A New Analytical Model for SIM–OFDM Contradicts the Previously Claimed Features

Ahmed N. Jabbar, Samir J. Almuraab, and Abdulkareem A. Kadhim, *Senior Member, IEEE*

***Abstract*—The Subcarrier Index Modulation OFDM (SIM–OFDM) appeared in 2009 promising a –3 dB transmitted power reduction without affecting the system performance. Therefore, it became an attractive choice to upgrade the communication systems with researchers' increasing interest. Despite the research efforts in SIM–OFDM field, there was no in-depth investigation for such transmitted power reduction or the system's performance. The claimed power reduction relies on probabilistic assumptions that were not validated considering system operation concepts. This paper provides a new analytical model that characterizes the actual SIM–OFDM behavior. The contribution of this model is the inclusion of the majority condition in the derivation of 1's *pmf* which modifies the 1's *pmf* into a complex nonlinear function that is always higher than 1/2. The new *pmf* effect upon the power reduction, synchronization, and the overall Bit Error Rate (*BER*) is investigated. The new analytical model shows that the –3 dB power reduction cannot be achieved. Also, no successful synchronization can be established unless extra subcarrier is added that will create a frame like communication system. Such scheme increases *BER* if the carrier is falsely detected creating a Frame Error Rate (*FER*) which might lead to serious problem.**

***Index terms*—OFDM, SIM–OFDM, Enhanced SIM–OFDM, BER OFDM, OFDM Synchronization.**

I. Introduction

The modern communication systems are competing to provide the highest possible bit rates with the best Quality of Service (QoS) level. However, high data rates require advanced techniques, sophisticated equipment, and high transmission power to ensure high Signal to Noise Ratio (SNR) to meet the designated QoS. The Orthogonal Frequency Division Multiplexing (OFDM) played a major role in the modern communication systems. It provided the highest bandwidth efficiency leading to maximum transmission rates. The OFDM simply divides the high bit rate serial data stream into multi-parallel low–rate data streams. These streams are orthogonalized using Inverse Fast Fourier Transform (IFFT) before modulating the data using the RF subcarrier [1, 2].

Manuscript received March 21, 2022; revised June 2, 2022. Date of publication July 5, 2022. Date of current version July 5, 2022. The associate editor prof. Joško Radić has been coordinating the review of this manuscript and approved it for publication.

A. N. Jabbar and S. J. Almuraab are with the College of Engineering, University of Babylon, Iraq (emails: Ahmed_AlJafari@yahoo.com, Dr.samiralmuraab@uobabylon.edu.iq).

A. A. Kadhem is with the Al-Nahrain University, College of Information Engineering, Computer Networks Engineering Department, Iraq (email: abdulkareem.a@coie-nahrain.edu.iq).

Digital Object Identifier (DOI): 10.24138/jcomss-2021-0198

The recent developments in communication networks promised data rates as high as few Giga bits for the users [3–6]. Such rates rely on favour channel environments and high SNR. Such considerations employ close distance base stations and high transmission power resulting in health risk and may violate government regulations. Such effects can be mitigated by adopting the so-called *Green Communication* [7, 8]. The green communication is a term that embraces all the attempts, developments, upgrades and new techniques aiming to reduce the consumed or transmitted power to an eco-friendly level. The eco-friendly means that the power should not impact the health of the fauna and/or the flora not to mention the human health.

The techniques that were adopted to reduce the power are either spatial or temporal modulation or, recently, the Subcarrier Index Modulation (SIM) [9]. The spatial or temporal modulation, such as the Multiple Input Multiple Output (MIMO), tries to divide the power into multi parallel paths or time slots in order to reduce the power intensity per path. The MIMO focuses on upgrading the RF physical layer of the network.

Subcarrier Index Modulation–Orthogonal Frequency Division Multiplexing (SIM–OFDM) is considered as two-dimensional modulation where part of the data is modulated as a conventional OFDM and the other part is used to switch ON and OFF the subcarriers like the ON/OFF Keying (OOK) [10]. The authors of [10] showed that this technique can ensure a –3 dB reduction in the transmitted power without significantly sacrificing the *BER* performance. However, implementing this technique faces some issues that were not referred to by the authors of [10] nor the researchers implemented SIM–OFDM in their papers. These issues can be summarized as follows: the –3 dB power reduction if it is true, the synchronization between the transmitter and receiver and the *BER* level. Such issues will dramatically affect the SIM–OFDM performance. This research is addressing and discussing these issues to build an exact model describing the SIM–OFDM behaviour using mathematical equations. The final model is presented in order that the researchers should be aware of how these issues may compromise the system performance while implementing SIM–OFDM.

This research provides a new analytical model for the behaviour of SIM–OFDM for the first time. The analytical model showed that the previously claimed features of the SIM–OFDM are not accurate due to the rough estimation for the 1's and 0's probability mass function (*pmf*). Thus, this new model shows that the claimed –3 dB power reduction cannot be accomplished practically only with partial success. Also, the previously published *BER* expression describing the SIM–OFDM performance did not consider the synchronization effect. When the synchronization is considered, a new error source will emerge and may dominate all other *BER* sources. Therefore, the new analytical model advices the researchers to carefully consider these problems before developing their SIM–OFDM based systems in the future for correct operation.

The remaining parts of the paper are organized as follows; the description of SIM–OFDM system is given in Section II covering both the conventional SIM–OFDM and its modified versions. Section III shows the complete analysis to determine the correct 0's and 1's *pmf*, the synchronization problem and the Frame Error Rate (*FER*). The conclusion and remarks are presented in Section IV.

