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A RASCH MODELING APPROACH TO ANALYZING STUDENTS' INCORRECT ANSWERS ON MULTIPLE-CHOICE QUESTIONS: AN EXAMPLE FROM WAVE OPTICS

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An analysis of students' difficulties for a curricular topic may help the educator to gain better insight into students' reasoning about that topic which is a prerequisite for high-quality teaching. The purpose of this study was to demonstrate how distractor analysis may be used for identifying students' difficulties in a certain topic. In order to be in position to perform invariant measurement and to easily relate students' difficulties to their achievement levels, we decided to take a Rasch modeling approach. Our study included 14 wave optics items and 286 students from five universities in Slovenia, Croatia, and Bosnia and Herzegovina. Rasch modeling was used to estimate item and student measures, as well as to create option probability curves which allowed us to relate students' achievement levels to the choice of individual distractors. It has been found that all 14 included items function in line with the Rasch model. In 10 out of 14 items there were distractors chosen by at least 25% of students. For several out of these 10 items, the option probability curves indicated that attractiveness of individual distractors depended on students' ability levels. We could conclude that the Rasch-based distractor analysis may provide very useful information for differentiation of physics instruction.

Keywords: *Rasch modeling, distractor analysis, option probability curves, wave optics*

Introduction

Measurement and interpretation of students' latent traits should be based on test theories. Traditionally, classical test theory has been most often used in earlier research, but in the past few decades probabilistic test theory becomes increasingly popular (Liu, 2010). In probabilistic test models the probability of a student's answer is modeled to be a function of the measured trait and of certain characteristic(s) of the item (Hambleton *et al.*, 1991). One of the simplest probabilistic test models is the dichotomous (simple) Rasch model (Bond *et al.*, 2015). In the dichotomous Rasch model, the probability that a student will respond to an item correctly is determined by the difference in the student's achievement level and the difficulty of the item, according to the following equation (Hambleton *et al.*, 1991):

$$P_i(\theta) = \frac{e^{(\theta - b_i)}}{1 + e^{(\theta - b_i)}}$$

where $P_i(\theta)$ is the probability that a randomly chosen student of ability θ correctly responds to item i and b_i is the difficulty parameter for item i.

We can see that in the Rasch model there is only one item measure which in interaction with the person measure predicts the probability of correctly solving the item. The person measure is an estimate of student ability on the tested construct, and the item measure is an estimate of item difficulty. Both, the person and item measure, are expressed on the same interval scale, and are measured in logits.

One distinguished feature of the Rasch measurement is its specific objectivity which only holds if data fit the Rasch model and boils down to the idea that the difference between two persons' abilities does not depend on which set of items these two persons solve, i.e. a person who has a higher ability should always have a higher probability to solve any question which is related to this ability (Wallace and Bailey, 2010). That is why it is often asserted that Rasch modeling is the best way to perform objective, invariant measurement.

On the other hand, high-quality measurement of students' learning outcomes may provide precious feedback to the teacher and help her/him to improve the quality of the teaching and learning process. Concretely, in physics education it is particularly important to continuously track students' learning and to identify potential conceptual difficulties, because difficulties that are not taken into account may largely hinder new learning. By analyzing students' difficulties on a certain topic, we may get some vivid insight into students' reasoning about that topic which is a prerequisite for improving the quality of instruction.

An effective and practical approach to analysis of students' misconceptions and other types of students' difficulties is the Rasch analysis of misconceptions distractor-driven multiple choice (MDDMC) items (Herman-Abell and DeBoer, 2011; Wind and Gale, 2015).

In MDDMC items the distractors are systematically developed, based on earlier empirical research on students' difficulties for the given topic area. Unlike in a basic Rasch analysis, where we are interested in the probability of obtaining correct (1's) or incorrect answers (0's), here the Rasch analysis is conducted with the database which contains complete information on which answering option has been selected by the respondents for each test item.

