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What Is and What Is Not Mathematical Modelling in Primary School: Opinions of Slovenian and Croatian Primary School Teachers

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Abstract

Every day we encounter various obstacles and tasks that we solve with the help of mathematics, consciously or unconsciously. Mathematical modelling in teaching is an important interactive process in which students are introduced to problems from the real world and their environment. All education is slowly striving for students' better education on problem-solving, including mathematical modelling, but this is not yet the practice in the lower grades of elementary schools. Our research describes what mathematical modelling actually is in primary school and its advantages and difficulties, all from the point of view of the primary school teachers themselves. The research was conducted on 1000 primary school teachers from Croatia and Slovenia. The study results show almost no differences according to the originating country even though we expected some since Slovenia had previously introduced mathematical modelling into its curricula. Results also indicate that teachers are fairly uneducated on the topic of mathematical modelling, and the difficulties that could arise from its introduction into the classroom. It can also be reported that teachers are eager and ready for education on a given topic, and in the vast majority show interest in additional much-needed education.

Key words: *cross-country comparison; mathematical models; mathematics; teacher education; primary school.*

Introduction

Modelling is a strategy of teaching mathematics that promotes the connection of the real world with the abstract world of mathematics. Mathematical models help many sciences to solve their problems, therefore the competence of mathematical modelling is highly desirable in the modern world (Merrit et al., 2017). Teaching leading to the development of this competence has been heavily studied over the last decade (e. g. Artigue & Blomhøj, 2013). Modelling ideas are often intertwined with classroom strategies, which include problem-based, inquiry-based, and project-based teaching. Modelling can be considered an effective way to ensure permanent learning in many topics of mathematic (Erdem et al., 2015). Because modelling is based on problem-based teaching, its applications are easier when the student has already acquired a certain mathematical knowledge level. Therefore, research is focused primarily on the faculty environment (e.g. medicine) and partly also on the secondary school environment (Blum et al., 2007). It has been found that a change in the traditional teaching paradigm is required for the effectiveness of this strategy (Kirschner et al., 2010).

For the purpose of our research, we decided on the definition of mathematical modelling given by Stholmann and Albarracin, "Mathematical modelling is an iterative process that involves the open-ended, real-world, practical problems that students make sense of with mathematics using assumptions, approximations, and multiple representations. Other sources of knowledge besides mathematics can be used as well" (Stohlmann & Albarracín, 2016, p.1). There are many definitions of mathematical modelling in the literature, but we singled out this definition because we believe that it understandably represents mathematical modelling in teaching. Mathematical modelling in teaching is a very important interactive process in which students become acquainted with problems from the real world and the environment in which they find themselves. Students solve such problems with assumptions, their own experiences, and different presentations. The main goal of mathematical modelling is not always to find a unique solution. The goal of mathematical modelling is to understand problems and think when we encounter real-world problems. For mathematical modelling tasks, it is important to examine the text as a whole, not just the individual parts. It is also important to look critically at the problem's text to identify essential and irrelevant variables to solve the problem itself. Processes such as constructing, reasoning, predicting, guessing, organizing data, quantifying, etc. are becoming increasingly important in every individual's life. Mathematical modelling, which is rarely used in teaching in secondary schools, much less in primary education, offers a rich source of opportunities to develop these important processes in students (English & Watters, 2004). However, the basics of mathematical modelling can begin and should begin as early as the primary school level, as children already have the basic competencies on the basis of which they can develop modelling (English & Watters, 2004). Traditionally, mathematical modelling at primary school was misunderstood. It

was focused on arithmetic problems with words (tasks with words) in which concrete materials were presented that would then model the more abstract operating rules (English, 2003). Solving such tasks requires a link between problem structure and symbolic mathematics. For example: *Amy saved $ 15. Emily saved six times as many as Amy. How much did Emily save?* The solution is modelled with a number sentence. Frequent problem-solving is not a modelling task for students, but relies on keywords or phrases in the problem, such as times, more, less, and so on (English, 2003). Solutions to such way of interpreting problems are very often unambiguous and unique, and therefore students are involved in limited mathematical thinking. Although we do not dispute the importance of this type of math tasks, they do not address the sufficient mathematical knowledge, processes, fluency, and social skills that our students need in the 21st century (English, 2003).

From a didactic point of view, modelling is related to mathematics teaching approaches that promote the connection of the real world with the abstract world of mathematics. Modelling is also mainly related to non-mathematical contexts. Consequently, a teacher can easily find himself in an awkward situation as he can not be an expert on all possible modelling fields, such as natural sciences, computing, economics, art, sports, etc. The same goes for the students. Therefore, it is important to choose the context of the model for which both the teacher and the students have sufficient knowledge. The teacher must encourage students to use that knowledge, which does not have to be closely related to mathematical knowledge. It also needs the teacher to be aware that a specific problem can lead to different models (Spandaw & Zwaneveld, 2009). Mathematical modelling is inextricably linked to other mathematical competencies such as design, reading, communication and application of problem-solving strategies, which emiphasize high cognitive skills. Most primary school teachers assume that a large proportion of students find modelling difficult or challenging so they rarely apply it in the classroom (Asempapa, 2015).

In Slovenia and Croatia, mathematical modelling is already included in some curricular documents in secondary and primary school; there are also some resources available for teachers in Slovene primary and secondary schools. We also found two examples that are appropriate for the lower grades of primary school, but the emphasis is on the higher grades of primary school. Still, in both curricula, it is more relevant for upper grades of primary school (from 6th grade to 9th grade in Slovenia, and from 5th grade to 8th grade in Croatia). In contrast, it is not mentioned for lower grades (from 1st grade to 5th grade in Slovenia and from 1st grade to 4th grade in Croatia). Also, most textbooks written for primary and secondary schools place less emphasis on mathematical modelling activities.

Our research's guiding aim was to find out whether primary school teachers understand the concept of mathematical modelling and how to incorporate it into mathematics teaching. Furthermore, it also aimed at finding out if they are familiar with the advantages and disadvantages of mathematical modelling.

Methodology

Research design

An exploratory study using a self-constructed questionnaire delivered to teachers in both counties was used. The quantitative empirical pedagogical research methods we used are descriptive and causal non-experimental methods. The questions were set in Croatian for teachers from Croatia and in Slovenian language for teachers from Slovenia. In Slovenia, we were interested in the knowledge of teachers who teach 1st to 5th grade of primary school, because of the nine-year primary school system of education, and in Croatia, in the knowledge of teachers who teach 1st to 4th grade of primary school, because of the eight-year primary school system.

A questionnaire was designed with several types of questions: a) questions about teachers' basic data (gender, years of work experience in the classroom), b) general questions about mathematical modelling, c) specific tasks involving mathematical modelling followed by questions related to the given tasks, d) claims regarding mathematical modelling given with a five-degree Likert scale of agreement/disagreement, and e) questions about teachers' opinions regarding introducing mathematical modelling into the classroom.

Sample

The survey was carried out based on completed questionnaires from a convenience sample of 1000 primary school teachers in Croatia and Slovenia. The questionnaire was distributed via email addresses. We present the sample structure in Table 1.

Table 1
Sample structure

		f	f %
N		1000	100,0
Country	Slovenia	475	47.5 %
	Croatia	525	52.5 %

Teachers expressed their opinions on several statements regarding their general attitudes towards teaching mathematics and, more specifically, opinions on statements regarding teaching mathematical modelling. We must note that not all respondents gave an answer in some statements or questions, so the analyzed sample will be somewhat reduced, as we did not take into account the claims / questions to which respondents did not give an answer or their opinion. In the sample description we present the results (in Table 2) on two statements that are not specifically related to mathematical modelling.

Statement A: I love teaching math.

Statement B: The teacher's ability to find multiple solutions to a task is an important indicator of the student's abilities.

Table 2
Claims regarding teaching mathematics

		Statement A		Statement B	
		f	f %	f	f %
Slovenia	Entirely disagree	3	1.5	10	5.0
	Partially disagree	6	3.0	8	4.0
	Neither agree nor disagree	39	19.5	14	7.0
	Partially agree	152	76.0	85	42.5
	Entirely agree	3	1.5	83	41.5
Croatia	Entirely disagree	30	7.4	12	3.0
	Partially disagree	7	1.7	26	6.5
	Neither agree nor disagree	33	8.1	75	18.6
	Partially agree	87	21.4	155	38.5
	Entirely agree	250	61.4	135	33.5

The majority, i.e. 87 % of teachers from Croatia and Slovenia either entirely or partially agreed with statement A. However, we see that the vast majority of Slovenian teachers, 76 %, partially agreed, while only 1.5 % of them entirely agreed with the statement. However, with Croatian teachers the situation is reversed; 21.4 % of them partially agreed with the statement and 61.4 % of them entirely. The results related to statement B are similar: 72 % of Croatian and 84 % of Slovenian primary school teachers either entirely or partially agreed with the statement that the teacher's ability to find multiple solutions to a task is a very important indicator of the student's abilities.

