PROPAGATION PREDICTION FOR INDOOR WIRELESS COMMUNICATION BASED ON NEURAL NETWORKS

Predviđanje rasprostiranja elektromagnetskog polja u bežičnim komunikacijama zatvorenog prostora zasnovano na neuronskim mrežama

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Abstract

The installation of indoor radio systems requires rather detailed propagation characteristics for any arbitrary configuration, so appropriate wave propagation model must be established. In spite of a number proposed solutions for prediction of the propagation characteristics in WLAN environment, it is difficult to say that we have completely satisfied solution. A neural network propagation model that was trained for particular environment was developed. The network architecture is based on the multilayer perceptron. The neural network results are additionally compared with the numerical results obtained by the deterministic 3-D ray tracing model. The ray tracing model includes three reflected rays from the walls and other obstacles what was enough accurate for the given environment. The neural network is used to absorb the knowledge about given environment through training with three access points. Using such obtained knowledge the network is used to predict signal strength at any spot of space under consideration. The various training algorithms were applied to the network to achieve the best convergence results and best possible network model behavior. The network model was trained by Scaled Conjugate Gradient (SCG), Conjugate Gradient of Fletcher-Reeves (CGF), Quasi-Newton (QN), and Levenberg-Marquardt (LM) algorithms. The comparison of the obtained results is presented.

Key words: indoor propagation model, 3-D tracing model, WLAN, multilayer perception, training algorithm

Sažetak

Uvođenje bežičnih komunikacijskih sustava u bilo kakav prostor zahtijeva prilično detaljno poznavanje propagacijskih karakteristika, tako da je potrebno izraditi odgovarajući model rasprostiranja elektromagnetskoga polja. Unatoč većem broju do sada predloženih rješenja za predviđanje propagacijskih karakteristika u bežičnim lokalnim mrežama (WLAN), teško je reći da postoji potpuno zadovoljavajuće rješenje. Razvijen je propagacijski model zasnovan na neuronskoj mreži, koja je obučena za određeni okoliš. Arhitektura je mreže zasnovana na višeslojnom perceptronu. Rezultati dobiveni s pomoću neuronskoga modela uspoređeni su s rezultatima postignutima s determinističkim trodimenzionalnim modelom zasnovanim na metodi slijeđenja zrake. Metoda slijeđenja zrake koristi se trima reflektiranim zrakama od zidova, što osigurava dostatnu točnost za zadani prostor. Neuronska je mreža upotrijebljena za prikupljanje znanja povezanoga s propagacijskim karakteristikama

određenog prostora, i to za tri priključne točke. Uporabom tako prikupljenog znanja mreža je upotrijebljena za predviđanje snage signala u bilo kojoj točki razmatranoga prostora. Neuronska je mreža obučavana s nekoliko različitih algoritama kako bi se postigla najbolja konvergencija, to jest model s najboljim karakteristikama. Upotrijebljeni su algoritmi: Konjugirani gradijent (SCG), Konugirani gradijent Fletcher-Reevesa, Quasi_newton (QN) i Leveberg-Marquardt. Prikazani su postignuti usporedni rezultati.

Ključne riječi: propagacijski model zatvorenog prostora, trodimenzionalno slijeđenje zrake, bežična lokalna mreža, višeslojni perceptron, algoritam za učenje.

INTRODUCTION / *Uvod*

The popularity of indoor wireless communication systems - phones, hand-held terminals, various PDA devices - are constantly increasing. These portable devices tend to be mobile and in principle can be located anywhere, while access points need to provide good link to the communications backbone of the system. The base stations need to be positioned carefully so that they cover the building with adequate signal level. Generally problem can be reduced to given building, where we need to answer to questions like how many access points will be needed, at which positions they will be placed to cover the building with minimum power level.

Prediction of the signal strength for indoor propagation environments is faced with effects of multipath propagation, such as signal attenuation, reflection, diffraction, and interference, due to diversity of building geometrical and construction characteristics [1],[2],[3],[4]. The Maxwell's equations with the relevant boundary conditions
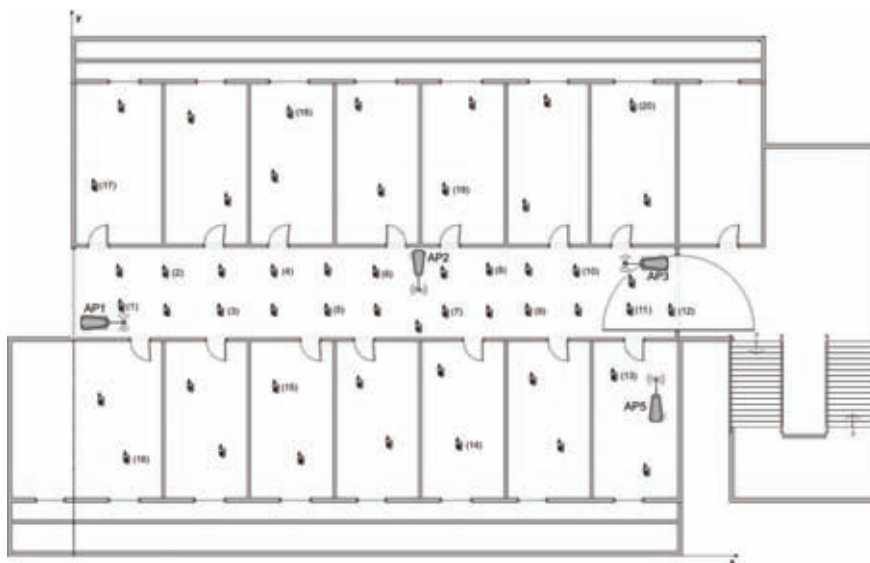