Distractor analysis plots are created by plotting estimates of Rasch achievement measures (low to high) along the x-axis, and the proportion of students selecting each answer option along the y-axis (see Figure 1). These distractor analysis plots can be seen as diagnostic tools that summarize patterns of students' difficulties. The mere students' difficulties may be analyzed from the perspective of different cognitive theories. Within the physics education research community particularly prominent are the misconceptions-based perspective and the resources-based perspective (Hammer *et al*, 2005; Hammer, 1996; Sabella and Redish, 2007). While the misconceptions-based perspective focuses on the idea "that students' prior knowledge includes quite reasonable conceptions that are not consistent with expert understanding" (Hammer, 1996, 99), the resources-based perspective points out that each time students undertake a cognitive task, they activate a set "of their available cognitive resources, in a way that is context-dependent" which may result with ideas that are not consistent with experts' understanding (Buteler and Coleoni, 2009, 101). When student understanding is generally poor and phenomena are mostly far away from their everyday experiences, students' answering behaviour is most often governed by the context-dependent activation of cognitive resources rather than by the pre-existing firm ideas. Therefore, in such situations it is more appropriate to apply the cognitive resources framework for analyzing students' answers to conceptual questions. Hammer *et al.* (2005) distinguish two types of resources: conceptual and epistemological resources. Conceptual resources refer to tools and processes that, when activated, allow the students to reason about physical situations and to construct new knowledge. Epistemological resources, on the other hand, are control structures that primarily handle the activation of the mere conceptual resources. One type of conceptual resources which are often observed in students' reasoning about physics are the phenomenological primitives or p-prims (diSessa, 1993; Hammer, 1996). P-prims are "basic statements about the functioning of the physical world that a student considers obvious and irreducible" (Redish, 2004, 20; Sabella and Redish, 2007, 1018). They are called 'phenomenological' because they are typically abstracted from everyday phenomena. For example, the "closer is stronger" p-prim may be abstracted from the experience with open fire or with loudspeakers. Obviously, in many contexts the mentioned p-prim allows one to come to useful conclusions. However, when this same p-prim is applied to thinking about the Earth-sun system, it may result with the errorneous idea that in summer it is warmer at the northern hemisphere because the Earth is closer to the Sun. In other words, the p-prim itself may not be considered as right or wrong, but its application to various contexts may be more or less consistent with experts' understanding.

Research aim

Taking into account that the benefits of Rasch-based analyses of distractors have not been sufficiently covered in earlier research, we decided to demonstrate this methodological approach within the context of our wave optics item bank (Mešić *et al.*, 2019a; Salibašić Glamočić *et al.*, 2021). Taking into account that the databases from our bank development studies included answers from students studying at five different universities from Slovenia, Croatia, and Bosnia and Herzegovina, as well as the fact that our item banks contain MDDMC items (Mešić *et al.*, 2019a), we strongly believe that Rasch-based analyses of these items' distractors may provide us with a valuable insight into university students' difficulties in the area of wave optics. This insight may help the university physics educators from the mentioned three countries to improve their wave optics classes. From a theoretical perspective, this study may also help us to relate difficulty patterns to students' levels of achievement which would be more difficult to do within the framework of classical test theory. This information about differential difficulty patterns may be useful for differentiating wave optics instruction, i.e. for tailoring the instruction in line with the specific needs of students at various ability levels (Tomlinson, 2001). Differentiated instruction is particularly needed when students or student subgroups exhibit different types of difficulties with a given topic.

Methodology

Sample of participants

Our initial Rasch analysis included all 294 physics and engineering students who participated in our earlier wave optics item bank development studies (Mešić *et al*., 2019a; Salibašić Glamočić *et al*., 2021). These students came from five different universities: University of Maribor, University of Zagreb, University of Split, University of Rijeka, and University of Sarajevo. 180 of them were male and 114 were female. Among the sampled students, there were first year students, but also final year students. All of them had lectures on wave optics at least in one university course, before being administered with our wave optics test.

Curriculum

Although the sampled universities substantially differed when it comes to coverage of wave optics in their curricula, for most of them holds that wave optics is more widely covered than in typical introductory courses which follow textbooks such as *Physics for Scientists and Engineers with Modern Physics* by Serway and Jewett. Concretely, topics such as: double-slit interference, single-slit diffraction, grating diffraction and thin-film interference were covered in all sampled curricula. In addition, an important feature of wave optics instruction at all sampled universities is related to the mathematized approach to wave optics and insufficient focus on development of conceptual understanding. More details about the sampled wave physics curricula can be found in the paper by Mešić *et al*. (2019a).

Sample of items

Taking into account that Rasch analyses are more reliable when conducted with larger student samples, we decided to include into our initial Rasch analysis all 294 students from our bank development databases. In addition, we decided to include into our analysis only the 14 items which were used as common items in our bank development studies. In our opinion, these 14 items are a good item set for demonstrating the Rasch-based approach to distractor analysis, because they proved to be well-functioning in earlier studies (Mešić *et al*., 2019a; Salibašić Glamočić *et al*., 2021). Furthermore, they represent well the wave optics curricula from the sampled universities (Salibašić Glamočić *et al*., 2021) which allows us to come to informative conclusions about university students' learning outcomes for the topic of wave optics. Taking into account that the distractors for these items were developed based on an explicit, extensive list of empirically identified student difficulties, we can consider the sampled 14 items to be MDDMC items which makes them suitable for Rasch-based distractor analysis (Wind and Gale, 2015). Here it is also important to note that in all of our items there was a single correct answer and three incorrect answer choices (distractors).

Taking into account that for each item we had offered four answering choices, the probability of choosing the correct answer by mere guessing was 25%. Therefore, we decided to consider the distractors chosen by at least 25% of the respondents, as indicators of pronounced students' difficulties. Initial analysis showed that there were ten items with distractors chosen by at least 25% of respondents.

Procedures

First, we prepared the statistical database to include all 294 students and 14 wave optics items. The database included information on the answer option each student chose on each of the items.