Methods of obtaining and analyzing data

Before conducting the survey, we conducted a pilot survey with 20 primary school teachers in Slovenia and Croatia. Based on the feedback received from the pilot survey, we saw that the survey was well and clearly composed, and there was no ambiguity regarding the questions and tasks. We conducted the survey online. Teachers from all over Croatia and Slovenia were represented. For the needs of Slovene primary school teachers, the survey was written in Slovene, and in Croatian language for teachers in Croatia. The survey was anonymous and was conducted from February 2019 to December 2019. The obtained data were processed and analyzed with the help of the IBM SPSS statistics 22 program.

Results

General questions

The first set of questions regarding mathematical modelling aimed at finding out if the teachers were acquainted with the concept of mathematical modelling. If teachers

answered positively, we asked them to write what they thought mathematical modelling was. The next question was whether they were doing tasks that include mathematical modelling in math classes. The last question from the first set was of the open type. We asked the teachers to write an example of a task/activity that, according to them, includes mathematical modelling. The results are presented in Table 3.

Table 3
General questions about mathematical modelling

		f	f %
Are you acquainted with the concept of mathematical modelling?			
Slovenia	yes	43	9.3
	no	364	78.6
	barely	56	12.1
	total	463	100.0
Croatia	yes	41	7.8
	no	425	81.0
	barely	59	11.2
	total	525	100.0
Do you incorporate tasks/activities in your math class that include mathematical modelling?			
Slovenia	yes	35	9.2
	no	291	76.2
	sometimes	56	14.7
	total	382	100.0
Croatia	yes	44	8.4
	no	400	76.2
	sometimes	81	15.4
	total	525	100.0

We can see from the results that most Slovenian primary school teachers, 78.6 % of them, are not acquainted with the concept of mathematical modelling. The same goes for 81 % of Croatian primary school teachers. When we look at both countries' overall results, we see that the vast majority of teachers (79.9 %) are not familiar with mathematical modelling. Those who were acquainted with the concept of mathematical modelling were afterwards asked to describe what mathematical modelling is. The answers were related to solving equations, connecting mathematics with everyday life, modelling geometric figures, problem tasks, logical tasks, textual tasks, programming, creative problem solving, solving with the presentation of concrete models, etc. However, most of the answers tended to be solving problems from everyday life with the help of mathematics and other knowledge, not only mathematical. We can notice that teachers familiar with mathematical modelling (8.5 %) had a good idea of what mathematical modelling is, of course, with some exceptions.

More than three quarters (76.2 %) of Slovenian and Croatian primary school teachers answered no to a question asking whether they did activities/tasks including mathematical

modelling. The result was not surprising given the results of the previous question. As teachers are not even familiar with mathematical modelling, they do not include such tasks in their math classes. As answers to the open-ended question about what mathematical modelling is for them, teachers should have given an example of a task which, in their opinion, includes mathematical modelling. Most left this question blank, and some of the answers were: *estimate the length of the playground before measuring; The price of one ice cream is 1 kn. At the winter sale, we can buy 6 ice creams for 5 kn. How much more ice cream can we buy for 36 kn?; Subtract their difference from the sum of the numbers xx; Marko purchased the ball for 54 kunas. He paid with a 100 kuna bill. How many kunas does the saleswoman have to give him?; Clay shape geometric bodies; Matko is 7 years old. The sister is 5 years older than him. His dad is 5 years older than his mother. The mother is 3 times older than Matko's sister. How old is his sister, mom, dad? How old will he be in 10 years?*, etc. No task involving actual mathematical modelling was offered by either Croatian or Slovenian primary school teachers. Some teachers did not provide a specific task, but only wrote that these were problem tasks that involved problems from everyday life, and these answers were the closest to the correct one.

Since we expected similar results regarding the conducted pilot survey, we provided teachers with the definition of mathematical modelling from Stohlmann and Albarracín, given earlier in the article.

Specific mathematical modelling task

The second set of questions was related to a given specific task involving mathematical modelling. The *Beans problem* was:

Farmer Jake is trying to decide which light conditions are best for growing beans. To help Farmer Jake make his decision, he is growing bean plants using two different light conditions. The two light conditions are: growing beans in full sun with no shade at all, and growing beans in the shade.

Sunlight				**Shade**			
Butter Bean Plants	Week 8	Week 9	Week 10	Butter Bean Plants	Week 8	Week 9	Week 10
Row 1	9 kg	12 kg	13 kg	Row 1	5 kg	9 kg	15 kg
Row 2	8 kg	11 kg	14 kg	Row 2	5 kg	8 kg	14 kg
Row 3	9 kg	14 kg	18 kg	Row 3	6 kg	9 kg	12 kg
Row 4	10 kg	11 kg	17 kg	Row 4	6 kg	10 kg	13 kg

Figure 1. Table with data on the growth of beans in the sun and in the shade

Using the data above, determine which of the light conditions is suited to growing beans to produce the most excellent crop. In a letter to Farmer Jake, outline your recommendation of the light condition and explain how you arrived at this decision. Second, predict the weight of butter beans produced on week 12 for each type of light. Explain how you made your prediction so that Farmer Jake can use it for other similar situations (English, 2003).

After the *Beans problem*, the questions followed: Does the task have an unambiguous solution; What would be the best way to evaluate such a task; Would you do such a task in your math class? The last question was of an open type in which they had to describe how they would solve the task. The results are shown in Table 4.

Table 4
Question about a given specific mathematical modelling task

		f	f %
Given task has an unambiguous solution.			
Slovenia	Entirely disagree	89	41.2
	Partially disagree	44	20.4
	Partially agree	69	31.9
	Entirely agree	14	6.5
Croatia	Entirely disagree	125	31.6
	Partially disagree	88	22.2
	Partially agree	157	39.6
	Entirely agree	26	6.6
Which is the best way to evaluate the task?			
Slovenia	Oral exam	87	39.9
	Written exam	21	9.6
	Group work	82	37.6
	Homework assignment	8	3.7
	This task can't be evaluated	5	2.3
	Other	15	6.9
Croatia	Oral exam	123	30.1
	Written exam	36	8.8
	Group work	183	44.7
	Homework assignment	22	5.4
	This task can't be evaluated	26	6.4
	Other	19	4.6
I would do such task in my math class.			
Slovenia	Entirely disagree	18	8.3
	Partially disagree	23	10.6
	Partially agree	112	51.4
	Entirely agree	64	29.4
Croatia	Entirely disagree	44	10.8
	Partially disagree	57	13.9
	Partially agree	226	55.3
	Entirely agree	82	20.0

From the answers to the first question, we see that primary school teachers are divided, with the majority (56.6 %) entirely or partially disagreeing with the claim that the given task has a straightforward solution, which we are actually looking for. In comparison, as many as 43.4 % partially or entirely consider that the task has an unambiguous solution.

When asked what would be the best way to evaluate the *Beans problem*, teachers agreed that written examination might not be the best solution. Still, 42.3 % voted for group work, followed by 33.5 % who advocated oral examination. Under the option *other*, some of the answers were: conversation, self-evaluation, practical work, all of the above, research task, and some answered that they would not evaluate such task at all. 22.7 % of teachers would not include such task in their math classes.

Regarding the question establishing whether they would do such a task in their math class, most primary school teachers from both countries partially agreed (54 %), while even 23.3 % of them entirely agreed with the statement, that is, they would undoubtedly do such a task in their math classes.

The last question related to the *Bean problem* was to describe how they would solve that task. Unfortunately, most teachers did not answer this question (81.2 % from Croatia, 74.7 % from Slovenia). Among those who responded, some of the following answers could be found: *I would estimate by comparing previous weeks; I would look at the difference in kg for each previous and next week; I would find a crop increase model and anticipate solutions; Add up to calculate how many kg of beans each bed gave. A sunny bed gives significantly more yields. Conclusion: Beans is a plant that requires sunny terrain; I would calculate the mean outcome. I would recommend growing type 3 beans in a sunny bed as it gives the best yield; I would add up the kg on the sunny and shady side; I would look for a pattern and predict.*

Claims regarding mathematical modelling

The next set were claims regarding mathematical modelling. We presented 6 statements to the teachers, which we will present below, and show the results in Tables 5 and 6. For each statement teachers had the option of marking: I entirely disagree/I partially disagree/I neither agree nor disagree/I partially agree/I agree entirely. The results regarding statements C, D and E are shown in Table 5.