II. Subcarrier Index Modulation OFDM System

The conventional OFDM system is shown in Fig. 1. This system contains N mappers feeding the N points IFFT. Each mapper requires M bits to generate 2^M possible symbols. Then, at every instance the system is fed by $M \cdot N$ bits block as shown in Fig. 1.

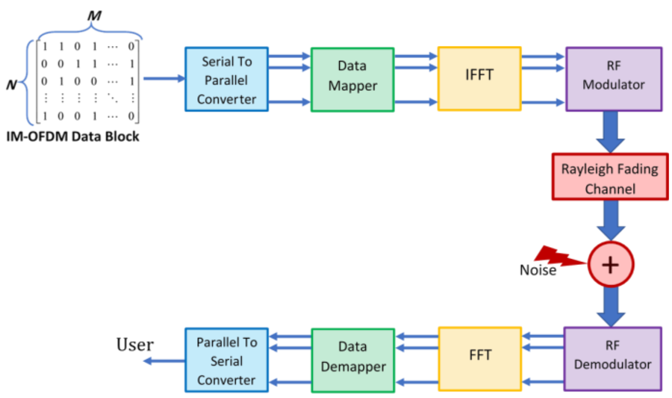


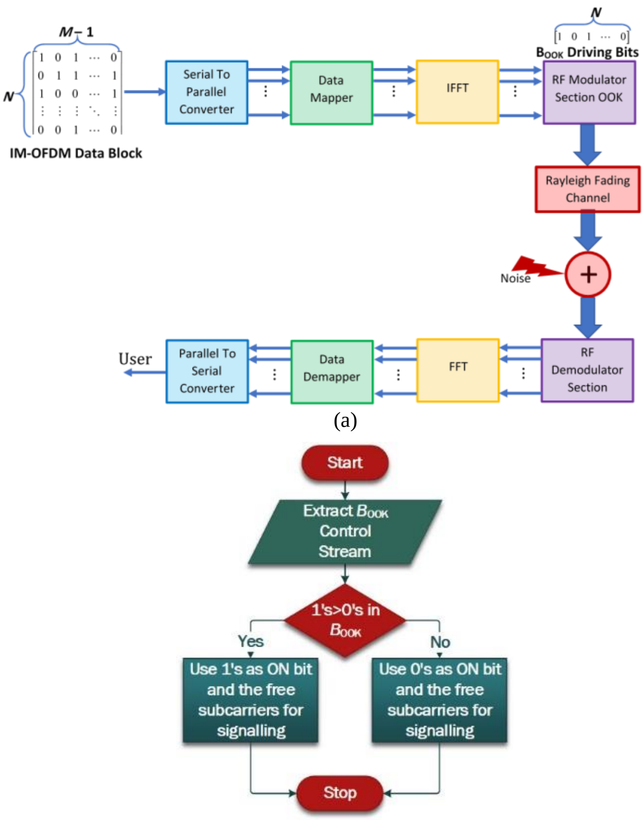
Fig. 1. Conventional OFDM system.

The proposed SIM-OFDM in [10] suggested that N bits are extracted to switch ON/OFF the subcarriers. These N bits are called B_{OOK}. After extracting the N bits, the remaining block bits are $(M-1)\cdot N$. This can be interpreted in two ways, either that all the mappers' symbols are halved, $2^M/2$ keeping N constant, or the number of mappers is halved, $N/2$, keeping the mappers with 2^M symbols. The latter way, $N/2$, is implemented in the SIM–OFDM to reduce the transmitted power by a half [10]. This implies that the B_{OOK} should contain at least $N/2$ 1's to switch ON the subcarriers. Hence the condition in (1) is checked at every modulation instance

$$N_{\mathrm{maj}} = \max\left\{N_{\mathrm{ones}}^{B_{\mathrm{OOK}}}, \left(N_{\mathrm{FFT}} - N_{\mathrm{ones}}^{B_{\mathrm{OOK}}}\right)\right\} \qquad (1)$$

where $N_{\mathrm{ones}}^{B_{\mathrm{OOK}}}$ is the number of 1's and $\left(N_{\mathrm{FFT}} - N_{\mathrm{ones}}^{B_{\mathrm{OOK}}}\right)$ is the number of 0's. The majority N_{maj} in (1) will count the number of 1's in B_{OOK}. If the number of 1's is less than the number of 0's then the B_{OOK} bits are inverted. Otherwise, the B_{OOK} is passed without any inversion to the OOK stage. This condition ensures that the system will switch ON the required number of subcarriers to satisfy the number of mappers. The SIM–OFDM system is shown in Fig. 2(a) and the N_{maj} flowchart is in Fig. 2(b).

The authors in [10] showed that the system can take advantage of the power difference to operate in two modes that are Power Re-allocation Policy (PRP) and Power Saving Policy (PSP). In PRP, the average symbol's power can be doubled to increase the SNR. Using PRP SIM–OFDM, the transmitted power is equivalent to the conventional OFDM as given by (2) [10]



(a) (b)

Fig. 2. (a) SIM-OFDM communication system, (b) N_{maj} flowchart

$$\gamma_{\mathrm{PRP}} = 10\log_{10}\left(\frac{P_S}{E\left[N_{\mathrm{maj}}\right]}\right) - 10\log_{10}\left(\sigma^2\right), \quad \mathrm{dB} \qquad (2)$$

where P_S is the average symbol power, and σ^2 is the Additive White Gaussian Noise (AWGN) variance per subcarrier. The value of $E\left[N_{\mathrm{maj}}\right]$ depends on the expected occurrences of the 1's during the transmission. Since $E\left[N_{\mathrm{maj}}\right] \approx N/2$ which, according to (2), clarifies that P_S can be doubled without exceeding the conventional OFDM transmitted power.