Rasch analysis of this database has been performed with the WINSTEPS 3.72 software (Linacre, 2011). For purposes of performing the distractor analysis, for each item we had to specify what answering option is correct and what options are distractors. Our initial Rasch analysis resulted in certain estimates of Rasch person and item measures, as well as with certain statistics describing person and item fit. For all items the mean-square (MNSQ) item infit statistics was below 1.2 which indicates excellent fit. However, for eight persons the answering patterns (as measured by person MNSQ outfit and infit statistics) did not fit the Rasch model. Consequently, these persons were dropped from the database. Thereafter, the Rasch analysis has been repeated and this time all item and person fit statistics were inside the recommended boundaries which is a prerequisite for invariant measurement. Thus, the final database for this study contained information on how each of the 286 students (173 male and 113 female) responded to each of the 14 items. Person reliability amounted to 0.53 which is acceptable given the relatively low number of items, and item reliability amounted to 0.96 which is excellent.

WINSTEPS 3.72 software (Linacre, 2011) has been used with that final database to estimate reliability measures and to generate option probability curves for the 10 items in which at least one distractor has been chosen by at least 25% of students. In most of these items the distribution of distractor choices differed across the ability subgroups. For purposes of keeping our analysis concise, out of the identified ten items, we finally picked six items which were particularly convenient for demonstrating the similarities and differences between the classical and Rasch-based distractor analysis (see Appendix). The option probability curves were analyzed only for these six items. Generally, the option probability curves show how for a given item, the probability of selecting a particular answer changes as a function of students' ability level. These curves were the primary focus of our study and are discussed in the next section.

Results and discussion

Our analysis of students' incorrect answers will be based on option probability curves and resources-based framework. For purposes of facilitating analyses, along with probability curves a table with frequencies of individual answering choices will be presented. In the tables, the correct answer is marked with an asterisk, and the distractors chosen by at least 25% of the respondents are marked with an exclamation mark.

In ITEM 2 students were expected to interpret a wavefront representation of two wave interference. Concretely, they were expected to answer what type of interference (constructive/destructive) occurs at the points A and B which both lie on anti-nodal lines, but B is located between two subsequent wavefronts and A is located at the intersection of two wavefronts. Here the correct answer was that at all points on an anti-nodal line the path difference corresponds to an integer multiple of λ and constructive interference will be observed at A as well as at B.

	OPTION		DATA	
			COUNT	%
ITEM 2	!	*a*	77	26.9
	*	*b*	81	28.3
		c	52	18.2
	!	*d*	75	26.2
		no answer	1	0.3

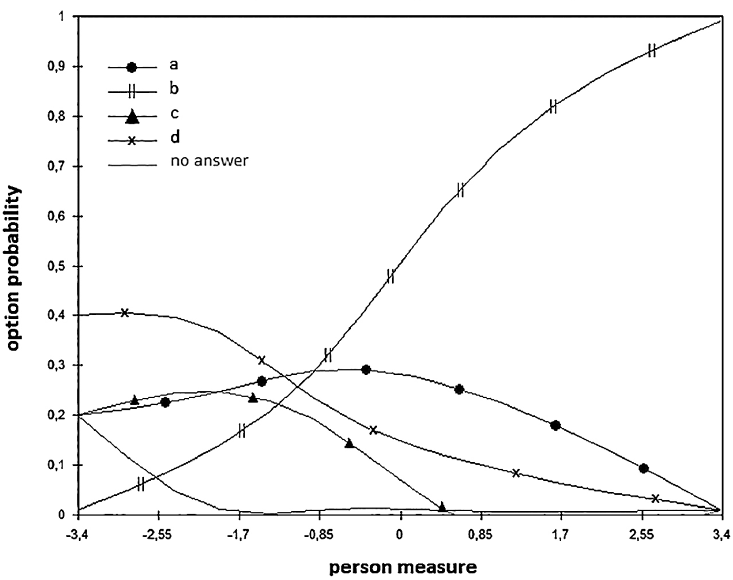

Figure 1. Frequency of response options and option probability curves for ITEM 2

Although the classic distractor analysis shows that distractor "a" was most attractive to the students, Rasch-based analysis provides us with another interesting information: the probability of selecting option "d" is clearly the most attractive option for under-average achievers (Figure 1). In fact, it seems that these students already fail at understanding the concept of maximum-order: it seems that they associate the ordinal number of the wavefronts with the maximum-order which results in the erroneous idea that second-order maximum occurs at the place where the second wavefront from one light source meets the second wavefront from the other light source. On the other hand, option "a" was a more attractive distractor to average achievers who correctly recognized that point A is a zero-order (not a second-order) maximum, but seemed to wrongly believe that at a location exactly between two subsequent wavefronts a minimum has to occur. Concretely, it seems that these students knew that for one single wave there is a through exactly between two subsequent crests, and they seem to have wrongly associated the existence of a through in a single wave situation with the occurrence of a minimum in a two-wave situation. The truth is that the occurrence of minima and maxima only depends on the phase difference of the two coherent, interfering waves.