Statement C: *I think mathematical modelling is not appropriate for primary school, but has its place in high schools.*

Statement D: *Mathematical modelling is an exact, formal process, or a collection of formulas and rules that have to be applied.*

Statement E: *Although mathematical modelling activities improve students' ability to solve problems, I find that there are too many obstacles to incorporate such activities into my math classes.*

Table 5
Claims regarding mathematical modelling

		Statement C		Statement D		Statement E	
		f	f %	f	f %	f	f %
Slovenia	Entirely disagree	84	41.8	43	21.3	57	28.2
	Partially disagree	31	15.4	34	16.8	41	20.3
	Neither agree nor disagree	29	14.4	45	22.3	34	16.8
	Partially agree	52	25.9	66	32.7	65	32.2
	Entirely agree	5	2.5	14	6.9	5	2.5
Croatia	Entirely disagree	119	29.6	89	22.5	76	18.8
	Partially disagree	72	17.9	56	14.2	70	17.3
	Neither agree nor disagree	78	19.4	122	30.9	96	23.7
	Partially agree	122	30.3	109	27.6	144	35.6
	Entirely agree	11	2.7	19	4.8	19	4.7

In statement C, where it is stated that mathematical modelling is nevertheless more appropriate for high schools, teachers were not unanimous. 50.8 % of them from both countries did not either partially or entirely agree with the statement, which means that they think that mathematical modelling also has its place in primary school. A slightly smaller percentage of teachers, 31.6 %, believed that mathematical modelling is not for primary schools. Plenty of teachers, almost 18 %, did not opt for any of these. One of the reasons for different opinions among teachers and unwillingness to include mathematical modelling in primary schools is possibly due to the results of the question determining whether they have ever encountered the concept of mathematical modelling. Almost 80 % answered no, so due to lack of education about mathematical modelling itself, they are not ready to include it in primary education. The results for statement D are astounding in a negative sense. Namely, although we stated in the survey the exact definition of mathematical modelling, more than a third of teachers from both countries (34.8 %) either partially or entirely agreed with the statement, which is contradictory to the very definition of mathematical modelling, and 28 % of them neither agreed or disagreed with the statement. We also notice that 34.7 % of Slovenian teachers and 40.3 % of Croatian teachers partially agreed or entirely agreed with statement E, although we know that mathematical modelling improves many student capabilities, such as solving problems. One reason for such a result may also be insufficient education about mathematical modelling.

The results regarding statements F, G and H are shown in Table 6.

Statement F: *I think mathematical modelling is a necessary skill in the 21st century for every student.*

Statement G: *Mathematical modelling can be taught in specific environments, despite all the difficulties associated with learning and teaching modelling.*

Statement H: *Mathematical modelling is a creative process relevant to society, through which an individual seeks the best solution to a real-life problem.*

Table 6
Claims regarding mathematical modelling

		Statement F		Statement G		Statement H	
		f	f %	f	f %	f	f %
Slovenia	Entirely disagree	2	1.0	8	4.0	1	0.5
	Partially disagree	6	3.0	18	9.0	4	2.0
	Neither agree nor disagree	22	11.1	28	14.1	14	7.0
	Partially agree	88	44.2	107	53.8	82	41.0
	Entirely agree	81	40.7	38	19.1	99	49.5
Croatia	Entirely disagree	9	2.2	12	3.0	10	2.5
	Partially disagree	12	3.0	22	5.5	9	2.2
	Neither agree nor disagree	43	10.6	82	20.5	32	8.0
	Partially agree	175	43.1	185	46.3	160	39.9
	Entirely agree	167	41.1	99	24.8	190	47.4

From the results, we see that 84.9 % of Slovenian and 84.2 % of Croatian primary school teachers partially or entirely agree with statement F, which says that mathematical modelling is a necessary skill in the 21st century for every student, which is, of course, an encouraging result. Primary school teachers from both countries also agreed on statement G, which says that mathematical modelling can be taught in specific environments, despite all the difficulties associated with learning and teaching modelling. 48.7 % partially agreed with the statement while 22.9 % of them entirely agreed. At the last statement, H, which says that mathematical modelling is a creative process relevant to the society where an individual seeks the best solution to a real-life problem, 87.3 % of Croatian primary school teachers and 90.5 % of Slovenian teachers entirely or partially agreed.

Introducing mathematical modelling into the classroom

Last questions in the research were about teachers' opinions regarding introducing mathematical modelling into the classroom. We have presented the results in Table 7.

Table 7
Teachers' opinions regarding introducing mathematical modelling into the classroom

		f	f %
I believe that I am sufficiently acquainted and educated to teach mathematical modelling in elementary math classes.			
Slovenia	yes	46	23.0
	no	154	77.0
Croatia	yes	105	26.1
	no	298	73.9
I think that more tasks/activities that include mathematical modelling should be introduced in elementary math classes.			
Slovenia	yes	180	90.9
	no	18	9.1
Croatia	yes	328	84.3
	no	61	15.7
Would you do mathematical modelling in your math class if the hours were accurately provided for in the curriculum?			
Slovenia	yes	175	88.8
	no	22	11.2
Croatia	yes	357	91.3
	no	34	8.7
If there were training/workshops for primary school teachers on mathematical modelling, would you attend them?			
Slovenia	yes	181	92.8
	no	14	7.2
Croatia	yes	358	90.6
	no	37	9.4

For the first question, *I think that I am sufficiently acquainted and educated to teach mathematical modelling in elementary math classes*, given the previous research results, the result did not surprise us. 25 % of teachers from Croatia and Slovenia answered yes, while the vast majority, 75 %, answered no. We expected such result because of the results from an earlier question in the survey. Namely, 79.9 % of teachers stated that they had never even encountered the concept of mathematical modelling. The next open-ended question followed up on the previous question. We asked the teachers who answered no to the previous question to write the reason why this is so. Some of the answers were: *I do not have enough experience, I have never encountered it before, an insufficient amount of examples/tasks of this type, too little educational and teaching content for teachers on this topic.* Answers of both Croatian and Slovenian primary school teachers did not differ in that they had never heard of mathematical modelling, they did not have the opportunity to be educated on this topic, or they do not even know where to find or look for such education or resources that they could include in their math classes. The next question was whether they felt that more tasks/activities involving mathematical modelling should be introduced in the elementary

math classes. 86.5 % of teachers from Croatia and Slovenia agreed that more such tasks/activities should be introduced. Only a small proportion of teachers, 13.5 %, answered this question negatively. Since we know that mathematical modelling is minimally included in the curricula, and even fewer classes are provided for such activities, we asked them if they would do mathematical modelling in a mathematics class if the hours were accurately provided for that in the curriculum. Teachers from both countries agreed on the issue, with 90.5 % saying they would do the modelling in that case. Only 9.5 % of them, despite the planned hours, would not do mathematical modelling activities or tasks. We were also interested in what difficulties they encounter when doing tasks/activities that involve mathematical modelling or what they think the difficulties would be if they do not currently implement such tasks. Since we know that mathematical modelling activities are not easy to do in the classroom, we were interested in what would bother our teachers the most. Some of the answers we got were: *too few hours of math classes, students' lack of interest in thinking and no motivation for such tasks, significant differences in knowledge among students, lack of teaching materials on this topic, problems in critical thinking in students, insufficient education of teachers themselves, too many students in the classroom.* For most teachers from both Croatia and Slovenia, the main problem was lack of time, which was not surprising. Furthermore, they were most bothered by students' lack of interest in such tasks, namely they stated that students *find it difficult to concentrate for a long time, find it harder to think, and most always look for a unique solution*. One of the more significant problems is the impossibility of finding such problems or the lack of education for creating and designing such activities/tasks. Another problem they cite is different prior knowledge and the level of students' mathematics knowledge. They believe that only „better" students would be able to solve such tasks. The last question of the research was whether they would attend if there were training workshops for primary school teachers on mathematical modelling. 90.6 % of Croatian and 92.8 % of Slovene primary school teachers answered that they would attend, while 9.4 % of Croatian and 7.2 % of Slovene primary school teachers would not. We think that this result is excellent given that the vast majority of the teachers did not know until our research what mathematical modelling in teaching is.

Discussion

The results of the research speak for themselves. It is evident that participants are entirely unfamiliar with mathematical modelling. We have seen that almost 80 % of primary school teachers from Croatia and Slovenia have never been acquainted with the concept of mathematical modelling, while almost 12 % of them have been barely acquainted with the concept of mathematical modelling. Slightly more than three quarters of primary school teachers from Slovenia and Croatia admitted that they do not do tasks/activities that include mathematical modelling at all, which was not surprising due to the result of the previous question, where they answered that they do not know what

mathematical modelling in primary education is. We also saw that participants who responded to the task of giving one example activity involving mathematical modelling for the most part wrote plain textual tasks requiring the application of some basic arithmetic operation, without the need to use any mathematical modelling. The results are in accordance with similar studies in other countries. Most mathematics teachers have misconceptions about mathematical modelling and the modelling process due to their lack of knowledge about mathematical modelling (Blum, 2012). Additionally, most teachers may have had limited exposure to mathematical modelling and have unclear ideas of what is meant by the term mathematical modelling. Teachers also appear to know less about mathematical processes, tasks, pedagogies, and assessments associated with mathematical modelling (Gaston & Lawrence, 2015).

