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PREVALENCE OF DIVISION MODEL AND ITS IMPLEMENTATION IN MATHEMATICAL TEXTBOOKS

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A b s t r a c t

Children have more dificulties with division than with other arithmetic operatios. One of the causes could be a dual nature of division, which comprehends partition and quotition as models of division. The study investigates the problem of predominance of a division model in different age groups, from childhood to adulthood. Moreover, this predominance was looked at also in relation to gender, socio-economic status, learning ability, type of school/environment and mathematical textbooks. The results have shown that the partitive division model is prevalent in all age groups regardless of the variables. We also presented a short analysis of mathematical textbooks for primary school in our country, as we believe that the school practices should take the partitive model into account, if pupils' experience is to be taken into consideration. Practical guidelines for good school practice are also given.
Keywords: *division; division concept; division model; partition; quotition; teaching division*

1. The arithmetic operation of division – theoretical framework

Division is required for many of the processes used in everyday situations and it is also critical in problem solving. Therefore, a secure understanding of division, along with other forms of multiplicative thinking, is essential for work with fractions, ratios, algebra and further mathematics that marks the transition from the arithmetical thinking of the primary school to the more advanced thinking of the secondary curriculum and beyond (Lamb and Booker, 2003). For fully understanding of arithmetic operation of division, it is necessary to realize that there is connection between multiplication and division. Therefore, multiplication and division are inverse operations. One of the properties of multiplication and division is the order property (commutative property). In general that means, the order of the numbers makes no difference. When we multiply, it is obvious that 4 8 is the same as 8 4. Product is in both examples 24. But there are cases for which multiplication is "psyhologically non-commutative" (Greer, 1987). In these cases the number of groups and the number of objects in each group play quite different roles. As a consequence, two types/models of division are distinguishable. Division by the number of groups is called partition[1] (fair-sharing), while division by the number in each group is called quotition[2] (measurment or sometimes repeated subtraction). In each of the multiplication situations, one of the quantities can be identified as the multiplicand and the other as the multiplier. Therefore, the concepts of partitive and quotitive division have been extended by defining partition generally as division by the multiplier and quotition as division by multiplicand (Greer, 1987). We can say, that symbols representing division have one meaning in quotitive and another in partitive division.

Vergnaud (1983, cited in Neuman, 1999, p. 103) demonstrates how multiplication concerns relations within one or between two measurement variables, showing that it is more common to think within one variable than between two. It is similar with division; quotitive division concerns one measurement variable and partitive division two. In the quotitive division problems, the dividend as well as the divisor concern one thing. On the other hand, in partitive division, the dividend and divisor are two different things. It could be said, that quotitive as well as partitive division have a dual partitive-quotitive caracter (Neuman, 1999). Children think of both kinds of division as partitive, but deal with both kinds in a quotitive way. The reason why they prefer quotitive model for computation may be related to their first meeting with arithmetic at school when dealing with one variable (Vergnaud, 1983).

Many researchers agree tht division concepts begin developing in students as early as age seven but are not clearly understood by most students until age 18. Children's early multiplication and division knowledge results from cognitive reorganisation of their counting, addition and subtraction strategies, and builds on number word sequences, combining and partitioning. However it differs from addition and subtraction mainly because the former incorporates the ability to use equal groups as "abstract composite units" (Steffe, 1992). This means that the child focuses on the unit structure of a numerical composite e.g. one ten, rather than on the unit items e.g. ten ones. The development of multiplication and division as inverse processes forms the basis of a developmental model of composite structure (Mulligan and Wright, 2000). It has been found that division is not necessarily more difficult than multipication. For example, it may be easier for a child to share counters into equal groups and count the number of groups rather than keep track of a largere number of composite groups for multiplication. Teaching children to share and group small numbers into equal parts can also faciliate the development of multiplication and division strategies (Mitchelmore and Mulligan, 1996; Mulligan and Wright, 2000).

Some researches investigated young children's intuitive models of division. Mulligan and Mitchelmore (1996) identified four intuitive models for division (direct counting, repeated subtraction, repeated addition and multiplicative operations) and differentiated intuitive models from sematic structures of word problems or situations. Students use a different set of intuitive models, which they can apply to both multiplication and

[1] We have 21 apples and 7 baskets. How many apples can come in one basket?

[2] We have 21 apples. We put them into baskets containing 7. How many baskets did we use?

division problems of various semantic structures. The reason for that appears to lie in the fact in every multiplication and division situation, "there must be equal–sized groups".

Research on primitive models of multiplication and division was also done by Fischbein, Deri, Nello and Marino (1985). These authors presented, that one primitive model is associated with multiplication (repeated addition), but two with division: The partitive ("sharing") model and the quotitive ("measurement") model. Yet, after more careful analysis of their results, the authors came to the conclusion that the quotitive model is acquired with instruction, and that there is only one primitive model for division: the partitive. Other studies (Murray, Olivier and Human, 1991; Mulligan 1992) did not confirm the upper thesis. Mulligan (1992) showed that the young pupils in her study preferred an additive building-up model for partitive as well as for quotitive division. Even if not related to repeated subtraction, this model is a quotitive measurement model.