In ITEM 5 students were expected to predict how adding new (identical) slits to a double-slit mask would affect the original interference pattern if the whole process preserves the between-slit separation. The answer is that only the fringes would become narrower (and more intense), because when the number of narrow slits (i.e., interfering waves) increases, absolute destructive interference occurs for phase differences lower than 180°, which means that first (and higher) minima will occur at lower diffraction angles. Classic distractor analysis tells us only that distractor "b" was most attractive to the students (Figure 2).

	OPTION		DATA	
			COUNT	%
ITEM 5		*a*	41	14.3
	!	*b*	78	27.3
		c	38	13.3
	*	*d*	122	42.7
		no answer	7	2.4

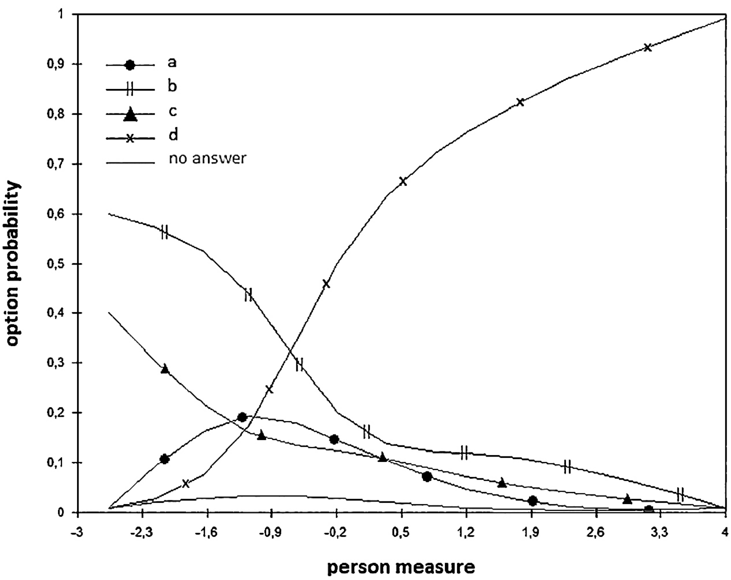

Figure 2. Frequency of response options and option probability curves for ITEM 5

However, Rasch-based distractor analysis additionally tells us that "b" was the by far most popular option for under-average achievers, whereas for average achievers option "a" was similarly attractive as option "b". The key difference between "a" and "b" was as follows: in option "a" it was stated that the width of fringes increases and (consequently) their number decreases, while in option "b" it was stated that the width of fringes, as well as their number increases. It seems that under-average achievers seem to more often think that increasing the number of slits results in an increasing number of fringes which means that fringes are viewed as some kind of slit images (Mešić *et al*., 2019b). In other words, under-average achievers seem to more often activate resources from geometrical optics while trying to solve tasks from wave optics.

In ITEM 6 students were expected to explain what processes occur at a slit of width 20 μm and height 2 cm when it is illuminated with red laser light. Simply, here we can apply the Huygens-Fresnel principle to conclude that all points within the slit which are "hit" by laser light become sources of secondary waves which superimpose to give alternating bright and dark diffraction fringes on the screen. Classic distractor analysis shows that for students from all ability levels the most attractive distractor was "a" which reflects the misconception that only the two edges of the slit become sources of secondary waves (Ambrose *et al.*, 1999) (Figure 3).

	OPTION		DATA	
			COUNT	%
ITEM 6	!	*a*	79	27.6
	*	*b*	123	43.0
		c	36	12.6
		d	41	14.3
		no answer	7	2.4