After the given definition of mathematical modelling and a specific task that includes mathematical modelling, the results showed that slightly more than half of participants from Slovenia and Croatia believe that the given task does not have a unique solution, whereas a little more than 43 % believe that it has a unique solution. Answer that a task wich includes mathematical modelling has a unique solution is contrary to the very goal of mathematical modelling. With the help of this type of assignment, students can exchange ideas about possible solutions, since the task is not limited by a unique solution. When asked what would be the best way to evaluate such a task, somewhat more than 40 % of participants from Slovenia and Croatia agreed that it would be group work, while a little more than a third of participants opted for oral examination. Interestingly, almost 5 % of participants opted for the impossibility of evaluating such a task. The obtained results are in correlation to other studies: for example, Spandaw and Zwaneveld (2009) state that commonly written tests are not well suited to evaluating higher skills such as modelling. It would be better to use other alternatives, such as group work, homework or oral exams. When asked if they would do such a task in their math classes, little more than three-quarters of the participants decided that they would, but somewhat less than a quarter of them would not do such a task. One of the encouraging results of the research was that just under 90 % of participants from both states like to teach math. The next result was not as expected or encouraging. Although they had a written definition of mathematical modelling, more than a third of participants from Slovenia and Croatia agreed with the statement that mathematical modelling is an exact, formal process, or a collection of formulas and rules that have to be applied, while a little less than a third of them neither agreed nor disagreed with the statement. On the claim of whether mathematical modelling has a place in primary school or if it is only for high schools, half of the participants agreed that mathematical modelling should be included in primary school. This is in line with the findings by English and Watters (2004) who state that the basics of mathematical modelling should and can begin in primary school where students already possess the basic competencies on which modelling can be developed. In a statement that says how taught mathematical modelling activities improve students'

ability to solve problems, *I find that there are too many obstacles to incorporate such activities into my math classes*, about 40 % of participants agreed with the statement and as many disagreed. Such result could be due to the fact that they are not even closely enough educated about mathematical modelling and therefore feel a kind of aversion to doing or incorporating such tasks in their math classes. A better result was found for the claim that mathematical modelling is a necessary skill in the 21st century for every student, where more than four-fifths of participants agreed with that statement. We obtained a similar result with the statement that the teacher's ability to find multiple solutions to a task is an important indicator of the student's abilities. Just over three-quarters of primary school teachers agreed with the statement, as evidenced by the literature. As Brown and Ikeda (2019) state, teacher plays an important role in supporting student engagement in mathematical modelling, reflection on it, decisions related to technology use and mathematical modelling and the interactions between these. Despite the discouraging results of some previous questions in the research, where teachers did not really show a willingness to include mathematical modelling in their classrooms, almost 90 % of the participants agreed with the claim that mathematical modelling is a creative process relevant to society, through which an individual seeks the best solution to a real-life problem. A similar result is also often cited in the literature, where, for example, Zbiek & Conner (2006) state that mathematical modelling allows students to connect classroom mathematics with the real world, showing the applicability of mathematical ideas. Bahmaei (2011) also states that given a real problem, students need to understand the real-world situation and make assumptions to devise a mathematical method to solve the problem. The results of questions about whether the participants thought they were educated enough to be able to introduce mathematical modelling into their classes were expected. Three-quarters of participats responded that they thought not. Blum (2012) states that one way of providing future teachers with the necessary professional knowledge is to offer specific modelling seminars already at the university, with compulsory own teaching experiences. However, although the participants are of the opinion that they themselves are not sufficiently educated, almost 90 % of them think that more tasks/activities including mathematical modelling should be implemented in primary education. Time was named as one of the reasons for which they feel it would be difficult to incorporate mathematical modelling into math classes. Kang and Noh (2012) state that, according to the curriculum, mathematical modelling in teaching is given too little or no time. Due to the lack of time for mathematical modelling, teachers avoid mathematical modelling in the classroom. The second most common reason against mathematical modelling was that they think that students were not ready for such types of tasks or activities. Such results often appear in the literature. It is known that teachers (falsely) assume that a vast majority of students find modelling challenging or difficult so teachers rarely include modelling activities or tasks in their classroom (Spandaw & Zwaneveld, 2009; Asempapa, 2015; Brown, 2019). The result on the

last question is definitely the first step of progress in incorporating mathematical modelling into primary education. Over 90 % of participants from both countries would like to attend workshops/trainings on mathematical modelling in primary education. It therefore appears from the results that teachers are eager and ready for education on mathematical modelling, but they simply do not have the opportunities and possibilities at this moment.

Conclusion

Mathematical modelling and its implementation are important processes, however demanding. This implies great efforts in order to make the modelling accessible to students (Blum, 2012). Although Slovenia and Croatia are neighbors with a common past, there are significant differences in teaching mathematics (Sabo & Lipovec, 2017). Therefore, we expected differences in teachers‘ knowledge, but as the results showed, teachers from both countries are at the same level when it comes to the topic of mathematical modelling. The results showed that participants do not know what exactly mathematical modelling in primary school is and therefore do not use or do tasks/activites that include mathematical modelling in their math classes. Some of the reason why teachers so rarely do such tasks, as Borromeo Ferri (2010) states, are time constraints and their perception that mathematical modelling tasks are to demanding or complex. Research shows that when students have opportunities to do mathematical modelling tasks, they are able to reason mathematically, improve their problem-solving abilities and make connections to real-world problems (Bleiler-Baxter et al., 2017). Mathematical modelling is undoubtedly challenging and complicated for teachers, but many studies show and point to its benefits for students. Along these lines, we can also report optimistic results as participating teachers are very interested in education on mathematical modelling and its integration into mathematics classes: 91.4 % of them would like to attend mathematical modelling training if it existed. This attitude alone is already the first step towards greater involvement of mathematical modelling in lower grades of primary school.

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Što jest, a što nije matematičko modeliranje u razrednoj nastavi: mišljenja slovenskih i hrvatskih učitelja razredne nastave

Sažetak

Svakodnevno se susrećemo s raznim preprekama i zadatcima koje svjesno ili nesvjesno rješavamo uz pomoć matematike. Matematičko modeliranje u nastavi važan je interaktivan proces u kojem se učenici upoznavaju s problemima iz stvarnoga svijeta i svojega okruženja. Čitavo obrazovanje polako teži boljem obrazovanju učenika o temi rješavanja problema, uključujući matematičko modeliranje, ali to još nije praksa u nižim razredima osnovnih škola. Naše istraživanje opisuje što je zapravo matematičko modeliranje u osnovnoj školi te njegove prednosti i poteškoće, sa stajališta učitelja razredne nastave. Istraživanje je provedeno na 1000 učitelja razredne nastave iz Hrvatske i Slovenije. Rezultati istraživanja ne pokazuju gotovo nikakve razlike prema zemlji podrijetla, iako smo ih očekivali budući da je Slovenija prethodno uvela matematičko modeliranje u svoje kurikule. Rezultati također ukazuju na to da su učitelji prilično neobrazovani po pitanju matematičkoga modeliranja i poteškoća koje bi mogle nastati njegovim uvođenjem u učionice. Također se može izvijestiti da su učitelji željni i spremni za obrazovanje o navedenoj temi te u većini pokazuju interes za dodatno, prijeko potrebno, obrazovanje o matematičkom modeliranju.

Ključne riječi: *matematika; matematički modeli; međudržavna usporedba; obrazovanje učitelja; osnovna škola.*

Uvod

Modeliranje je strategija poučavanja matematike koja promiče povezanost stvarnoga svijeta s apstraktnim svijetom matematike. Matematički modeli pomažu mnogim znanostima u rješavanju njihovih problema, stoga je kompetentnost matematičkoga modeliranja vrlo poželjna u suvremenom svijetu (Merrit i sur., 2017). Poučavanje koje je dovelo do razvoja te kompetencije jako je proučavano u posljednjem desetljeću (npr. Artigue i Blomhøj, 2013). Ideje kod modeliranja često su isprepletene sa strategijama u učionici koje uključuju nastavu zasnovanu na problemima, istraživanjima i projektima. Modeliranje se može smatrati učinkovitim načinom osiguravanja trajnoga učenja u mnogim matematičkim temama (Erdem i sur., 2015). Budući da se modeliranje temelji na problemsko osnovanoj nastavi, njegova primjena je lakša ako je učenik već stekao

određenu razinu matematičkoga znanja. Stoga je istraživanje usmjereno prvenstveno na fakultetsko okruženje (npr. medicinu), a djelomično i na srednjoškolsko okruženje (Blum i sur., 2007). Utvrđeno je da je za učinkovitost ove strategije potrebna promjena tradicionalne nastavne paradigme (Kirschner i sur., 2010).

