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Some Facts and Challenges in Array Antenna Synthesis Problems

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Many efforts have been done in recent years to solve array synthesis problems in a way as effective as possible. Stochastic optimization schemes have been widely applied to this end, notwithstanding their performance rapidly lowers as the size of the problem increases. This trend has led to neglect mathematical properties of the problem, as convexity or partial convexity, which may be very useful to devise really effective synthesis procedures. In this paper we show that in a number of array synthesis problems of significant practical interest global optimisation is not required at all, or can be limited to a subset of the unknowns. This is by no means an academic point, as we show that a proper exploitation of the convexity allows to achieve, in a much shorter time, design solutions significantly better than those obtained by using general purpose global optimization techniques.

Key words: Convex Optimisation, Array Synthesis

1 INTRODUCTION AND RATIONALE

In the last years, global stochastic optimization techniques, as Simulated Annealing, Genetic Algorithms and Particle Swarm Optimization, have been widely proposed and adopted as a flexible and convenient way to solve antenna, and in particular array, synthesis problems [1–15]. The diffused enthusiasm for these »physically inspired« optimization techniques has induced to neglect the fact that all global optimization algorithms are limited in their performances by the computational cost required to get, *within a given precision*, the solution. This cost grows very rapidly with the number of unknowns [16], i. e., with the antenna size. As a consequence, in large scale problems, due to the necessity of stopping the search after a given amount of flops, it is likely that only sub-optimal solutions will be generally achieved, which can be significantly worse than the actual optimal ones. Moreover, not only general global algorithms are computationally heavy: they are also *all essentially equivalent*, as implied by the so called *No Free Lunch Theorems* [17]. Loosely speaking, these theorems state that a truly general-purpose universal optimization strategy *does not exist*: on average the performances of any two optimization algorithms are the same across *all* possible optimization problems. Hence, for any algorithm, an elevated performance over one class of problems is exactly paid for in performance over another class. Now, for a given sufficiently general algorithm, neither it is practically possible to characterize the class of problems to which it is fitted, nor we can blindly refer to results obtained in a other area. And so, the only way to devise an effective algorithm is to exploit the properties of the specific class of problems under consideration.

Obviously, the most useful property is *convexity*, which ensures the uniqueness of the minimum, thus allowing the use of *local*, much more effective, optimization techniques. Unfortunately, as it is well known, synthesis problems are, in general, not convex. However, *there are* many problems, of significant practical interest, which are convex or partially convex. In these cases very effective techniques can and have been recently devised [18–23].

In the light of above considerations, this paper has a twofold aim. The first is to review these last mentioned results under a general framework, in order to illustrate when and how convexity can be exploited. The second is to show that in these cases the use of global optimization algorithms is not only a waste of computational resources, but can, indeed, prevent the attainment of the solution.

In Section 2, the array synthesis problem is briefly reviewed under a general and unitary setting, in order to put in evidence the origin and the relevance of the trapping problem, the role of convexity and the necessity of reducing as much as possible the size of the problem to which global optimizers must be applied. Then, we consider a class of array synthesis problems which are convex, hence do not require global optimization at all (Section 3), or partially convex, thus allowing to adopt an hybrid approach, reducing the need for global optimization (Section 4). In Section 5 the capabilities and performances of this hybrid approach are demonstrated with reference to problems of practical interest. It is shown that the achievable results outperform those reported in the recent literature. Conclusions follows.

2 THE ARRAY ANTENNA SYNTHESIS PROBLEM AND ITS DIFFICULTIES

In its general formulation, the array synthesis problem can be stated as follows:

Given a set of design specifications concerning:

1) the radiated pattern (or patterns, in the case of scanning or reconfigurable beam antennas);
2) the radiating elements and the array structure;

determine:

– the array structure and excitation (or excitations, in the case of scanning or reconfigurable beam antennas);
– the feeding network required to provide such excitation;

so that the design requirements are fulfilled.

Leaving aside the beam forming network, the antenna system can be schematically represented as shown in Figure 1. The inputs x and p are the array excitations and a suitable set of parameters specifying its geometrical and electromagnetic properties, respectively, and y is the corresponding radiated field. From the mathematical viewpoint, x, p and y can be assumed to belong to Banach or Hilbert spaces, say X, P and Y, finite (X and P) or infinite (Y) dimensional [24]. The system is represented by a (frequency dependent) continuous operator, S, which is *linear* in x, but usually *not linear* in p.