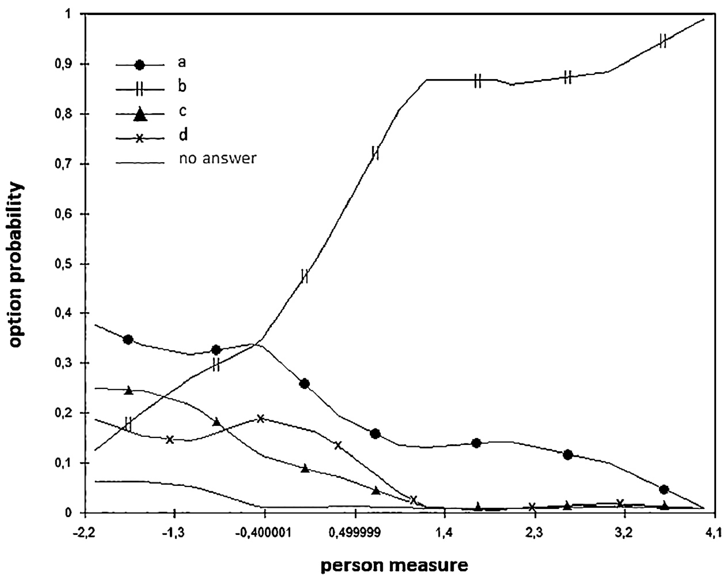

Figure 3. Frequency of response options and option probability curves for ITEM 6

Taking into account this result it is not surprising that students are confused because of different conditions for minima in single-slit diffraction and double-slit interference. In fact, they often think that in both cases we have interference of two waves and that is why they are confused with different conditions for minima. Rasch-based distractor analysis additionally shows us that the second most attractive distractor differed for under-average and average achievers: under-average achievers more often tended to choose distractor “c” while average achievers more often chose distractor “d”. The key difference between “c” and “d” is again related to the fact that “c” reflects mixing of wave optics and geometric optics models (Ambrose *et al*., 1999). Concretely, they predict the occurrence of only one secondary wave and one corresponding fringe that occurs at the center of the screen, i.e., they see the fringe as some kind of slit image. The distractor “d” reflected the belief that there are only three secondary waves: two at the edges of the slit, and one at its center.

In ITEM 7 students were expected to differentiate between factors which affect interference/diffraction aspects of a pattern in combined interference and diffraction. Concretely, for double-slit diffraction students were asked to predict what will happen with the central diffraction maximum if we increased the distance between the two slits. Truth is that diffraction aspects of the given pattern do not depend on separation between the two slits (this separation affects interference aspects of the pattern), and the answer is that the width of the central diffraction fringe would not change. We could add that the central diffraction maximum would now contain more interference fringes. When it comes to the individual distractors, Rasch based analysis shows that “b” was by far most attractive for under-average achievers while “c” was somewhat more attractive than “b” for average achievers (Figure 4).

	OPTION		DATA	
			COUNT	%
ITEM 7	*	*a*	55	19.2
	!	*b*	99	34.6
	!	*c*	90	31.5
		d	39	13.6
		no answer	3	1.0

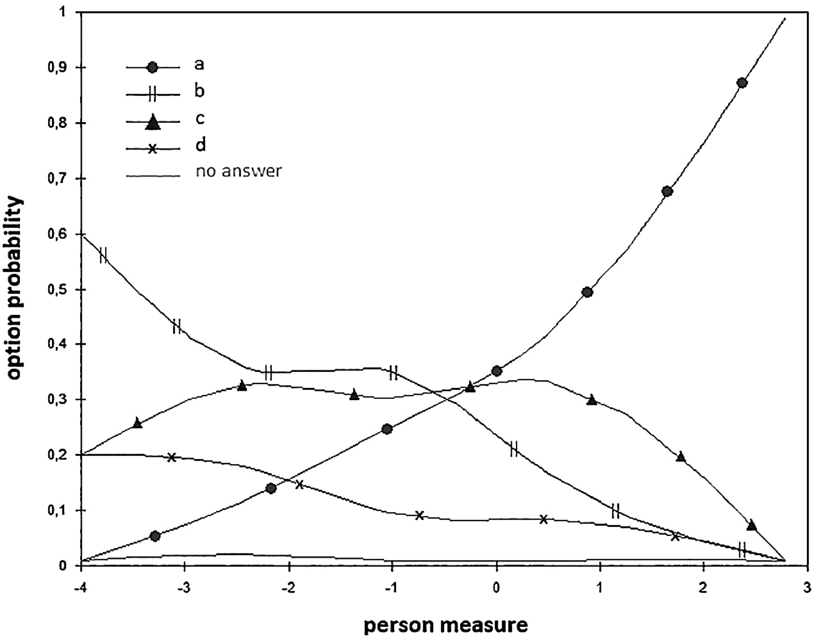

Figure 4. Frequency of response options and option probability curves for ITEM 7

Option "b" reflects the believe that increasing slit distance also increases distance between all fringes in the pattern which, they think, also increases the width of the diffraction fringe. Such a reasoning again indicates activation of the geometrical optics resources in the "wrong context". Average achievers more often chose "c" which states that increasing slit separation would decrease width of the diffraction fringe. The choice of this option reflects not mixing of wave and geometric models, but mixing of interference and diffraction effects. In fact, increasing slit separation decreases width of interference fringes.

In ITEM 9, students were expected to apply their knowledge about the concept of coherence. Concretely, among the given choices they had to choose the optimal procedure for increasing the degree of coherence of light. In fact, spatial coherence may be increased by passing light through a narrow aperture, while temporal coherence may be increased by increasing light's monochromaticity, i.e., by passing the light through a color filter. Already classic distractor analysis shows that the most frequently chosen distractor was "d" reflecting the believe that passing light through a convergent lens is the best option for increasing the coherence of light (Figure 5).