U svrhu našega istraživanja odlučili smo se za definiciju matematičkoga modeliranja po Stholmann i Albarracín:„Matematičko modeliranje je iterativni proces koji uključuje otvorene, stvarne, praktične probleme koje učenici razumiju pomoću matematike koristeći pretpostavke , aproksimacije i višestruke prikaze. Mogu se koristiti i drugi izvori znanja osim matematike. „(Stohlmann i Albarracín, 2016., str. 1). U literaturi postoji mnogo definicija matematičkoga modeliranja, ali smo tu definiciju izdvojili jer smatramo da ona razumljivo predstavlja matematičko modeliranje u nastavi. Matematičko modeliranje u nastavi vrlo je važan interaktivan proces u kojem se učenici upoznaju s problemima iz stvarnoga svijeta i okruženja u kojem se nalaze. Učenici rješavaju takve probleme pretpostavkama, vlastitim iskustvima i različitim prezentacijama. Glavni cilj matematičkoga modeliranja nije uvijek pronaći jedinstveno rješenje. Cilj je matematičkoga modeliranja razumjeti probleme i razmišljati kada naiđemo na probleme u stvarnom svijetu. Za zadatke matematičkoga modeliranja važno je pregledati tekst u cjelini, a ne samo pojedine dijelove. Također je važno kritički pogledati tekst problema kako bi se identificirale bitne i nevažne varijable za rješavanje samoga problema. Procesi poput konstruiranja, zaključivanja, predviđanja, nagađanja, organiziranja podataka, kvantificiranja itd. postaju sve važniji procesi u životu svakog pojedinca. Matematičko modeliranje, koje se rijetko koristi u nastavi u srednjim školama, a još manje u osnovnoškolskom obrazovanju, nudi bogat izvor mogućnosti za razvoj ovih važnih procesa kod učenika (English i Watters, 2004). Osnove matematičkoga modeliranja mogu započeti, a također bi trebale započeti već na razini osnovne škole jer djeca već imaju osnovne kompetencije na temelju kojih mogu razviti modeliranje (English i Watters, 2004). Tradicionalno matematičko modeliranje u osnovnoj školi pogrešno je shvaćeno. Bilo je fokusirano na aritmetičke probleme s riječima (zadatci s riječima) u kojima su prezentirani konkretni materijali koji bi zatim modelirali apstraktnija operativna pravila (English, 2003.). Za rješavanje takvih zadataka potrebna je veza između strukture problema i simboličke matematike. Na primjer: Amy je uštedjela 15 dolara. Emily je uštedjela šest puta više od Amy. Koliko je Emily uštedjela? Rješenje je modelirano brojčanom rečenicom 6 x 15 = 90. Često rješavanje takvih problema nije zadatak modeliranja, već se oslanja na ključne riječi ili fraze u problemu, kao što su puta, više, manje itd. (English, 2003.). Rješenja takvoga načina tumačenja problema vrlo su često jednoznačna i jedinstvena, pa su učenici uključeni u ograničeno matematičko razmišljanje. Iako ne osporavamo važnost ove vrste matematičkih zadataka, oni se ne bave dovoljnim matematičkim znanjem, procesima, tečnošću i društvenim vještinama koje su učenicima potrebne u 21. stoljeću. (English, 2003.).

S didaktičkoga gledišta, modeliranje je povezano s pristupima nastavi Matematike koji promiču povezanost stvarnoga svijeta s apstraktnim svijetom matematike. Također

se modeliranje uglavnom odnosi na nematematičke kontekste. Slijedom toga, učitelj se lako može naći u neugodnoj situaciji jer ne može biti stručnjak na svim mogućim područjima modeliranja, poput prirodnih znanosti, računarstva, ekonomije, umjetnosti, sporta itd. Isto vrijedi i za učenike. Stoga je važno odabrati kontekst modela za koji učitelj i učenici imaju dovoljno znanja. Učitelj mora potaknuti učenike da koriste to znanje, koje ne mora biti u bliskoj vezi s matematičkim znanjem. Učitelj također treba biti svjestan da određeni problem može dovesti do različitih modela (Spandaw i Zwaneveld, 2009). Matematičko modeliranje neraskidivo je povezano s drugim matematičkim kompetencijama poput dizajna, čitanja, komunikacije i primjene strategija rješavanja problema, što naglašava visoke kognitivne vještine. Većina učitelja osnovnih škola pretpostavlja da je velikom broju učenika modeliranje teško ili izazovno, pa ga rijetko primjenjuju u razredu (Asempapa, 2015).

U Sloveniji i Hrvatskoj matematičko modeliranje već je uključeno u neke kurikulne dokumente u srednjoj i osnovnoj školi. U Sloveniji postoje i neki resursi dostupni nastavnicima u srednjim školama i učiteljima u osnovnim školama. Pronašli smo i dva primjera primjerena nižim razredima osnovne škole, no naglasak je na višim razredima osnovne škole. Ipak, u oba nastavna programa relevantniji je za više razrede osnovne škole (od 6. do 9. razreda u Sloveniji, a od 5. do 8. razreda u Hrvatskoj). Nasuprot tome, ne spominje se za niže razrede (od 1. razreda do 5. razreda u Sloveniji i od 1. razreda do 4. razreda u Hrvatskoj). Također, većina udžbenika napisanih za osnovne i srednje škole stavljaju manji naglasak na aktivnosti matematičkoga modeliranja.

Vodeći cilj našega istraživanja bio je saznati razumiju li učitelji razredne nastave koncept matematičkoga modeliranja i kako ga uključiti u nastavu Matematike. Nadalje, jesu li upoznati s prednostima i nedostatcima matematičkoga modeliranja?

Metodologija

Dizajn istraživanja

Korištena je istraživačka studija pomoću upitnika dostavljenoga učiteljima razredne nastave u obje države. Kvantitativne empirijske pedagoške metode istraživanja koje smo koristili su deskriptivne i uzročno neeksperimentalne metode. Pitanja su postavljena na hrvatskom jeziku za učitelje iz Hrvatske i na slovenskom jeziku za učitelje iz Slovenije. U Sloveniji nas je zanimalo znanje učitelja koji predaju od 1. do 5. razreda osnovne škole zbog devetogodišnjega osnovnoškolskog sustava obrazovanja, a u Hrvatskoj znanje učitelja koji predaju od 1. do 4. razreda osnovne škole, zbog osmogodišnjega osnovnoškolskog sustava.

Upitnik je bio sastavljen od nekoliko vrsta pitanja: a) pitanja o osnovnim podatcima učitelja (spol, godine radnoga iskustva), b) opća pitanja o matematičkom modeliranju, c) zadan specifični zadatak koji uključuje matematičko modeliranje, nakon čega slijede pitanja na zadani zadatak d) tvrdnje u vezi s matematičkim modeliranjem dane s pet Likertovih skala slaganja/neslaganja e) pitanja o mišljenjima učitelja o uvođenju matematičkoga modeliranja u učionicu.

Uzorak

Istraživanje je provedeno na temelju ispunjenih upitnika na uzorku od 1000 učitelja razredne nastave u Hrvatskoj i Sloveniji. Upitnik je distribuiran putem e-adresa. Strukturu uzorka prikazujemo u Tablici 1.

Tablica 1.

Učitelji su izrazili svoja mišljenja o nekoliko izjava u vezi s njihovim općim stavovima prema nastavi Matematike, a konkretnije mišljenja o izjavama u vezi s poučavanjem matematičkoga modeliranja. Moramo napomenuti da u nekim izjavama ili pitanjima nisu svi ispitanici dali odgovor, pa će analizirani uzorak biti donekle smanjen, jer nismo uzeli u obzir tvrdnje / pitanja na koja ispitanici nisu dali odgovor, niti njihovo mišljenje. U opis opisa uzorka (u Tablici 2) uvrštavamo dvije tvrdnje koje nisu posebno povezane s matematičkim modeliranjem.

Izjava A: Volim poučavati matematiku.

Izjava B: Sposobnost učitelja da pronađe višestruka rješenja nekog zadatka važan je pokazatelj učenikovih sposobnosti.

Tablica 2.

Većina, 87 % učitelja iz Hrvatske i Slovenije, ili se u cijelosti ili djelomično složilo s izjavom A. Međutim, vidimo da se među slovenskim učiteljima većina, 76 %, djelomično složila, dok se samo 1,5 % njih u potpunosti složilo s izjavom. Međutim, s hrvatskim učiteljima situacija je obrnuta: njih 21,4 % djelomično se složilo s tvrdnjom, a 61,4 % njih u potpunosti. Rezultati koji se odnose na tvrdnju B su slični. 72 % hrvatskih i 84 % slovenskih učitelja razredne nastave u cijelosti se ili djelomično slažu s tvrdnjom da je učiteljeva sposobnost da pronađe višestruka rješenja za neki zadatak vrlo važan pokazatelj učenikovih sposobnosti.

Metode prikupljanja i analize podataka

Prije provođenja istraživanja proveli smo pokusnu anketu s 20 učitelja razredne nastave u Sloveniji i Hrvatskoj. Na temelju povratnih informacija dobivenih iz pokusnoga istraživanja, vidjeli smo da je anketa dobro i jasno sastavljena te da nema nedoumica u vezi s pitanjima i zadatcima. Anketu smo proveli na internetu. Bili su zastupljeni učitelji iz cijele Hrvatske i Slovenije. Za potrebe slovenskih učitelja razredne nastave, anketa je napisana na slovenskom, a za učitelje u Hrvatskoj na hrvatskom jeziku. Istraživanje je bilo anonimno i provedeno je od veljače 2019. do prosinca 2019. Dobiveni su podatci obrađeni i analizirani uz pomoć programa IBM SPSS statistics 22.

