Prediction of shear wave velocity and modification of Castagna and Carroll relationships in one of the Iranian oil fields

Mohammad Sadegh Mahmoudian; Yousef Shiri; Ahmad Vaezian

1,2 Shahrood University of Technology; Faculty of Mining, Petroleum and Geophysics Engineering, Shahrood, Iran, P.O. Box: 36199-95161
1 Email: mahmoudian.mohammad65@gmail.com, https://orcid.org/0000-0001-9831-8663
2 Email: yousefshiri@shahroodut.ac.ir; https://orcid.org/0000-0002-4003-8017
3 Email: avaezian@yahoo.com, https://orcid.org/0000-0001-8669-4100

Abstract
Shear wave velocity is one of the essential parameters for describing hydrocarbon reservoirs that have several applications in petrophysical, geophysical, and geomechanical studies. Shear wave velocity usually does not exist in all wells, especially in old oil fields. In the current study, two equations of Carroll and Castagna have been modified, and linear and nonlinear multi-regressions were used to estimate shear wave velocity in an oil reservoir in southwestern Iran. Initially, compressional wave velocity and porosity were determined as the most effective wire-line logs on shear wave velocity by comparing their correlations. Then, two equations of Carroll and Castagna were modified. In addition, new equations based on porosity and compressional wave velocity for estimating the shear wave velocity were obtained. Shear wave velocity was estimated by new exponential equations in the wells of the current oil field with excellent goodness of fit by determination coefficients of 0.80 in the whole well, 0.72 in the Ghar-Shale-1, and 0.78 in Ghar-Shale-3 in X-07 well.

Keywords:
shear wave velocity estimation; wire-line log data; multi-regression; Carroll equation; Castagna equation

1. Introduction

The complexity of oil reservoirs causes an enormous challenge for geoscientists and the lack of reliable data leads to insufficient understanding of reservoir behaviours (Nikravesh, 2003). Shear wave velocity ($V_s$) is one of the most important parameters for evaluating and describing hydrocarbon reservoirs in the oil industry. $V_s$ together with compressional wave velocity ($V_p$) data leads to vital petrophysical (Shi and Zhang, 2021), geophysical (Mirkhesheni et al., 2022), and geomechanical (Bratton, 2021; Ogunkunle et al., 2022) information for studying reservoirs.

Compressional wave velocity and $V_p$ are affected by the density and elasticity of rock and $V_p$ has more sensitivity to fluid changes than $V_p$ (Saemi et al., 2007). Direct measurement of $V_p$ and $V_p$ in oil wells using a dipole sonic imager (DSI, downhole image logging tools with high resolution) is costly and only available in 5% of all wells. For this reason, using empirical equations for estimating $V_p$ from conventional petrophysical logs is very valuable (Akhundi et al., 2014). DSI can determine primary and secondary porosity, lithology, permeability, and fractures. Sonic logging tools in open-hole wells can have two sources, unipolar and bipolar. Some modern sonic log tools have two perpendicular dipole sources and correspondingly two dipolar receivers, therefore, $V_p$ is recorded in two directions (Kazemi, 2014). Conventional wire-line log data, such as porosity and $V_p$ are taken in most oil wells (Lim, 2005). In 1992, Greenberg and Castagna offer an empirical relationship for $V_s$ estimation based on lithology and $V_p$. In this equation, the coefficients belong to a unique lithology with 100% water saturation; otherwise, it should be corrected based on Gassmann equations (Greenberg and Castagna, 1992). The Wiley relationship is a linear equation between the porosity and $V_p$ (Wyllie et al., 1956). Raymer et al. proposed a new equation between porosity and $V_p$ for low and high porosities (Raymer et al., 1980).