	OPTION		DATA	
			COUNT	%
ITEM 9		*a*	36	12.6
		b	47	16.4
	*	*c*	80	28.0
	!	*d*	118	41.3
		no answer	5	1.7

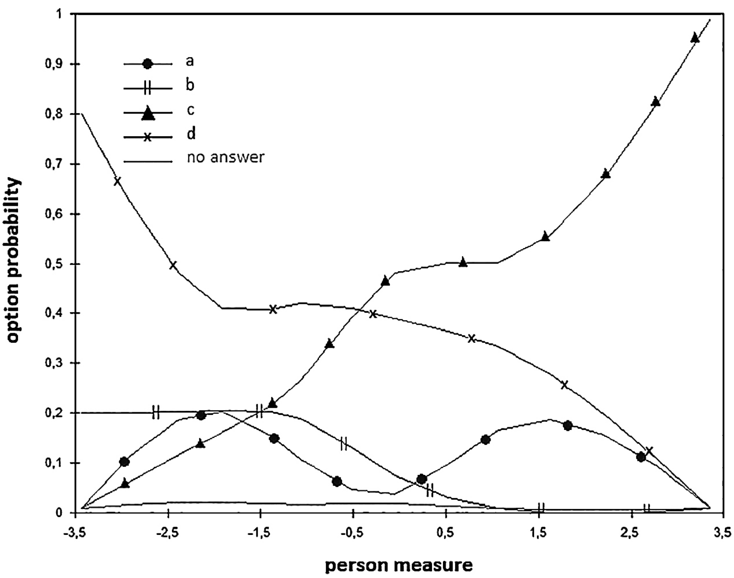

Figure 5. Frequency of response options and option probability curves for ITEM 9

These results are in line with the finding from earlier research that students often do not understand the role of lenses in the context of wave optics phenomena (Colin and Viennot, 2001). In the context of wave optics experiments, lenses are mostly used for: observing far-field diffraction on the focal plane of the lens, enlarging fringes and in certain contexts they are also used for collimating light. However, convergent lenses alone are not as effective as a combination of narrow aperture and color filter when it comes to increasing spatial and temporal coherence of light. Rasch-based distractor analysis reveals an additional interesting effect: compared to average achievers, distractor "a" was more frequently chosen not only by under-average achievers, but also by above-average achievers. In both these groups many students believed that passing light through a thin film may increase its coherence. In fact, in practice thin films are not used to change coherence of light, but thin films are used because each light is characterized by finite (mostly small) coherence length and if film thickness is larger than that coherence length, we cannot observe stable, non-uniform interference patterns. Taking into account that even the high-achievers exhibit many difficulties with this question, we can conclude that there is space for substantial improvement when it comes to covering temporal and spatial coherence in the sampled curricula.

In ITEM 11 the students were told that a double-slit mask has been illuminated with laser light and they were expected to reason about the intensity of laser light in the region of space between the double-slit mask and the screen. Truth is that two cylindrical secondary waves propagate and superimpose in that region of space. Taking into account that these waves are mutually coherent, the intensity of the resulting wave varies in this region of space, and at some points it is zero and at other points it is at its maximum value.

	OPTION		DATA	
			COUNT	%
ITEM 11		*a*	27	9.4
	!	*b*	89	31.1
		c	30	10.5
	*	*d*	123	43.0
		no answer	17	5.9

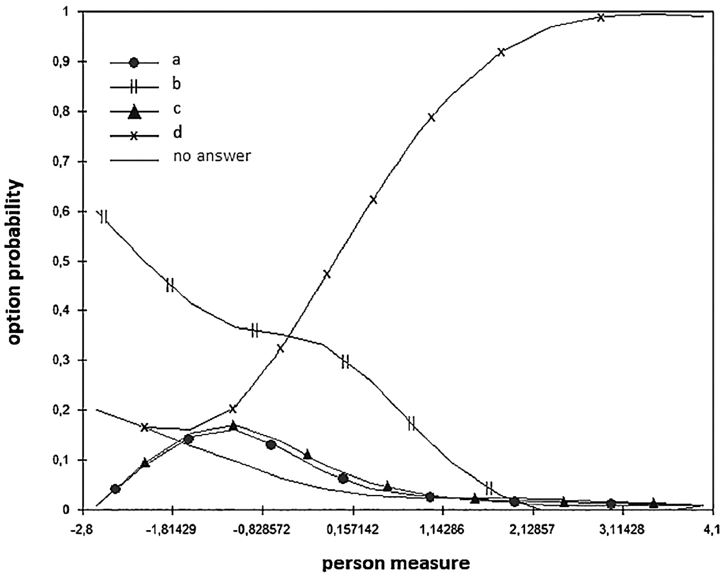

Figure 6. Frequency of response options and option probability curves for ITEM 11

The ranking of distractor frequencies was similar for all ability groups which means that classic distractor analysis is approximately equally effective as Rasch-based analysis in this context (Figure 6). Concretely, for students from all ability groups the idea that intensity of laser light is constant and non-zero proved to be most attractive. Similarly, as has been observed in research on learning geometrical optics (Goldberg and McDermott, 1987), it seems that many students do not distinguish between “existence of interference pattern” and “seeing an interference pattern”, i.e., they often believe that the interference pattern exists only on the screen.