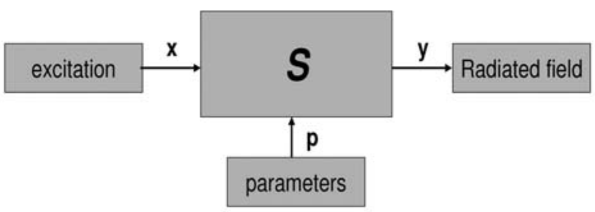

Fig. 1 Mathematical model of an antenna system

Concerning the design specifications, they can be subdivided in two classes:

– sharp constraints (e.g., array and element sizes, excitation phase and/or amplitude quantization, excitation dynamics, minimum gain, maximum side-lobes level,…) which *must* be satisfied;
– quality criteria or loose-constants, (e.g., Q factor, efficiency, etc.) which enforce the solution to enjoy as much as possible some appealing feature.

Sharp constraints specify in the excitation, parameter and radiated field spaces corresponding subsets X_c, P_c and Y_c, respectively, to which the solution must belong. Quality criteria define corresponding functionals in one or more of the spaces, which the solution should minimize (or maximize). And so, to solve the synthesis problems amounts to find a point $(x,p)\,\varepsilon\, X_c x P_c$ such that its image $S(x,p)$ belongs to Y_c, i.e., a point of the set $(X_c \times Y_c) \cap S^{-1}(Y_c)$, or, equivalently, of $S(X_c \times P_c) \cap Y_c = Y_s \cap Y_c$. If a solution does exist, i.e., if the intersection is not void, quality criteria could be used to pick up a »best« solution.

And so, any synthesis problem is essentially an intersection finding one, which can be always reduced [24] to that of minimizing over $X_c x P_c$ the functional $\phi(x,p) = d^2(S(x,p),Y_c)$ wherein $d^2(y,Y_c)$ is the squared distance between y and the set Y_c, i.e.,

$$d^2(y,Y_c) = \inf_{y_c \in Y_c} \|y - y_c\|^2 , \qquad (1)$$

$\|\cdot\|$ denoting the norm in the output space. Accordingly, solving an array antenna synthesis problem amounts to find a minimum of the functional (1), i.e., to find a point such that the distance of its image from the set Y_c attains its (absolute) minimum. Therefore, any synthesis algorithm is, in its essence, a minimization algorithm, providing a sequence of points $(x_n,p_n) = z_n \in X_c \times P_c$ such that the corresponding sequence of squared distances $d^2(S(z_n),Y_c)$ is not increasing and converging to the absolute infimum of the functional (1). Note that if this minimum is not zero, strictly speaking the synthesis problem does not have a solution. However, the corresponding minimum point is, in the chosen norm, the best choice we can adopt, unless some constraint is relaxed.

In order to be practically significant, the solution must not be too *sensitive*, otherwise small, unavoidable, *relative* errors in its realization could induce large *relative* errors in the corresponding radiated field, quite surely moving it outside Y_c.

To avoid a large sensitiveness, the problem must be *well conditioned*, i.e., for a given *relative* variation of $(x_n,p_n) \in X_c \times P_c$, a comparable *relative* variation of the radiated field should correspond.

To face this issue, we can exploit the concept of degrees of freedom of the radiated field. As shown in [25, 26], due to the properties of the radiation operator, the fields radiated (or scattered) by any bounded source can be uniformly approximated, by elements of a *finite dimensional* linear space with arbitrarily small error. Moreover, for large sources, the behaviour of the approximation error as a function of the number of dimensions is step-like, and goes very rapidly to zero as the number of dimension exceeds a critical value, N_0 say. This means that, for any given precision, the set of all radiated fields can be embedded in a finite dimensional linear space, with (complex) dimension slightly larger than N_0. A tight upper bound for N_0 can be obtained, and it turns out that [26]

$$N_0 \cong 2\frac{\mathrm{Area}(\Sigma)}{\left(\frac{\lambda}{2}\right)^2} \tag{2}$$

wherein Σ is the surface of the source convex hull, and λ the wavelength. Note that expression (2) refers to the case of a domain fully encircling the sources. However, analogous results can be obtained in the case of truncated domain [26].

From above results, we get immediately that if the (complex) dimension of the input space $X\times P$ significantly exceeds N_0, the problem will be certainly ill conditioned, unless the constraints set $X_c\times P_c$ has a sufficiently small-diameter. As a matter of fact, in such a case the operator S would have a quasi null-space, which obviously implies ill-conditioning. On the other hand, if the dimension of $X\times P$ is significantly lower than N_0, we will not fully exploit the potentiality of a source with the given size and shape.