Several methods, predominantly empirical equations by multi-regression methods and Artificial Intelligent techniques, have been recently used for estimating $V_s$ from wire-line logs (Posavec et al., 2017; Torkan et al., 2021). Empirical equations were the most used method, such as Carroll and Castagna relationships (Carroll, 1969; Castagna et al., 1985). Also, linear and nonlinear multi-regressions analysis by a variety of well log data are used to estimate $V_p$ (Eskandari et al., 2004). Wang and Peng (2019), Akhundi (2014), and Animangely (2017) used artificial intelligent techniques to predict $V_p$ in hydrocarbon reservoirs and the accuracy increased dramatically.
(Akhundi et al., 2014; Anemangely et al., 2017; Wang and Peng, 2019). In 2022, Wang et al. predicted S-wave velocity by using a deep hybrid neural network (combining the classical convolution neural network (CNN) with the long short-term memory (LSTM) network). This method was convenient and has considerable academic and application implications (Wang et al., 2022). In addition, $V_s$ can be predicted by rock physics relationships. In 2020, Shiri and Falahat used a hybrid rock physics method to estimate pressure changes from acoustic impedance (Shiri and Falahat, 2020). Later, Dalvand and Falahat used modified Gassmann’s relationships to estimate $V_s$ from wire-line log data (Dalvand and Falahat, 2021).

Boadu used artificial neural networks and multivariate regression methods for estimating the $V_s$ in a carbonate reservoir in southwestern Iran (Boadu, 2001). Eskandari et al. used multivariate regression and artificial neural network methods for predicting $V_s$ (Eskandari and Rezaee, 2003; Eskandari et al., 2004). Rezaee et al. and Rajabi et al. used fuzzy and neural-fuzzy logic methods to predict $V_s$ (Rezaee et al., 2007; Rajabi et al., 2010). Bagheripour and Maleki et al. used SVR and NN-BP methods to estimate $V_s$ from petrophysical data. Their research indicates the superiority of the SVR method with fast and accurate results (Maleki et al., 2014; Bagheripour et al., 2015).

In this study, linear and non-linear multi-regressions were used to estimate $V_s$ in an Iranian carbonate/sandstone oil reservoir located in southwestern Iran. In this oil reservoir, only one of seven oil wells had $V_s$, and it was necessary to estimate it for simulation and petrophysical modelling. Conventional wire-line well logs including caliper, gamma-ray, resistivity, $V_p$, porosity, and density logs were analysed, $V_s$ and porosity were used for $V_s$ estimation.

2. Methodology

Regression analysis is one of the most common statistical techniques for studying and modelling the linear and non-linear relationships between dependent and independent variables (Kraemer and Blasey, 2017). In this statistical method, one or more dependent variables (such as $V_s$) are predicted from several independent variables (such as conventional wire-line well logs). Generally, a multi-regression analysis is based on Equation 1 as (Srivastava, 2002):

$$V_s = a_0 + a_1 x_1 + ... + a_n x_n + e$$ (1)

Where $V_s$ is the response variable which is shear wave velocity in this study. It is a dependent outcome from $n$ independent variables of $x_1, x_2, x_3, ..., x_n$ with weights of $a_0, a_1, a_2, a_3, ..., a_n$ as regression parameters. The prediction value is $V_{s^*}$ as defined by Equation 2 as:

$$V_{s^*} = x(x^T x)^{-1} x^T$$ (2)

Root Mean Square Error formula (RMSE of Least square method), Equation 3 is used to minimize the error of fitting a straight line with these data as:

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (V_w - V_{w^*})^2}$$ (3)

Where:
- $V_w$ – the shear wave velocity,
- $V_{w^*}$ – the prediction value of shear wave velocity,
- $e$ – the residual values,
- $x$ – the matrix of independent variables,
- $a$ – the weighting coefficient,
- $N$ – the number of samples.

In this study, seven vertical wells were drilled in the carbonate/sandstone reservoir of the oil field. These wells had compressional wave sonic (DT log), gamma-ray (GR Log), density (RHOB Log), porosity (NPHI Log), resistivity (RLLD Log), and caliper logs. The X-07 well was selected as the main well for data evaluation and used to estimate $V_s$ in other wells. In this way, two equations of Carroll and Castagna were modified and new linear and nonlinear relationships were obtained.

3. Introducing Data and Geology of the Study Area

The studied oil field was an anticline with a north-south trend. There are seven vertical wells in this oil field that all have conventional well log data but $V_s$ only exists in the X-07 well. In the X-07 well, the Ghar sandstone reservoir depth is 856 meters and the oil-water contact depth is 876 meters below sea level.

