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# An Optimization Approach for Pricing of Sherpa Target Redemption Notes

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**Abstract:** Based on the one-factor CIR interest rate model, the pricing of Sherpa Target Redemption Notes (STARN) with early-excise features is investigated in this paper. Firstly, the characteristics of Sherpa target redemption notes were described and the partial differential equation was proposed. Secondly, both non-arbitrage jump conditions on the coupon date and early-excise policy on the redemption date were provided; furthermore, the boundary conditions of partial differential equations were also discussed. Thirdly, a numerical method for solving the partial differential equation was obtained based on the control volume in the theory of finite volume by making use of the upwind weighting scheme to avoid the numerical oscillation phenomenon. Finally, the sensitivity of the model parameters was analyzed. The results show that the STARN value decreases rapidly with the increase in short-term interest rates, furthermore, when short-term interest rates reached a turning point the rate of decline slowed. As volatility increases, the value of the Notes is increased; increasingly as the proportion redeemed is large, STARN value increases.

**Keywords:** one-factor CIR interest rate model; sherpa target redemption notes

## 1 INTRODUCTION

Sherpa Target Redemption Note [1] is an American-style financial derivative. It is a target redemption note and the holders of STARN can redeem part of the principal of the notes for early termination of the contract on the agreed non-coupon dates before the expiration. A Target Redemption Note is similar to an Inverse Floating Rate Note. In contrast, it adds a condition that if the payment coupon exceeded a certain limit, the contract ended prematurely. For example, STARN, issued by HSBC (Hongkong and Shanghai Banking Corporation Limited), its maturity is 10 years. Besides, its interest coupon is paid quarterly. And its coupon payment is the face value of STARN at 11% in the first quarter; the remaining quarter is up to a reverse floating rate calculation formula associated with a stock market index. If the accumulated coupon rate reached 11.01% of the target cap, STARN would be terminated. Then the holder of STARN will recover the principal which is equal to the face value. The date of the payment of the principal is uncertain and it could be earlier than the expiration. The holders of STARN can also apply for HSBC to terminate the STARN immediately at the end of the non-payment of the month, but the STARN holders may only get part of the principal. When the interest rates are at a low level, with the attractive initial coupon and the characteristics of the ability to recover the principal in a short time, the target redemption notes are particularly attractive to Asian investors in the early 21st century [2]. When the interest rates decline, the time to recover the coupon and principal payments is shorter, and the time value of cash flows for investors has higher returns. However, in another extreme case, the interest rate continues to rise and is always higher than a certain level, investors can only hold the target redemption notes to maturity and receive the principal and remaining coupons of the notes at maturity. When the interest rates go in the negative direction, the STARN holders can choose to terminate the contract in advance. Thus the Sherpa Target Redemption Note has greater flexibility than other notes.

Fluctuations in interest rates led to the number of coupons that STARN holders received in coupon date uncertain, but it also determines the uncertainty of termination time. When the payment accumulated coupon amount reaches the target level, STARN will cease (Knock-out feature), and when interest rates rise, STARN holders can also take the initiative to redeem the principal amount and terminate the STARN contract (Early excise feature). Therefore, the value of STARN is designed by two random state variables, one is the interest rate and the other is the path-dependent cumulative coupon amount.