Rezultati

Opća pitanja

Prvi skup pitanja u vezi s matematičkim modeliranjem bilo je propitivanje jesu li učitelji upoznati s pojmom matematičkoga modeliranja. Ako su učitelji odgovorili pozitivno, zamolili smo ih da napišu što misle da je matematičko modeliranje. Sljedeće

pitanje bilo je rade li oni na satima Matematike zadatke koji uključuju matematičko modeliranje. Posljednje pitanje iz prvoga skupa bilo je otvorenoga tipa. Zamolili smo učitelje da napišu primjer zadatka/aktivnosti koji prema njihovom mišljenju uključuje matematičko modeliranje. Rezultati su prikazani u Tablici 3.

Tablica 3.

Iz rezultata možemo vidjeti da većina slovenskih učitelja, 78,6 % njih, nije upoznata s pojmom matematičkoga modeliranja. Isto vrijedi i za 81 % hrvatskih učitelja razredne nastave. Kada pogledamo ukupne rezultate obje zemlje, vidimo da većina učitelja (79,9 %) nije upoznata s matematičkim modeliranjem. Oni koji su bili upoznati s pojmom matematičkoga modeliranja kasnije su zamoljeni da opišu što je prema njihovu mišljenju matematičko modeliranje. Odgovori su se odnosili na rješavanje jednadžbi, povezivanje matematike sa svakodnevnim životom, modeliranje geometrijskih figura, problemskih zadataka, logičkih zadataka, tekstualnih zadataka, programiranje, kreativno rješavanje problema, rješavanje prezentacijom konkretnih modela itd. Međutim, većina odgovora težila je rješavanju problema iz svakodnevnoga života pomoću matematike i drugih znanja, ne samo matematičkoga. Možemo primijetiti da su učitelji upoznati s matematičkim modeliranjem (8,5 %) imali dobru predodžbu o tome što je matematičko modeliranje, naravno, uz neke iznimke.

Više od tri četvrtine (76,2 %) slovenskih i hrvatskih učitelja razredne nastave odgovorilo je negativno jesu li radili aktivnosti/zadatke koji uključuju matematičko modeliranje. Rezultat nije bio iznenađujući s obzirom na rezultate prethodnoga pitanja. Kako učitelji nisu ni upoznati s matematičkim modeliranjem, ne uključuju takve zadatke u svoje sate Matematike. Rezultati otvorenoga pitanja bili su povezani s odgovorima na pitanje što za njih predstavlja matematičko modeliranje. Učitelji su trebali dati primjer zadatka koji po njihovu mišljenju uključuje matematičko modeliranje. Većina na ovo pitanje nije odgovorila, no neki od odgovora bili su: *Procijenite duljinu igrališta prije mjerenja; Cijena jednog sladoleda je 1 kn. Na zimskom sniženju možemo kupiti 6 sladoleda za 5 kn. Koliko sladoleda možemo kupiti za 36 kn ?; Oduzmite njihovu razliku od zbroja brojeva xx; Marko je loptu kupio za 54 kune. Platio je novčanicom od 100 kuna. Koliko kuna mu prodavačica mora vratiti?; Glinom oblikujte geometrijska tijela; Matko ima 7 godina. Sestra je 5 godina starija od njega. Tata je 5 godina stariji od majke. Majka je 3 puta starija od Matkove sestre. Koliko godina imaju sestra, mama, tata? Koliko će imati godina za 10 godina? itd.* Ni hrvatski, ni slovenski učitelji razredne nastave nisu ponudili zadatak koji uključuje stvarno matematičko modeliranje. Neki učitelji nisu dali određeni zadatak, već su samo napisali da se radi o problemskim zadatcima koji uključuju probleme iz svakodnevnoga života, a ti su odgovori bili najbliži točnom.

Budući da smo očekivali slične rezultate zbog provedenoga pokusnoga istraživanja, učiteljima smo dali definiciju matematičkoga modeliranja po Stohlmann i Albarracín, danu ranije u članku.

Zadatak matematičkoga modeliranja

Drugi skup pitanja odnosio se na zadani zadatak koji uključuje matematičko modeliranje. *Problem graha* glasi:

Farmer Jake pokušava odlučiti koji su svjetlosni uvjeti najbolji za uzgoj graha. Kako bi pomogao farmeru Jakeu u donošenju odluke, on uzgaja biljke graha koristeći dva različita svjetlosna uvjeta. Dva svjetlosna uvjeta su: Uzgoj graha na suncu bez ikakve hladovine i uzgoj graha u sjeni.

Slika 1.

Koristeći gornje podatke, odredite koji su svjetlosni uvjeti prikladni za uzgoj graha kako bi se dobio najuspješniji urod. U pismu farmeru Jakeu iznesite svoju preporuku o svjetlosnom stanju i objasnite kako ste došli do te odluke. Nadalje, predvidite težinu graha proizvedenoga 12. tjedna za svaku vrstu svjetla. Objasnite kako ste napravili svoje predviđanje kako bi ga farmer Jake mogao upotrijebiti u drugim sličnim situacijama (English, 2003.).

Nakon problema s grahom uslijedila su pitanja: Ima li zadatak jedinstveno rješenje; Koji bi bio najbolji način za evaluaciju takvoga zadatka; Biste li radili takav zadatak na svojem satu Matematike. Posljednje pitanje bilo je otvorenoga tipa u kojem su morali opisati kako bi oni riješili dani zadatak. Rezultati su prikazani u Tablici 4.

Tablica 4.

Iz rezultata prvoga pitanja vidimo da su učitelji razredne nastave podijeljeni, pri čemu se većina (56,6 %) u cijelosti ili djelomično ne slaže da zadani zadatak ima jedinstevno rješenje, što mi zapravo tražimo. Za usporedbu, čak 43,4 % djelomično se ili potpuno slaže da zadatak ima jedinstveno rješenje.

Na pitanje kako bi bilo najbolje ocijeniti *Problem graha*, učitelji su se složili da pismeni ispit možda nije najbolje rješenje. Ipak, 42,3 % bilo je za grupni rad, a zatim 33,5 % za usmeni ispit. U okviru druge opcije neki od odgovora bili su: razgovor, samovrednovanje, praktični rad, sve gore navedeno, istraživački zadatak, a neki su odgovorili da takav zadatak uopće ne bi ocijenili. 22,7 % učitelja razredne nastave ne bi uključilo takav zadatak u svoje sate Matematike.

S izjavom da bi na satu Matematike radili takav zadatak, većina učitelja razredne nastave iz obje zemlje djelomično se složila (54 %), dok se čak 23,3 % njih u potpunosti složilo s tvrdnjom, odnosno nesumnjivo bi radili takav zadatak na satima Matematike.

Posljednje pitanje vezano uz *Problem graha* bilo je opisati kako bi oni riješili taj zadatak. Nažalost, većina učitelja nije odgovorila na ovo pitanje (81,2 % iz Hrvatske, 74,7 % iz Slovenije). Među onima koji su odgovorili mogli su se pronaći neki od sljedećih odgovora: *Procijenio bih usporedbom prethodnih tjedana; Pogledao bih razliku u kg za svaki prethodni i sljedeći tjedan; Našao bih model povećanja usjeva i predvidio rješenja; Zbrojite kako biste izračunali koliko je kg graha dao svaki red. Sunčana strana daje znatno više prinosa. Zaključak: Grah je biljka koja zahtijeva sunčan teren; Ja bih izračunao srednju vrijednost. Preporučio bih uzgoj graha na sunčanoj gredici jer daje najbolji urod; Zbrojio bih kg sa sunčane i sjenovite strane; Tražio bih obrazac i predvidio.*

Tvrdnje u svezi s matematičkim modeliranjem

Sljedeći skup su tvrdnje u vezi s matematičkim modeliranjem. Učiteljima smo predstavili 6 tvrdnji koje ćemo predstaviti u nastavku, a rezultate prikazujemo u tablicama 5 i 6. Za svaku tvrdnju učitelji su imali mogućnost označavanja: u cijelosti se ne slažem/ djelomično se ne slažem/ niti se slažem niti se ne slažem/ djelomično se slažem/ u cijelosti se slažem. Rezultati u vezi s tvrdnjama C, D i E prikazani su u Tablici 5.

Tvrdnja C*: Mislim da matematičko modeliranje nije prikladno za osnovnu školu, već mu je mjesto u srednjim školama.*

Tvrdnja D*: Matematičko modeliranje je točan, formalan proces ili zbirka formula i pravila koja se moraju primijeniti.*

Tvrdnja E: *Iako aktivnosti matematičkoga modeliranja poboljšavaju sposobnost učenika za rješavanje problema, smatram da postoji previše prepreka za uključivanje takvih aktivnosti u moje sate matematike.*

Tablica 5.