The lithology of this oil field consists of Aghajari, Mishan, Gachsaran (with three parts of upper Gachsaran, salt member, and Ghar cap rock), Asmari (with three layers of Ghar, Asmari-A, and Asmari-B), Jahrom, Pabdeh-Jahrom, Pabdeh, Ilam-Gurpi, Sarvak, and Kozhdehmi (with two members of shale and sandstone) formations. Figure 1 shows some available wire-line log data for estimating $V_s$ in X-07 well.

**Gachsaran**: the Gachsaran Formation belongs to Oligocene-Mioseen and has three members, including Upper Gachsaran, Salt, and Ghar cap rock.
- Upper Gachsaran: The thickness of this formation is 287 meters, measured depth (MD) is 1759 meters and True Vertical Depth (TVD) is 1733.87 meters. It contains anhydrite, marl and dolomite.
- Salt: The thickness of this section is 166 meters; MD is 2046 meters and TVD is 2020 meters. It contains salt, anhydrite, marl, and shale.
- Ghar cap rock: The thickness of this section is 86 meters; MD is 2212 meters and TVD is 2187 meters. It contains anhydrite, marl, and shale.

**Asmari**: Asmari Formation is the reservoir of the current oil field with the Oligocene age. Based on petrophysical information and using the DSI log, it is divided into three parts of Ghar, Asmari-A, and Asmari-B.

**Ghar**: this member has a thickness of 94 meters, MD of 2280 meters, and TVD of 2254. It is consisting of Ghar-zone-1, Ghar-Shale-1, Ghar-zone-2, Ghar-zone-3,
and Ghar-Shale-3 layers. The lithologies of these layers are sand, dolomite, anhydrite, shale, and graphite. Sandstone lithology is well-rounded grains and relatively high porosity. In the X-07 well, the depth of Ghar-Shale-1 is from 2301.3 to 2304.2 meters, and Ghar-Shale-3 is from 2369 to 2373.

**Asmari - A:** this layer has a thickness of 33.5 meters, MD of 2374 meters, and TVD of 2349 meters. It contains dolomite and anhydrite.

**Asmari - B:** this layer has a thickness of 82.5 meters, MD of 2407.5 meters, and TVD of 2382.37 meters. It contains dolomite, anhydrite, and marl.

**Figure 1:** Wire-line log data in the X-07 well
4. Results and Discussion

Regression analysis, especially multi-regression is one of the popular methods for estimating valuable outcomes (e.g. \( V_s \)) from relevant data (e.g. conventional wire-line logs). Several empirical relationships for estimating \( V_s \) from \( V_p \) have been proposed recently (Pickett, 1963; Castagna et al., 1985; Han and Nur, 1986; Greenberg and Castagna, 1992; Brocher, 2005). However, most of them, such as the two equations of Castagna and Carroll, are valid for special lithology (Carroll, 1969; Castagna et al., 1985; Greenberg and Castagna, 1992), and they were modified in this study. \( V_s \) only were available for the X-07 well and in other wells of the oil field, it was necessary for further petrophysical and geomechanical analysis. There are some assumptions for applying regression analysis in a specific oil field, such as homogenous sedimentation, which was implemented in the current study. Based on the linear and nonlinear multi-regression analysis, \( V_s \) and porosity well log data showed the highest correlation with \( V_s \) and were selected for \( V_s \) estimation. The results are shown in Table 1, Table 2, and Table 3 and are described below.

4.1. Investigation of Linear Relationships Between \( V_s \) and Conventional Wire-line Well Logs

According to the results of correlation analysis in the X-07 well, \( V_p \) and porosity log were selected as the best candidates to estimate \( V_s \) among all wire-line logs. The determination coefficient between \( V_s \) and conventional well logs were: \( R^2 \) = 0.8 for \( V_p \), \( R^2 \) = 0.67 for porosity, \( R^2 \) = 0.34 for density, \( R^2 \) = 0.13 for electrical resistivity, and \( R^2 \) = 0.09 for gamma-ray. The extracted equations with their determination coefficients are given in Table 1.