STARN is one of the structured note products which has great flexibility to respond effectively according to markets changing in a dynamic. Chen and Kensinger (1990), Chen and Sears (1990), and Baubonis et al. (1993) firstly analyzed the US stock-linked products (equity-linked products) on the market [3-5]. Wasserfallen and Schunk (1996), Barth et al. (2001), Wilkins et al. (2003), Grünbichler and Wohlwend (2005) studied the SPs on the Swiss and German markets respectively [6-9]. Stoimenov and Wilkins (2005) found that the additional revenue decreased with the approaching expiration of products. He proposed the "Life Cycle Hypothesis" (life cycle hypothesis) [10]. Muck (2006, 2007), Wilkens and Stoimenov (2007), aiming at the first generation of leveraged products (leverage products) which have a knock characteristic (knock-out) for the German market, found that the actual prices of these products were higher than their theoretical price [11-13]. Muck (2007), however, thought that the risk of asset price jump may partly explain the reasonableness of such additional costs, which indicates that the so-called "life-cycle hypothesis" does not exist. Entrop et al. (2009), researching the second generation of leveraged products, get a similar conclusion [14]. Kang and Zheng (2005) used the BDT model to price Xiamen foreign exchange structured deposit [15]. Ren and Li (2005) studied "the Australian dollar into gold" which is a financial product design and pricing mechanism held by Bank of China Beijing Branch, and give an explicit pricing solution using the method of partial differential [16], similar studies including Cui Hairong, He Jianmin and Hu Xiaoping (2012) [17] and so on. Shefrin and Statman (1993) analyze the design of innovative products from the perspective of behavioral finance theory [18]. Breuer and Perst (2007) estimated the attractiveness of discount bonds and reverse convertible bonds to investors using cumulative prospect theory [19]. Hens and Rieger (2009) studied the relationship between the reason of investors and the attractiveness of SPs [20].

## 2 PRICING MODEL
### 2.1 Introduction STARN

The number of coupon tickets and date of receipt of principal which the holders of STARN received on every coupon date is uncertain. In addition, STARN holders could be initiated to terminate the contract prematurely by a point agreed to get part of the principal. Therefore, STARN can be considered as a Contingent Claim whose underlying stochastic state variables are the amount of accumulated coupon of interest rates and path dependence. As used herein, short-term interest rates as a state variable, the short-term interest rate is assumed to obey a one-factor CIR (Cox-Ingersoll-Ross, 1985) model. The CIR model is an example of a "one-factor model" because it describes interest movements as driven by a sole source of market risk. It is used as a method to forecast interest rates and is based on a stochastic differential equation. The CIR model can be utilized, among other things, to calculate prices for bonds and value interest rate derivatives.

$$\mathrm{d}r = a(b-r)\mathrm{d}t + \sigma\sqrt{r}dW \tag{1}$$

$a$, $b$, $\sigma$ is a constant greater than zero. $dW$ is one dimension Brownian motion.

Let $t_0 = 0$ be the start date of the STARN, $t_k$ is the $k$-th polling day, $k = 1, 2, \ldots, K$, $t_k = T$, $T$ is the maturity date of the STARN. We assume that the interval between two coupon dates is a constant $\tau$, namely $t_k = k\tau$. $N$ is notional of START. $t_k^{\tau} = k\tau + m\Delta$ is coupon date. $m$-th STARN holders can take the initiative in advance to maturity of the contract between $t_k, t_{k+1}$, $M = 1,2,\ldots,M-1$, $M\Delta = \tau$. In this case, noteholders may only receive a portion of the principal of $N_\rho, 0 < \rho < 1$, $\rho$ is recovery ratio. As same as widely traded goals redeem the notes, in the coupon data when the contract has not been terminated. The coupon amount which the holders of STARN received is designed by the Inverse floating rate formula as follows:

$$C(t_k, r) = \hat{N}\tau(f - sL(t_k, r; \tau))^+ \tag{2}$$

Here, $(x)^+ = \max(x, 0)$; $f$, $s$ are the positive constants; $L(t_k, r; \tau)$ is $\tau$ - the interest index at $t_k$ (like LIBOR). $A(t)$ represents the accumulated coupon amount that the holders of STARN received up to $t$, assuming that STARN is still viable up to coupon data $t_k$. So,

$$\begin{aligned} &A(t) = \hat{N}\tau\sum_{i=1}^{k}(f - sL(r, t_i; \tau))^+, \\ &t_k^+ < t < t_{k+1}^-, k = 1,2,\ldots,k-1, \\ &A(t) = 0, 0 < t < t_1. \end{aligned} \tag{3}$$

