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Distributed Adaptive Control for a Class of Heterogeneous Nonlinear Multi-Agent Systems with Nonidentical Dimensions

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Abstract: A novel feedback distributed adaptive control strategy based on radial basis neural network (RBFNN) is proposed for the consensus control of a class of leaderless heterogeneous nonlinear multi-agent systems with the same and different dimensions. The distributed control, which consists of a sequence of comparable matrices or vectors, can make that all the states of each agent to attain consensus dynamic behaviors are defined with similar parameters of each agent with nonidentical dimensions. The coupling weight adaptation laws and the feedback management of neural network weights ensure that all signals in the closed-loop system are uniformly ultimately bounded. Finally, two simulation examples are carried out to validate the effectiveness of the suggested control design strategy.

Keywords: distributed control; heterogeneous multi-agent systems; radial basis function neural network (RBFNN); uniformly ultimately bounded (UUB)

1 INTRODUCTION

Due to the increasing usage of MASs in a variety of engineering applications, cooperative control for multi-agent systems (MASs) has become a hot research topic in various disciplines [1-7]. The consensus among MASs is that forcing all followers to converge on the leader's states is a key issue in cooperative control. Extant works on MASs consensus include sampling data consensus [8, 9], event-triggered consensus [10, 11], finite-time consensus [12], linear MASs [13, 14], and nonlinear MASs [15-18]. It is worth noting that the consensus in these papers is based on homogeneous MASs, which means that each agent's dynamic behaviors are identical. As a result, the proposed controls can only be utilized to obtain an agreement among agents with the same dynamical behavior.

Because the nonidentity of dynamic agents has piqued interest in the consensus of heterogeneous MASs, it is critical to design some control techniques to achieve MASs consensus. In recent years, more control approaches for linear heterogeneous MASs have been developed in [19]-[24]. In the consensus problem of uncertain nonlinear heterogeneous MASs, a distributed adaptive fuzzy control was developed for a class of heterogeneous second-order MASs with unknown nonlinear terms in [25]. The fuzzy adaptive control introduced in [26] can guarantee the output consensus of heterogeneous MASs with unknown nonlinear functions.

For the consensus and synchronization of a family of higher-order nonlinear MASs, the authors established a robust neural network adaptive control for cooperative tracking in [27]. Two distributed learning control methods for the consensus problem of heterogeneous high-order nonlinear MASs with output constraints were designed by applying a general form of barrier Lyapunov function in [28]. In [29, 30], in order to achieve the consistency or synchronization for two types of high-order nonlinear MASs with time-varying actuator faults and high-order nonlinear MASs with unknown state dependent control effects, the authors proposed a cooperative adaptive fuzzy tracking control and a distributed adaptive neural network control.

However, because the composition of heterogeneous MASs was supposed to be identical in [27-30], the works provided in these results were only valid for a few classes of peculiar triangular heterogeneous MASs. Furthermore, in these works, the dimensions of heterogeneous MASs were all the same, making the offered control concepts ineffective for establishing the consensus of heterogeneous MASs with varying dimensions. As a result, it is required to investigate a number of distinct consensus controls that must be used to variety of MASs with the same or different dimensions.

Based on an analysis of existing consensus controls for several categories of heterogeneous MASs, we aim to close the gap in consensus control processes across heterogeneous MASs with the same and different dimensions in this research. Each agent is viewed as a node in a network topology graph from the perspective of the MASs workspace's connection structure, where the spatial dimension of each node is the same or different. In this study, similar qualities were introduced to comparable agents with nonidentical dimensions based on the experiences of the similar distinguishing feature of large-scale systems in [31-36]. A unique consensus RBFNN adaptive control based on the comparable parameters among each agent with various dimensions is examined by adding the characteristics of similar heterogeneous agents. For the consensus of heterogeneous MASs with the same or different dimensions, as well as identical MASs, the proposed control is used.

The proposed control is employed for the consensus of heterogeneous MASs with the same or different dimensions. Comparing with the existing works on the consistency of heterogeneous MASs, the main contributions of the control protocol proposed in this paper are as follows:

1) Similar definition and properties among each agent are presented, and similar parameters such as vectors or matrices can be obtained by the properties of heterogeneous MASs with identical or non-identical dimensions.

2) For the consensus problem of heterogeneous MASs, a distributed adaptive RBFNN feedback control with similar parameters is designed, which may be employed for cooperative control of heterogeneous nonlinear MASs with the same or different composition.

The rest of the paper is summarized as follows: Section 2 contains preliminary information and system descriptions. The distributed adaptive RBFNN control approach for heterogeneous nonlinear multi-agent systems is investigated in Section 3. To demonstrate the validity of theoretical results, Section 4 gives two types of cases, one with the same dimension and the other with different dimensions. Section 5 concludes with some recommendations.