The best determination coefficient by linear regressions was in the whole well, and the Ghar-Shale-1 and Ghar-Shale-3 (see Table 1). Porosity and \( V_p \) were selected as the best candidates for estimating \( V_s \). The relationship between \( V_s \) and \( V_p \) in the entire well data with a determination coefficient of 0.8 is shown in Figure 2 and Equation 4 as follows:

\[
V_s = 0.4711V_p + 326.82
\] (4)

Table 1: Linear regression analysis for existing formations in well X-07

<table>
<thead>
<tr>
<th>Relationship</th>
<th>Estimation Interval</th>
<th>Equation</th>
<th>Determination coefficient ((R^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear regression ((V_s \text{ versus } V_p))</td>
<td>The whole well</td>
<td>(V_s = 0.4711V_p + 326.82)</td>
<td>0.8</td>
</tr>
<tr>
<td>Ghar – Shale-1</td>
<td>(V_s = 0.3544V_p + 901.27)</td>
<td>0.65</td>
<td></td>
</tr>
<tr>
<td>Ghar – Shale-3</td>
<td>(V_s = 0.4979V_p + 310.25)</td>
<td>0.67</td>
<td></td>
</tr>
<tr>
<td>Linear regression ((V_s \text{ versus porosity log}))</td>
<td>The whole well</td>
<td>(V_s = -25.956\phi + 2916.1)</td>
<td>0.22</td>
</tr>
<tr>
<td>Ghar – Shale-1</td>
<td>(V_s = -34.921\phi + 2915)</td>
<td>0.71</td>
<td></td>
</tr>
<tr>
<td>Ghar – Shale-3</td>
<td>(V_s = -46.367\phi + 3226.3)</td>
<td>0.7</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2: Regression analysis between observes and estimated \( V_s \) in the X-07 well

Rudarsko-geološko-naftni zbornik i autori (The Mining-Geology-Petroleum Engineering Bulletin and the authors) ©, 2022, pp. 137-144, DOI: 10.17794/rgn.2022.4.11
Where:

\[ V_s \] – the shear wave velocity [km/s],
\[ V_p \] – the compressional wave velocity [km/s].

4.2. Castagna and Carroll's Relationships and Proposing New Equations

Castagna et al. proposed several empirical relationships for the prediction of \( V_s \) (Castagna et al., 1985; Greenberg and Castagna, 1992). Equation 5 is one of the most common relationships to predict \( V_s \) for the limestone as:

\[
V_s = -0.05509V_p^2 + 1.0168V_p - 1.0305
\]  (5)

Where:

\[ V_s \] – the shear wave velocity [km/s],
\[ V_p \] – the compressional wave velocity [km/s].

Comparing the Castagna relationship with observed data at the X-07 well showed this empirical equation did not have an acceptable outcome for shale layers with a determination coefficient of 0.48. The coefficients of the original Castagna relationship were adjusted and extracted from the observed data of the current oil field and the determination coefficient reached 0.67. In addition, the determination coefficient was 0.79 by a new equation based on the porosity log (see Table 2).

Where \( V_s \) is the shear wave velocity (m/s), \( V_p \) is the compressional wave velocity (m/s), and \( \phi \) is the porosity (%).

Carroll proposed an empirical relationship between compressional and shear wave velocities as Equation 6 (Carroll, 1969), and it was necessary to modify it based on new lithology and environments.

\[
V_s = 1.099V_p^{0.9258}
\]  (6)

Comparing Carroll’s relationship with \( V_s \) data of the X-07 well showed this relationship did not have a good result with the determination coefficient of 0.71. The coefficients of the original Carroll’s relationship were adjusted and extracted from the observed data of the current oil field and modified with the determination coefficient of 0.8 as Equation 7:

\[
V_s = 1.86V_p^{0.65}
\]  (7)

Multiple linear and nonlinear regression analysis results between \( V_s \), \( V_p \), and porosity log of X-07 well are shown in Table 3. The exponential relationship between \( V_s \) and \( V_p \) for Ghar-Shale-1 with a determination coefficient of 0.72 (see Figure 3) is as Equation 8:

Table 2: New and modified Castagna (1985) relationships with their coefficients of determinations in well X-07

<table>
<thead>
<tr>
<th>Relationship</th>
<th>Estimation Interval</th>
<th>Equation</th>
<th>Determination coefficient ((R^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modified Castagna’s equation ((V_s \text{ versus } V_p))</td>
<td>The whole well</td>
<td>[ V_s = 1 \times 10^{-2}V_p^2 + 0.3742V_p + 535.53 ]</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>Ghar – Shale-1</td>
<td>[ V_s = 0.0001V_p^2 + 0.774V_p - 3150.1 ]</td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td>Ghar – Shale-3</td>
<td>[ V_s = 9 \times 10^{-4}V_p^2 + 0.4316V_p - 433.2 ]</td>
<td>0.67</td>
</tr>
<tr>
<td>New equations ((V_s \text{ versus porosity log}))</td>
<td>The whole well</td>
<td>[ V_s = 2.0077\phi^2 - 99.554\phi + 3536.8 ]</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>Ghar – Shale-1</td>
<td>[ V_s = 3.3126\phi^2 - 149.83\phi + 3855.8 ]</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>Ghar – Shale-3</td>
<td>[ V_s = 3.2865\phi^2 - 211.49\phi + 5203 ]</td>
<td>0.79</td>
</tr>
</tbody>
</table>

Table 3: Multiple and nonlinear regression between \( V_p \) porosity, and \( V_s \) in well X-07

<table>
<thead>
<tr>
<th>Relationship</th>
<th>Estimation Period</th>
<th>Equation</th>
<th>Determination coefficient ((R^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential equations ((V_s \text{ versus } V_p))</td>
<td>The whole well</td>
<td>[ V_s = 987.42e^{0.0002V_p} ]</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>Ghar – Shale-1</td>
<td>[ V_s = 1266.48e^{0.0002V_p} ]</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>Ghar – Shale-3</td>
<td>[ V_s = 912.87e^{0.0002V_p} ]</td>
<td>0.67</td>
</tr>
<tr>
<td>Exponential equation ((V_s \text{ versus porosity log}))</td>
<td>The whole well</td>
<td>[ V_s = 2940.4e^{-0.015\phi} ]</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>Ghar – Shale-1</td>
<td>[ V_s = 2983.64e^{-0.018\phi} ]</td>
<td>0.72</td>
</tr>
<tr>
<td></td>
<td>Ghar – Shale-3</td>
<td>[ V_s = 3539.35e^{-0.021\phi} ]</td>
<td>0.73</td>
</tr>
<tr>
<td>Multiple linear regression ((V_s \text{ versus porosity log and } V_p))</td>
<td>Ghar – Shale-1</td>
<td>[ V_s = 0.06V_p^{2} - 29.3\phi + 2561.85 ]</td>
<td>0.71</td>
</tr>
<tr>
<td></td>
<td>Ghar – Shale-3</td>
<td>[ V_s = 0.25V_p^{2} - 28.09\phi + 1879.38 ]</td>
<td>0.76</td>
</tr>
<tr>
<td>Exponential equation ((V_s \text{ versus porosity log and } V_p))</td>
<td>Ghar – Shale-1</td>
<td>[ V_s = 2611.5e^{0.0002V_p} ]</td>
<td>0.72</td>
</tr>
<tr>
<td></td>
<td>Ghar – Shale-3</td>
<td>[ V_s = 2017.88e^{0.0001V_p} ]</td>
<td>0.78</td>
</tr>
</tbody>
</table>
Also, the exponential relationship between $V_s$, $V_p$ and porosity for Ghar-Shale-3 with a determination coefficient of 0.78 (see Figure 4) is as Equation 9:

$$V_s = 2017.88e^{(0.00104605V_p-0.04198321\phi)}$$  (9)

Where:
- $V_s$ – the shear wave velocity [km/s],
- $V_p$ – the compressional wave velocity [km/s],
- $\phi$ – the porosity [%].