Let $C_{\text{cap}}$ be the target cap of the STARN coupon. The sum of the coupons which the holder of STARN received is $\hat{N}C_{\text{cap}}$. Therefore, $A(t)$ is upper bounded and the upper bound is $\hat{N}C_{\text{cap}}$. STARN would not be ended until the sum of the coupon which is received gets $\hat{N}C_{\text{cap}}$. Assuming that STARN would be ended at coupon data $t_k$, the coupon which the holder received is $\hat{N}C_{\text{cap}} - A(t_k^-)$. Let $C(t_k, r)$ be the amount of the coupons at $t_k$, $K = 1,2,\ldots,K$. So,

$$C(t_k, r) = \begin{cases} \hat{N}\tau(f - sL(t_k, r; \tau))^+, & \text{not terminated at time } t_k \\ \hat{N}C_{\text{cap}} - A(t_k^-), & \text{terminated at time } t_k \end{cases} \tag{4}$$

Let $P_t^T$ be units of par value of default-free zero-coupon bond value at the time $t$ with the maturity date of $T$. $L(t_k, r; \tau)$ in the period of $\tau$ has the following relationship between its value and discounted bond expired at the time $t+\tau$:

$$L(t_k, r; \tau) = \frac{1}{\tau}\left(\frac{1}{P_t^{t+\tau}} - 1\right) \tag{5}$$

In the CIR interest rate model in (1), it is easy to deduce that the price of units of par value of default-free zero-coupon bonds is

$$P(t, T) = A(t, T)\mathrm{e}^{-B(t,T)r_t} \tag{6}$$

where

$$A(t,T) = \left(\frac{2h\mathrm{e}^{(h+a)(T-t)/2}}{(h+a)(\mathrm{e}^{h(T-t)} - 1) + 2h}\right)^{\frac{2ab}{\sigma^2}}$$

and $B(t,T) = \dfrac{2(\mathrm{e}^{h(T-t)} - 1)}{(h+a)(\mathrm{e}^{h(T-t)} - 1) + 2h}, h = \sqrt{a^2 + 2\sigma^2}$.

$\hat{N}$ - the face value of STARN is assumed to be 1.

### 2.2 Pricing Partial Differential Equation

STARN is essentially an interest rate derivative, and the pricing of interest rate derivatives differs from the pricing of financial derivatives where the underlying asset can be traded directly. Interest rate derivatives due to their subject matter (Underling) are not traded interest rate, no other financial derivatives pricing based on replication method, so use the Wilmott (2006) [22] based on the market prices of interest rate risk (Market Price of Interest-rate Risk), the concept of pricing in the objective probability measure.

Because its subject matter is the interest rate cannot be traded, the interest rate derivatives cannot be priced by common methods for other financial derivatives. Therefore, based on the market price of interest-rate risk introduced by Wilmott, pricing under the objective probability measure is proposed. $V(t, r; A)$ is the value of STARN, depending on the time $t$ and the interest rate $r$. Because $A$ - the cumulative sum of the coupon changed only in the coupon date, the dependent variable $A$ will not be found in the pricing differential equation of STARN.

In the non-coupon days and redemption days, based on Ito's Lemma, we can get the pricing partial differential equations

$$\frac{\partial V}{\partial t}+\frac{\sigma^2}{2}r\frac{\partial^2 V}{\partial r^2}+(a(b-r)+\sigma r^{1/2}\lambda(t,r))\frac{\partial V}{\partial t}-rV=0, \quad t_k^{m+}<t<t_k^{m+1-}, k=0,1,\dots,K-1, m=0,1,\dots,M-1 \tag{7}$$

where $\lambda(t,r)$ is the market price of interest-rate risk. We assumed that $\lambda(t,r)$ is a deterministic function of time and interest rates. The estimation method of $\lambda(t,r)$ was introduced in Wilmott (2006) [22]. On coupon date $t_k$, $A$ changed variable based on the rule

$$A^+(t_k)=A^-(t_k)+C(t_k,r)$$

where $A^-(t_k), A^+(t_k)$ are for the previous value and the changed value of $A$ respectively on coupon day $t_k$.