Fig. 1. Plan of the second floor university building
Sl. 1. Tlocrt drugog kata sveučilišne zgrade

enable the most accurate solving indoor propagation problems, but with extreme calculation complexity. To avoid this complexity a lot of empirical propagation models have been developed. The ray tracing model based on geometric optics is enough accurate when include more then one reflected ray, and also diffraction effects. This model requires detailed information about building characteristics and too much computation time, so it can't be feasible for real buildings.

Artificial neural networks can be used as an alternative to various deterministic propagation prediction methods. Several authors have already proposed such solutions [5], [6] with different approaches and neural network architectures.

Very good input-output mapping make these networks useful in signal strength prediction with the same accuracy as other deterministic methods. Through the learning process the relevant network has possibility to absorb the knowledge about propagation characteristics for given indoor space, based on the relationship between input and output.

The network is trained with measured data, and tested with different data, also obtained by measurement. Additionally, the adequacy of using neural networks in indoor propagation prediction problems is proved by comparison with ray tracing results.

In WLAN frequency band of 2.4GHz or 5GHz the diffraction influence to signal strength can be neglected, and the sum

of directed ray and the reflected rays is enough accurate to describe behavior of channel propagation. Hence, main task is to describe main obstructions and the surfaces that affect the signal propagation. This description includes geometric characteristics of the propagation environment as well as electromagnetic parameters of surfaces to determine the surface reflection coefficients. In this paper the 3-D ray-optical model is presented to describe the WLAN signal propagation in an indoor environment. Using the model, signal strengths are calculated in various points of indoor environment. This analytical model is verified by neural network simulation and measurement. The predicted values obtained by the ray-tracing technique and neural network model are compared with measurement and it is found that they follow the same trend.

The multilayer perception (MLP) is trained with different algorithms to achieve the best convergence results and the best possible model behavior in signal strength prediction. The comparison is made for algorithms: Scaled Conjugate Gradient (SCG), Conjugate Gradient of Fletcher-Reeves (CGF), Quasi-Newton (QN), and Levenberg-Marquardt (LM), and Bayesian regularization (BS). The models are compared based on the mean, root mean squared error (RMSE) and standard deviation.

PROPAGATION MODEL / *Propagacijski model*

The second floor of Dubrovnik University B building is chosen for simulation environment. The dimensions of the floor are 33x11x2.40 m^3, as it is shown in Fig. 1 with origin of coordinate system in left lower corner and locations of base stations for neural network training purposes. The environment under consideration ends with folding door. The access points are CISCO Aironet 1100 series for WLAN 802.11b standard. Coordinates of access points are shown in the Table 1. The walls are made of the bricks with wooden doors, while the ceiling and floor are made of the concrete.

Measurements of the received signal strength for the various locations of the receiver and each base station (Fig.1) have been made in the first step. The each WLAN access point was operating on the 7th channel at 2.437 GHz (100mW), and transmitter antenna gain was 8.5 dBi. The signal strength measurements were made by a laptop computer with PCMCIA wireless card positioned 1.2 m above the floor. The measurements were performed for 98 receiving points (locations) that were 1 m apart from each other. There were made three measurements for each location and mean value was saved with location coordinates. These values will be used in the training and testing of the neural network, as well as for comparison with the results obtained by the ray tracing technique.

Table 1. The Coordinates of the access points
Tablica 1. Koordinate prijemnih točaka

Acess points	x	y	z
AP1	0.0	4.85	2.2
AP2	17.0	7.65	2.2
AP3	33.0	7.65	2.2
AP5	30.0	2.3	2.2

SIGNAL STRENGTH PREDICTION BY RAY TRACING MODEL / *Predviđanje snage signala s pomoću modela slijeđenja zrake*

The transmitter was the access point one, denoted by AP1 and receiving points are marked as mobile phones in the Fig. 1. The computation begins with the line of sight path (the corridor), followed with the receiving points in the side rooms. The reflection and transmission coefficients are calculated for each surface (wall, floor, ceiling, door or window). We assumed that all reflecting surfaces are orthogonal, what was really true in our case.