But why is that pupils have more difficulties with division than with other arithmetic operations? The purpose of a division problem is to work out the quotient: somebody trying to solve a division problem must understand that s/he has to divide the dividend by the divisior to produce the quotient. This does not imply an understanding of the terms dividend, divisor and quotient (Squire and Bryant, 2002). What is also harder when dividind than dealing with other arithmetic operations, are two models of division. The division is the only operation which concept consists of two different models. Good undestanding of only one of the models is not enough for dealing with division. The results of researches on relative difficulty of partitive and quotitive division problems are mixed. Some suggest that quotitive problems are easier (Burgeois and Nelson, 1977; Gunderson, 1955; Zweng, 1964, cited in Squire and Bryant, 2003, p. 367), others (Correa et al., 1998; cited in Squire and Bryant, 2003, p. 367) found that partitive problems were easier and Squire and Bryant (2002b) found no difference between the two types of division. On the other hand, regardless of relative difficulty of division problems, it is important for children to experience the difference between two types of division and if not, teacher should draw their attention to it.

2. Method

Aim and research questions

The aim of the present study was to define and explore predominance of partitive and quotitive division in different age groups, from childhood to adulthood. Moreover, this predominance was looked at also in relation to gender, socio-economic status, learning ability, type of school/environment and mathematical textbooks.

Three research questions were formulated:

1. Which model of division (partitive or quotitive) is prevalent in different age groups?

2. Do variables like gender, socio-economic status, learning ability, type of school/environment and mathematical textbooks have influence on prevalence of partitive or quotitive division?

3. Is partitive and quotitive division equally present in mathematical textbooks and if not, which model of division (partitive or quotitive) is prevalent?

Participants

Five different age groups participated. First consisted of 70 pupils grade 3-6 (age 8 to 11), second consisted of 39 pupils grade 7-9 (age 12 to 15), third group consisted out of 27 students in secondary education (age 16 to 18), 28 students in college (age 22 to 25) were joined in fourth group and the fifth group consisted of adults age 30 to 60. Altogether 189 participants were interviewed. All participants were chosen by chance.

Design of the present study

Each participant was interviewed individually. Main task, was to write a story problem, which will result division computation. For younger pupils (age 8) we made a task a little easier, so they were asked to write a story in accordance with computation 15 : 3 = 5. Participants were also asked some other questions regarding:

- *education of their parents* (for pupils and students) or *their education* (for adults)
- *type of primary school* – rural, suburban or urban or *type of secondary school* – technical and vocational high school or upper secondary school or *type of environment* – rural or urban (for adults)
- *final mark in mathematics in previous school year* (for pupils and seconday school students) or *mark in final examination in mathematics* (for college students and adults)
- *mathematical textbook*, which they are using (only for children in grade 3 to 6)

All story problems were analysed and classified by type of division (partition or quotition).

3. Results

Prevalence of division model in connection with some variables

Analysis of story problems showed prevalence of partitive division model (figure 1). The majority of participants (90,5 %) wrote a story problem with partitive division. It is clear that participants when dealing with division, think of partitive division. This could be so, because partition is more closely linked to everyday experiences regarding division.

Figure 1 *Prevalence of division model*

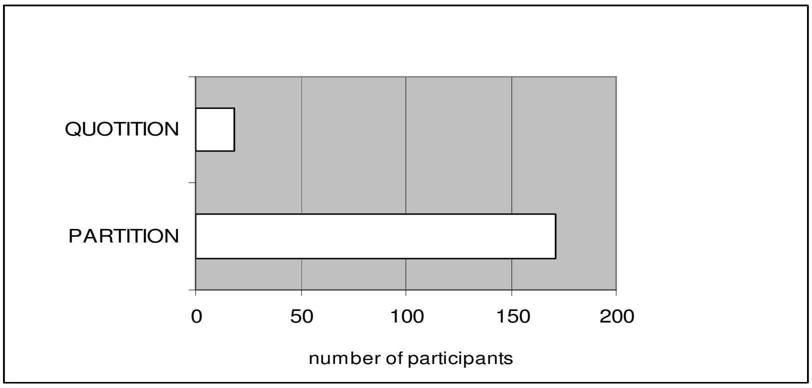
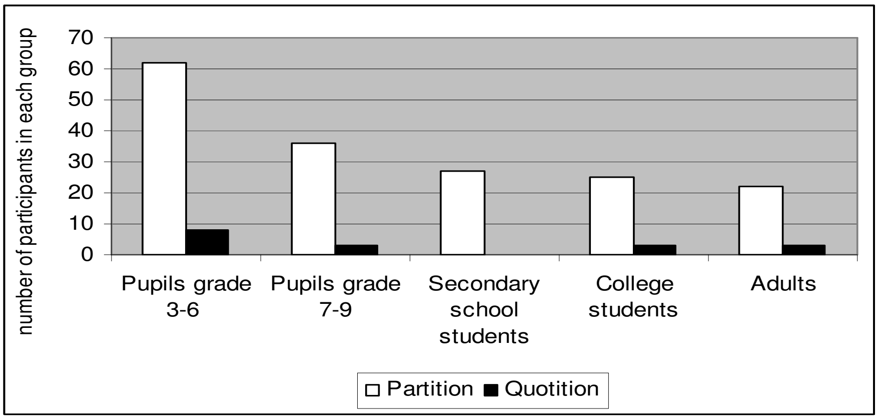
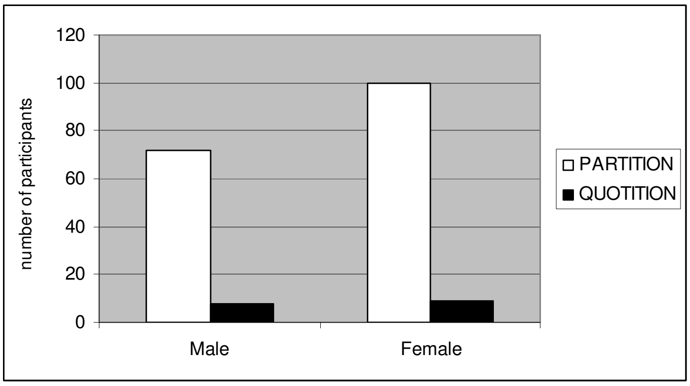