On the coupon dates $t_k$, when $A^+(t_k)=C_{\text{cap}}$, the contract of STARN will terminate. The sum of the face value of the ticket and the remaining unpaid coupons - $C_{cap}-A^-(t_k)$ will be paid on the termination of the contract with the following equation

$$V^-(t_k,t)=1+C_{\text{cap}}-A^-(t_k) \tag{8}$$

STARN will change upon the no-arbitrage jump condition on the coupon dates when premature knock-out did not happen, with the following equation

$$V^+(t_k,r)=V^-(t_k,r)+C(t_k,r), A^+(t_k)<C_{\text{cap}} \tag{9}$$

On the maturity date $T$, the payment for STARN depends on $A^-(T)$. If premature knock-out or redemption did not happen, the remaining portion of the total value of the guarantee coupon and the face value should be paid to the STARN holders. Thus, right before the maturity date, the value of STARN is

$$V^-(T,r)=1+C_{\text{cap}}-A^-(T), A^-(T)<C_{\text{cap}} \tag{10}$$

On the redemption dates $t_k^m, m=1,2,...,M-1$, the STARN holders can choose the early termination of the contract by applying for part of the face value. At this time, STARN changes as the following equation

$$V^+(t_k^m,r)=\max(V^-(t_k^m,r),\rho) \tag{11}$$

In the pricing equation of STARN described in Eqs. (7) to (11), the dependent variable $A$ does not explicitly appear in the partial differential Eq. (7). So the pricing of STARN is implemented by solving the partial differential equations coupled with a different value $A$.

### 2.3 Boundary Condition

For the single-factor CIR model described in Eq. (1), the short-term interest rate $r$ ranges in value $[0,+\infty)$. When $r\to+\infty$ based on Halluin (2001)[23], the boundary condition is

$$V(t,+\infty)=0 \tag{12}$$

When $r\to 0$ the condition will be relatively complicated. Oleinik & Radkevich (1973) pointed out that, for the partial differential equations with the form like

$$f_t=\tilde{a}(r)f_{rr}+\tilde{b}(r)f_r+\tilde{c}(r)f \tag{13}$$

as long as $\lim_{r\to 0}(\widehat{a}(r)-\widehat{b}(r))\geq 0$, the boundary conditions were not required at $r = 0$. Houston et al. (2000) [23] promoted the above conclusion. They proved that, under the single-factor CIR model, as long as

$$\frac{2ab}{\sigma^2}\geq 1 \tag{14}$$

there is no need to add the boundary conditions at $r = 0$ in Eq. (7).

## 3 A NUMERICAL METHOD BASED ON THE THEORY OF FINITE VOLUME ELEMENT

### 3.1 Equations on non Cancellable Date

For STARN, the coupon payments, a possible knockout, and early redemption all occur at the discrete point. The numerical method for solving the pricing equation was obtained based on the control volume in the theory of finite volume proposed by Zvan (2001) [24] which was used to price for the discrete monitored path-dependent options. The pricing algorithm can be seen as a problem by a series of sub-structure. Each sub-path problem was associated with a given path-dependent state variable. These state variables are the total amount of accrued coupon received. Only on coupon payment date, can communication occur between these independent sub-problems.

Let $z=T-t$, for $T-t_k^m<z<T-t_k^{m-1}, r>0$ and $0\leq A<C_{\text{cap}}$, where $k=1,2,\dots,K$, $m=1,2,\dots,M$, there be

$$\frac{\partial V}{\partial z}=\frac{1}{2}\sigma^2 r\frac{\partial^2 V}{\partial r^2}+(a(b-r)+\sigma r^{1/2}\lambda(z,r))\frac{\partial V}{\partial z}-rV \tag{15}$$

with the initial conditions

$$V(0^+,r;A)=1+C_{\text{cap}}-A \tag{16}$$

When $A=C_{\text{cap}}$,

$$V(z,r)=1 \tag{17}$$

For each given $A$, one dimensional subproblem described by Eq. (15) can be solved independently. Uniform-size grid point was used in this article. Calculated fields are decided by grid point $(Z_n, A_i, r_j), n = 0,1,\ldots,n_{\max}, i = 0,1,\ldots,i_{\max}, j = 0,1,\ldots,j_{\max}$, where $\Delta Z = \dfrac{T}{n_{\max}}, \Delta A = \dfrac{C_{\text{cap}}}{i_{\max}}, \Delta r = \dfrac{r_{\max}}{j_{\max}}$