Conclusion

In this study, we conducted a Rasch-based distractor analysis on a sample of MDDMC wave optics items. All items proved to fit the Rasch model which allowed us to do our analysis within a framework of invariant measurement. The Rasch-based distractor analyses showed that sometimes students from different ability subgroups experience different types of difficulties with a given physics topic and in other occasions the distribution of difficulties is the same across all subgroups. If students from different subgroups experience different types of difficulties differentiated instruction is strongly recommended. Instruction may be differentiated through offering multiple learning pathways which, for example, differ with respect to the choice of content, choice of activities, types of products and rate of learning progress (Tomlinson, 2001). One way of organizing differentiated instruction is to occasionally perform ability grouping in the regular classroom.

It should be noted that classical distractor analysis typically identifies only the difficulty which is most prevalent for the whole group of students. In other words, it does not provide diagnostic information across the different academic ability levels, which makes it a less effective choice for planning differentiated instruction. The conclusions obtained from classical distractor analysis are similar to conclusions from Rasch-based distractor analyses only for those items for which a similar distribution of difficulties across all ability levels is observed.

For example, it has been shown that students from all ability levels often have difficulties with distinguishing between "seeing of an interference pattern" and "existence of an interference pattern". To overcome this difficulty, the teacher may organize similar learning activities for students from all academic ability subgroups. A particularly effective approach may be to carefully introduce the wavefront representation, because unlike the sinusoidal representation it nicely shows how waves superimpose in the space between the slits and screen and not only on the screen.

We also identified difficulties which were somewhat more prominent for certain ability subgroups than for others. An example of a difficulty which proved to be particularly prominent for underachievers is related to activating resources from geometrical optics when reasoning about wave optics phenomena. A possible way for overcome this difficulty would be to strengthen the students' understanding about the wave model of light, so that it has an increased probability for being activated in a wider range of contexts. Some researchers suggested that this can be effectively done by drawing analogies between light waves and water waves, supported by intensive hands-on work and external visualizations (Wosilait *et al*., 1999).

Finally, it should be also noted that some items proved to be very demanding for our sample of students. For example, in items 2 and 9 the percentage of correct answers was below 30%, although all sampled students learned about wave optics in at least one university course. This may indicate that at the sampled universities more attention should be paid to developing students understanding about the wavefront representation of interference, as well as about the concept of spatial and temporal coherence. In fact, in some countries (e.g., Germany), students have the opportunity to learn about spatial and temporal coherence already at the high-school level (Dorn and Bader, 2010).

When it comes to main limitations of this study, it would be very useful to corroborate some of our interpretations through in-depth, oral interviews. Also, the findings about students' difficulties in wave optics would be certainly richer if they were based on a larger number of items. In our future research, we plan to conduct large scale studies in order to investigate how the structure of misconceptions differs for various ability groups in different areas of the physics curriculum.

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PRIMJENA RASCH MODELIRANJA U ANALIZI NETOČNIH ODGOVORA STUDENATA NA PITANJA VIŠESTRUKOG IZBORA: PRIMJER IZ VALNE OPTIKE

Džana Salibašić Glamočić, Vanes Mešić

Analiza studentskih poteškoća vezanih za određenu nastavnu temu može pomoći predavaču u stjecanju boljeg uvida u studentsko razumijevanje određene teme, što je preduvjet za visokokvalitetnu nastavu. Svrha ovog istraživanja sastojala se u demonstriranju načina na koji analize ometača mogu biti iskorištene za identificiranje studentskih poteškoća u određenom području. Kako bismo mogli provoditi invarijantna mjerenja i lako povezati studentske poteškoće s razinama njihovih postignuća, odlučili smo se studiju provesti u okviru Rasch formalizma. Naše istraživanje obuhvatilo je 14 zadataka iz područja valne optike i 286 studenata s pet sveučilišta u Sloveniji, Hrvatskoj te Bosni i Hercegovini. Rasch modeliranje korišteno je za procjenu težine zadataka i sposobnosti studenata, kao i za kreiranje krivulja vjerojatnosti ponuđenih opcija što nam je omogućilo povezivanje razine *razumijevanja studenata s izborom pojedinačnih ometača. Utvrđeno je da svih 14 zadataka funkcionira u skladu s Rasch modelom. Za 10 zadataka imali smo ometače koje je odabralo najmanje 25 % studenata. Kod nekoliko od ovih 10 zadataka krivulje vjerojatnosti ponuđenih opcija ukazuju na to da atraktivnost pojedinačnih ometača ovisi o razinama sposobnosti učenika. Možemo zaključiti da analiza ometača primjenom Rasch modeliranja može dati vrlo korisne informacije za diferencijaciju nastave fizike.*

Ključne riječi: *Rasch modeliranje, analiza ometača, krivulje vjerojatnosti ponuđenih opcija, valna optika*

Appendix: Final sample of items that has been used for demonstrating the Rasch-based approach to distractor analysis

Item 2

Let us imagine that two identical, mutually coherent point sources 1 and 2 start to emit light waves at the same instant. Which of the following best describes the situation at points A and B of Figure 1?