Figure 2 *Prevalence of division model and age*



Furthermore, the analysis showed that partition is prevalent in all age groups, which means that age has no influence on prevalence of division model [2↑ = 6,784 < χ^2 (α = P = 0,05; g = 4) = 9,488]. When analysing pupils in grade 3-6, prevalence of quotitive division was shown in 11,4% of pupils. Further analysis showed that share of pupils in grade 3-6 with prevalent partitive division is greatest in pupils in grade 3 and 4 (year 8 and 9). All secondary school students wrote story problems with partitive division. When looking at figure 2, we can see that the share of participants with prevalent quotitive division declines with age and is lowest in secondary school students, afterwards this amount increases in college students (10,7%) and reaches the top in adults (12%). The main reason for prevalence of partitive division in all age groups are everyday experiences regarding divison. This experiences are obiviously the same regardless of age.

Figure 3 *Prevalence of division model and gender*



When looking at gender in connection with prevalence of division model (figure 3), results are the same – gender does not influence on prevalence of division model [$\chi^2 = 0{,}171 < \chi^2$ (α = P = 0,05; g = 1) = 3,841]. Partition is division model male and female think of when dealing with division. This could also mean that everyday experiences with division are the same regardless of gender.

Our interest was also to research if socio-economic status, learning ability, type of school/environment and mathematical textbooks have influence on prevalence of partitive or quotitive division. Data showed that selected variables do not have influence on prevalence of division model. It was always partitive division which was prevalent.

From figure 4, we can see that partition is prevalent division model regardless of education of participants or their parents [$\chi^2 = 5{,}34 < \chi^2$ (α = P = 0,05; g = 3) = 7,815]. If we can say that partition is more closely linked to everyday experiences and quotition is not, than we can also assume that quotition requires more advanced and flexible thinking. Therefore it is interesting that the share of participants with only primary education and prevalent quotition model is quite high (16,67%).

Figure 4 *Prevalence of division model and socio-economic status*

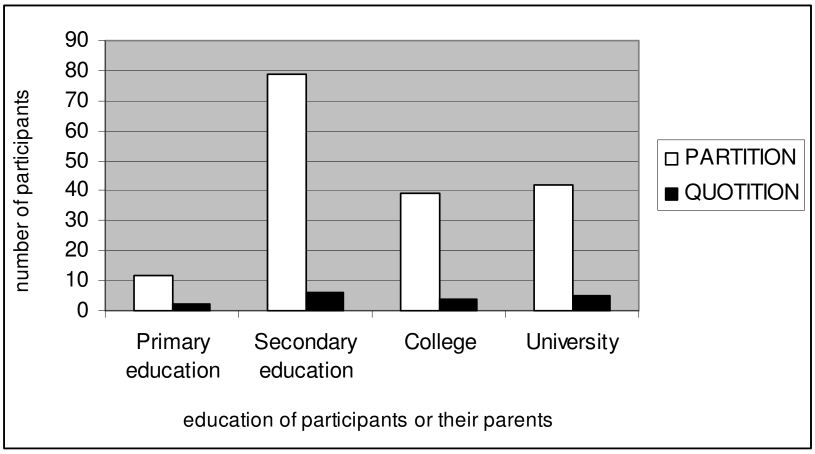
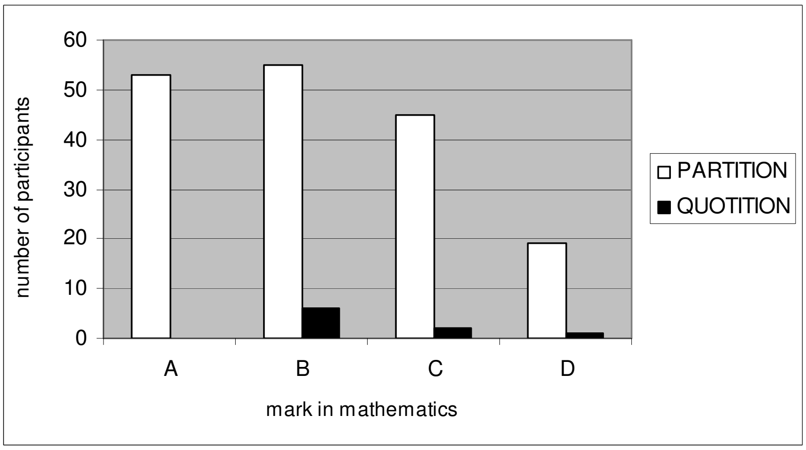


