the limitation of the moment matching technique that there is no single state multinomial tree with more than 5 states.

The underlying asset price follows the exponential Lévy process under the risk-neutral probability measure.

$$S_t = S_0 \mathrm{e}^{(rt-\mu t+X_t)} \tag{6}$$

Where S_0 is the stock price at $t=0, r>0$ is the risk-free interest rate, X_t follows the Lévy process, μ is the constant that makes the expected return rate on assets (6) equal to the risk-free rate. $T > 0$ is maturity. $\delta = \frac{T}{N}$. Due to properties of independent stationary increment of Lévy process, we have:

$$\log\left(S_{n\delta}\right) - \log\left(S_{(n-1)\delta}\right) = \left(r-\mu\right)\delta + X_\delta \tag{7}$$

Without citing confusion, we denote $S_n := S_{n\delta}, X_n := X_{n\delta}$ and X_δ as approximation Lévy distribution in [0, δ] of $L = 2^k$ state multinomial tree. The truncation interval of X_δ is $[-a, a]$. Given $\Delta = \frac{2a}{L-1}$, we have:

$$x_{1,j} = -a + \Delta j, j = 0, 1, 2, ..., L-1 \tag{8}$$

There are $(L-1)n+1$ nodes in the phase $n(n \le N)$ of L - state recombined multinomial tree. And the j-th node follows:

$$x_{n,j} = -na + \Delta j, j = 0, 1, 2, ..., (L-1)n \tag{9}$$

In the n-th period, the truncation interval of X_n is $[-na, na]$. Considering:

$$\begin{aligned} X_{n+1,j+k} &= -(n+1)a + (j+k)\Delta \\ &= -na + j\Delta - a + k\Delta \\ &= X_{n,j} + X_{1,k} \end{aligned} \tag{10}$$

where $k = 0, 1, 2, ..., L-1$. Thus, we denote $X_{n+1,j}, ..., X_{n+1,j+L-1}$ as the L-th node in the $(n + 1)$th period from n-th node in the period n. The price of the underlying asset at the j-th node in the period n is:

$$\begin{aligned} S_{n,j} &= S_0 \mathrm{e}^{((r-\mu)\delta n + X_{n,j})} \\ &= S_0 \mathrm{e}^{((r-\mu)\delta n - na + j\Delta)}, j = 0, 1, 2, ..., (L-1)n \end{aligned} \tag{11}$$

$\Phi(u)$ is the characteristic function in $[0, \delta]$ of the Lévy process. And the probability density Eq. (3) is discretized as:

$$\begin{aligned} f(x) &= \frac{1}{\pi}\int_0^{+\infty} \Re\left(\mathrm{e}^{-ixu}\Phi(u)\right)\mathrm{d}u \\ &\approx \frac{1}{\pi}\sum_{j=0}^{L-1} \Re\left(\mathrm{e}^{-ixu_j}\Phi(u_j)\right)\eta \end{aligned} \tag{12}$$

The upper bound of the integral Eq. (12) is $b = L\eta$, with $\Delta\eta = \frac{2\pi}{L}$, so that the FFT can be used to calculate the Eq. (12). Values x can be obtained with the use of FFT. Given:

$$x_j = -a + \Delta j, j = 0, 1, 2,. .., L-1 \tag{13}$$

$x_j \in [-a, a]$. Let $u_j = \eta j$ further, then the Eq. (12) can be expressed as:

$$f(x_k) \approx \frac{1}{\pi} \sum_{j=0}^{L-1} \Re\left(e^{-i\Delta\eta kj} \Phi(u_j) \eta w_j\right) \tag{14}$$

Where $k = 0, 1, 2, ..., L-1$, w_j is the weight coefficient based on different numerical integration methods, for example, the regular weight coefficient of the Trapezoidal rule $w_j = \frac{1}{2}$, when $j = 0, L-1$, otherwise $w_{j=1}$, the weight coefficient of Simpson's rule based on cubic polynomial is:

$$w_j = \begin{cases} \frac{1}{3}, \ j = 0, L-1 \\ \frac{4}{3}, \ j = 1, 3, 5, ..., L-3 \\ \frac{2}{3}, j = 2, 4, 6, ..., L-2 \end{cases} \tag{15}$$

Using FFT, we can get the approximate value of the probability density function at the inner-equidistant grid point $[-a, a]$.