2 SOME PRELIMINARIES AND SYSTEM DESCRIPTIONS

Assuming that the network contains N agents, a weighted directed graph $G = \{V, E, A\}$ is used to model the interaction of these agents, where $V = \{1, 2, \ldots, N\}$ is the node set, $E \subseteq V \times V$ is the edge set, matrix $A = (a_{ij}) \in \mathbb{R}^{N\times N}$ denotes the weighted adjacency matrix. If there is an edge between two agents, then they are adjacent. Assuming that the graph has no self-loop or repetition ratio, it is called as a simple graph. An edge from node j to the end of node i is marked as (v_j, v_i) with $a_{ij} > 0$, $(v_j, v_i) \in E$, otherwise $a_{ij} = 0$. The set of neighbor nodes of node i is denoted as $N_i = \{j \mid (v_j, v_i) \in E\}$. The degree of node i is defined as $d_{in}(i) = \sum_{j=1}^{N} a_{ji}$, and the degree matrix is $D = \text{diag}\{d_{in}\} \in \mathbb{R}^{N\times N}$. The Laplacian matrix is $L = D - A$. In a directed graph, the directed path from node i to node j is composed of a series of edges in the form of $\{(v_i, v_l), (v_l, v_r), \ldots, (v_r, v_j)\}$.

Assuming that in this graph there is any node that can be used as a leader, define its adjacency matrix as $\Lambda_k = \text{diag}(a_{1,k}, a_{2,k}, \ldots, a_{n,k})$, and the following lemma is proposed.

Lemma 1 [23]: The matrix $H = L + \Lambda_k$ is symmetric and positive definite, where $a_{ik} > 0$, if and only if the i-th follower can access the leader's state information, otherwise $a_{ik} = 0$.

The MASs with N agents is described as:

$$\dot{x}_i(t) = A_i x_i(t) + B_i[u_i(t) + f_i(x_i)] \tag{1}$$

where $x_i = [x_{i1}, x_{i2}, ..., x_{in_i}] \in \mathbb{R}^{n_i\times 1}$ is the state vector of the i-th agent, $A_i \in \mathbb{R}^{n_i\times n_i}$, $B_i \in \mathbb{R}^{n_i\times m_i}$ are two known matrices with appropriate dimension, $u_i(t) \in \mathbb{R}^{m_i\times 1}$ is control input that will be designed, $f_i(x_i(t)) \in \mathbb{R}^{m_i\times 1}$ represents the unknown nonlinear function.

Definition 1: Consider the i-th and j-th follower agent systems such as agent system (1), if there exists matrix $F_i \in \mathbb{R}^{n_j\times n_i} (F_i \neq 0)$, matrix $K_i \in \mathbb{R}^{m_i\times n_i}$, and a known matrix $J_j \in \mathbb{R}^{m_j\times m_i}$ satisfying the following condition:

$$\begin{cases} F_i(A_i + B_i K_i) = (A_j + B_j K_j) F_i \\ F_i B_i = B_j J_j \end{cases} \tag{2}$$

Definition 2: In (2), the matrices F_i, K_i and J_j are named as similar parameters.

Assumption 1: For any uncertain continuous nonlinear function $f_i(x_i)$ on compact set $\Omega \in \mathbb{R}^{n_i}$, it can be approximated by the universal approximation property of the following RBF neural network.

$$f_i(x_i) = W_i^T \varphi_i(x_i) + \varepsilon_i, \forall x_i \in \Omega \tag{3}$$

where $W_i \in \mathbb{R}^{s_i\times q_i}$ represents the ideal weight matrix that can be updated online automatically. $\varepsilon_i \in \mathbb{R}^{q_i\times 1}$ denotes the approximation accuracy with bounded value $\bar{\varepsilon}_i$, and satisfies the condition $\|\varepsilon_i\| < \bar{\varepsilon}_i$, in which, $\bar{\varepsilon}_i$ is a known positive constant. Ω is approximation domain that is commonly chosen to be large enough. $\varphi_i(x_i) = [\varphi_{i1}(x_i), \varphi_{i2}(x_i), \ldots, \varphi_{is_i}(x_i)] \in \mathbb{R}^{s_i\times 1}$ is the RBFNN vector with s_i being the number of basic functions. Generally speaking, the Gaussian function is selected as:

$$\varphi_i(x_i) = \exp\left(-\frac{\left\|x_i - \mu_{is_i}\right\|^2}{2\sigma_{is_i}^2}\right) \tag{4}$$

where $\|\bullet\|$ represents the 2-norm, $\mu_{is} = [\mu_{is_1}, \mu_{is_2}, \cdots, \mu_{is_{n_i}}]^T$ is the center of the RBFNN, and σ_{is_i} represents the width of the Gaussian function.