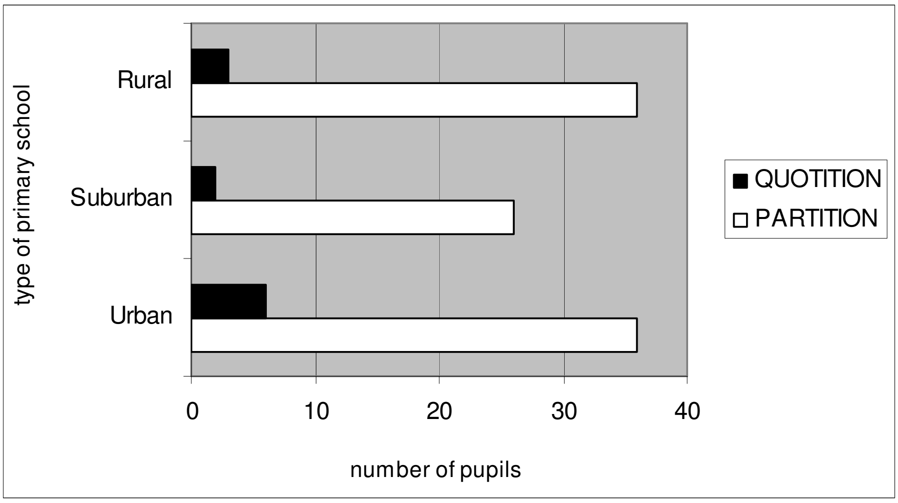
Figure 5 *Prevalence of division model and learning ability*



Results have shown that learning ability, which was measured with mark in mathematics, does not have influence on prevalence of division model [2↑ = 2,574 < χ^2 (α = P = 0,05; g = 3) = 7,815]. Partitive division model is prevalent regardless of the mark. There is a bit higher share of participants with grade A (15,1%) or B (10,9 %) and prevalent quotitive division model. If we earlier said that prevalence of quotition could indicate an advanced and more flexible thinking, then we can say that this kind of ability is present also at participants with lower grade in mathematics.

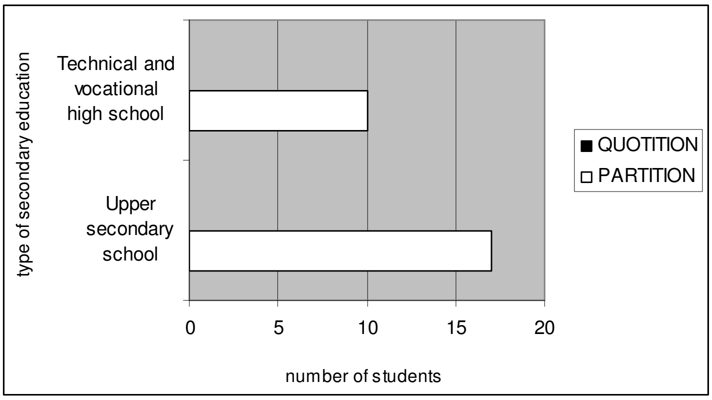
We also wanted to know, if there is some kind of connection between prevalence of division model and type of school which pupils and secondary school students are attending.

Figure 6 *Prevalence of division model and type of primary school*



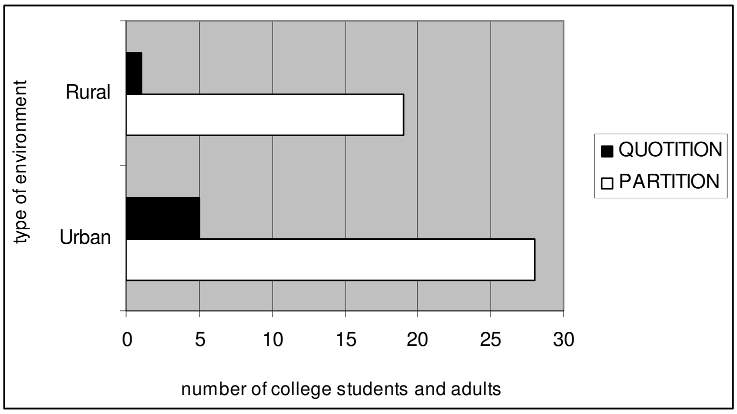
In figure 6, we can see prevalence of partitive division model regardless of the type of primary school [2↑ = 1,296 < χ^2 (α = P = 0,05; g = 2) = 5,991]. There is also higher share of pupils with prevalent quotitive division, which are attending urban or suburban primary school.