In any practical instance the interspacing between the array elements is at least equal to $\lambda/2$. Accordingly, the synthesis problem can be safely assumed well conditioned in the case of a *fixed geometry* array. However, in the general case wherein also the elements positions can be varied, the problem can become ill conditioned. This is by no means a drawback, as the extra degrees of freedom can be exploited, for instance, to reduce the number of the array elements, without significantly affecting the array performance.

Let us now turn to the main difficulty of the array synthesis problem.

According to (1), solving the synthesis problem amounts to find a point of the set Y_s nearest to Y_c. Now in most practical instances, both sets turn out to be *non-convex*. This implies that, apart from the points of absolute minimum, there can be, and usually there are, also points of relative minimum of the functional (1). As a consequence, the minimization algorithm can be trapped in a local minimum, and we get a false solution. Because we are not able to distinguish between absolute and relative minima and the number of secondary minima increases rapidly with the dimension of problem (i.e., the antenna size), the trapping problem is a crucial one, which must be explicitly faced for developing effective synthesis techniques. This is the fundamental reason which leads to the adoption of global minimization algorithms. However, as has been stressed in the Introduction, they in practice can not guarantee the attainment of the optimum, unless the size of the problem is sufficiently small. Accordingly, full exploitation of any property, such as convexity or partial convexity, which allows to avoid or reduce as much as possible the need for global algorithms appears mandatory in order to develop effective synthesis techniques for large size problems.

3 A CLASS OF PROBLEM NOT REQUIRING GLOBAL OPTIMIZATIONS

In this Section we consider a class of convex synthesis problems, which, accordingly, does not require global optimization [18, 19, 21].

For a fixed geometry array, let us consider the problem of choosing the complex excitation coefficients in such a way to maximize the field in a given direction, while enforcing an arbitrary upper level outside a given main lobe region.

Let us refer, by the sake of simplicity, to the case wherein one can introduce an array factor $AF(\theta,\varphi)$. Then, by choosing the reference phase in such a way that $AF(\theta_0,\varphi_0) = \pi$ in the target direction (θ_0,φ_0), the problem can be formulated as [18]:

$$\underset{(I_1,\ldots I_N)}{Min}\ \mathrm{Re}[AF(\theta_0,\varphi_0] \tag{3}$$

subject to

$$\mathrm{Im}[AF(\theta_0,\varphi_0)] = 0 \tag{4}$$

$$|AF(\vartheta_1,\varphi_1)|^2 \le SLL(\vartheta_1,\varphi_1)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots \tag{5}$$

$$|AF(\vartheta_M,\varphi_M)|^2 \le SLL(\vartheta_M,\varphi_M)$$

wherein $I_1,\ldots,I_N$ are the complex excitations and $\{\theta_1,\varphi_1\},..,\{\theta_M,\varphi_M\}$, is a sufficiently fine discretization of the sidelobe region. Then, as (3) is linear in terms of the unknowns, and constraints (4), (5) define convex sets, the overall problem is reduced to the minimization of a linear function on a convex set. This is a **C**onvex **P**rogramming (CP) problem, which admits a single minimum, so that global optimization is not required. The same statement stays valid if an array factor can not be defined (as in the case of conformal arrays), if mutual coupling is taken into account, as well as if other constraints as:

– near field constraints,

– non super-directivity constraints,

– constraints on excitations variations,

or any combination of them are added. Also, under the same kind of constraints, the same general conclusions also hold true in case one wants to optimize directivity (which is readily achieved by minimizing the radiated power for a given maximum) or designing an optimal difference pattern [21]. Notably, in the case of centro-symmetric (linear or planar) arrays the overall focusing (or difference pattern) problem reduces to a simpler **L**inear **P**rogramming (LP) one [19, 21], which can be solved by using simple and effective local optimization routines (as LINPROG in MATLAB). As a consequence of all the above, solving optimal focusing problems using global optimization procedures is questionable, as *local* approaches will find better solutions in a shorter time.

As an example, let us first consider the problem considered in [11], i. e., the synthesis of a $\lambda/2$ spaced 40 elements linear array in such a way to maximize the radiated field in the broadside direction, while enforcing a desired mask for the sidelobes, as reported in Figure 2 (a-line). Due to the centro-symmetric geometry of the considered array, the optimum radiation pattern is real [19] and the synthesis problem reduces to a LP one. The synthesized radiation pattern and the corresponding currents distribution are reported in Figure 2. The result is about 1 dB better than that achieved in [11] by means of time-consuming global optimizations.