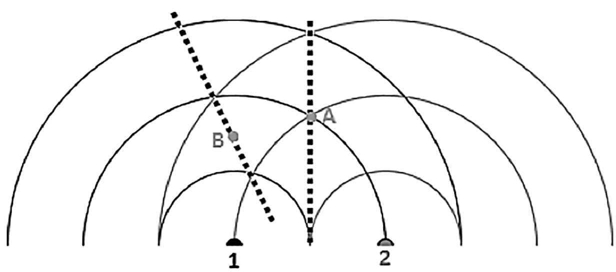

Figure 1

a) At point A there would be a zero-order maximum and at point B there would be a first-order minimum.
b) At point A there would be a zero-order maximum and at point B there would be a first-order maximum.
c) At point A there would be a second-order maximum, whereas at point B the waves from the two sources would not superpose, at all.
d) At point A there would be a second-order maximum, whereas at point B there would be a second-order minimum.

Item 5

A laser beam of wavelength 650 nm is perpendicularly incident on a double slit mask which results in the occurrence of alternating bright and dark fringes on the opposite screen. The width of each of the slits is 0.5 μm, and their mutual separation is *d*=50 μm. Then we start adding new 0.5 μm slits to the mask, while holding constant the original separation of adjacent slits *d*. How would this process affect the appearance of the interference pattern?

a) The number of primary maxima would decrease, and their width would increase.
b) The number of primary maxima would increase, as well as their width.
c) The number of primary maxima would decrease and they would become narrower.
d) The number of primary maxima would not change, but they would become narrower.

Item 6

A laser beam of wavelength 650 nm is perpendicularly incident on a slit of width 20 μm and height 2 cm. Which processes will occur after the light encounters the slit?

a) The edges of the slit will become sources of secondary waves, whose superposition will result in the occurrence of bright and dark fringes on the screen.
b) All the points between the two edges of the slit will become sources of secondary waves, whose superposition will result in the occurrence of bright and dark fringes on the screen.
c) The point at the centre of the slit will become a source of secondary waves which will result in the occurrence of one bright, vertical fringe on the screen.
d) The edges of the slit, as well as the point at the centre of the slit will become sources of secondary waves whose superposition will result in the occurrence of bright and dark fringes on the screen.

Item 7

A laser beam of wavelength 500 nm is perpendicularly incident on a double slit mask which results in the occurrence of multiple interference minima within the central diffraction maximum (see Figure 2). The width of each of the slits is 30 μm and their mutual separation is 150 μm. What would happen to the central diffraction maximum if we doubled the distance between the slits, while holding the number of slits and their width constant?

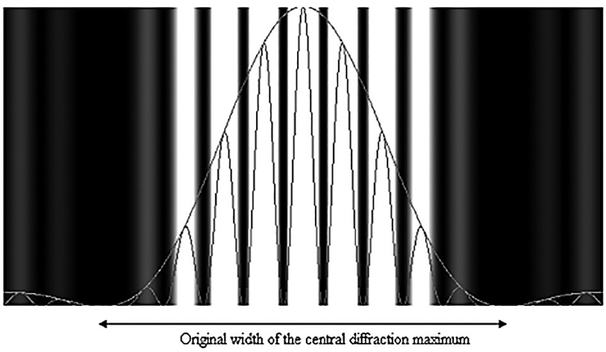

Figure 2

a) The width of the central diffraction maximum would not change.
b) The width would increase by a factor of two.
c) The witdh would decrease by a factor of two.
d) The witdh would decrease by a factor of four.

Item 9

Which of the following procedures would be the best choice if our aim is to increase the coherence of low-coherence light?

a) Forcing the light beam to pass through a very thin film.
b) Forcing the light beam to pass through a prism.
c) Forcing the light beam to pass through a very narrow aperture, before passing through a color filter.
d) Forcing the light beam to pass through a convergent lens.

In item 11 a laser beam of wavelength 650 nm is incident on a double slit mask which results in the occurrence of bright and dark fringes on the screen located L=2 m behind the mask. The width a of each slit is 500 nm (i.e. smaller than the wavelength), their height h is 2 cm, and their mutual separation d is 50 μm (see Figure 3).

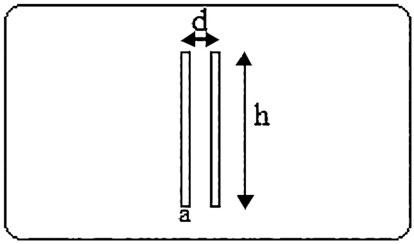

Figure 3. Double slit mask

Item 11

Which of the following is true regarding the intensity of laser light in the space between the mask and the screen?

a) The intensity of laser light is zero, because interference of laser light only occurs at fixed points of the screen.
b) There is a constant and non-zero intensity of laser light.
c) The intensity of laser light can be calculated by using Malus' law.
d) The intensity of laser light is changing from point to point according to a regular pattern.