Figure 7 *Prevalence of division model and type of secondary school*



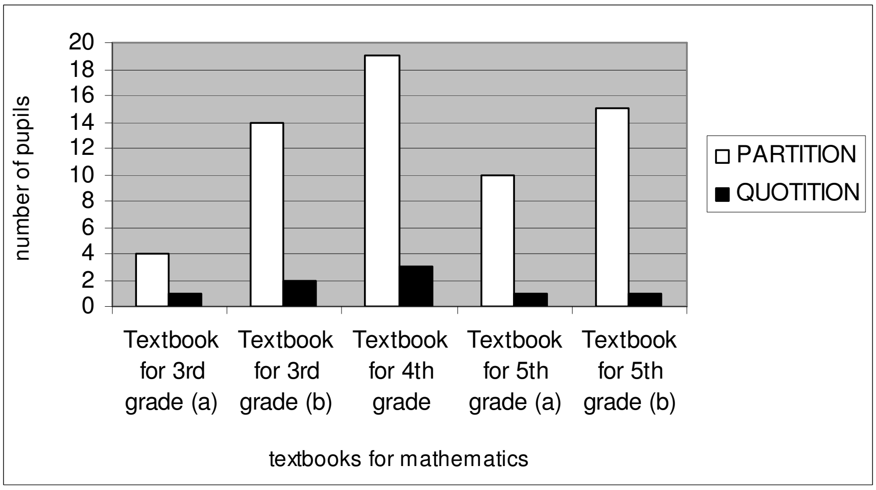
It is very interesting that secondary school students does not show any prevalence of quotitive division model. Partition is prevalent to all students in secondary education regardless of the type of school (technical, vocational or upper secondary school) [2↑ = 0 < χ^2 (α = P = 0,05; g = 1) = 3,841].

Figure 8 *Prevalence of division model and type of environment*



We asked college students and other adults where they live, so we could see, if there is any connection between type of environment and prevalence of division model. We got the same results as with other variables. Partition is prevalent regardless of the type of environment [2↑ = 1,424 < χ^2 (α = P = 0,05; g = 1) = 3,841]. = P = 0,05; g = 4) = 9,488].

Figure 9 *Prevalence of division model and textbooks for mathematics*



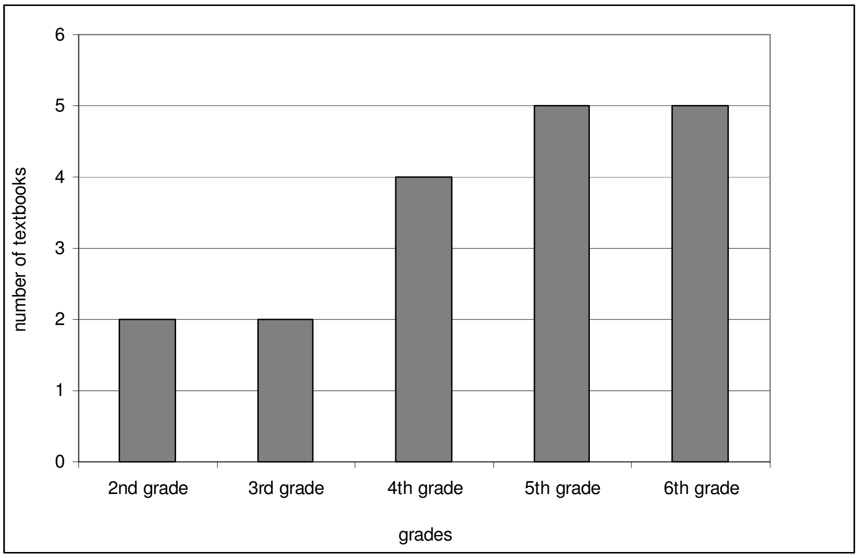
We can also see high prevalence of partitive division model in connection with Slovenian textbooks for mathematics (figure 8). When we introduce pupils into arithmetic operation of division, we do it with quotitive division model. Therefore it is interesting a high share of pupils in third grade with prevalent partitive division model. On the other hand, high share of third grade pupils with prevalent quotition is expected.

This variable shows, how important are everyday experiences with division; so important that they outshine the impact of textbooks and formal schooling. We can say that textbooks do not have an impact on prevalence of division model [2↑ = 0,99 < χ^2 (α

Implementation of division concept in Slovenian textbooks for mathematics

Among other things, our study pointed out that mathematical textbooks do not have an influence on prevalence of division model. Regardless of mathematical textbooks, partitive division model was prevalent to majority of pupils. In connection with these results, our interest was also *if partitive and quotitive division are equally present in mathematical textbooks and if not, which model of division (partitive or quotitive) is prevalent in mathematical textbooks*. For this purpose, we reviewed all verified Slovenian textbooks for mathematics in grades 2 to 6 for school year 2007/2008. We centered on chapters that deal with division and story problems that appear in these chapters.