3 DESIGN OF DISTRIBUTED FEEDBACK ADAPTIVE RBFNN CONTROLLER WITH SIMILAR PARAMETERS

With Definition 1 of similar heterogeneous MASs (1), the consensus tracking error is defined as $\delta_i(t) = F_i x_i(t) - F_j x_j(t)$, $e_i \in \mathbb{R}^{n_i}$ stands for the local cooperative tracking error denoted as the following equation form:

$$e_i(t) = \sum_{j=1}^{N} a_{ij}(F_i x_i(t) - F_j x_j(t)) \tag{5}$$

The distributed feedback adaptive RBFNN control with similar parameters is designed as

$$\begin{aligned} u_i = {} & c_i \bar{K} \sum_{j=1}^{N} a_{ij}[F_i x_i(t) - F_j x_j(t)] + K_i x_i(t) + \\ & + \bar{K}[F_i x_i(t) - F_j x_j(t)] - \bar{W}_i^T \varphi_i(x_i) \end{aligned} \tag{6}$$

where $\bar{W}_i$ is the estimated value of W_i , $\tilde{W}_i = \bar{W}_i - W_i$ is the estimated error. The adaptive law for W_i estimation is constructed as the following expression:

$$\dot{\bar{W}}_i = -\eta_{\omega i} \bar{W}_i + \theta_{\omega i} \varphi_i(x_i)(PB_j J_j)^T e_i(t) \tag{7}$$

The adaptive update law of the coupling weight c_i is designed as

$$\dot{c}_i = -\eta_{ci} c_i - \theta_{ci} e_i^T(t)(PB_j J_j)\bar{K} e_i(t) \tag{8}$$

where the positive parameters $\eta_{\omega i}$, $\eta_{\omega i}$, η_{ci}, and θ_{ci} are given by the user.

For any given positive matrix Q, the control gain $\bar{K}$ and the positive symmetric matrix P can be obtained by solving the following linear matrix inequality (LMI)

$$\begin{bmatrix} \Delta & X^T \\ * & -\bar{Q} \end{bmatrix} \le 0 \tag{9}$$

where $X > 0$, $\bar{Q} > 0$, $\Delta = A_j X + X A_j^T + D_j X + X D_j^T + E_j M + M^T E_j^T$, control gain $\bar{K} = M X^{-1}$ and matrix $P = X^{-1}$ are designed.

Let $\bar{x}_i(t) = F_i x_i(t)$, according to Definition 1 and applying control protocol (6) with adaptive laws (7) to (8), the following transformation can be obtained as:

$$\begin{aligned} \dot{\bar{x}}_i(t) = {} & (A_j + B_j K_j) F_i x_i(t) + B_j J_j \bar{K} \delta_i + B_j J_j \bar{K} c_i e_i + \\ & + B_j J_j (-\tilde{W}_i^T \varphi_i(x_i) + \varepsilon_i) \end{aligned} \tag{10}$$

Based on (10), the derivative of the consensus tracking error can be expressed as

$$\begin{aligned} \dot{\delta}_i(t) = {} & \dot{\bar{x}}_i(t) - \dot{\bar{x}}_j(t) = (A_j + B_j K_j + B_j J_j \bar{K}) \delta_i(t) + \\ & + B_j J_j \bar{K} c_i e_i + B_j J_j (-\tilde{W}_i^T \varphi_i(x_i) + \varepsilon_i) \end{aligned} \tag{11}$$

Eq. (11) can be further rewritten as

$$\begin{aligned} \dot{\delta}(t) = {} & [I_N \otimes (A_j + B_j K_j + B_j J_j \bar{K}) + cH \otimes B_j J_j \bar{K}] \cdot \\ & \cdot \delta(t) + \sum_{i=1}^{N} B_j J_j [-\tilde{W}_i^T \varphi_i(x_i) + \varepsilon_i] \end{aligned} \tag{12}$$

where $\delta(t) = [\delta_1^T(t), \cdots, \delta_N^T(t)]^T$, $c = [c_1^T, \cdots, c_N^T]^T$.

For the consensus stability control task of heterogeneous nonlinear MASs with different dimensions, Theorem 1 is proposed.