As a second example, let us refer to the problem considered in [13], wherein an hybrid stochastic scheme has been adopted for optimizing the weights of a $\lambda/2$-spaced 100 elements linear array in order to best meet a specified far-field requirement, with a 60dB notch on one side (see a-line, Fig. 3). Also in this case, the problem can solved by means of a simple LP based procedure.

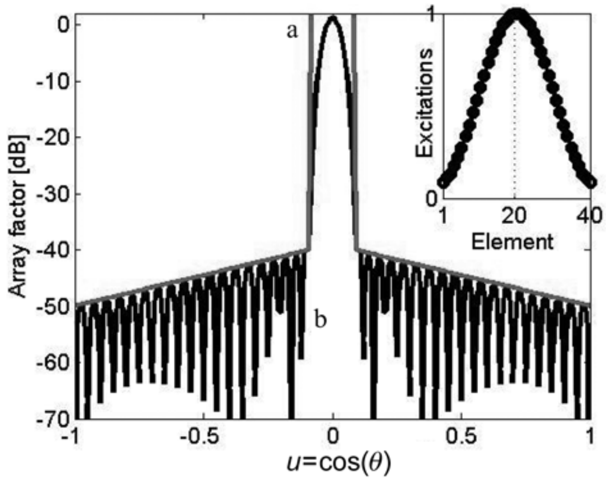

Fig. 2 Synthesized array factor of a 40-elements λ/2 spaced linear array optimized by using the LP approach. The achieved current distribution is also reported

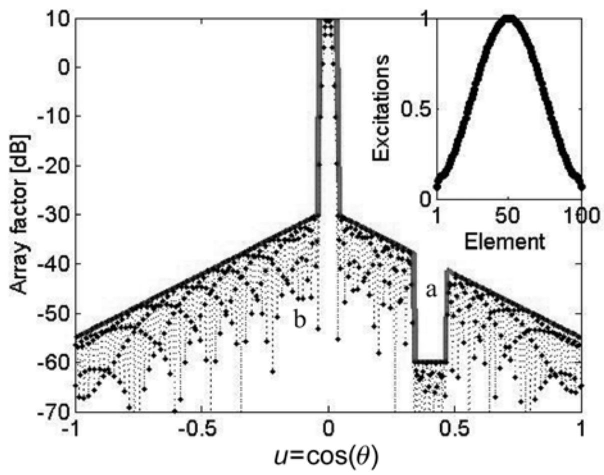

Fig. 3 Synthesized array factor of a 100-elements λ/2 spaced linear array optimized by using the LP. The excitations distribution is also reported

The optimal pattern and the synthesized weights are reported in Figure 3. It must be stressed that the achieved maximum is 10 dB higher than that obtained in [13], which dramatically shows that in this case, due to the relatively large number of unknowns, the optimum solution could not be reached by using global optimization based schemes.

4 A CLASS OF PROBLEMS WHEREIN PARTIAL CONVEXITY CAN BE EXPLOITED

When the geometry of the array is not fixed in advance, so that also the locations the radiating elements must be determined, the above synthesis problems are no more convex. However, they are still convex with respect to the excitations for each given set of tentative locations. Then, it makes sense to consider the following class of problems. Let us denote by $\mathbf{X} = [x_1,...,x_N]$ a subset of variables to be determined, and by $\mathbf{Y} = [y_1,...y_M]$ the complementary one. Let us suppose that the synthesis problem can be formulated as [20, 22]:

$$\underset{(x_1,\ldots,x_N,y_1,\ldots,y_M)}{Min} [F(\mathbf{X},\mathbf{Y})] \tag{6.1}$$

$$\begin{aligned} G_1(\mathbf{X},\mathbf{Y}) &\leq 0 \\ &\cdots \\ G_P(\mathbf{X},\mathbf{Y}) &\leq 0. \end{aligned} \tag{6.2}$$