Figure 10 *Number of verified textbooks by grades*



In our country arithmetical operation of division is introduced in second grade and in connection with repeated subtraction. Strategy of repeated subtraction is only possible with quotitive story problems. There is another reason for indroducing pupils first with quotitive division. When teaching younger children, concrete materials like marbles are used. When drawing a picture of quotitive distribtion, we encircle items so that we get equal subsets. This way of making a drawing is easier that making a drawing for partitive division task.

All mathematical textbooks for second and third grade indroduce division only with quotitive story problems. Here is an example: Mark has 12 apples. He put them into bags containing 3 apples each. How many bags did Mark use? Then pupils explain the quotitive situation with marbles and the movement of them. They make four groups of tree marbles. Then they usually draw their processes of distribution on paper. After encircling marbles, they get four units containing three individual units. Students are expected to interpret distribution activity with a picture signifying the result of the distribution. The answer of the task is nuber of units. At last, pupils write a computation, based on repeated subtraction. For example: 12 – 3 – 3 – 3 – 3 = 0. Then they count, how many times the whole (12) is measured of in sets of three. The answer is 4.

In third grade knowledge of division expands. Pupils learn multipilcation tabel and along with it, they divide. When they learn mathematical expression »12 divided by 3« and method for calculation as multipilcation, they stop using repeated subtraction. Partitive division tasks appear in third grade, because pupils are not dependent on making drawings or on strategy of repeated subtraction. However, quotition and partition tasks are still not equally present. Partition tasks appear rarely in textbooks for third grade, except in chapters "fair-sharing", which clearly indicates on partition problems. Therefore we must ask ourselves, why it is, that partitive divison model is prevalent for pupils in third grade. It is clear, that only early experiences with division have influence on prevalence of division model and we already excluded influence of textbooks. Regardless of story problems present in lessons, partitive division model in present in mental structures of children. If formal education in mathematics could have influence on prevalence of division model, this would be seen here, in third grade. In spite of all that, we can not make an assertion that formal education could not change the present state. This might be possible with consistently present both models of division and with explaining differences between quotition and partition (Lutovac, 2007).

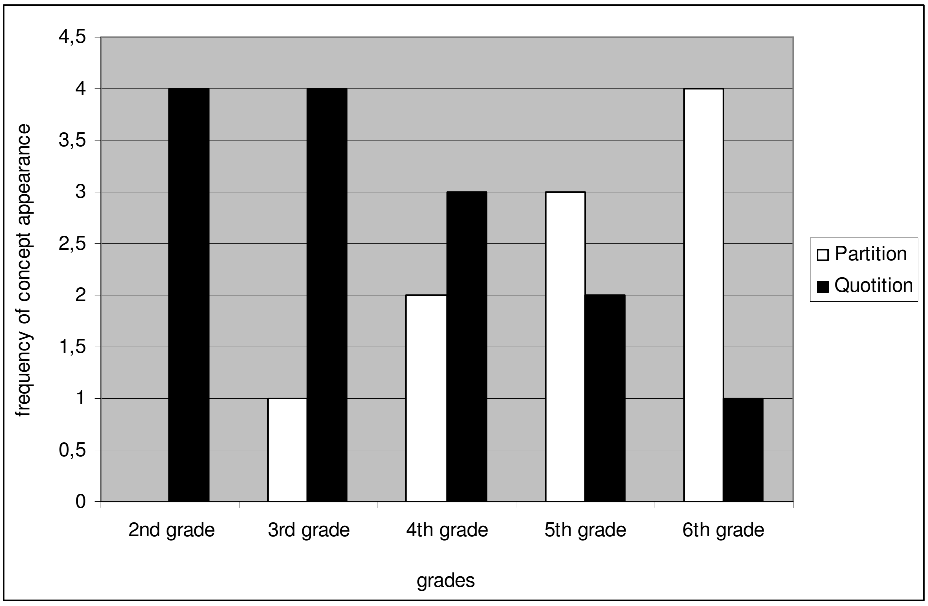
In fourth grade pupils start to learn written algorithm for division. Stoy problems appear rarely. There is always a quotitive task as an indroduction in chapter of division, later on partitive division tasks begin to apper more. If we can say, that partition is based on experiences of pupils and is a part of their prior knowledge, therefore school practice considers this only when pupils attend fourt grade. But on the other hand, in fourth grade school practise starts to neglect quotitive model of division which leads to uncomplete knowledge and understanding of division concept.

In fifth grade more complex tasks allready appear. They usually require more than one arithmetic operation. Beside division, there can also be one of the other three operations or perhaps all of them in one task. Some tasks contain two questions and both of the models. For example: Jill is reading a book that has 590 pages. How many days will she read it, if she reads 20 pages per day? How many pages per day will she read, if she will finish reading the book in 8 days. These kind of tasks show explicitly the difference between both of the models (Lutovac, 2007). Pupils can clearly see that in both cases we operate with division only structure is different. In textbooks for fifth grade, partition is prevalent. Quotition tasks start to dissapear from fifth grade on.