The methodology of finding the reflection or the transmission points is illustrated in the Fig. 2 in two-dimensional space. The points denoted with T and R are the locations of the transmitter and the receiver, respectively. The two reflection ray is considered. It is easy to realize that coordinates of the points S_3 and S_{34} are ($2L_3 - x_t$, y_t) and ($2L_3 - x_t$, $2L_4 - y_t$), respectively . According to that the coordinates of the points of reflection can be obtained as intersection of the line T-S_{34} with wall *4*, and intersection of the line r_1-S_3 with the wall *3*, respectively.
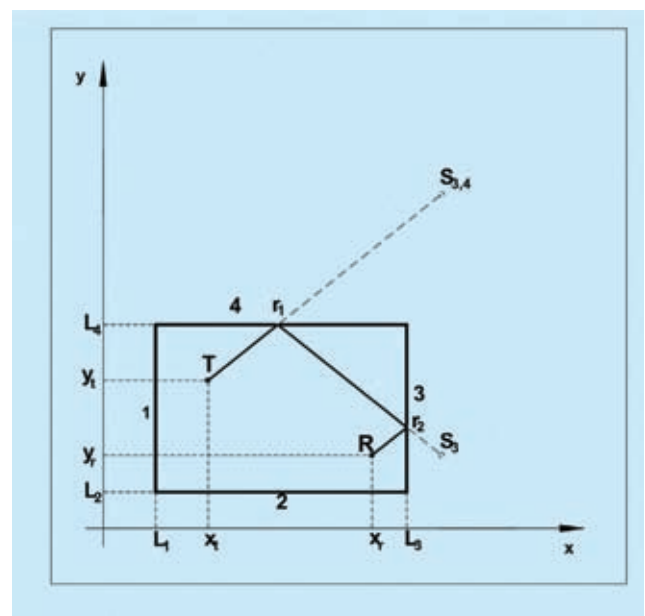

Fig. 2. Ray tracing methodology
Sl. 2. Metodologija slijeđenja zrake

The signal strength in the arbitrary receiving point for the rays with one and two reflections is given as

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi}\right)^2 \left| \frac{1}{d_0} + \sum_{i=1}^{N} \frac{R_i}{d_i} e^{-j\Delta\varphi_i} + \sum_{k=1}^{M} \frac{R_{1k} R_{2k}}{d_k} e^{-j\Delta\varphi_k} \right|^2 \quad (1)$$

$$\Delta\varphi_i = \frac{2\pi}{\lambda}(d_i - d_0) \qquad \Delta\varphi_k = \frac{2\pi}{\lambda}(d_k - d_0)$$

d_0 = length of the direct path

d_i, d_k = length of the paths with one and two reflections, respectively

R_i = reflection coefficient for the path with one reflection

R_{1k}, R_{2k} = reflection coefficients for the paths with one and two reflections, respectively

$\Delta\varphi_i$, $\Delta\varphi_k$ = phase diferences between direct and reflected paths

The reflection coefficients were computed by the Fresnel's equation for the vertical polarization

$$R = \frac{\cos\theta_i - \sqrt{\varepsilon_r - \sin^2\theta_i}}{\cos\theta_i + \sqrt{\varepsilon_r - \sin^2\theta_i}} \quad (2)$$

where i is the angle of incidence. The receiving signal strength is computed for one direct component and rays with maximum 3 reflections. The non-uniformities of the reflected surface materials; such that can produce scattering were neglected, because their contribution to signal strength is insignificant in the environments such is the environment under consideration. Further reflected rays are not taken into account because of the computing time. It is useful to investigate their contribution to total signal strength. The vertical polarization has been assumed. Received power (dBm) was calculated for 93 points that have been 1m apart and 1.5 m above floor. Adequate computer software is developed for the ray tracing model calculation. The geometrical and construction characteristics of the environment under consideration were included in the computation from the appropriate database.