Finally, let us suppose that both the objective function F and the constraint sets determined by the functions $G_1,\ldots,G_P$ are convex with respect to $\mathbf{X}$ for each value of $\mathbf{Y}$. Then, instead of using global optimization on all variables, let us consider a hybrid procedure, trying to take advantage from the convexity of the problem with respect to $\mathbf{X}$. To this end, let us define the auxiliary function $f(\mathbf{Y})$, defined as

$$f(\mathbf{Y}) = \underset{(x_1,\ldots x_N)\in C_Y}{Min} [F(\mathbf{X},\mathbf{Y})] \tag{7}$$

wherein, for each fixed $\mathbf{Y}$, C_Y is the convex set defined by the constraints (6.2). Note that $f(\mathbf{Y})$ is a function of the second subset of variables. While not being generally available in explicit form, the function $f(\mathbf{Y})$ can be computed as the solution of a CP problem. Its solution will provide not only the (unique) minimum value of $F(\mathbf{X},\mathbf{Y})$ for the given value of $\mathbf{Y}$, but also a value of $\mathbf{X}$ where such a minimum is achieved [Note the minimum can be achieved either at a single point or on a convex subset of C_Y]. Then, the overall problem can be conveniently formulated as the global optimization of the function $f(\mathbf{Y})$. In this way the number of unknowns in the global optimization process is reduced with respect to the simpler and largely adopted solution of performing the global optimization simultaneously on all variables. This latter would involve $N+M$ instead of M variables. As a consequence, global optimization tools have to deal with a reduced number of unknowns, thus saving computational times and/or finding better solutions. As expected, much better performances have been verified when N is significantly smaller than the overall number of unknowns and in all those cases wherein this number is very large [20, 22, 23].

On the other side, the proposed approach also has a draw-back. In fact, as it requires the solution of an auxiliary CP problem, the evaluation of the objective function $f(\mathbf{Y})$ is usually by far more cumbersome than the evaluation of $F(\mathbf{X},\mathbf{Y})$, which has also to be taken into account. This circumstance affects the choice of the global optimization one has to adopt. In particular, Genetic Algorithms [3], which require computation of the objective function for each element of the population (at each generation), appear less appealing than Simulated Annealing [1].

5 APPLICATIONS OF THE HYBRID APPROACH TO SOME ARRAY ANTENNA SYNTHESIS PROBLEMS

In order to illustrate by means of examples the above considered approach, in this Section we present some results achieved in the synthesis of pencil beams and difference patterns by means of sparse linear and planar arrays. As a further contribution, very recent results in the synthesis of radiating systems capable to simultaneously produce »optimal sum and difference patterns« are also reported.

5.1 Synthesis of sparse and weighted arrays

In order to exemplify the approach, let us first discuss the problem of determining the *excitations* and *locations* of a linear array in such a way to maximize the field in a given direction while keeping the sidelobes below a given arbitrary mask [18, 19, 20, 23]. In such a case, by defining $\mathbf{X} = [I_1,...,I_N]$, where $I_1,...,I_N$ are the (complex) excitations of the array, and $\mathbf{Y} = [d_1,...,d_N]$, where $d_1,...,d_N$ are the unknown locations of the elements of the array, it can be easily shown that the synthesis problem can be reduced to a CP for any fixed set of locations, so that the overall formulation proposed above can be applied. As an example, let us herein consider the problem of determining the locations and weights of a sparse linear array with 25 elements and a length of 50 λ, such to maximize the radiated field in the broadside direction, while enforcing a desired mask for the sidelobes, as described in [5, 6]. By adopting the hybrid approach, fixing the *same array dimension* and *the same beam-width* considered in [5, 6], we achieve the result reported in Figure 4 (a-line), to be compared with the best result reported in [5, 6] (b-line), which has a SSL of 4 B higher. The synthesized locations and weights are reported in Figure 5. Note that all the antennas

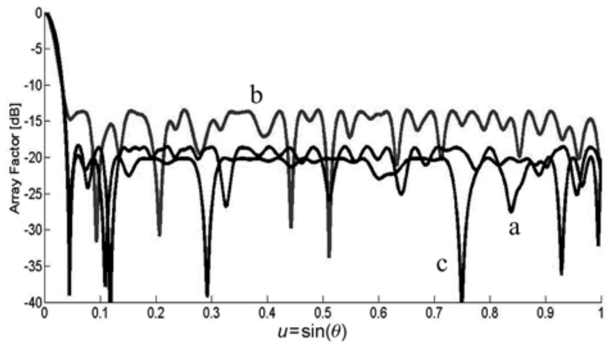

Figure 4 Synthesized array factors for a sparse linear array with 25 isotropic elements. a-line: 50 λ array, [10]. b-line: 50 λ array, hybrid approach. c-line: 26 λ array, hybrid approach