Decimal numbers are introduced in sixth grade. This means that pupils are often dealing with computations and less with story problems. Out of a few story problems, partitive division is prevalent. Quotition almost completely dissapears from six grade textbooks. We can sum up the results of implementation of division model in Slovenian textbooks for mathematics with figure.

In second grade we see complete prevalence of quotitive division model in mathematical textbooks. State in third grade is not much different, but in fourth grade partitive division model is slowly becoming prevalent. Till sixth grade, partition becomes completely prevalent division model. Therefore, we can say, that both models are not equally present in textbooks.

Figure 11 *Textbooks for mathematics and frequency of model appearance*



0: Division model is not present
1: Division model appears rarely (up to 20% of all division tasks)
2: Division model appears on avrage (between 20 and 50% of all division tasks)
3: Division model appears often (over 50% of all division tasks)
4: Division model is prevalent (over 90% of all division tasks)

4. Discussion

The first research question asked about prevalence of division model (partition or quotition) in connection with different age groups. The answer to this was that *partitive division model is highly prevalent in all age groups and regardless of choosen variables like gender, socio-economic status, learning ability, type of school/environment and mathematical textbooks*. The letter is also the answer to the second research question about some variables and their influence on prevalence of division model. The main reason for this kind of results could be early experiences with division. From early childhood, parents accustom their children to sharing, which then becomes everyday activity. They are conscious of social effect of these kinde of actions, but not of effect on mathematical understanding. On this basis, children's early understanding of division may form. Informal, everyday activity sharing may be important in children's initial understanding of division (Squire and Bryant, 2002). Even children in the reception class (5-year-olds) are quite familiar with phrase "share out", some studies also demonstrated that most 5-year-old children know how to share out quantities in a distributive (one for A, one for B) manner, and that by 5 years they tend to understand quite a lot about basis of this procedure (Carpenter, Ansell, Franke, Fennema and Weisbeck, 1993; Mulligan, 1992). Children's proficiency with sharing means that this activity is good candidate for the "schema of action[3]" (Piaget, 1977) from which an understanding of division might develop. Our study showed that some resonable variables do not have influence on prevalence of division model, so our assumption that only experiences with sharing influence on prevalence of partitive division model fits well with upper notion, therefore we could say that sharing is the "schema of action" which later on develops a mental model of division, partition. Psychologists agree (Squire and Bryant, 2002b; Fischbein, Deri, Nello and Marino, 1985; Greer, 1987 and 1992) that most of children think of partition when dealing with division. This model holds an enduring position in pupil's intuitive thinking. Furthermore, shown by Silver (1987, cited in Neuman, 1999, p.104), when asked to write stories based on division sums, pupils seem to tell stories involving partitive division whenever that is possible. In their study, Berenson and Vidakovic (1995) also discussed about mental models of division and pointed out that the majority of students selected the partitive model as their model of choice. On the other hand, younger students selected quotitive model more than older students, which was also shown in our study. 8-year-old children in our study wrote quotitive story problems more that older students. Textbooks and the type of story problems in it, may be the cause of that (Lutovac, 2007).

In our country, *division is introduced with quotitive division model, which means that majority of story problems in second and third grade are problems with quotitive division.* In contrary to textbooks, our curriculum for mathematics alleges partitive division as model for introducucing arithmetic operation of division. Although, it was demonstrated that *textbooks do not have influence on prevalence of division model*, there is possibility that exposure to quotitive division problems on everyday basis could cause prevalence of that model. Children often use memorizing technique, when studying mathematics. They simply learn the pattern of mathematical tasks and they successfully use this pattern until it is possible. From this aspect, the certain amount of exposure to quotitive division model, could cause a prevalence of it. Our study showed that *mathematical textbooks from fourth grade on give in the forefront partitive division problems.* At the same time, it was shown quite significant decline of pupils with prevalent quo-

[3] "Schemas of action" are familiar actions that might provide a first understanding of arithmetic operations, because the logical requirements and relationships that must be kept constant in arithmetical operations also have to be invariant in the child's shema of action. (Piaget, 1977)

titive division model in upper grades. Partition as primitive model of division evidently anchors in our mental models and stays there regardless of the formal education. However, we can not neglect formal education. Good educational implications and good practise are inevitable, if we want to change the present state.