Theorem 1: Consider the MASs (1), where the similar parameters among each agent can be obtained by using Definition 1, the feedback control (6) with adaptive laws (7) to (8) with similar matrices K_i, F_i, K_j and J_j can ensure that all signals in the closed-loop system (13) are UUB, the consensus tracking errors satisfy the following bound.

$$\lim_{t \to \infty} \|\delta\| \le \sqrt{\frac{\beta / \alpha}{\lambda_{\min}(I_N \otimes P)}} = \gamma \tag{13}$$

Proof: Supposing the candidate Lyapunov function is defined as

$$\begin{aligned} V(t) = {} & \frac{1}{2} \delta^T(t)(H \otimes P)\delta(t) + \frac{1}{2} \theta_c^{-1} c^T c + \\ & + \frac{1}{2} \sum_{i=1}^{N} \tilde{W}_i^T(t) \theta_{\omega i}^{-1} \tilde{W}_i(t) \end{aligned} \tag{14}$$

Derivation of (14) is obtained as:

$$\begin{aligned} \dot{V}(t) = {} & \frac{1}{2} \delta^T(t) \{ H \otimes [P(A_j + B_j K_j + B_j J_j \bar{K}) + \\ & + (A_j + B_j K_j + B_j J_j \bar{K})^T P] \} \delta(t) + \\ & + \delta^T(t)(H \otimes PB_j J_j)\varepsilon + \sum_{i=1}^{N} [\theta_{ci}^{-1} c_i^T \dot{c}_i + e_i^T c_i PB_j J_j \bar{K} e_i + \\ & + \tilde{W}_i^T(t) \theta_{\omega i}^{-1} \dot{\tilde{W}}_i(t) - e_i^T PB_j J_j \tilde{W}_i^T \varphi_i(x_i)] \end{aligned} \tag{15}$$

where $\varepsilon = [\varepsilon_1^T, \varepsilon_2^T, \ldots, \varepsilon_N^T]^T$. From the update laws (7) and (8), the inequality (15) is further simplified to

$$\begin{aligned} \dot{V}(t) \le {} & \frac{1}{2} \delta^T(t) \{ H \otimes [P(A_j + B_j K_j + B_j J_j \bar{K}) + \\ & + (A_j + B_j K_j + B_j J_j \bar{K})^T P] \} \delta(t) + \\ & + \delta^T(t)(H \otimes PB_j J_j)\bar{\varepsilon} - \sum_{i=1}^{N} \frac{\eta_{ci}}{\theta_{ci}} c_i^2 - \sum_{i=1}^{N} \frac{\eta_{\omega i}}{\theta_{\omega i}} \tilde{W}_i^T \bar{W}_i \end{aligned} \tag{16}$$

Owing to the following inequalities hold:

$$-\frac{\eta_{\omega i}}{\theta_{\omega i}} \tilde{W}_i^T \bar{W}_i \le -\frac{1}{2} \frac{\eta_{\omega i}}{\theta_{\omega i}} \tilde{W}_i^T \tilde{W}_i + \frac{1}{2} \frac{\eta_{\omega i}}{\theta_{\omega i}} W_i^T W_i \tag{17}$$

Then Eq. (16) is equal to the following inequality:

$$\begin{aligned} \dot{V}(t) \le {} & \frac{1}{2} \delta^T(t) \{ H \otimes [P(A_j + B_j K_j + B_j J_j \bar{K}) + \\ & + (A_j + B_j K_j + B_j J_j \bar{K})^T P] \} \delta(t) - \frac{\eta_c}{\theta_c} c^T c - \\ & - \frac{1}{2} \frac{\eta_\omega}{\theta_\omega} \tilde{W}^T \tilde{W} + \delta^T(t)(H \otimes PB_j J_j)\bar{\varepsilon} + \frac{1}{2} \frac{\eta_\omega}{\theta_\omega} W^T W \end{aligned} \tag{18}$$

If we denote $\alpha = \min\left\{ \frac{\lambda_{\min}(H \otimes Q)}{\lambda_{\min}(H \otimes P)}, 2\eta_c, \eta_\omega \right\}$, $\beta = \|\delta\| \cdot \|H \otimes PB_j J_j\| \bar{\varepsilon} + \frac{1}{2} \frac{\eta_\omega}{\theta_\omega} W^T W$, inequality (18) is equivalent to the following format:

$$\dot{V}(t) \le -\alpha V(t) + \beta \tag{19}$$

Integrating both sides of the inequality (19), then it follows that

$$V(t) \le \left(V(0) - \frac{\beta}{\alpha}\right) e^{-\alpha t} + \frac{\beta}{\alpha} \tag{20}$$

According to the result shown as (20), it easily can be seen that $V(t)$ is bounded with $t \to 0$, so all signals in the closed-loop system can satisfy semi-globally UUB. In terms of the Lyapunov function (14), the following inequality is also established:

$$\frac{1}{2}\lambda_{\min}(H\otimes P)\|\delta\|^2 \le \frac{1}{2}\delta^T(H\otimes P)\delta \le V(t) \tag{21}$$

Therefore, the following formula holds:

$$\lim_{t\to\infty}\|\delta\| \le \sqrt{\frac{\beta/\alpha}{\lambda_{\min}(I_N\otimes P)}} = \gamma \tag{22}$$

The norm of consensus tracking errors δ satisfies the boundary value γ, which completes the proof of Theorem 1.