U tvrdnji C, gdje se navodi da je matematičko modeliranje ipak prikladnije za srednje škole, nastavnici nisu bili jednoglasni. Njih 50,8 % iz obje zemlje nije se djelomično ili u cijelosti složilo s tvrdnjom, što znači da misle da matematičko modeliranje također ima svoje mjesto u osnovnoj školi. Nešto manji postotak učitelja, 31,6 %, smatra da matematičko modeliranje nije za osnovne škole. Dosta učitelja, gotovo 18 %, nije se odlučilo ni za jednu opciju. Jedan od razloga za različita mišljenja među učiteljima i nespremnost da se matematičko modeliranje uključi u osnovne škole vjerojatno je posljedica rezultata pitanja jesu li se ikada susreli s konceptom matematičkoga modeliranja. Gotovo 80 % odgovorilo je negativno, pa zbog nedostatka obrazovanja o samom matematičkom modeliranju nisu ga spremni uključiti u osnovno obrazovanje. Rezultati za tvrdnju D zapanjujući su u negativnom smislu. Naime, iako smo u istraživanju naveli točnu definiciju matematičkoga modeliranja, više od trećine nastavnika iz obje zemlje (34,8 %) ili se djelomično ili u cijelosti složilo s tvrdnjom, što je kontradiktorno samoj definiciji matematičkoga modeliranja, a 28 % njih se niti slaže niti se ne slaže s danom tvrdnjom. Također primjećujemo da se 34,7 % slovenskih učitelja i 40,3 % hrvatskih učitelja razredne nastave djelomično ili u cijelosti slaže s tvrdnjom E, iako znamo da matematičko modeliranje poboljšava mnoge sposobnosti učenika, poput rješavanja problema. Jedan od razloga za takav rezultat može biti i nedovoljno obrazovanje o matematičkom modeliranju.

Rezultati u vezi s tvrdnjama F, G i H prikazani su u Tablici 6.

Tvrdnja F: *Mislim da je matematičko modeliranje neophodna vještina u 21. stoljeću za svakog učenika.*

Tvrdnja G: *Matematičko se modeliranje može poučavati u posebnim okruženjima, unatoč svim poteškoćama povezanim s učenjem i poučavanjem modeliranja.*

Tvrdnja H: *Matematičko modeliranje kreativan je proces relevantan za društvo, kroz koji pojedinac traži najbolje rješenje za problem iz stvarnoga života.*

Tablica 6.

Iz rezultata vidimo da se 84,9 % slovenskih i 84,2 % hrvatskih učitelja razredne nastave djelomično ili u cijelosti slaže s tvrdnjom F koja kaže da je matematičko modeliranje nužna vještina u 21. stoljeću za svakoga učenika, što je, naravno, ohrabrujući rezultat. Učitelji razredne nastave iz obje zemlje također su se složili s tvrdnjom G koja kaže da se matematičko modeliranje može poučavati u specifičnim okruženjima, unatoč svim poteškoćama povezanim s učenjem i modeliranjem nastave. 48,7 % se djelomično složilo s tvrdnjom, dok se njih 22,9 % u cijelosti složilo. Na posljednju tvrdnju H koja kaže da je matematičko modeliranje kreativan proces relevantan za društvo u kojem pojedinac traži najbolje rješenje za problem iz stvarnoga života, 87,3 % hrvatskih učitelja razredne nastave i 90,5 % slovenskih učitelja složilo se u cijelosti ili djelomično .

Uvođenje matematičkoga modeliranja u učionicu

Posljednja pitanja našega istraživanja bila su pitanja o mišljenjima učitelja o uvođenju matematičkoga modeliranja u učionicu. Rezultate smo prikazali u Tablici 7.

Tablica 7.

Za prvo pitanje: Mislim da sam dovoljno upoznat i obrazovan da poučavam matematičko modeliranje u razrednoj nastavi Matematike, s obzirom na prethodne rezultate istraživanja, rezultat nas nije iznenadio. 25 % učitelja iz Hrvatske i Slovenije odgovorilo je potvrdno, dok je većina, 75 % odgovorila negativno. Očekivali smo takav rezultat zbog rezultata iz ranijih odgovora u anketi. Naime, 79,9 % učitelja izjavilo je da se nikada nisu ni susreli s konceptom matematičkoga modeliranja. Sljedeće otvoreno pitanje nadovezalo se na prethodno pitanje. Zamolili smo učitelje koji su na prethodno pitanje odgovorili ne, da napišu razlog zašto je tome tako. Neki od odgovora bili su: *Nemam dovoljno iskustva, nikada se s tim nisam susretao, nedovoljna količina primjera/zadataka ove vrste, premalo obrazovnih i nastavnih sadržaja za nastavnike o ovoj temi.* Odgovori hrvatskih i slovenskih učitelja razredne nastave nisu se razlikovali, te su bili: nikada nisu čuli za matematičko modeliranje; nisu imali priliku educirati se o ovoj temi; ili čak ne znaju gdje pronaći ili tražiti takve edukacije ili resurse koje bi mogli uključiti u nastavu Matematike. Sljedeće je pitanje bilo smatraju li da bi u razrednu nastavu Matematike trebalo uvesti više zadataka/aktivnosti koji uključuju matematičko modeliranje. 86,5 % učitelja iz Hrvatske i Slovenije složilo se da bi trebalo uvesti više takvih zadataka/aktivnosti. Samo mali dio učitelja, 13,5 %, odgovorilo je negativno na ovo pitanje. Budući da znamo da je matematičko modeliranje minimalno uključeno u nastavne programe, a za takve aktivnosti predviđeno je još manje nastave, pitali smo ih bi li se bavili matematičkim modeliranjem na satu Matematike ako bi za to imali točno predviđene sate nastavnim planom i programom. Učitelji iz obje zemlje složili su se o tom pitanju, a 90,5 % je reklo da bi u tom slučaju radili matematičko

modeliranje. Samo 9,5 % njih, unatoč planiranim satima, ne bi radilo aktivnosti ili zadatke matematičkoga modeliranja. Zanimalo nas je i s kakvim se poteškoćama susreću pri obavljanju zadataka/aktivnosti koje uključuju matematičko modeliranje ili ako ne rade takve zadatke/aktivnosti, što misle da bi bile poteškoće s kojima bi se susreli. Budući da znamo da aktivnosti matematičkoga modeliranja nije lako izvesti u razredu, zanimalo nas je što bi najviše smetalo našim učiteljima. Neki od odgovora koje smo dobili bili su: *premalo sati nastave Matematike, nedostatak interesa učenika za razmišljanje i nedostatak motivacije za takve zadatke, značajne razlike u znanju među učenicima, nedostatak nastavnoga materijala za tu temu, problemi u kritičkom razmišljanju učenika, nedovoljno obrazovanje samih učitelja, previše učenika u razredu*. Za većinu učitelja iz Hrvatske i Slovenije glavni je problem bio nedostatak vremena, što nije iznenađujuće. Nadalje, najviše ih je zasmetao nedostatak interesa učenika za takve zadatke, naime izjavili su da se učenici teško koncentriraju na duže vrijeme, teže razmišljaju, a većina uvijek traži jedinstveno rješenje. Jedan od značajnijih problema je nemogućnost pronalaženja takvih problema/zadataka ili nedostatak obrazovanja za kreiranje i osmišljavanje takvih aktivnosti/zadataka. Drugi problem koji navode je različito predznanje i razina znanja učenika iz matematike. Smatraju da bi samo „bolji" učenici mogli riješiti takve zadatke. Posljednje pitanje istraživanja bilo je bi li pohađali radionice za učitelje razredne nastave o matematičkom modeliranju kad bi one bile dostupne. 90,6 % hrvatskih i 92,8 % slovenskih učitelja odgovorilo je pozitivno, dok 9,4 % hrvatskih i 7,2 % slovenskih učitelja razredne nastave ne bi pohađalo. Smatramo da je ovaj rezultat odličan, s obzirom na to da većina učitelja do našega istraživanja nije čula za matematičko modeliranje u nastavi.

Diskusija

Rezultati istraživanja govore sami za sebe. Očigledno je da sudionici u potpunosti nisu upoznati s matematičkim modeliranjem. Vidjeli smo da gotovo 80 % učitelja razredne nastave iz Hrvatske i Slovenije nikada nije bilo upoznato s pojmom matematičkoga modeliranja, dok je gotovo 12 % njih jedva upoznato s pojmom matematičkoga modeliranja. Nešto više od tri četvrtine učitelja razredne nastave iz Slovenije i Hrvatske priznalo je da uopće ne radi zadatke/aktivnosti koje uključuju matematičko modeliranje, što nije bilo iznenađujuće, zbog rezultata prethodnoga pitanja, gdje su odgovorili da ne znaju što je matematičko modeliranje u razrednoj nastavi. Također smo vidjeli da su sudionici koji su dali jedan primjer zadatka koji uključuje matematičko modeliranje, većinom napisali jednostavne tekstualne zadatke koji zahtijevaju primjenu neke osnovne aritmetičke operacije, bez potrebe za korištenjem bilo kakvoga matematičkoga modeliranja. Rezultati su u skladu sa sličnim istraživanjima u drugim zemljama. Većina učitelja matematike u osnovnim školama ima pogrešna shvaćanja o matematičkom modeliranju i procesu modeliranja zbog nedostatka znanja o matematičkom modeliranju (Blum, 2012). Nadalje, većina učitelja možda je imala ograničenu izloženost matematičkom modeliranju i imaju nejasne

ideje o tome što se podrazumijeva pod pojmom matematičko modeliranje. Čini se da učitelji također manje znaju o matematičkim procesima, zadatcima, pedagogiji i procjenama povezanim s matematičkim modeliranjem (Gaston i Lawrence, 2015).