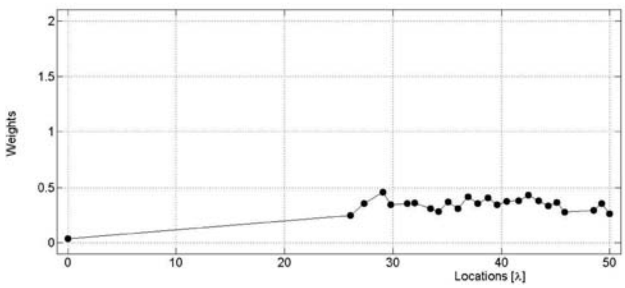

Fig. 5 Synthesized weights and locations by using the hybrid approach for the 50 λ array of Figure 4

with non negligible weights are located in a reduced interval of about 26 λ. This circumstance suggests the possibility of reducing the overall dimension of the synthesized array from 50 λ to 26 λ. By still adopting the hybrid approach, but without constraining the array length to 50 λ, we achieve the result shown in Figure 4 (c-line), with a further decrease of SLL of more than 1 dB. As it can be seen, it is possible to reduce the final SLL, simultaneously reducing the overall dimension of the array, preserving the same beam-width of the original pattern.

As a second example, let us consider the synthesis of sum patterns by means of *sparse planar arrays*. In this case the set of unknowns **Y** contain a couple of real numbers for each element of the array, as both coordinates of the plane have to be determined. The set $\mathbf{X} = [I_1,..,I_N]$ still contains the elements weights. In order to keep the paper in its length, the results achieved by using the hybrid approach mentioned above will not be reported in the following. The interesting reader is addressed to [22] for the analysis and discussion of the achieved results and the comparison with the literature. Also in this more complex case, we get solutions better than the ones achieved when the same of global optimizer acts on all the involved unknowns, as in [4, 7]. In particular, with reference to a sparse planar array with elements not uniformly located on a rectangle of $4\lambda \times 3\lambda$ [7], by using only 29 elements the hybrid method is able to achieve a SLL = −19 dB, while the best results reported in [7] achieves a SLL = −16.5 dB with 41 elements.

5.2 Optimal compromise amongst sum and difference pattern problems problem

In radar applications, both sum and difference patterns are simultaneously required. In order to avoid a full duplication of the feeding network (one network for the pencil beam and another one for the difference pattern), it has been suggested to design the feeding network in such a way to optimize the sum pattern, while subdividing the array into sub-arrays to get the difference pattern. A proper clustering of the N array elements into sub-arrays, and a proper choice of the weight of each sub-array, allow to get a »good« difference pattern, with the maximum possible slope and subject to the same upper bounds on the sidelobes [10, 14, 15, 23]. This kind of synthesis problems can be dealt with by means of the hybrid approach proposed above. In this case, $\mathbf{X} = [g_1,...,g_R]$, wherein R is the number of adopted sub-arrays, and $g_1,...,g_R$ their weights. Moreover, $\mathbf{Y} = [h_1,...,h_N]$ is an array of integer unknowns, with $h_i \in [0, R]$, wherein $h_i = k$ means that the i-th element of the array belongs to the k-th sub-array, whereas $h_i = 0$ means that the i-th element does not belong to any cluster.

Addressing the interested reader to [22, 23] for more details, we present in the following a comparison with a very recent result, reported in [15], relative to a linear array of 20 elements and 8 sub-arrays. The result achieved with the hybrid approach choosing the same beam-width (as estimated from the Figure 3 of [15]), is reported in Figure 6. We get a SLL of −43 dB, which is 1 dB lower than the result of [15]. The synthesized clustering and weights are

$$\mathbf{Y} = [2\ 8\ 5\ 3\ 6\ 7\ 7\ 4\ 5\ 1\ 1\ 5\ 4\ 7\ 7\ 6\ 3\ 5\ 8\ 2]$$

and

$$\mathbf{X} = [2.90\ 0.78\ 10.75\ 11.45\ 7.81\ 13.05\ 13.74\ 4.56],$$

respectively. On the other side, it must be noted that the approach of [15], thanks to a clever exploration of the unknowns space, is more effective from the computational time and memory require-

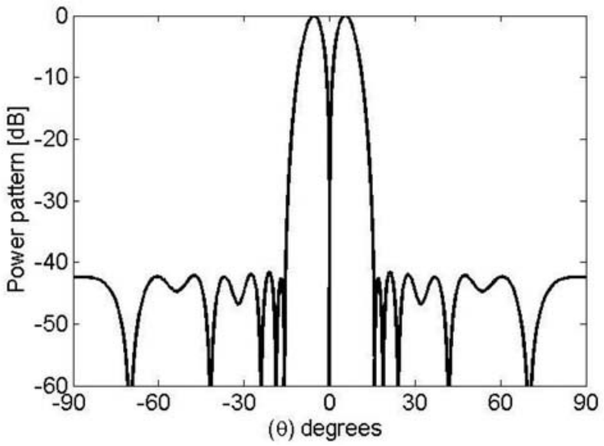

Fig. 6 Synthesized difference patterns for N = 20 and R = 8

ments point of view. It is also worth to note that the best performance one can achieve by independently feeding all the elements according to the optimal synthesis procedure described in [27] is a SLL of –45dB.