Remark 1: The inequality $P(A_j + B_jK_j + B_jJ_j\bar{K}) + +(A_j + B_jK_j + B_jJ_j\bar{K})^T P \le -Q$ is a nonlinear inequality with unknown matrices $P>0$, $Q>0$, and $\bar{K}$. Denoting $D_j = B_jK_j$, $E_j = B_jJ_j$, $M = \bar{K}X$, $\bar{Q} = Q^{-1}$, then multiplying the left and the right sides of this inequality by X, it can get the following equation

$$A_jX + XA_j^T + D_jX + XD_j^T + E_jM + M^TE_j^T + X^TQX \le 0 \tag{23}$$

By employing Schur's complement Lemma, inequality (23) is equivalent to LMI (9).

4 SIMULATION EXAMPLES

In this section, two classes of multi-agent systems with the same dimension and different dimensions are provided to demonstrate the effectiveness of the proposed control method.

Example 1: The network topology diagram with 5 agents is shown as in Fig. 1.

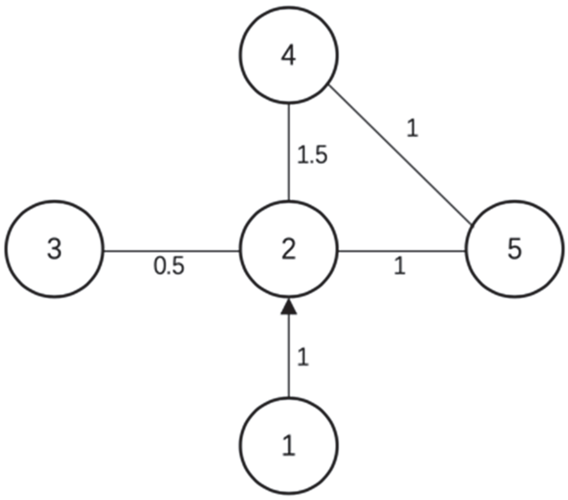


Figure 1 Communication topology of five agent systems

From Fig.1, it can be easily obtained that the Laplace matrix is written as

$$L = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 4 & -0.5 & -1.5 & -1 \\ 0 & -0.5 & 0.5 & 0 & 0 \\ 0 & -1.5 & 0 & 2.5 & -1 \\ 0 & -1 & 0 & -1 & 2 \end{bmatrix}$$

All agents in the nonlinear MASs described by Eq. (1) have the same dimension.

In this example, as shown in Fig. 1, in order to compare with other controllers, the dynamics equation in [7] is considered as follows

$$\begin{cases} \dot{s}_i(t) = v_i(t) \\ \dot{v}_i(t) = f_i(t, s_i(t), v_i(t)) + u_i(t) + g_i(t) \end{cases} \tag{24}$$

where i = 1, …, 5, s_i and v_i are the position and velocity states of agent i, respectively. $f_i(t, s_i(t), v_i(t)) = \sin(s_i(t))$ denotes the unknown nonlinear function and $g_i(t)$ is the external disturbance. For the leader system, $u_1(t) = 0$ and $g_1(t) = 0$. If we define $s_i(t) = x_{i1}(t)$, $v_i(t) = x_{i2}(t)$ and denote state vector $x_i(t) = [x_{i1}(t), x_{i2}(t)]^T$, then (24) becomes

$$\dot{x}_i(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_i(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (f_i(t) + u_i(t) + g_i(t)) \tag{25}$$

If all matrices in the system equations of each agent (25) are the same that are represented by: $A_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. By using Definition 1 and solving LMI (9), similar parameters and control gain can be obtained as $K_i = [-1, -2]$, $F_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $P = \begin{bmatrix} 3.2945 & 1.9767 \\ 1.9767 & 3.2945 \end{bmatrix}$, $\bar{K} = [-1.9688, -1.2812]$.

The initial values of coupling weight in (6) are chosen as $c_i(0)$ = 1, the initial states of the agents are selected as $x_{i1}(0) = [0, 5, -3, 6, -7]^T$ and $x_{i2}(0) = [0, 2, -1, 2, 1]^T$ .The initial values of adaptive estimation parameters $\bar{W}_i(t)$ are given as:

$\bar{W}_1(0) = [0.49, 0.45, 0.41, 0.46, 0.39]^T$,

$\bar{W}_2(0) = [0.09, 0.22, 0.10, 0.13, 0.05]^T$,

$\bar{W}_3(0) = [0.10, 0.27, 0.31, 0.11, 0.21]^T$,

$\bar{W}_4(0) = [0.19, 0.15, 0.17, 0.14, 0.21]^T$,

$\bar{W}_5(0) = [0.15, 0.12, 0.25, 0.21, 0.31]^T$,

$\bar{W}_6(0) = [0.06, 0.03, 0.07, 0.02, 0.09]^T$.