The probability of occurrence of a state j in a state tree L is:

$$P_j = \frac{f(x_j)}{\sum_{k=0}^{L-1} f(x_k)} \tag{16}$$

The denominator in Eq. (16) guarantees that all states in the multinomial tree L can form a complete probability space.

Theorem 1: When $a \to +\infty, L \to +\infty$ discrete random variables $X_{1,j}$ converge in distribution continuous distribution X_δ.

Proof: The density function of X_δ is $f(x), x \in (-\infty, +\infty)$, $X_{1,j}$ represents the L aliquot grid points within $[-a, a]$, we can extend the discrete random variables $X_{1,j}$ to be continuous distribution in $[-a, a]$, the cumulative distribution function is:

$$\tilde{F}(x) = \begin{cases} P_0, x \le -a \\ \sum_{k=0}^{j} P_k, \ -a + \Delta j < x \le -a + \Delta(j+1) \\ 1, x > a \end{cases} \tag{17}$$

i) If $x \le -a$, because of monotonically increasing property of $F(x)$, we have:

$$\begin{aligned} \left|F(x) - \tilde{F}(x)\right| &\le F(x) + \tilde{F}(x) \\ &\le F(-a) + \tilde{F}(-a) \\ &\le F(-a) + f(-a) \end{aligned}$$

$$\because \lim_{-a \to -\infty} F(-a) = \lim_{-a \to -\infty} f(-a) = 0$$

$$\therefore \lim_{\substack{-a \to -\infty \\ x \le -a}} \left|F(x) - \tilde{F}(x)\right| = 0$$

ii) If $x \ge a$, $\left|F(x) - \tilde{F}(x)\right| = \left|F(x) - 1\right|$

$$\because \lim_{a \to +\infty} F(a) = 1$$

$$\therefore \lim_{\substack{a \to +\infty \\ x \ge a}} \left|F(x) - \tilde{F}(x)\right| = 0$$

iii) If $-a + \Delta j < x \le -a + \Delta(j+1)$, $j = 1, 2, ..., L-1$,

$$\tilde{F}(x) = \sum_{k=0}^{j} P_k = \frac{\sum_{k=0}^{j} f(x_k)}{\sum_{i=0}^{L-1} f(x_i)} = \frac{\sum_{k=0}^{j} f(x_k)\Delta}{\sum_{i=0}^{L-1} f(x_i)\Delta}$$

$\because$ If a is fixed, $L \to +\infty$, $\Delta = \frac{2a}{L-1} \to 0$, $x_j \to x$

$$\therefore \lim_{L \to +\infty} \tilde{F}(x) = \frac{F(x) - F(-a)}{F(a) - F(-a)}$$

$\because$ When $a \to +\infty, F(-a) \to 0, F(a) \to 1$

$$\therefore \lim_{a \to +\infty} \lim_{L \to +\infty} \tilde{F}(x) = F(x)$$

Integrated i), ii) and iii), $\forall x \in (-\infty, +\infty)$, we have $\lim_{\substack{L \to +\infty \\ a \to +\infty}} \tilde{F}(x) = F(x)$.

That is, the distribution of the discrete random variable $X_{1,j}$ converges to the continuous distribution X_δ.