Nakon zadane definicije matematičkoga modeliranja i specifičnoga zadatka koji uključuje matematičko modeliranje, rezultati su pokazali da nešto više od polovice sudionika iz Slovenije i Hrvatske smatra da zadani zadatak nema jedinstveno rješenje, ali nešto više od 43 % vjeruje da ima jedinstveno rješenje. Odgovor da zadatak koji uključuje matematičko modeliranje ima jedinstveno rješenje suprotno je cilju matematičkoga modeliranja. Uz pomoć ove vrste zadataka učenici mogu razmjenjivati ideje o mogućim rješenjima budući da zadatak nije ograničen jedinstvenim rješenjem. Na pitanje koji bi bio najbolji način evaluacije takvoga zadatka, nešto više od 40 % sudionika iz Slovenije i Hrvatske složilo se da će to biti grupni rad, dok se nešto više od trećine sudionika odlučilo za usmeni ispit. Zanimljivo je da se gotovo 5 % sudionika odlučilo za opciju da se takav zadatak ne može ocijeniti. Takvi su rezultati u korelaciji s drugim studijama, na primjer Spandaw i Zwaneveld (2009) navode da uobičajeno napisani testovi nisu prikladni za vrednovanje viših vještina, poput modeliranja. Bilo bi bolje koristiti druge alternative, poput grupnoga rada, domaćih zadaća ili usmenih ispita. Na pitanje bi li radili takav zadatak na satima Matematike, nešto više od tri četvrtine sudionika odlučilo je da hoće, ali nešto manje od četvrtine njih to ne bi učinilo. Jedan od ohrabrujućih rezultata istraživanja bio je da nešto manje od 90 % sudionika iz obje države voli predavati matematiku. Sljedeći rezultat nije bio toliko očekivan i ohrabrujući. Iako su imali pisanu definiciju matematičkoga modeliranja, više od trećine sudionika iz Slovenije i Hrvatske složilo se s tvrdnjom da je matematičko modeliranje točan, formalan proces ili zbirka formula i pravila koja se moraju primijeniti, dok se malo manje od trećine njih nije složilo s tvrdnjom. Na tvrdnju ima li matematičko modeliranje mjesto u osnovnoj školi ili je to samo za srednje škole, polovica sudionika složila se da bi matematičko modeliranje trebalo uključiti u osnovnu školu. Kao što English and Watters (2004) navode da osnove matematičkoga modeliranja trebaju i mogu započeti u osnovnoj školi gdje učenici već posjeduju osnovne kompetencije na kojima se modeliranje može razviti. U tvrdnji koja kaže kako, *iako aktivnosti matematičkoga modeliranja poboljšavaju sposobnost učenika za rješavanje problema, smatram da postoji previše prepreka za uključivanje takvih aktivnosti u moje sate matematike*, oko 40 % sudionika složilo se s tvrdnjom, a isto toliko se nije složilo. Takav rezultat mogao bi biti posljedica činjenice da nisu dovoljno obrazovani o matematičkom modeliranju pa stoga osjećaju neku vrstu odbojnosti prema obavljanju ili uključivanju takvih zadataka u svoje sate Matematike. Bolji rezultat bio je s tvrdnjom da je matematičko modeliranje neophodna vještina u 21. stoljeću za svakog učenika, gdje se više od četiri petine sudionika složilo s tom tvrdnjom. Sličan smo rezultat dobili tvrdnjom da je učiteljeva sposobnost pronalaska više rješenja zadatka važan pokazatelj učenikovih sposobnosti. Nešto više od tri četvrtine učitelja razredne nastave složilo se s tom tvrdnjom, o čemu svjedoči literatura. Kao što Brown i Ikeda (2019) navode,

učitelj ima važnu ulogu u podupiranju učeničkoga angažmana u matematičkom modeliranju, razmišljanju o njemu, odlukama vezanim uz uporabu tehnologije i matematičko modeliranje te međudjelovanju među njima. Unatoč obeshrabrujućim rezultatima nekih prethodnih pitanja u istraživanju, gdje učitelji zapravo nisu pokazali spremnost da uključe matematičko modeliranje u svoje učionice, s tvrdnjom da je matematičko modeliranje kreativan proces relevantan za društvo, kroz koji pojedinac traži najbolje rješenje za problem iz stvarnoga života, gotovo 90 % sudionika se složilo. Sličan rezultat također se često navodi u literaturi, gdje se npr. Zbiek i Conner (2006) navode da matematičko modeliranje omogućuje učenicima povezivanje učioničke matematike sa stvarnim svijetom, pokazujući primjenjivost matematičkih ideja. Bahmaei (2011) također navodi da s obzirom na stvarni problem, učenici trebaju razumjeti stvarnu situaciju i donijeti pretpostavke kako bi osmislili matematičku metodu za rješavanje problema. Očekivani su rezultati pitanja o tome misle li sudionici da su dovoljno obrazovani da mogu uvesti matematičko modeliranje u razrede. Tri četvrtine ispitanika odgovorilo je da misli da ne. Blum (2012) navodi da je jedan od načina da se budućim učiteljima pruži potrebno stručno znanje nuđenjem specifičnih seminara o modeliranju koji se već nalaze na sveučilištu, uz obvezno vlastito iskustvo u nastavi. No, iako sudionici smatraju da ni sami nisu dovoljno obrazovani, gotovo 90 % njih smatra da bi u osnovno obrazovanje trebalo uključiti više zadataka/aktivnosti koji uključuju matematičko modeliranje. Kao jedan od razloga zašto misle da bi bilo teško uključiti matematičko modeliranje u sate Matematike bilo je vrijeme. Kang i Noh (2012) navode kako se prema nastavnom planu i programu matematičkom modeliranju u nastavi daje premalo ili nimalo vremena. Zbog nedostatka vremena za matematičko modeliranje, učitelji izbjegavaju matematičko modeliranje u učionici. Drugi najčešći razlog protiv matematičkoga modeliranja bio je to što misle da učenici nisu spremni za takve vrste zadataka ili aktivnosti. Takvi se rezultati često pojavljuju u literaturi. Poznato je da učitelji (lažno) pretpostavljaju da je većini učenika modeliranje izazovno ili teško, pa učitelji rijetko uključuju aktivnosti ili zadatke modeliranja u svoju učionicu (Spandaw i Zwaneveld, 2009; Asempapa, 2015; Brown, 2019). Rezultat posljednjega pitanja definitivno je prvi korak napretka u uključivanju matematičkoga modeliranja u razrednu nastavu. Preko 90 % sudionika iz obje zemlje željelo bi pohađati radionice/edukacije o matematičkom modeliranju u razrednoj nastavi. Dakle, iz rezultata se vidi da su učitelji željni i spremni za obrazovanje o matematičkom modeliranju, ali jednostavno nemaju mogućnosti u ovom trenutku.

Zaključak

Primjena i modeliranje su važni, a učenje i modeliranje zahtjevni. To implicira da se moraju uložiti posebno veliki napori kako bi se modeliranje učinilo dostupnim učenicima (Blum, 2012). Iako su Slovenija i Hrvatska susjedi sa zajedničkom prošlošću, postoje značajne razlike u nastavi Matematike (Sabo i Lipovec, 2017). Stoga smo očekivali razlike u znanju učitelja, ali kako su rezultati pokazali, o temi matematičkoga

modeliranja, učitelji iz obje zemlje na istoj su razini. Rezultati su pokazali da sudionici ne znaju što je točno matematičko modeliranje u razrednoj nastavi, pa stoga ne koriste i ne rade zadatke/aktivnosti koji uključuju matematičko modeliranje na sate Matematike. Neki od razloga zašto učitelji tako rijetko obavljaju takve zadatke kao što navodi Borromeo Ferri (2010) su vremenska ograničenja i njihova percepcija da su zadatci matematičkoga modeliranja zahtjevni ili složeni. Istraživanja pokazuju da kada učenici imaju priliku modelirati matematiku, oni su u stanju matematički razmišljati, poboljšati svoje sposobnosti rješavanja problema i povezati se s problemima u stvarnom svijetu (Bleiler-Baxter i sur., 2017.). Bez sumnje, matematičko modeliranje izazovno je i kompliciranо za učitelje, ali mnoge studije pokazuju i ukazuju na njegove prednosti za učenike. Možemo također izvijestiti o optimističnim rezultatima jer su učitelji koji su sudjelovali u istraživanju jako zainteresirani za obrazovanje o matematičkom modeliranju i njegovu integraciju u sate Matematike. Ipak, 91,4 % njih željelo bi pohađati neku vrstu edukacije iz matematičkoga modeliranja kad bi ona postojala. Taj je stav već prvi korak prema većem uključivanju matematičkoga modeliranja u niže razrede osnovne škole.