6 CONCLUSIONS AND CHALLENGES

The above discussion and examples clearly show that a proper exploitation of the characteristics of the different synthesis problems allows (much) improved design solutions with respect to »blind« approaches simply relying on global optimization procedures. As consequence of a careful discussion of the need, role and way to exploit global optimization, a different perspective has been presented here, which can be summarized in a sentence:

> *exploit carefully all the mathematical properties of the problem which allows to reduce as much as possible the need of using global optimizers.*

Of course, a number of challenging questions arise, as:

– are there other problems belonging to the two classes discussed above?

– Up to what extent, and how, can the above results be extended to antennas other than arrays?

– Is there any clever way to deal with more cumbersome synthesis problems, such as phase only or sparse array synthesis with constrained, in particular identical, excitations?

In the light of the very good results which have been achieved, these points certainly deserve a deep investigation.

REFERENCES

[1] S. Kirkpatrick, C. D. Gelatt Jr, M. P. Vecchi, **Optimization by Simulated Annealing.** Science, vol. 220, pp. 671–680, 1983.

[2] F. J. Ares-Pena, A. Rodriguez-Gonzalez, E. Villanueva-Lopez, S. R. Rengarajan, **Genetic Algorithms in the Design and Optimization of Antenna Array Patterns.** IEEE Trans. Antennas Propag., vol. 47, pp. 506–510, Mar. 1999.

[3] Y. Rahmat-Samii, E. Michielssen, **Electromagnetic Optimization by Genetic Algorithms.** New York: Wiley, 1999.

[4] A. Trucco, **Thinning and Weighting of Large Planar Arrays by Simulated Annealing.** IEEE Trans. Ultra. Ferr. Feq. Control, vol. 46, pp. 347–355, March 1999.

[5] A. Trucco, V. Murrino, **Stochastic Optimization of Linear Sparse Array.** IEEE Jour. Oce. Eng., vol. 24, 1999.

[6] Lommi, A. Massa, E. Storti, A. Trucco, **Sidelobe Reduction in Sparse Linear Arrays by Genetic Algorithms** Micr. Opt. Tech. Letters, 32, pp.194–196, Feb. 2002.

[7] S. Caorsi, A. Lommi, A. Massa, S. Piffer, A. Trucco, **Planar Antenna Array Design with a Multi-purpose GA-based Procedure.** Micr. Opt. Tec. Letters, vol. 35, 2002.

[8] D. W. Boeringer, D. H. Werner, **Particle Swarm Optimization Versus Genetic Algorithms for Phased Array Synthesis.** IEEE Trans. Antennas Propag., vol. 52, pp. 771–779, Feb. 2004.

[9] J. Robinson, Y. Rahmat-Samii, **Particle Swarm Optimization in Electromagnetics.** IEEE Trans. Antennas Propag., vol. 52, pp. 397–407, Feb. 2004.

[10] S. Caorsi, A. Massa, M. Pastorino, A. Randazzo **Optimization of the Difference Patterns for Monopulse Antennas by a Hybrid Real/Integer-coded Differential Evolution Method.** IEEE Trans. Ant. Propag., vol. 53, pp. 372–376, 2005.

[11] S. M. Mikki, A. A. Kishk, **Quantum Particle Swarm Optimization for Electromagnetics.** IEEE Trans. Ant. Propag., vol. 54, pp. 2764–2775, 2006.

[12] W. Weng, F. Yang, A. Elsherbeni, **Linear Antenna Array Synthesis Using Taguchi's Method: A Novel Optimization Technique in Electromagnetics.** IEEE Trans. Antennas Propag., vol. 55, pp. 723–730, March 2007.

[13] F. Grimacia, M. Musseta, R. Zich, **Genetical Swarm Optimization: Self-Adaptive Hybrid Evolutionary Algorithm for Electromagnetics.** IEEE Trans. Ant. Propag., vol. 54, pp. 781–785, March 2007.

[14] P. Rocca, A. Manica, A. Massa, **Synthesis of Monopulse Antennas through Iterative Contiguous Partition Method.** Electronic Letters, vol. 43, 16, pp. 854–856, August 2007.