In PSP, the symbol power is kept without alteration to ensure the transmission power is halved. The average power per carrier is given by (3):

$$\gamma_{\text{PSP}} = 10\log_{10}\left(\frac{P_S}{N}\right) - 10\log_{10}\left(\sigma^2\right), \qquad \text{dB} \tag{3}$$

The analytical solution of the portability of error (P_e) for the PRP and PSP is given by (4) and (5) respectively for 4-QAM constellation [10]

$$\begin{aligned} P_e^{\text{PRP}} &= \frac{1}{2}\left[\frac{1}{2}\left(1-\sqrt{\frac{\bar{\gamma}_S}{1+\bar{\gamma}_S}}\right)\right] \\ &+\frac{1}{2}\left[-\frac{1}{8}-\frac{1}{2}\sqrt{\frac{2\bar{\gamma}_S}{2+2\bar{\gamma}_S}}+\frac{1}{2\pi}\sqrt{\frac{2\bar{\gamma}_S}{2+2\bar{\gamma}_S}}\tan^{-1}\left(\sqrt{\frac{2+2\bar{\gamma}_S}{2\bar{\gamma}_S}}\right)\right] \end{aligned} \tag{4}$$

$$\begin{aligned} P_e^{\text{PSP}} &= \frac{1}{2}\left[\frac{1}{2}\left(1-\sqrt{\frac{0.5\bar{\gamma}_S}{1+0.5\bar{\gamma}_S}}\right)\right] \\ &+\frac{1}{2}\left[-\frac{1}{8}-\frac{1}{2}\sqrt{\frac{\bar{\gamma}_S}{2+\bar{\gamma}_S}}+\frac{1}{2\pi}\sqrt{\frac{\bar{\gamma}_S}{2+\bar{\gamma}_S}}\tan^{-1}\left(\sqrt{\frac{2+\bar{\gamma}_S}{\bar{\gamma}_S}}\right)\right] \end{aligned} \tag{5}$$

where: $\bar{\gamma}_S \triangleq \overline{\alpha^2}\dfrac{\bar{E}_S}{N_0^{\text{SC}}}$ is the SNR and α , $\bar{E}_S$ and N_0^{SC} are the Rayleigh fading magnitude coefficient, the average energy per symbol, and the noise power per symbol respectively.

The authors in [11] showed that the *BER* analysis was incomplete and that the SIM–OFDM suffers also from burst *BER* due to error propagation. The error propagation comes from the false detection of one bit or multi bits that form the B_{OOK}. Hence, an attempt by the authors in [11] to Enhance the SIM–OFDM performance by reducing this burst *BER* was introduced. The new method is called as Enhanced SIM–OFDM (ESIM–OFDM). They suggested that the B_{OOK} bits should not be used directly, instead each “1” is followed by a 0 and each “0” is followed by a “1”. This will eliminate the need for matched filters to reduce the burst error and prevent the confusion in the B_{OOK} bits detection. Figure 3 shows the suggested version of B_{OOK} bits in the ESIM–OFDM [11]

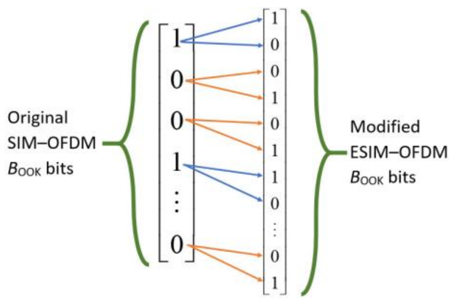


Fig. 3. Modified ESIM-OFDM B_{OOK} bits.

The new *BER* for ESIM-OFDM is shown in (6) [11]:

$$BER_{\text{EIM-OFDM}} = BER_{B_{\text{OOK}}} + BER_{\text{QAM}} \tag{6}$$

Despite that ESIM–OFDM solved the problem of error propagation, its disadvantage is a slight reduction in the spectral efficiency as given in (7) [11]:

$$\eta_{\text{EIM-OFDM}}^{\text{GENERAL}} = \frac{N_a\log_2(M)}{N} + \frac{\left\lfloor \log_2\left(\dfrac{N!}{N_a!(N-N_a)!}\right)\right\rfloor}{N} \tag{7}$$

where N_a is the number of active carriers and N is the B_{OOK} length.

III. Building The New SIM–OFDM Analytical Model

The following sections will describe the exact SIM–OFDM model. The issues covered by this model are the actual transmitted power reduction, the synchronization problem and the *BER*.

A. The BOOK Bits and the Exact –3dB Power Reduction Model

According to [10 and 11], the transmitted power can be reduced to half (–3dB). However, this claim can only be ensured if the process is completely independent random variable. In probability theory, every discrete independent mutually exclusive random process will have a uniform probability distribution (equiprobable elements). Assume a random process $\mathbf{X}$ having K possible outcomes $\mathbf{X} = [x_1 \quad x_2 \quad \cdots \quad x_K]$ then the *pmf* of any element x_i will be as in (8):

$$pmf\ (x_i) = 1/K \tag{8}$$

In SIM–OFDM, each B_{OOK} bit can be either “0” or “1” with equal probability (*i.e.*, $pmf\ (0) = pmf\ (1) = 1/2$). Thus, for a long transmission period, each subcarrier will have a probability of $1/2$ to be ON. This means that only half the subcarriers will be ON and the other half is OFF on average. This explains the –3 dB reduction in the transmitted power.