As can be seen, these two equations are the best relationships with this view of using both porosity and $V_p$ for $V_s$ estimation. Comparing the determination coefficient of the new exponential result with observed data and modified Castagna and Carroll equations at X-07 well showed an increase in the consistency and accuracy of $V_s$ estimation.

Where $V_s$ is shear wave velocity (m/s), $V_p$ is compressional wave velocity (m/s), and $\phi$ is porosity (%).

5. Conclusion

Compressional wave velocity and porosity log have good relationships with shear wave velocity, which can be used for petrophysical and geomechanical studies. $V_s$ due to economic issues is not measured in all wells. Empirical relationships like the two equations of Castagna and Carroll only used $V_p$ with non-adjusted coefficients for estimating $V_s$. This research aimed to use multi-regression analysis and add more relevant variables to estimate the $V_s$ of reservoir formation in a southwestern Iranian oil field. In this way, the old empirical equations were modified, and new relationships were obtained.

The results showed $V_p$ and porosity logs were recognized as the most effective logs for estimating $V_s$. After performing multiple linear and nonlinear regressions for $V_s$ estimation in X-07 well, two equations of Castagna and Carroll were modified, and three new equations with a maximum accuracy of $R^2=0.8$ were introduced.

It can be said that linear and nonlinear multi-regression, as a reliable method with wide application, has good accuracy in the indirect prediction of $V_s$. So, the proposed relationships can be used in all wells of the current oil field (X-01-06 wells) for petrophysical and geomechanical studies. This study can be improved by using more wire-line log data in various oil fields.

Acknowledgements

We appreciate the exploration management division of the National Iranian Oil Company (NIOC) for providing data and their cooperation. Thanks to Seyed Mohammad Javad Seyed Sabour for priceless assistance in drafting the manuscript and language editing.

6. References


Bratton, T. (2021): A Coupled Petrophysical and Geomechanical Workflow To Interpret Dipole Sonic Velocities for in-
Prediction of shear wave velocity and modification of Castagna and Carroll relationships...

Situ Stress. SPWLA 62nd Annual Logging Symposium. doi: 10.30632/spwla-2021-0116


Predviđanje brzine smičnoga vala i modifikacija Castagnine i Carrollove jednadžbe u slučaju jednoga iranskega naftnog polja

Brzina smičnoga vala je jedan od osnovnih parametara za opisivanje ležišta ugljikovodika te ima nekoliko primjena u petrofizičkim, geofizičkim i geomehaničkim istraživanjima. Brzina smičnoga vala obično se ne očitava u svim bušotinama, osobito ako je riječ o starih poljima. U ovome su istraživanju modificirane dvije jednadžbe, Carrollova i Castagnina, za procjenu brzine smičnoga vala u naftnome ležištu u jugozapadnom Iranu uporabom linearne i nelinearne višestruke regresije. Prvo je brzina smičnoga vala određena korelacijom brzine kompresijskoga vala i šupljikavosti iz karotažnih dijagrama. Zatim su prilagođene i Carrollova i Castagnina jednadžba te su dobivene nove za procjenu brzine takva vala, koje se temelje na podacima o šupljikavosti i brzini kompresijskoga vala. Brzina smičnoga vala procijenjena je novim eksponencijskim jednadžbama u bušotinama naftnog polja. Dobiveni su dobri koeficijenti determinacije od 0,80 u cijeloj bušotini, 0,72 u jedinici Ghar-Shale-1 i 0,78 u jedinici Ghar-Shale-3, sve to u bušotini X-07.

Ključne riječi:
procjena brzine posmičnoga vala, karotažni podatci, multiregresija, Carrollova jednadžba, Castagnina jednadžba

Author's contribution

Mohammad Sadegh Mahmoudian (M.Sc. Student, Petroleum Engineering) performed analyses and wrote the paper. Yousef Shiri (Assistant Professor, Ph.D., Petroleum Engineering) supervised the analyses and proofed the paper. Ahmad Vaezian (Assistant Professor, Ph.D., Mining Engineering) provided the interpretations and presentations of the results.