[15] P. Rocca, A. Manica, A. Massa, **Synthesis of Monopulse Antennas through Iterative Contiguous**

Partition Method. Electronic Letters, vol. 44, 2, pp. 75–76, January 2008.

[16] A. S. Nemirovsky, D. B. Yudin, **Problem Complexity and Method Efficiency in Optimization.** Wiley, Interscience Series in Discrete Mathematics, 1983.

[17] D. H. Wolpert, W. G. Macready, **No Free Lunch Theorems for Optimization.** IEEE Trans. on Evolutionary Computation, vol. 1, pp. 67–82, 1997.

[18] T. Isernia, P. Di Iorio, F. Soldovieri, **An Effective Approach for the Optimal Focusing of Array Fields Subject to Arbitrary Upper Bounds.** IEEE Trans. Ant. Propag, vol. 50, pp. 1837–1847, 2000.

[19] O. M. Bucci, L. Caccavale, T. Isernia, **Optimal Focusing of Uniformly Spaced Arrays Subject to Arbitrary Upper Bounds in Non-target Directions**. IEEE Trans. Ant. Propag, vol. 50, pp. 1549–1554, 2002.

[20] T. Isernia, F. J. Ares Pena, O. M. Bucci, M. D'Urso, J. F. Gomez, J. Rodriguez, **A hybrid Approach for the Optimal Synthesis of Pencil Beams Through Array Antennas.** IEEE Trans. Ant. Propag., vol. 52, pp. 2912–2918, 2004.

[21] O. M. Bucci, M. D'Urso, T. Isernia, **Optimal Synthesis of Difference Patterns Subject to Arbitrary Sidelobe Bounds by Using Arbitrary Array Antennas.** IEE Proc. Mic Ant. Propag., vol. 152, pp.129–137, 2005.

[22] M. D'Urso, T. Isernia, **Solving Some Array Synthesis Problems by Means of an Effective Hybrid Approach.** IEEE Trans. Ant. Propag., vol. 55, pp. 750–759 March 2007.

[23] M. D'Urso, T. Isernia, F. Meliadò, **An Effective Hybrid Approach for the Optimal Synthesis of Monopulse Antennas.** IEEE Trans. Ant. Propag., vol. 55, pp. 1059–1066, April 2007.

[24] O. M. Bucci, G. D'Elia, G. Mazzarella, G. Panariello, **Antenna Pattern Synthesis: A New General Approach.** Proceedings of IEEE, vol. 82, pp. 358–371, 1994.

[25] O. M. Bucci, G. Franceschetti, **On the Degrees of Freedom of Scattered Fields.** IEEE Trans. Antennas Propagat., vol. AP-37, pp. 918–926, 1989.

[26] O. M. Bucci, C. Gennarelli, C. Savarese, **Representation of Electromagnetic Fields over Arbitrary Surfaces by a Finite and Nonredundant Number of Samples.** IEEE Trans. Antennas Propagat., vol. 46, pp. 351–359, 1998.

[27] D. A. Mc Namara, **Direct Synthesis of Optimum Difference Patterns for Discrete Linear Array Using Zolotarev Distribution.** IEE Proc. H, vol. 140, no. 6, pp. 445–450, Dec. 1993.

Neke činjenice i izazovi u problemu sinteze antenskih nizova. Zadnjih godina učinjeni su mnogi napori kako bi se riješio problem sinteze antenskih nizova na najefikasniji mogući način. Često su primjenjivane stohastičke optimizacijske sheme premda performance metode drastično opadaju kako raste veličina problema. Ovaj trend je doveo do potpunog negiranja matematičkih svojstava promatranog problema, jer je negirano svojstvo konveksnosti odnosno djelomične konveksnosti koje se može iskoristiti za razvoj izrazito efikasne metode sinteze. U radu ćemo pokazati da mnogi problemi sinteze antenskih nizova od značajnog praktičnog interesa uopće ne zahtijevaju globalnu optimizacijsku metodu, ili se globalna optimizacijska metoda može primijeniti na podskup varijabli-nepoznanica. Ta činjenica je mnogo šira od akademske diskusije jer ispravno korištenje svojstva konveksnosti omogućuje dizajn antenskog niza, postignut u puno kraće vrijeme, sa značajno boljim svojstvima u odnosu na dizajn postignut globalnom optimizacijskom metodom.

Ključne riječi: konveksna optimizacija, sinteza antenskog niza

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