The obtained results are saved and processed to compare with calculated values. The results are shown in the Fig. 3. It is shown the change in signal strength with increasing transmitter-receiver separation. The differences between measured and calculated results are more significant in the proximity of the transmitter. The results show that reflected rays of higher order need to be taken into account, as well as, the influence of diffracted rays with more accurate model of the walls. Mean variation of calculated values is 3.74 dBm.
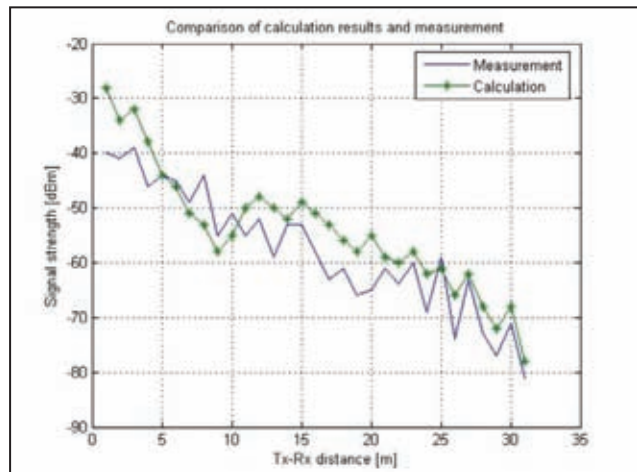

Fig. 3. Calculated and measured signal strength for ray tracing model

Sl. 3. Usporedba izmjerenih i izračunatih vrijednosti za model slijeđenja zrake

SIGNAL STRENGTH PREDICTION BY NEURAL MODEL / *Predviđanje snage signala uz pomoć neuronskoga modela*

The basic component in the neural network model is neuron. The network function and its position in the network architecture determine behavior of the each neuron. According to the recommendations from [7] we chose multilayer perception (MLP) for propagation simulation that is shown in the Fig. 4 with two hidden layers. The input layer as inputs receive location coordinates of access points and receiving points. The network has one neuron in output layer for relevant signal strength value. Such neural network architecture can be learned applying a set of labeled training samples that involve modification of the synaptic
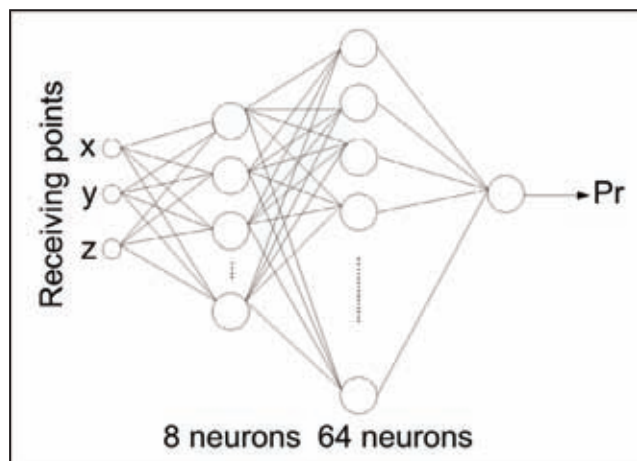

Fig. 4. Neural network architecture
Sl. 4. Arhitektura neuronske mreže

weights of neural network to produce corresponding (desired) output.

The training of the network is repeated for many input samples until the network reaches a steady state where there are no significant changes in the synaptic weights. After training phase the neural network is tested or simulated with input data from the set of examples but different of that used in the training, and if the outputs are reasonable the network generalizes well.

In this network construction the output of the each neuron of the kth layer is given by derivable nonlinear function f

$$y_i = f\left(\sum_j y_j w_{ji}\right)$$ (3)

The function f is called activation function and it can be of different type. In our case we choose sigmoidal hyperbolic tangent function

$$f(v,a) = \frac{1-e^{-av}}{1+e^{-av}}$$ (4)

The synaptic weights of the connection between the neuron j and neuron i are denoted as w_{ji}, while y_j is the output of the neuron j in the $(k-1)^{th}$ layer.

Appropriate initial values of synaptic weights (also called free parameters) and learning algorithm are crucial for learning phase, after the architecture of the network has been determined. As training rule we decided to compare several algorithms that update the weight and bias values producing minimal error between the network output and desired output. During the training phase the known input-output pairs are applied to the network. When the network has correctly learned the task specification, it can be used in the test phase with test samples as it is shown in the Fig. 5.

The way to make neural network training process more efficient some preprocessing steps need to be performed on the network inputs and targets (Fig. 5). The network inputs and targets scaling can be done by normalization of mean and standard deviation of the training set. The results of the normalization process are zero mean and unity standard deviation of the training set. The target data need to be in un-normalized form, so adequate opposite process need to be applied (Fig. 5).