Careful investigation of Fig. 2 and (2) implies that the actual *pmf* of “0” and “1” in the B_{OOK} bit pattern cannot be $1/2$ because the process is altered by the N_{maj} condition. In other words, the binary values are now correlated and the they are no longer independent. The resultant “1” *pmf* depends on N which can be either even or odd resulting in a different 1’s distribution. For example, assuming N=4 then the possible states of the B_{OOK} are equal to 2^4=16 as shown in Appendix A (Table A-I). The states in the table can be divided into two groups. The first group has the same number of 0’s and 1’s while the second group has different number of 0’s and 1’s. The first group will be left without any changes by the N_{maj} condition and has a recurrence of one. The second group with higher number of 0’s than 1’s will be inverted by the N_{maj} condition given by (1). These changed states will be identical to the unchanged states and that reduces the number of states of the second group to half. The inverted states are represented using red framed cell and they have twice recurrence. This conversion definitely increases the probability of “1” from $1/2$ to 0.6875 for N=4 SIM-OFDM system.

When N is odd then, there will be no states with equal number of 1’s and 0’s. This means that half of the states will be inverted causing the number of total states to be reduced to half. Table A-II in Appendix-A shows the possible states for N=5 before and after applying the N_{maj} condition with a $pmf\ (1) = 0.6875$.

To put the example shown in the appendix A, Table A-I and A-II, in mathematical equations, assume that there is a B_{OOK} bit block containing N bits. The total number of the elements for N bits is $N \cdot 2^N$. The probability of selecting the "1" equals to the ratio of the total number of 1's to the total number of elements. The number of 1's can be calculated using the combination theory multiplied by the number of 1's at every state as given by (9)

$$pmf\ (1) = \frac{\sum_{k=1}^{N}\left[k\left(\frac{N!}{k!(N-k)!}\right)\right]}{N\,2^N} = \frac{N\,\frac{2^N}{2}}{N\,2^N} = \frac{1}{2} \tag{9}$$

where k, is the summation counter and also the number of 1's in the current state. Equation (9) shows that for an independent mutually exclusive random process involving two outcomes process, like the bit, the *pmf* of both "1" and "0" is $1/2$.

After applying the condition given by (1), the number of 1's will be altered as shown by Tables A-I and A-II in the appendix. To find the new 1's *pmf* of the B_{OOK} bits, N should be treated as either even valued or odd valued. This is because, for N even there are states where the number of 1's equals the number of 0's; hence; the condition will not be applied. While for N is odd, there are no states with equal number of 0's and 1's. This fact affects the repetition of the state after the condition.

Starting with odd valued N then, the B_{OOK} pattern, after the N_{maj} condition in (1), will have 1's from $(N+1)/2$ to N. The 1's ratio or the *pmf* (1) is shown in (10)

$$\begin{aligned} pmf_{odd}(1) &= \frac{2\cdot\sum_{k=\frac{N+1}{2}}^{N}\left[k\left(\frac{N!}{k!(N-k)!}\right)\right]}{N\,2^N} \\ &= \frac{2\cdot \text{hypergeom}\left(\left[1, \frac{1-N}{2}\right], \frac{N+1}{2}, -1\right)(N-1)!}{2^N\left(\frac{N-1}{2}\right)!} \end{aligned} \tag{10}$$

Equation 10 shows that the 1's *pmf* depends on the value of N which reflects the effect of the condition unlike (9) which shows that the 1's *pmf* is a constant. The *pmf* given in (10) is a complicated function which can be plotted with respect to N as shown in Fig. 4. Figure 4 shows that the 1's *pmf* is always higher than $1/2$ and the value of $1/2$ is an asymptote which can be reached only when the value of $N \rightarrow \infty$. For example, given N =3, 5, 7 and 15 then 1's *pmf* are $3/4 = 0.75$, $11/16 = 0.6875$, $21/32 = 0.65625$ and $2477/4096 = 0.60474$ respectively.

Equation 10 can be converted to a power level in dB as in (11). This will give the real transmitted power reduction using SIM–OFDM technique

$$pmf_{odd}^{dB}(N) = 10\log\left(pmf_{odd}(1)\right) \tag{11}$$

The plot of (11) is shown in Fig. 5 also shows that the –3 dB reduction is reach at $N \rightarrow \infty$. The power above –3 dB is a wasted power.

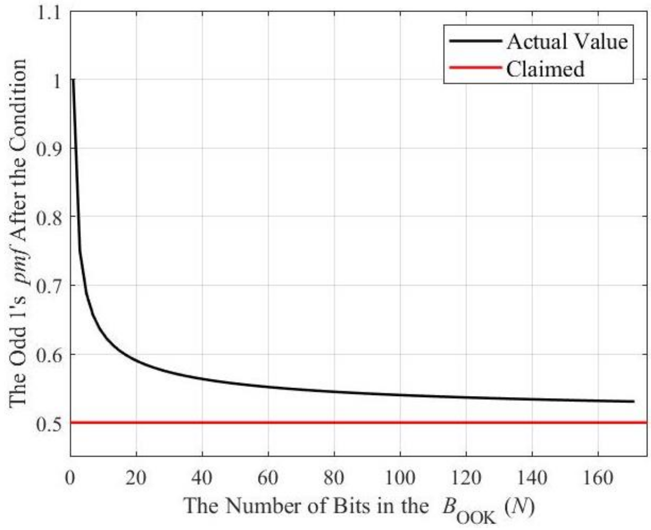


Fig. 4. Actual pmf_{odd} (1) for SIM– OFDM compared to $1/2$, *Eq.* (10)