The parameters in the adaptive laws (7) and (8) are selected as:

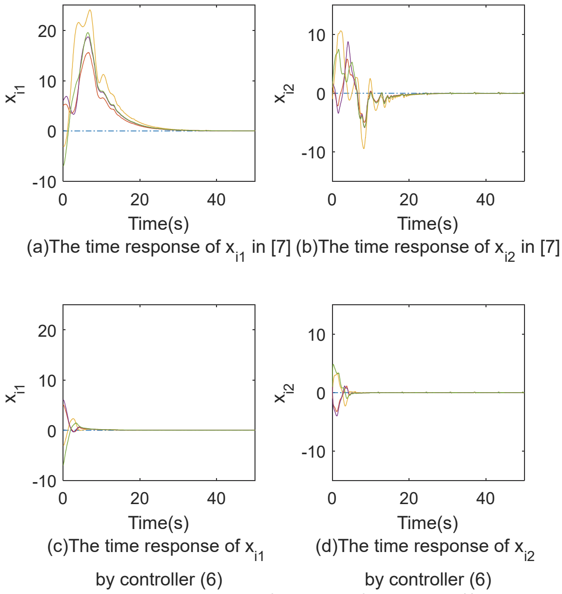
$$\eta_{\omega i} = [5.15, 5.13, 5.28, 5.14, 5.96, 5.33],$$

$$\theta_{\omega i} = [0.03, 0.06, 0.02, 0.05, 0.07, 0.01],$$

$$\eta_{ci} = [2.16, 2.23, 2.18, 2.24, 2.98, 2.36],$$

$$\theta_{ci} = [0.05, 0.02, 0.06, 0.04, 0.03, 0.07].$$

Fig. 2 shows the simulation results of the consensus tracking and errors of homogeneous MASs. The consensus stabilization of two states by using the proposed feedback control has very fast speed as shown in (c) and (d), but the time response of consensus by using the proposed control in [7] shows low velocity as shown in (a) and (b).



(a)The time response of x_{i1} in [7] (b)The time response of x_{i2} in [7]

(c)The time response of x_{i1} by controller (6) (d)The time response of x_{i2} by controller (6)

Figure 2 Trajectories of the state of all agents $x_i(t)$

The time response of estimation ideal weight and the coupling strength are shown in Fig. 3 and Fig. 4 respectively, which are guaranteed to be UUB. By comparing these results, it can be concluded that the control method proposed in this paper has advantages over the control in [7].

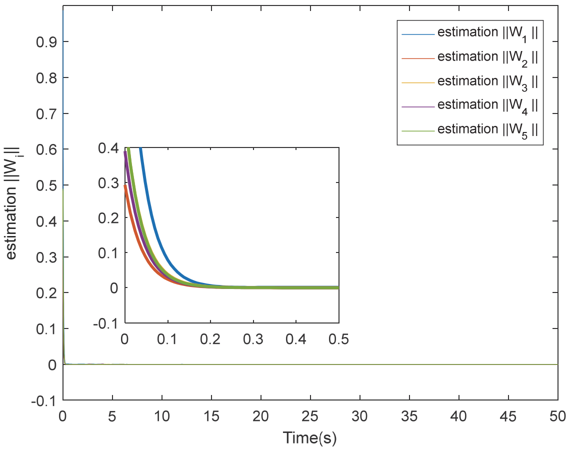


Figure 3 Trajectories of the adaptive estimation parameters $W_i(t)$

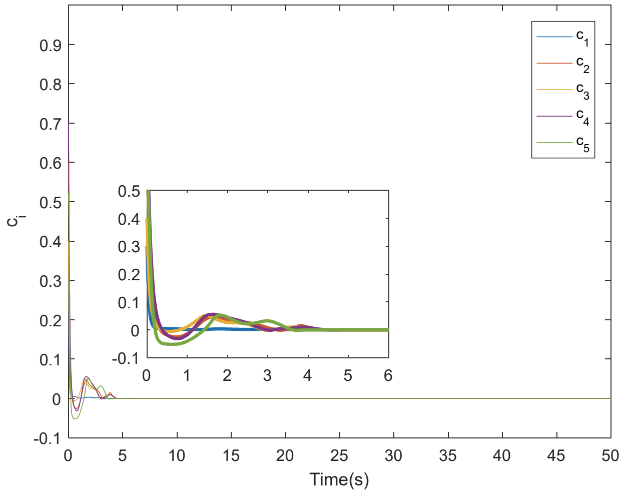


Figure 4 Trajectories of coupling strength among each agent

Example 2: The network topology diagram with 6 agents is shown as in Fig. 5.