The back propagation algorithm is usually used as training algorithm with multilayer perception [8]. There are several training rules for the neural model with the task to adjust the synaptic weights to optimize neural network configuration. The back propagation algorithm is based on the steepest descent gradient method applied to change the value of the each synaptic weight to minimize the output error [7]. During the training proces the synaptic weights are adjusted to minimize the sum of the squared differences between the desired and actual outputs expressed like

$$E = \frac{1}{2}\sum_j \left(v_{dj} - y_j\right)^2$$ (5)

where v_{dj} is the desired output value for the jth output neuron, and y_j is the output value obtained by the network for the same neuron. The changes in the synaptic weight value depend of the learning rule,
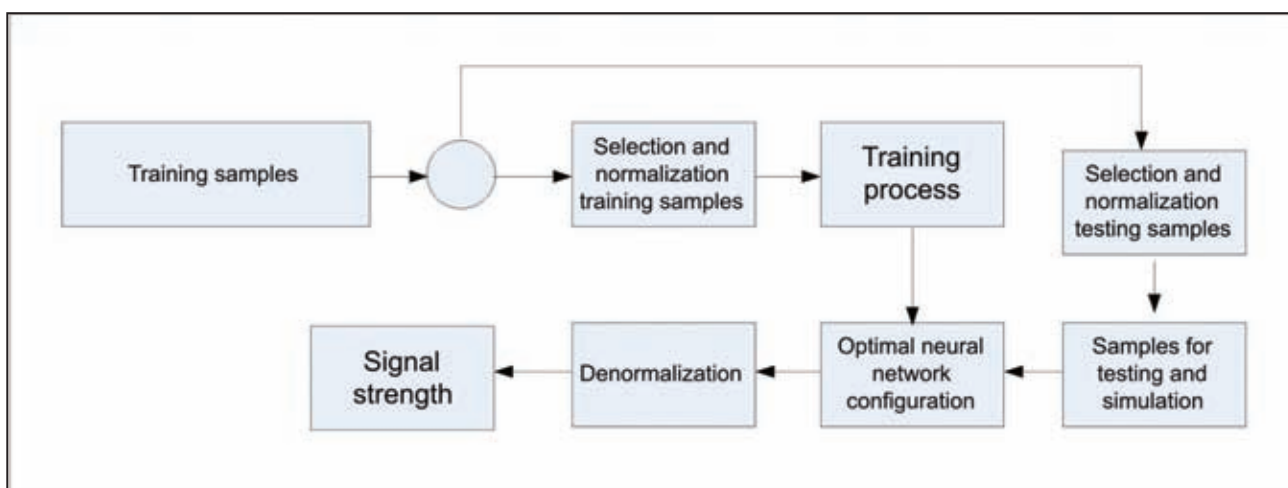

Fig. 5 Training and testing process for MLP
Sl. 5. Proces učenja i testiranja višeslojnog perceptrona

but generally, for the back propagation algorithm the weight change between i^{th} and j^{th} neuron at any layer can be expressed as

$$\Delta w_{ji} = -\mu \frac{\partial E}{\partial w_{ji}} \qquad (6)$$

where $\mu > 0$ is the learning rate. The minus sign is for *gradient descent* in weight space. There is seeking a direction for weight change to reduce the value of E [7]. The new value of the weight in k^{th} iteration is given by

$$w_{ji}(k+1) = w_{ji}(k) + \Delta w_{ji}(k) \qquad (7)$$

The training process starts with small random values of the weights. This initial values need to be sufficiently small so that training does not start from a point in the error space. This error space is connected with location on the curve of the activation function (Fig. 6). It can be seen in the Fig. 6 that for the argument values distant from the zero the value of the function is very small as well as the value of its first derivative.

As updating of the weights is directly dependable of the function's first derivative, so the learning rate is very slow in this case. The commonly used initial values of the weights are uniformly distributed random numbers in the interval from *-0.5/fan_in to 0.5/fan_in*, where *fan_in* is total number of neurons that are connected with these weights to preceding layer [8].