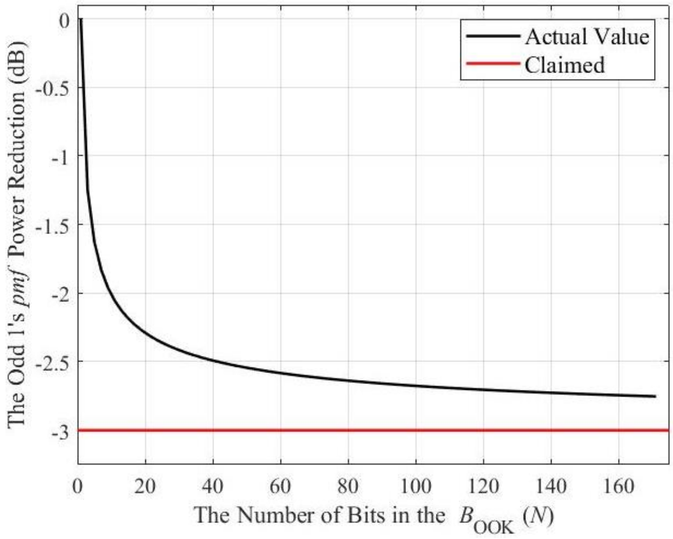


Fig. 5. Actual power reduction level for odd N in dB, *Eq.* (11)

The other value for N is the even value that behaves slightly different than the odd value for N. Hence, the 1's *pmf* for the even valued N is given by (12) where the number of 1's will always be from $N/2+1$ to N, see appendix Table A-II

$$\begin{aligned} pmf_{even}(1) &= \frac{2\cdot\sum_{k=\frac{N}{2}+1}^{N}\left[k\left(\frac{N!}{k!(N-k)!}\right)\right] + \frac{N}{2}\left(\frac{N!}{\left(\left(\frac{N}{2}\right)!\right)^2}\right)}{N\,2^N} \\ &= \frac{1}{2} + \frac{N!}{2^{N+1}\left(\left(\frac{N}{2}\right)!\right)^2} \end{aligned} \tag{12}$$

Equation 12 shows clearly that the 1's *pmf* is higher than $1/2$ and converges to $1/2$ when $N\to\infty$. As a numerical example, the *pmf* values for N equals 2, 8, 16 and 20 are $3/4=0.75$, $163/256=0.6367$, $39203/65536=0.5982$ and $308333/524288=0.5881$ respectively. The even valued N *pmf* for the 1's is plotted in Fig. 6.

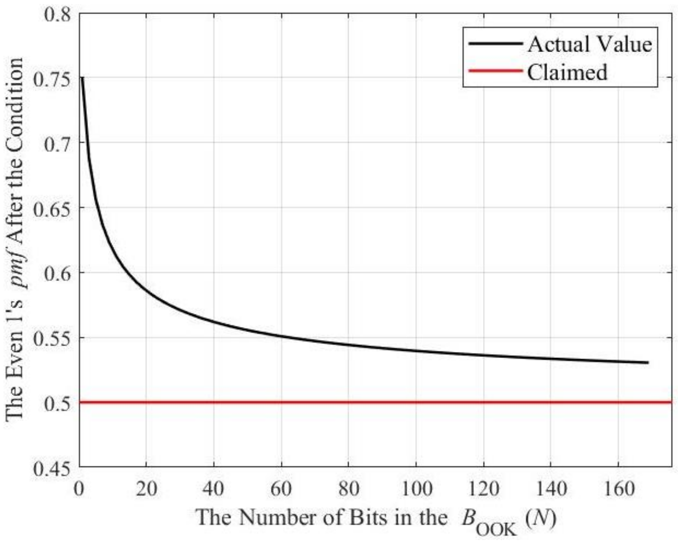


Fig. 6. Actual pmf_{even} (1) for SIM– OFDM compared to $1/2$, *Eq*. (12)

Figure 6 also shows that the $pmf=1/2$ can only be reached when $N\to\infty$. The amount of actual transmitted power reduction in dB is given by (13)

$$pmf_{\text{even}}^{\text{dB}}(1)=10\log\left(pmf_{\text{even}}(1)\right) \tag{13}$$

Plotting (13) will illustrate the actual transmitted power reduction as in Fig. 7

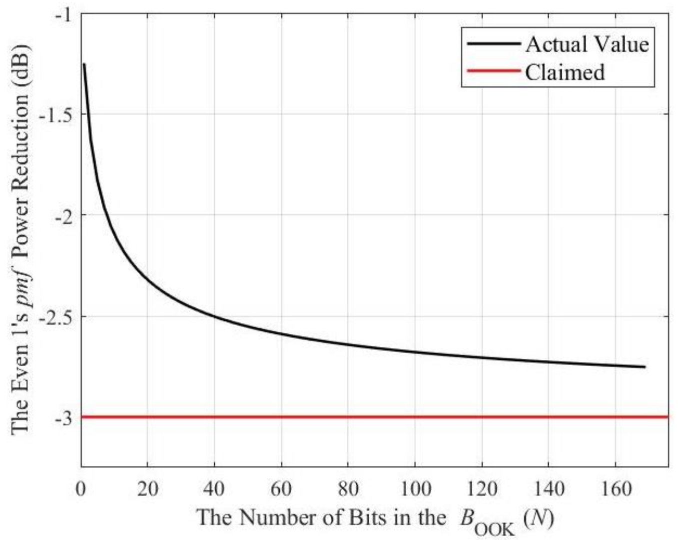


Fig. 7. Actual power reduction level for even N in dB, *Eq*. (13)

Figure 8 shows the *pmf* of the 1's for even and odd valued N. Figure 8 shows that there is a slight difference between the even and odd valued N at small values of N but they almost coincide at large N.

