In order to enhance objective interpretation of the architectural value of 257 selected Islamic architectural monuments, 268 approximate graphic analyses have been conducted via iteration of hypothetical geometric sequences in AutoCAD 2D software on digitized architectural drawings and 28 detailed analyses of architectural drawings as reconstructed according to measurements from the relevant literature. The analyses are contingent upon geometric harmonization derived from only a single starting length in all drawings at the same scale. This allowed for derivation of algebraic expressions with only one variable for all drawings at the same scale. Alternative geometric analyses have been conducted for specific monuments with proportions examined in the literature.

The only published scholarly book exclusively dedicated to geometric harmonization in Islamic architecture is Bulatov (1988), which focuses on geometric harmonization in Central Asian architecture based on the products of a square, equilateral triangle, half-square and the golden section. Geometric harmonization based on the square geometric sequence has been demonstrated in the Great Mosque of Córdoba by Fernández-Puertas (2000, 2008). The fundamental use of equilateral triangles in geometric harmonization has been demonstrated on monuments in the Maghreb by Ewert (1986). Many authors suggested harmonization based on a square diagonal by drawing, but with no elaborate demonstration. Geometric harmonization based on the golden section has been demonstrated on single monuments in several articles. The geometric complexity of muqarnas vaults has sparked considerable interest, resulting in the reconstruction of 3D variant models based on historical 2D drawings, but with no demonstration of the geometric harmonization.

Approximate analyses prove the golden section as the starting point and main tool of harmonization in Islamic architecture, based on an innovative breakdown of the square layout of the original Messenger’s Mosque in Medina, and done by dividing the half-side of the gross layout square in the golden section, thus forming the mosque courtyard by two golden rectangles. The golden rectangle appears as the first product of a starting length in numerous variations and compositions. The demonstrated harmonization of the golden section terms with the proximate terms of one or more other geometric sequences proved a praxis of combinations of the golden section with multiple symmetries as a harmonizational objective. Additional analyses demonstrate the frequent application of the sequence $\sqrt{2}/20^\circ$, supplementing the golden section as a source of harmonious intermediates.

The geometric harmonization of muqarnas vaults is executed only by the golden section terms coupled in most cases with a right angle division in a certain number of the same angles, wherein application of other symmetries manifests not before the second harmonization layer (i.e., details). Since precise measurements of the earliest extant monuments are unavailable, the hypotheses that sections of the rotational hyperboloid with rectilinear excerpts are the original and ideal case of kārbāndī vault structure, as opposed to the traditional construction of kārbāndī vaults on the curved pointed arch grid, remains speculative.

When defining 19 equal spans between transverse columns, the harmonization of the original Messenger’s Mosque applies the thus far unrecognized quality of the golden section, namely, the diagonal of the golden rectangle is equal in length to 19 tenths of the starting side, deviating a mere 1‰ and providing the geometric construction of the number 19, the declared guarantor of the Qur’an’s divine origin, and proving its arithmetic key, as a product of the golden section. The Great Mosque of Wasit, as the earliest extant, also applies transversal division in 19 equal spans. We have demonstrated the number 19 as a component of instruction by integers for the construction of the Ka‘ba’s original golden rectangular layout by Arabic steps coupled with the coincidence of the angle defined by the northward diagonal of the original Ka‘ba (19 Arabic steps) and the south-eastern wall (10 Arabic steps) with an azimuth determined by the first shadow cast by the sun on the summer solstice in Mecca.

All detailed analyses prove the respective geometric harmonisations of approximate analyses. There are six detailed analyses with only negligible (<1%) and minor (<5%) deviations. In total, there are 97 (41.4%) negligible, 107 (45.7%) minor, 24 (10.3%) significant and 6 (2.6%) major deviations. In principle, negligible deviations appear in the primary elements of composition, while an increase in deviation values appears parallel to the development of greater detail in the composition.

The conviction that design in Islamic architecture is preferably based on proportions of the square diagonal has proven baseless. 75 approximate and 9 respective detailed analyses have demonstrated the application of products of the square diagonal, but with no possibility of defining all necessary architectural elements without the application of the golden section, except in four late cases. Bulatov’s (1988) demonstration of geometric harmonization in Central Asian architecture based on the products of a square, equilateral triangle and half-square and the golden section can thus be generalized for the entirety of Islamic architecture, but only partially, because the golden section is the starting point and main instrument of geometric harmonization in virtually all examined monuments since the beginning of Islamic architecture.

The dissertation proves that the geometry of the golden section harmonized with other geometric sequences based on products of square division forms an efficient designer’s tool for achieving unified expression in all layers of a design throughout the history of Islamic architecture and it is deductive to the algorithmic form of algebraic expressions with only one variable per analysis, which is the starting length of the respective geometric harmonization.

Further research is proposed within the scope of detailed surveys of extant monuments, notably kārbāndī vaults and the foundations of the Ka‘ba. The geometric harmonizations applied in ancient architectural traditions encountered by the advance of Islam and in the European traditions contemporary to classical Islam and later, and also in modern and recent Islamic architectural practice, should also be researched.