It is obvious that during back propagation algorithm running the saturated values of the activation function derivative should be avoided. This can be done by decreasing the slope of the curves (Fig. 6). The slope decreasing makes the network more linear, what diminishes the multilayer effect. In the linear case one layer is enough. It is possible to find an optimum value for the activation function slope to satisfy learning rate and network mapping capabilities. The parameter *a*, as it is visible from (4) can be used for function slope adjusting. The function slope can be different from neuron to neuron, even in the same layer. The structure of MLP is very complex, so it is practically impossible to
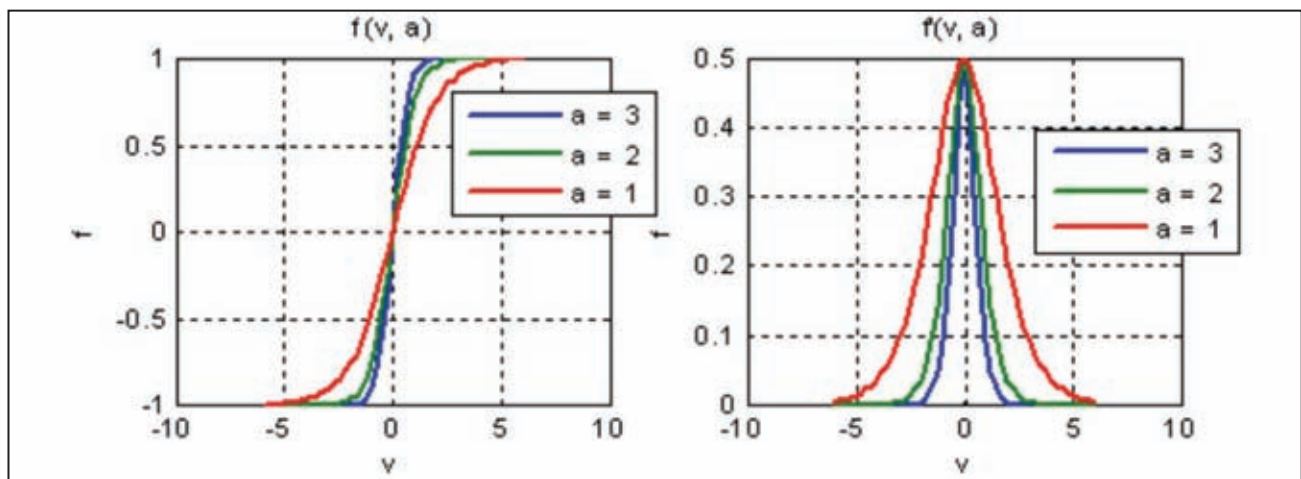

Fig. 6 Sigmoidal hyperbolic tangent function and it's first derivative
Sl. 6. Sigmoidalna funkcija hiperbolnog tangensa i njezina derivacija

determine function slope before the training starts. For the change of the slope parameter a from (4) is applied the same approach as for the weight change and for i^{th} neuron in any layer is given by

$$a_i(k+1) = a_i(k) - \frac{\partial E}{\partial a_i} \qquad (8)$$

The parameter *a* mustn't be smaller than some predefined value a_{min} to avoid linearization of the network mapping, so if amin $> a_i(k+1)$ then $a_i(k+1)=a_{min}$.

In this work we compared several training algorithms. First, the absolute error between the measured and predicted signal strength is computed as

$$e_i = \left| P_{mi} - P_{pi} \right| \qquad (9)$$

where i denotes the number of the mesured sample. The absolute mean error is calculated by

$$m_e = \frac{1}{N} \sum_{i=1}^{N} e_i \qquad (10)$$

where N is total number of measured samples. These two errors leads to the standard deviation:

$$\sigma = \sqrt{\frac{1}{N-1} \left(\sum_{i=1}^{N} e_i^2 - N m_e^2 \right)} \qquad (11)$$

Finally, the RMS error is obtained with absolute mean error and standard deviation:

$$RMSE = \sqrt{m_e^2 + \sigma^2} \qquad (12)$$

For each training algorithm adequate computer software is developed. The algorithms are briefly described hereafter.

Scaled conjugate gradient (SCG)

The scaled conjugate gradient algorithm, develped by Moller, was designed to avoid the time consuming line search. The algorithm is rather complex to be explained in few lines, but basic idea was to combine the model-trust redgion approach with conjugate gradient approach [7].

Fletcher-Reeves conjugate gradient (CGF)

This algorithm starts with searching into the steepest descent direction (negative of the gradient). The line search is then performed to determine the optimal distance to move along the current search direction. The succeeding directions are determined so that it is conjugate to previous search directions. The norm square of the previous gradient and the norm square of the current gradient are used in a Fletcher-Reeves version of conjugate gradient to calculate the weights and biases [7].

Quasi-Newton method (QN)

This algorithm is based on Newton's method but it doesn't require the calculatin of second derivatives. They update an approximate Hessian Matrix at each iteration. The update is computed as a function of the gradient. It has more computations in each iteration than conjugate gradient algorithms, but usually converges very fast [7].

Resilient propagation (RP)

The main purpose of this backpropagation algiorithm is to avoid harmful effects of the size of the partial derivative on the weight upadte. This is simple batch mode training algorithm with fast convergence and minimal storage requirements [7].

Levenberg-Marquardt algorithm (LM)

The best features of the Gauss-Newton method and the steepest-descent method are combined in this algorithm avoid many of their limitation. Its main characteristic is fast convergence [7].