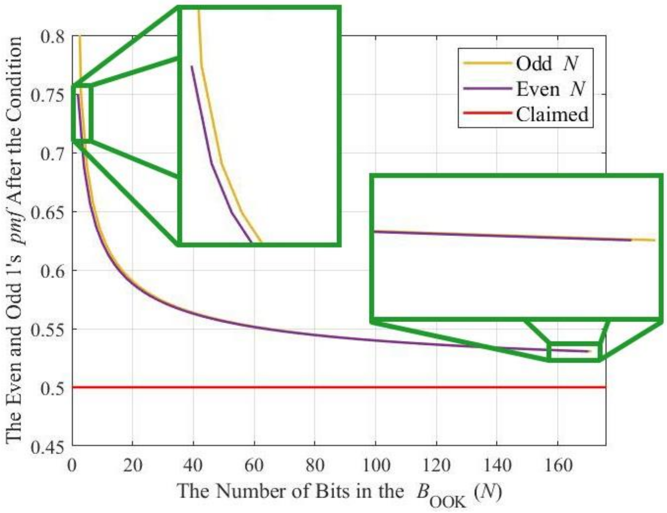


Fig. 8. The 1's *pmf* for even and odd valued N, *Eq*. (10) and (12)

The above analysis shows that the transmitted power cannot be reduced to half nor guaranteeing half of the carriers are ON. In consequence, the algorithm proposed for the ESIM–OFDM [11] will fail too as it requires half of the carriers at maximum to be ON to operate correctly.

B. The Synchronization Between the Transmitter and the Receiver

The synchronization between the transmitter and the receiver is a pivotal concept that ensures correct data delivery. The synchronization is a wide term but this research will only be concerned with *how the receiver knows that the condition in* (1) *is applied or not*. When the transmitter finds the number of 1's is less than the 0's then the B_{OOK} bits will be passed through NOT gates. When this process is executed, the receiver should be informed to reverse the process to retrieve the original data. Otherwise, the data will be lost and the *BER* will increase immensely.

In [10] and [11] there was no definite technique to inform the receiver that the NOT is applied or not. In [10] followed by [11] and other researchers [12–19], the suggested technique is to use the free carriers with no modulating data, or in other words the free carriers, to notify the receiver whether the condition is used or not. However, all the suggested techniques are based on the fact that the *pmf* is $1/2$ for the 1's. The previous section showed that this fact is no longer applicable when the B_{OOK} is passed through (1). Even so, all the researches did not show how to embed the notification when all the B_{OOK} bits are 1's. Not to mention that any carrier used for this purpose will create a bit pattern that is part of the 2^N states. Hence, the receiver will not differentiate between a notification and information. Nevertheless, the notification carrier should be modulated with a special value to avoid the confusion with the data. Choosing this special value for highly dense constellations is not an easy task. A practical way for notifying the receiver that the transmitter activated the condition or not is to add an extra carrier for this purpose. This arrangement will require the bandwidth (B) to be increased by an amount shown in (14) and illustrated by Fig. 9

$$B = (N+1)\cdot B_C \qquad \text{Hz} \tag{14}$$

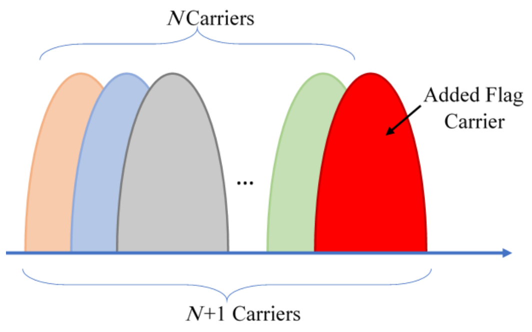


Fig. 9. The SIM–OFDM spectrum with added flag signal.

where: B_C is the carrier bandwidth and N is the number of carriers. The expansion ratio (E) in B will converge to "1" with the increase of N as seen in (15)

$$\begin{aligned} E &= \frac{(N+1)B_C}{N\cdot B_C} = \frac{N+1}{N} \\ \lim_{N\to\infty}(E) &= \lim_{N\to\infty}\left(\frac{N+1}{N}\right) = 1 \end{aligned} \tag{15}$$

The author of [20] showed that the SIM–OFDM and its derivatives will have a very low throughput that is ≤ 2 bit/sec/Hz. With the added flag carrier, the throughput will be reduced further more unless the number of carriers is large enough to maintain the 2 bit/sec/Hz.

C. The BER Model

The *BER* is an important measurement criterion that is used to evaluate the communication system performance. Equations 4, 5 and 6 in [10 and 11] shows the *BER* over Rayleigh fading AWGN channel for the SIM–OFDM and ESIM–OFDM respectively. These equations show that the *BER* is the sum of two independent processes which are the modulation technique, if QAM, and the B_{OOK} *BER* which is equivalent to ASK system. However, these equations are based on the assumption that the probability of "1" equals to $1/2$. Depending on the analysis in section III.*A* that showed the *pmf* (1) is dynamic depends on the value of N; then, the *BER* needs more review and reformation. Following the approach in [10 and 11] and including the synchronization problem solution suggested in section III.*B*, it can be seen that the *BER* comes from three sources instead of two which are QAM, B_{OOK} and the added flag carrier.

The false detection of the added flag carrier C_f, shown in Fig. 9, can be calculated as in (16) assuming the flag can take either "0" or "1" states

$$\begin{aligned} FER &= p\left(C_f^T = 1\right) p\left(C_f^D = 0 \middle| C_f^T = 1\right) \\ &+ p\left(C_f^T = 0\right) p\left(C_f^D = 1 \middle| C_f^T = 0\right) \end{aligned} \tag{16}$$

where *FER* is the Frame Error Rate, the superscript T refers to *transmitted*, D to *detected* and $p(\cdot)$ is the probability. The flag, as shown before, is directly related to the number of the B_{OOK} bits N which can be either even or odd. Starting with the odd N then the probability of activating the flag or not is equiprobable because half of the states are inverted and the others are kept without inversion. Therefore, the probability of the flag to be "1" or "0" equals to $1/2$. This means that (16) is equivalent to the probability of error in Binary Phase Shift Keying (BPSK) as given in (17) for *FER*

$$FER = Q\left(\sqrt{\frac{E_b}{N_0}}\right) \tag{17}$$

where E_b is the flag bit energy, N_0 is the AWGN power spectral density and $Q(\cdot)$ is the tail distribution function of the standard normal distribution.