5. Educational implications

When we introduce pupils with concepts of arithmetic operations, we should tend to better development of the terms, which is possible only with deeper understanding of strategies of arithmetical operations (in our case of division strategies). For teaching of division concept may be used symbolizing process on which basis are also designed mathematical manuals for teachers in our country. This process makes possible for children to progressively proceed from action to drawing pictures and from pictures to mathematical expressions and algorithms. We must also enable younger pupils to manipulate with concrete materials, like marbles and to externalise their internalized experiences by letting them discuss their drawings and notations. They will discover how informal ways of expressing ideas, for instance drawings, can be expressed in a more formal mathematical language. Along with teaching by symbolizing process, we must be aware of pupil's informal knowledge and their intuitive ideas. The letter should not be ignored or even suppressed, on the contrary it should be the starting-point of teaching division as well as other operations. Teachers should encourage and build on the base of children's informal knowledge. Some researces (Murray, Olivier and Human, 1991; Mitchelmore and Mulligan, 1996; Heirdsfield, Cooper, Mulligan and Irons, 1999; Kumagai, 2000) showed the great impact of early introduction of division strategies to development of division concept. Division strategies are numerous, so it is not good to border pupils on only one or even to impose certain strategy. The teacher's task is to aknowledge that pupils use a wide variety of strategies and to encourage them to expand their repertoire (Mitchelmore and Mulligan, 1996). Pupils should try and develop their own strategies, which suit them best, but they should also be able to explain them if necessery. On this basis, the connection between conceptual and procedural knowledge forms.

Elementary teachers often have uncomplete conceptual knowledge about division, which could also be one of the reasons for pupil's difficulties with division (Lamb and Booker, 2004). The study of Vinner and Linchevski (1988) showed that elementary teachers, as a group, lack basic mathematical understanding of arithmetic. Teacher's knowledge (or want of knowledge) has the impact on pupils, so proffesional development should not be neglected. The mirror of teachers's stage of proffesional perfection is also creativeness, which reflects through story problems. They should have understandable instructions, certain amount of originality and they should of course stimulate pupil's thinking. Pupils would benefit if teachers provided them with opportunities to solve multiplicative word problems as early as the first year of schooling, and if they linked multiplication and division much more closely.

From the view point of dual nature of division concept, it may be important to expose children to different problem representations and problem context in order to improve their ability to recognize the important variables in a problem, to deveop their conceptual understanding of multiplicative relations and encourage them to think flexibly in particular and diverse context (Squire and Bryant, 2002). When children's thinking moves from concrete representations to a more adequate and flexible understanding of relations between the different terms in division problems, they should eventually realize that partitive and quotitive problems are solved by the same arithmetic operation. Partitive and quotitive division problems should be equally present in mathematical textbooks, so pupils can clearly see the difference between the concepts of them and if not, teachers should explain it to them (Lutovac, 2007). Therefore it is necessary for teachers to have good conceptual understanding of division and to be able to present both concepts of division models so that pupils could understand them. Only knowledge and understanding of both models makes possible for pupils to fully understand the concept of division and to correctly use this arithmetic operation.

In conclusion, adjustment of curriculum for mathematics and textbooks is crucial. Curriculum for mathematics is fundamental guidance for teachers, therefore it must be clearly defined what to teach in terms of division and how to teach it. Texbooks, on the other hand, should follow curriculum instructions in order to provide a good practise (Lutovac, 2007).

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PREVLAST MODELA DIJELJENJA I NJEGOVA PRIMJENA U UDŽBENICIMA MATEMATIKE

Sažetak

Od svih aritmetičkih operacija djeca imaju najviše problema sa dijeljenjem. Jedan od mogućih razloga je dvostruka priroda dijeljenja , koja podrazumijeva podjelu i raščlanjivanje kao modele dijeljenja. Rad se bavi problemom prevladavanja modela dijeljenja u različitim dobnim skupinama, od djeteta do odraslih. Štoviše rad sagledava tu prevlast u odnosu na spol, društveno-ekonomski status, sposobnosti učenja, vrstu škole/okruženja i udžbenika matematike. Rezultati pokazuju da partitativni model dijeljenja prevladava u svim dobnim skupinama bez obzira na varijable. U radu smo prezentirali i kratku analizu udžbenika matematike za osnovnu školu u našoj zemlji, pošto smatramo da bi s obzirom na učeničko iskustvo u školskoj praksi trebalo primijeniti partitativni model. Rad sadrži i praktične smjernice primjene u školskoj praksi.

Ključne riječi: *dijeljenje, koncept dijeljenja, model dijeljenja, podjela, raščlanjivanje, nastava dijeljenja*

LA DOMINAZIONE DI UN MODELLO DI DIVISIONE E LA SUA APPLICAZIONE NEI MANUALI DI MATEMATICA

Riassunto

Fra tutte le operazioni aritmetiche i bambini hanno le difficoltà più frequenti con la divisione.Una delle possibili cause può essere la duplice natura della divisione che presenta come modelli di divisione la distribuzione e la partizione. Il saggio si occupa della dominazione di un unico modello di divisione nelle varie fasce d'età, dall'infanzia all'età adulta. Inoltre, il saggio prende in esame tale dominazione in relazione al sesso, allo status economico-sociale, alla capacità di apprendere, al tipo di scuola / o contesto e al libro di testo. I risultati indicano che il modello partitivo di divisione domina in tutte la fasce d'età, indipendentemente dalle variabili. Nel saggio abbiamo presentato anche una breve analisi di un manuale di matematica per la scuola elementare del nostro paese, in quanto siamo convinti che, considerata l'esperienza degli alunni nella realtà scolastica, andrebbe applicato il modello partitivo. Il saggio presenta anche indicazioni pratiche sull'applicazione di tale modello a scuola.