EXPERIMENTAL RESULTS / *Eksperimentalni rezultati*

As it is visible in the Fig. 1 base stations AP1, AP2, and AP3 are chosen for training and testing of the network. Randomly there are determined 78 receiving locations for training purpose and 20 for network testing of the total number of 98 receiving locations for which the measurement have already been made (Fig. 1). This has been made for each base station that for training results in 78x3 pairs of receiver coordinates - signal strength and 20x3 such groups for testing. Good network generalization is shown in the Fig. 7, where the change in signal strength with increasing transmitter-receiver separation is shown for neural network model, ray tracing calculation, and measured data. The differences between measured, simulated and calculated results are more significant in the proximity of the transmitter.
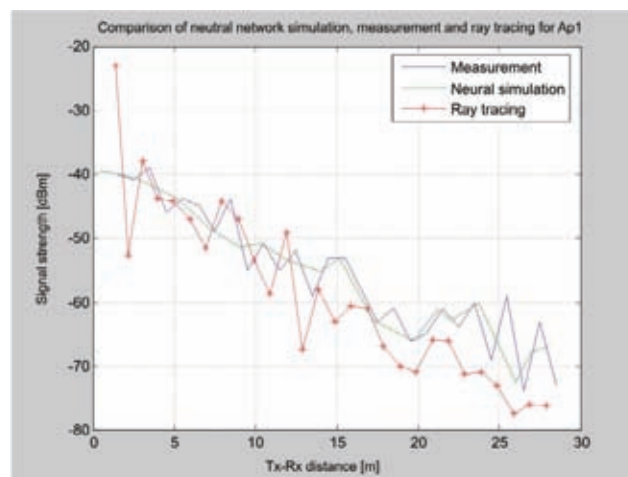

Fig. 7 Comparison of neural network simulation, measurement and ray tracing for base station AP1 and receiving points in main corridor

Sl. 7. Usporedba rezultata dobivenih s pomoću neuronske mreže, mjerenjem i metodom slijeđenja zrake

Neural network simulation results for base stations AP2 and AP3 are shown in Fig. 8 and 9 respectively. Receiving points denoted with numbers from 1 to 12 are located in main corridor, with beginning at x = 0, while the receiving points denoted with numbers from 13 to 20 are located in different rooms. We can see acceptable matching between neural network simulation results and measurement data for various testing locations of receiver according to the Fig. 1. The

overall mean variation of neural results in comparison with measured data was 3.8 dBm.

Additional testing is performed for base station AP5, that is not been participating in the training of the network. It is located at (30, 2.3) coordinates and results of comparison with measured data are shown in the Fig. 10. This is the worst case, so the mean variation between neural and measured data was little bit less than 10 dBm. In spite of this not encouraging result, we think that this method is still usable.

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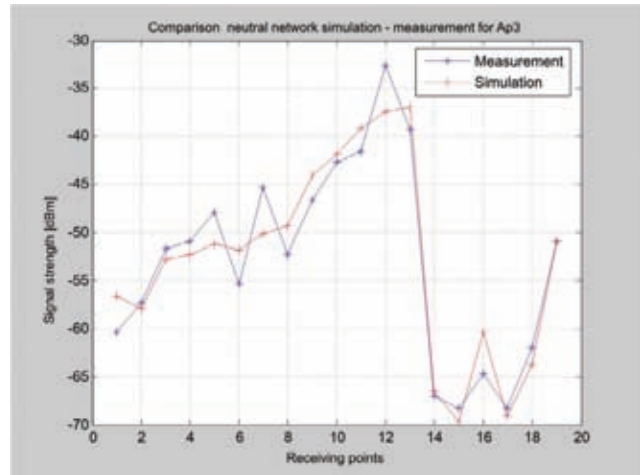
Fig. 9 Comparison of neural network simulation and measurement for base station AP3

Sl. 9. Usporedba rezultata dobivenih s pomoću neuronske mreže i mjerenjem za pristupnu točku AP3

Fig. 8 Comparison of neural network simulation and measurement for base station AP2

Sl. 8. Usporedba rezultata dobivenih s pomoću neuronske mreže i mjerenjem za pristupnu točku AP2

The comparison of six chosen training algorithms for MLP is expressed in terms of absolute mean error, root squared mean error and standard deviation. The achieved performance of the MPL network for mentioned training algorithms and ray-tracing results are presented in the Table2.