Recalling the SIM–OFDM system as described by [10 and 11] where an average $N/2$ of the subcarriers are active with a symbol's length M bits then, the average of error bits F_b in a transmitted instance due to *FER* is given by (18)

$$\begin{aligned} F_b &= M\cdot N\cdot pmf\ (1)\cdot FER \\ &= \frac{M\cdot N}{2} FER \end{aligned} \tag{18}$$

To find the actual number of bits that contribute in F_b, the *pmf* (1) is substituted by the actual 1's probability given by (10) and (12) in (18). Taking $N\to\infty$ then *pmf* (1) for both even and odd N will coincide on each other, see Fig. (8), and reaching $1/2$. Therefore, (18) can be written as in (19) even when (10) and (12) are substituted in (18)

$$\lim_{N\to\infty}\left(F_b^{\text{even, odd}}\right) = \frac{M\cdot N}{2} FER \tag{19}$$

Equation (19) shows that the *FER* effect can dominate the QAM and the B_{OOK} *BER*. This is due to the fact that F_b controls the complete frame bits unlike QAM that controls only M bits and the B_{OOK} controls only N bits. Consequently, *FER* should be treated with caution during the design. It is worth to mention that when *FER* occurs, the error in B_{OOK} and QAM will be meaningless as they will be included in the erroneous frame.

IV. Concluding Remarks

The SIM–OFDM is an elegant technique to reduce the transmitted power by incorporating the data to switch ON the carrier if "1" or OFF if the data is "0". This technique promised a –3 dB transmitted power reduction based on the probability of "1" which is $1/2$. However, by carefully examining the SIM–OFDM and applying the probability theories, it has been shown that the real situation differs significantly from what was promoted. This new model gives the actual description for the SIM–OFDM behaviour and performance. The analytical model showed that the 1's *pmf* depends on the length of IFFT N. Also, for all values of N, the reduction in the transmitted power is above –3 dB with appreciable power waste especially for low *N*. This waste reduces the bandwidth utilization and the system's efficiency. Another observed issue is the need for a flag carrier to signal the receiver when to apply the bits inversion to the B_{OOK}. This increases the needed bandwidth and further squanders the transmitted power without useful modulated data. This problem can be reduced and the system's efficiency can be increased by increasing *N* to approach its limit when $N \rightarrow \infty$. Although this might be a practical solution but increasing *N* will increase the *BER* due to the increment in frame's bits size. The increment in system's *BER* comes from the misinterpretation of the flag. This can have a catastrophic effect because the flag controls a complete frame rather than *N* bits by the B_{OOK} or *M* bits by the QAM. Hence, any error in detecting the flag will generate a burst error that overwhelms the other error sources. Therefore, careful management should be followed to balance the system's efficiency and the *BER* by choosing the appropriate IFFT length *N*. This balance can be achieved by applying the optimization algorithms for optimum performance to the analytical model presented in this research. The other point that should be referred to is that, the SIM–OFDM can work with best results in a high Signal to Noise Ratio (SNR) and not in hostile environments. This high SNR ensures the minimum error in detecting both the flag carrier and the B_{OOK} leading to a stable communication system. Another solution is to use a powerful encoding scheme to minimize the *FER*.

Appendix A

A numerical example illustrating the effect of *N* on the binary states before and after applying the N_{maj} condition.

TABLE A-I
THE 4 BITS B_{OOK} BIT PATTERNS

Value	B_{OOK} Bit Pattern	B_{OOK} Pattern After N_{MAJ}
0	0000	1111
1	0001	1110
2	0010	1101
3	0011	0011
4	0100	1011
5	0101	0101
6	0110	0110
7	0111	0111
8	1000	0111
9	1001	1001
10	1010	1010
11	1011	1011
12	1100	1100
13	1101	1101
14	1110	1110
15	1111	1111
pmf (1)	1/2	0.6875

Final B_{OOK} Bit Pattern	Repetitions
1111	2
1110	2
1101	2
1011	2
0111	2
0011	1
0101	1
0110	1
1001	1
1010	1
1100	1

TABLE A-II
THE 5 BITS B_{OOK} BIT PATTERNS

Value	B_{OOK} Bit Pattern	B_{OOK} Pattern After N_{MAJ}
0	00000	11111
1	00001	11110
2	00010	11011
3	00011	11100
4	00100	11011
5	00101	11010
6	00110	11001
7	00111	00111
8	01000	10111
9	01001	10110
10	01010	10101
11	01011	01011
12	01100	10011
13	01101	01101
14	01110	01110
15	01111	01111
16	10000	01111
17	10001	01110
18	10010	01101
19	10011	10011
20	10100	01011
21	10101	10101
22	10110	10110
23	10111	10111
24	11000	00111
25	11001	11001
26	11010	11010
27	11011	11011
28	11100	11100
29	11101	11101
30	11110	11110
31	11111	11111
pmf (1)	1/2	0.6875

Final B_{OOK} Bit Pattern	Repetitions
11111	2
11110	2
11101	2
11100	2
11011	2
11010	2
11001	2
00111	2
10111	2
10110	2
10101	2
01011	2
10011	2
01101	2
01110	2
01111	2

References

[1] K. D Rao, *Channel Coding Techniques for Wireless Communications*. India: Springer, 2015.

[2] R. Prasad, *OFDM for Wireless Communications Systems*, London: Artech House Inc., U.K., 2004.