5 OPTION PRICING

For the European option, the payoff function is $M(S_T)$. The following formula can be expressed to pricing options under recombined multi-tree with L states in N periods.

$$C = \mathrm{e}^{-rT} \sum_{k=0}^{(L-1)N} M(x_{N,k}) P_k^N \tag{18}$$

P_k^N is the probability of recombined multi-tree at node k in N-th period. Given:

$$P_k^N = C_k^N \prod_{m=1}^{N} P_{k_m} \quad (19)$$

Where $k_1 + k_2 + k_3 + ... + k_M = k$, $0 \le k_1, k_2, k_3, ..., k_M \le L-1$

$$C_k^N = \sum_{j=0}^{L-1} C_{k-L+1+j}^{N-1}$$
$$C_k^{N-1} = 0, k < 0 \text{ or } k > (L-1)(N-1) \quad (20)$$

The initial iteration condition is:

$$C_k^1 = 1, k = 0, ..., L-1 \quad (21)$$

When $k_1, k_2, k_3, ..., k_M$ satisfies one set equation $k_1 + k_2 + k_3 + ... + k_M = M + k - 1$. The following fast algorithm can be used.

$$k_i = \begin{cases} \left\lfloor \dfrac{k - \sum_{j=i+1}^{M} k_j}{i} \right\rfloor, i \le M-1 \\ \left\lfloor \dfrac{k}{M} \right\rfloor, i = M \end{cases} \quad (22)$$

European option can also be priced using the reverse iteration approach described below.

For path-dependent options, such as American options and barrier options, the reverse iteration method can be used to calculate the price of the option. The pricing of American options is the most representative. The following is an example of calculating the American option price process and steps by using the reverse option.

Step 1: When $i = N$, $H_{(N,j)} = M\left(S_{(N,j)}\right)$, $C_{(N,j)} = 0$, $j = 1, 2, ..., (L-1)N + 1$.

Step 2: The value of holding the American option at the node j in the i-th period is:

$$C_{(i,j)} = e^{-r\delta} \sum_{k=0}^{L-1} P_k H_{(i+1, j+k-1)}, j=1, 2, ..., (L-1)i + 1$$

The value of American options is:

$$H_{(i,j)} = \max\left(M\left(S_{(i,j)}\right), C_{(i,j)}\right)$$
$$i := i - 1$$

Step 3: If $i > 0$, turn Step 2. Else, $C = C_{(0,0)}$.

It is the price of American options at $t = 0$.

For Asian options, the option price depends on the historical path. In the multi-tree, the number of historical paths increases exponentially with the increase of the number of multi-tree periods. It is difficult to use recombined multi-tree pricing Asian options. However, path information can be seen as a new state variable, the number of calculation history paths can be reduced by using the interpolation method, and the price of the Asian option can also be calculated by the reverse iteration method. This article will not describe it in detail.

6 ILLUSTRATION

In the exponential Lévy stock price model, Merton's jump-diffuse model is the most widely used. The corresponding stochastic differential equation is:

$$\mathrm{d}S_t = \mu S_t \mathrm{d}t + \sigma S_t \mathrm{d}W_t + (J-1) S_t \mathrm{d}N(t) \quad (23)$$

Where $J \sim N\left(\mu_J, \sigma_J^2\right)$ is the size of the jump, $N(t)$ is a Poisson process with intensity λ. According to the ITO formula, we have:

$$\log(S_T) = \log(S_0) + \left(\mu - \frac{1}{2}\sigma^2\right)T +$$
$$+ \sigma\sqrt{T} N(0, 1) + \sum_{j=1}^{N(T)} \log J_j \quad (24)$$

The characteristic function of the log-Jump-diffuse process is:

$$\Phi_{X_t}(u) = \exp\left\{ t\left(i\mu u - \frac{1}{2}\sigma^2 u^2 + \lambda\left(e^{i\mu_J u - \frac{1}{2}\sigma_J^2 u^2} - 1 \right) \right) \right\} \quad (25)$$

Under the risk-neutral probability measure, Merton(1976) introduced a semi-analytical formula for European option pricing.

$$\sum_{n=0}^{\infty} \frac{\mathrm{e}^{-\lambda' T} (\lambda' T)^n}{n!} BS(S_0, \sigma_n, r_n, T, K) \quad (26)$$