The worst results are obtained for CSG algorithm, and best test results are achieved training with BR algorithm. The Fig. 11 shows signal strength for each testing receiving point for the worst and the best training algorithm comparing it with measured data.
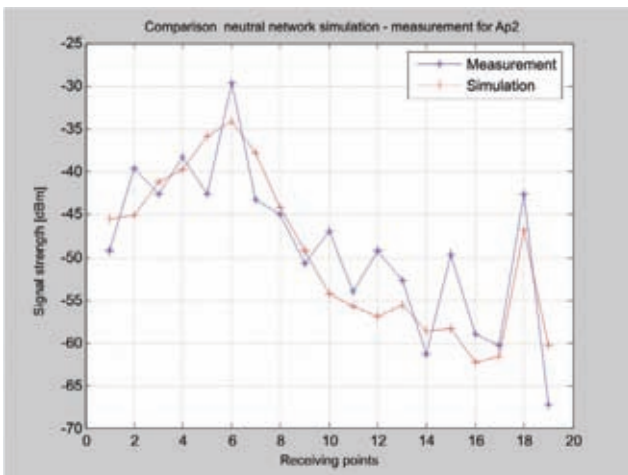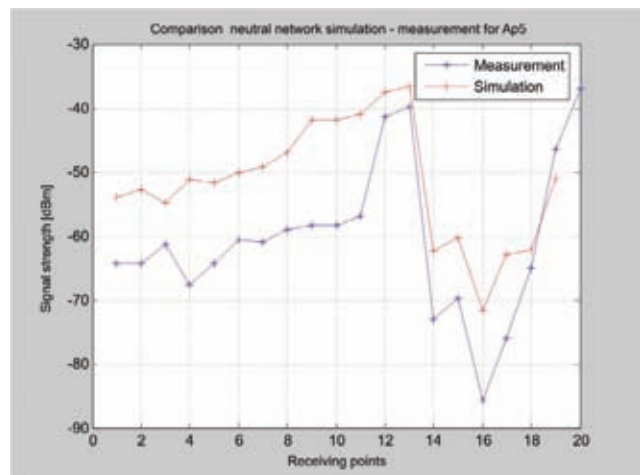

Fig. 10 Comparison of neural network simulation and measurement for base station AP5

Sl. 10. Usporedba rezultata dobivenih s pomoću neuronske mreže i mjerenjem za pristupnu točku AP5

Table 2. Error statistics for signal strength prediction by NN and ray-tracing models

Tablica 2. Statistika pogrešaka predviđanja snage signala za neuronske mreže i model slijeđenja zrake

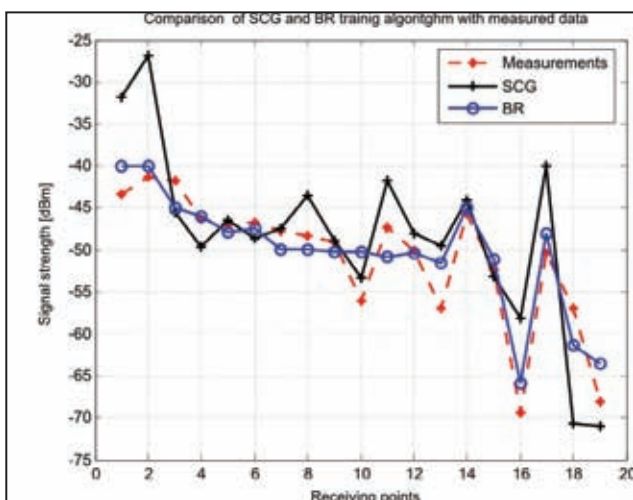| Prediction model	Mean error [dB]	RMSE [dB]	Standard drv. [dB]
MPL-SCG	6.015	7.845	5.036
MPL-CGF	3.640	4.406	2.843
MPL-QN	4.272	5.162	2.899
MPL-RP	5.141	6.516	5.139
MPL-LM	3.308	3.978	2.210
MPL-BR	2.451	2.967	1.672
Ray-tracing	4.474	5.237	2.722

Fig. 11 Comparison of the worst and the best algorithm case

Sl. 11. Usporedba najlošijeg i najboljeg slučaja

CONCLUSION / *Zaključak*

The contribution presented in this paper is that we incorporate a lot of propagation phenomena without complex and long last computations with practically equal accuracy as it is with more deterministic methods (like ray tracing method).

The two signal strength prediction models based on neural networks and ray tracing have been developed. The main advantage of the neural model is that it doesn't require any knowledge about dimensional or construction characteristics of the building under consideration, what is unknown in many cases. The training algorithms include adaptive activation function slopes; hence the speed of the training process is significantly increased. The overall process is relatively short. In any case it doesn't last longer then 10 minutes.

It is important to emphasize that the accuracy of the neural network model is comparable to the accuracy of the other propagation models. The behavior of several training algorithms has been investigated, and according to the obtained results, for the multilayer perception the best results show Bayesian regularization. This training algorithm has the smallest RMS error (2.97 dB). The obtained results for this algorithm are even better then ones obtained by ray tracing method. In the ray tracing method ideally smooth surfaces were assumed and some obstacles were neglected (window frames). The electromagnetic characteristics of the material of the walls and other obstacles are assumed in the ray tracing calculations. The more accurate values of the dielectric constants can be obtained by measurements.

The introduced model can be used for improving the performances of existing indoor wireless networks, and it can serve as a good tool for wireless network planning in general. The future work need to introduce other neural network configurations, like Radial Basis Function networks (RBF). The training process is an optimization process, so some other optimization methods can be introduced in this process, like Particle Swarm Optimization algorithm.

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