[3] M. Wen, C. Xiang and L. Yang, *Index Modulation for 5G Wireless Communications*, Springer, 2017.

[4] A. Zaidi et. al., *5G Physical Layer Principles, Models and Technology Components*, Academic Press, U.K., 2018.

[5] S. Z Asif, *5G Mobile Communications Concepts and Technologies*, CRC Press, U.S.A., 2019.

[6] A. M. Langer, *Analysis and Design of Next-Generation Software Architectures 5G, IoT, Blockchain, and Quantum Computing*, Springer Nature Switzerland, 2020.

[7] S. Khan, and J. L. Mauri. *Green Networking and Communications ICT for Sustainability*, CRC Press, U.S.A., 2014.

[8] Edited by H. A. Suraweera, J. Yang, A. Zappone and J. S. Thompson, *Green Communications for Energy-Efficient Wireless Systems and Networks*, The Institution of Engineering and Technology, U.K. 2021.

[9] G. Kaddoum and E. Soujeri, "On the Comparison Between Code-Index Modulation and Spatial Modulation Techniques", *International Conference on Information and Communication Technology Research (ICTRC2015)*, 2015.

[10] R. Abu-Alhiga and H. Haas, "Subcarrier-Index Modulation OFDM", *IEEE 20th International Symposium on Personal, Indoor and Mobile Radio Communications*, Tokyo, Japan, 2009.
[11] D. Tsonev, S. Sinanovic and H. Haas, "Enhanced Subcarrier Index Modulation (SIM) OFDM", *IEEE GLOBECOM Workshops (GC Wkshps)*, Houston, TX, USA, 2011.
[12] E. Basar, U. Aygolu, and E. Panayirci, "Orthogonal Frequency Division Multiplexing with Index Modulation in the Presence of High Mobility", *First International Black Sea Conference on Communications and Networking (BlackSeaCom)*, 2013.
[13] Y. Xiao et. al., "OFDM With Interleaved Subcarrier-Index Modulation", *IEEE Communications Letters*, Vol. 18, No. 8, 2014.
[14] E. Basar, "OFDM with Index Modulation Using Coordinate Interleaving", *IEEE Wireless Communications Letters*, Vol. 4, Issue 4, 2015.
[15] T. Van Luong and Y. Ko., "A Closed-Form Symbol Error Probability for MCIK-OFDM with Frequency Diversity", *IEEE 18th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, 2017.
[16] S. Dogan, A. Tusha and H. Arslan, "OFDM with Index Modulation for Asynchronous mMTC Networks", *Sensors*, Vol. 18, No. 4, 2018.
[17] Y. Shi, "Genetic Algorithm aided OFDM with All Index Modulation", *IEEE Communications Letters*, Vol. 23, Issue 12, 2019.
[18] M. Zhou et. al., "A Novel DNN Based Channel Estimator for Underwater Acoustic Communications with IMOFDM", *IEEE International Conference on Signal Processing, Communications and Computing (ICSPCC)*, 2020.
[19] Z. Qasem, H. A. Leftah, H. Sun, J. Qi and H. Esmaiel. "X-Transform Time-Domain Synchronous IM-OFDM-SS for Underwater Acoustic Communication", *IEEE Systems Journal (Early Access)*, 2021.
[20] N. Ishikawa, S. Sugiura, and L. Hanzo, "Subcarrier-Index Modulation Aided OFDM – Will It Work?", *IEEE Access*, Vol. 4, 2016.

Ahmed N. Jabbar (Kudos) received the B. Sc. In Electronic and Communication Eng. in 1994 and M. Sc. In 1997 from Al-Nahrain University Baghdad, Iraq. He received the title of lecturer in 2009, associated professor in 2012 and professor title in 2018 from the University of Babylon. Currently, he is a professor and Ph. D. student in University of Babylon, College of Eng., Electronic and Communication Eng. Babylon city, Iraq since 2019. The field of interest is High Speed Communication Systems, Microwave System Design and Analysis, Data Mining, Parallel Hardware System Design using FPGA and DSP for AI in Communication Systems.

Samir J. Almuraab was born in Hilla, Babylon, Iraq, in 9 Oct. 1959. He received the B.Sc. degree in Electrical Engineering/Electronics and Communications in 1981 from University of Sulaymaniyah, and the M.Sc. degree in Electrical Engineering/Electronics and Communications in 1986 from University of Baghdad and Ph.D. in communication engineering at the Department of technical Education- Electrical Engineering, University of Technology. Currently He works as a professor and Head of the Scientific Composition at the Electrical Department at the Faculty of Engineering, University of Babylon. His main interest is Wireless Communication, Spread spectrum systems, Digital video broadcasting (T.S.&C), Coding, Wireless Sensor Network Applications, Bioinformatics, Signal Processing, Healthcare System.

Abdulkareem Abdulrahman Kadhim () was born in Baghdad, Iraq, in 1958. He received his B.Sc. degree in Electrical and Electronics Engineering in 1981 from MEC, Iraq, and M.Sc. and Ph.D. degrees from Loughborough University of Technology, UK, in 1984 and 1989, respectively, in Digital Communication Systems. He is an IEEE Senior Member and Member of ACM. Currently, he is a professor of Digital Communications in the College of Information Engineering, Al-Nahrain University, Iraq. He has published 66 papers in international and national journals and scientific conferences. He successfully supervised 11 Ph.D. dissertations and 62 M.Sc. theses. His research interests include Modern Error Correction Codes for Next Generation Networks, Detection of Coded and Modulated Signals, Low Complexity Decoders, Millimetre Wave Channel Modelling, Network Coding, Software Defined Environments, and Efficient Routing for Wireless Sensor Networks (WSNs) and IoT Networks.