Where $BS(S_0, \sigma_n, r_n, T, K)$ is the Black-Scholes European option pricing formula with the given parameters. It follows:

$$\sigma_n = \sqrt{\sigma^2 + n{\sigma_J}^2 / T}$$
$$r_n = r - \lambda(\mu_J - 1) + n \log \mu_J / T$$
$$\lambda' = \lambda \mu_J$$

Given $\begin{matrix} S_0 = 100, \sigma = 0.2, \ r = 0.0075, \\ \lambda = 0.01, \mu_J = -0.02, \sigma_J = 0.6 \end{matrix}$, $L = 1024$, $T = 0.5$, $K = 25 : 0.5 : 175$. The lower bound of the multinomial tree is $-a = 5.12$. The first 50 items of semi-analytical solution in Eq. (26) should be calculated. The number of states of the multi-tree is $L = 1024$. The number of periods is $N = 5$. The relative error between the price of the European call option obtained by calculating the multinomial tree method and the price of the European call option price obtained by Eq. (26) can be obtained. The relationship between error and execution price is shown in Fig. 2.

It can be seen from Fig. 2 that the relative error of the option price is close to zero in the whole calculation interval, and the semi-analytic pricing formula does not have superiority in program running time.

The number of FFT-network nodes is fixed, while the nodes of the recombinant multinomial tree increase with the number of periods, thus, the computational complexity of the recombinant multinomial tree method is significantly reduced with the same computational accuracy, especially for options with short expiration times.

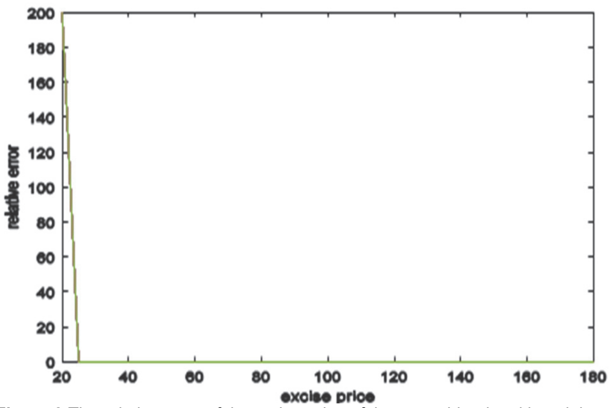

Figure 2 The relative error of the option price of the recombined multinomial tree method based on FFT

7 CONCLUSION

In this paper, we use the characteristic function of the Lévy process to obtain the function value of the probability density function in a given interval, and directly use the probability density function value at these lattice points to generate the risk-neutral probability. It is proved that the discrete random variables corresponding to the multinomial tree converge to the Lévy distribution continuous random variable. And the pricing formula of the European option is obtained. The option pricing process and steps of the iterative method are introduced for the American options and barrier options. By using the Jump-diffuse Lévy model with an existing semi-analytic solution of European options, the validity of the recombined multinomial tree is described. The results show that the FFT-based recombined multinomial tree method has the advantage of high computational efficiency, and overcomes the difficulties of constructing the multinomial tree by using the moment matching technique, which can be used to construct high order recombined multinomial tree.

Although recombined multinomial tree improves computational efficiency in the period of underlying asset price fluctuating slightly. However, with the increase of the number of multinomial tree periods, the number of nodes increases linearly, and the computation interval of FFT recombined multinomial tree also increases linearly. In order to recombine the multinomial tree, the calculation interval of each option multinomial tree is the same, sometimes it will lead to the calculation interval being too large, and reduce the calculation efficiency. In the whole process of Markov chain, the number of nodes remains unchanged, so the efficiency of the Markov chain is low in the period when the price of the underlying asset fluctuates a little. Therefore, the Lévy model of option pricing method based on the FFT, combining with the respective advantages of recombined multinomial lattices and the Markov chains, will be studied in the future.

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