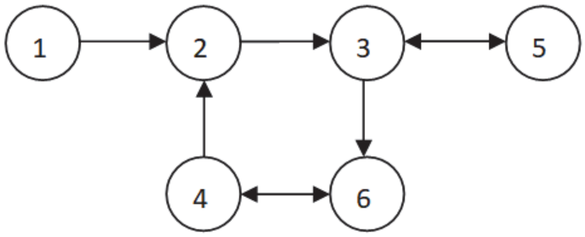


Figure 5 Communication topology of six agent systems

From Fig. 5, it can be easily obtained that the Laplace matrix is written as

$$L = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & -1 & -1 \\ 0 & -1 & 0 & 2 & 0 & -1 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}.$$

Different from example 1 about nonlinear heterogeneous MASs with the same dimension, each agent in the nonlinear MASs described by Eq. (1) has different dimensions, and the matrices such as the following are defined:

$$A_1 = \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix},\ A_2 = \begin{bmatrix} 0 & 1 & 0 \\ -3 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -4 & 3 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix},\ A_4 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -5 & 4 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & -3 \end{bmatrix},$$

$$A_5 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -6 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & -4 \end{bmatrix},$$

$$A_6 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -7 & 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -5 \end{bmatrix},$$

$$B_1 = [0,0;0,1],\ B_2 = [0,0;0,1;0,0],$$

$$B_3 = [0,0;0,1;O_{2\times1},O_{2\times1}],\ B_4 = [0,0;0,1;O_{3\times1},O_{3\times1}],$$

$$B_5 = [0,0;0,1;O_{4\times1},O_{4\times1}],\ B_6 = [0,0;0,1;O_{5\times1},O_{5\times1}].$$

where O denotes a matrix with every zero element, $J_j = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ can be obtained by using Definition 1, and the similar parameters are received as:

$$K_1 = \begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix},\ K_2 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & -3 & 0 \end{bmatrix},\ F_1 = I_{2\times2},$$

$$K_3 = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 2 & -4 & 0 & 0 \end{bmatrix},\ K_4 = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 3 & -5 & 0 & 0 & 0 \end{bmatrix},$$

$$F_2 = \begin{bmatrix} I_{2\times2} & O_{2\times1} \end{bmatrix},\ F_3 = \begin{bmatrix} I_{2\times2} & O_{2\times2} \end{bmatrix},$$

$$K_5 = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 \\ 4 & -6 & 0 & 0 & 0 & 0 \end{bmatrix},\ F_4 = \begin{bmatrix} I_{2\times2} & O_{2\times3} \end{bmatrix},$$

$$K_6 = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 5 & -7 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},\ F_5 = \begin{bmatrix} I_{2\times2} & O_{2\times4} \end{bmatrix},$$

$$F_6 = \begin{bmatrix} I_{2\times2} & O_{2\times5} \end{bmatrix}.$$

By solving LMI (9), Control gain matrix is $\bar{K} = \begin{bmatrix} 0.3750 & 0.6250 \\ -0.9687 & -2.2812 \end{bmatrix}$, and the positive matrix obtained as $P = \begin{bmatrix} 3.2945 & 1.9767 \\ 1.9767 & 3.2945 \end{bmatrix}$.

The nonlinear function is described by: $f_i(x_i) = [-x_{i1}\sin(x_{i2}) + x_{i2}\sin(x_{i1})\cos(x_{i2}), x_{i1}\sin(x_{i2})]^T$

The initial states of each agent are selected as:

$x_1(0) = [2.0, 1.8]^T$, $x_2(0) = [0.3, -1.6, 1.0]^T$,

$x_3(0) = [1.9, 2.4, 1.5, 0.2]^T$,

$x_4(0) = [1.1, 2.9, -1.0, 1.6, 2.4]^T$,

$x_5(0) = [-0.9, 0.6, 2.1, -1.1, -1.3, 2.2]^T$,

$x_6(0) = [2.5, 0.2, 0.1, 2.4, 1.0, 1.1, 2.0]^T$.

The original values of the coupling strength and adaptive parameters and the parameters in adaptive laws (7) and (8) are the same as in case 2. The simulation results are shown as Fig. 6, Fig. 7, Fig. 8.

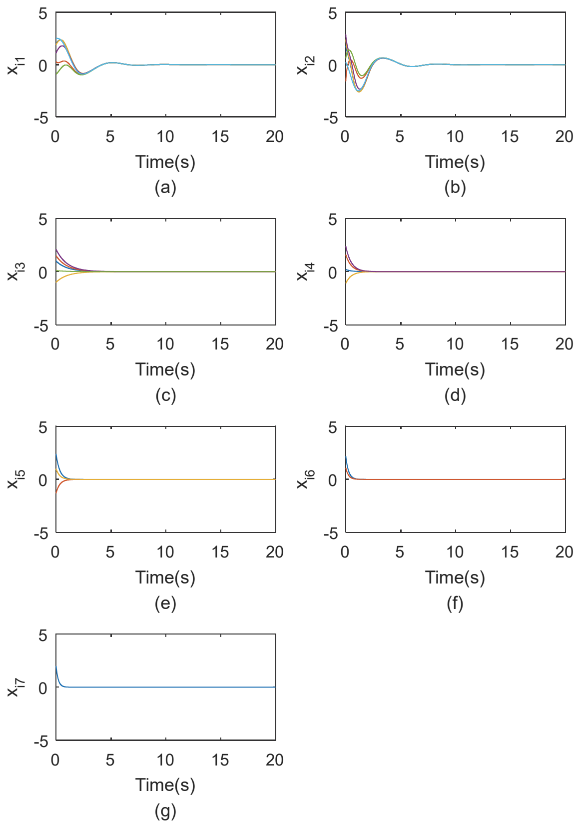


Figure 6 Consensus of states x_i of heterogeneous MASs with different dimensions

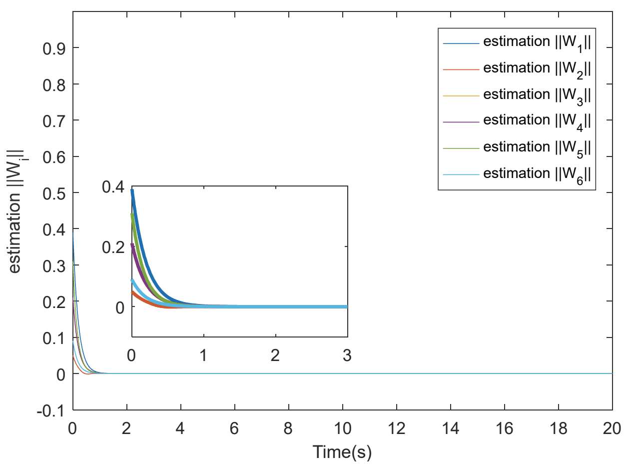


Figure 7 Trajectories of the adaptive estimation parameters $W_i(t)$

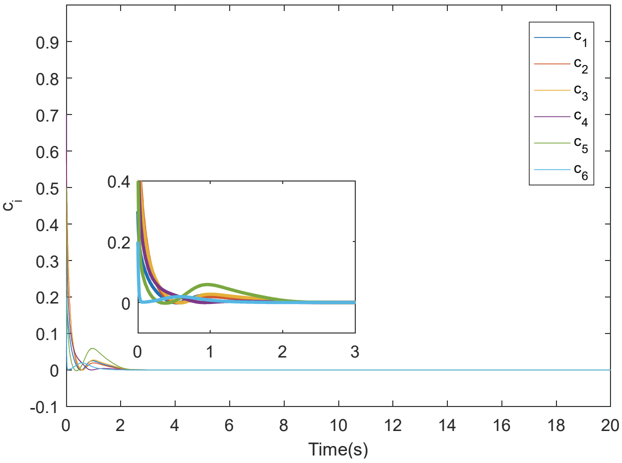


Figure 8 Trajectories of coupling strength among each agent

Each agent in the heterogeneous MASs has distinct dimensions, as shown in Fig. 6. The proposed distributed feedback RBFNN adaptive control protocol allows all agents' states to reach a consistent response rapidly. The RBFNN's coupling weights in Fig. 7 can ultimately be consistent and constrained. In Fig. 8, the time response of the determined coupling strengths is bounded.

5 CONCLUSIONS

A distributed RBFNN control scheme with similar parameters is being developed. Each agent in the heterogeneous nonlinear MASs with similar characteristics can be used as a leader, and other agents can track the leader's dynamic behaviors using the proposed control scheme, and all signals in closed-loop systems are guaranteed to be UUB. The effectiveness and advantage of the control design are demonstrated in two examples with comparison results. It should be noted that the similar condition in Definition only can be used to a class of heterogeneous MASs with canonical form. Our future research work will focus on design consensus control for some heterogeneous nonlinear multi-agent with strict feedback form and different dimensions.

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