Let $V_{i,j}^n$ be the numerical solution of STARN at grid point $(Z_n, A_i, r_j)$. For each fixed $r$, there is an initial condition

$$V_{i,j}^0 = 1 + C_{\text{cap}} - A_i, \forall j \tag{18}$$

Consider the control volume on the interval $\left[r_{j-\frac{1}{2}}, r_{j+\frac{1}{2}}\right]$, where $r_{j-\frac{1}{2}}$ is the midpoint of $r_{j-1}, r_j$ ,and $r_{j+\frac{1}{2}}$ is the midpoint of $r_j, r_{j+1}$. Solve the quadrature of the Eq. (15). Based on Zvan (2001), [24], we can get the implicit numerical equation

$$\begin{aligned}\frac{V_{i,j}^{n+1} - V_{i,j}^n}{\Delta Z} &= \frac{\sigma^2 r_j}{2\Delta r^2}(V_{i,j+1}^{n+1} - 2V_{i,j}^{n+1} + V_{i,j-1}^{n+1}) + \\ &+ \frac{a(b-r_j) + \lambda(z_n, r_j)\sigma r^{1/2}}{\Delta r}\left(V_{i,j+\frac{1}{2}}^{n+1} - V_{i,j-\frac{1}{2}}^{n+1}\right) - \\ &-r_j V_{i,j}^{n+1}\end{aligned} \tag{19}$$

To avoid spurious oscillations in numerical computation, we use an upstream weighting scheme to calculate $V_{i,j+\frac{1}{2}}^{n+1}$, which is defined as follows

$$V_{i,j+\frac{1}{2}}^{n+1} = \begin{cases} V_{i,j}^n, a(b-r_j) + \lambda(z_n, r_j)\sigma r_j^{1/2} < 0 \\ V_{i,j+1}^n, a(b-r_j) + \lambda(z_n, r_j)\sigma r_j^{1/2} > 0 \end{cases} \tag{20}$$

The initial and boundary conditions are as follows

$$\begin{aligned} &V_{i,j}^0 = 1 + C_{\text{cap}} - A_i, \forall i, j \\ &V_{i_{\max},j}^n = 1, \forall n, j \\ &V_{i,j_{\max}}^n = 0 \end{aligned} \tag{21}$$

Referring to Kwok and Kuen (2006) [2], we specified a time-weighted factor $\theta, 0 \le \theta \le 1$ and assessed the value of the spatial discretization at the old and the new points. Then we get the following finite volume element scheme:

$$\begin{aligned}\frac{V_{i,j}^{n+1} - V_{i,j}^n}{\Delta z} = \theta &\Bigg[\frac{\sigma^2 r_j}{2\Delta r^2}(V_{i,j+1}^{n+1} - 2V_{i,j}^{n+1} + V_{i,j-1}^{n+1}) \\ &+ \frac{a(b-r_j) + \sigma r_j^{1/2}\lambda(z_{n+1}, r_j)}{\Delta r}\left(V_{i,j+\frac{1}{2}}^{n+1} - V_{i,j-\frac{1}{2}}^{n+1}\right) \\ &-r_j V_{i,j}^{n+1}\Bigg] \\ (1-\theta)&\Bigg[\frac{\sigma^2 r_j}{2\Delta r^2}(V_{i,j+1}^n - 2V_{i,j}^n + V_{i,j-1}^n) \\ &+ \frac{a(b-r_j) + \sigma r_j^{1/2}\lambda(z_n, r_j)}{\Delta r}\left(V_{i,j+\frac{1}{2}}^n - V_{i,j-\frac{1}{2}}^n\right) \\ &-r_j V_{i,j}^n\Bigg]\end{aligned} \tag{22}$$

When $\theta = 0, \dfrac{1}{2}$, and 1, the Eq. (22) respectively corresponds to fully explicit format, Crank-Nicolson format, and fully implicit format. Crank-Nicolson format and fully implicit format are unconditionally stable. But only when the time step is small enough relative to the spatial discretization step, the fully explicit format is stable. The advantage of the Crank-Nicolson format is that it can reach the second-order accuracy in the time dimension. And yet fully explicit format and fully implicit format can only achieve the first-order accuracy in the time dimension. And considering the oscillation of the algorithm, in the fully implicit format oscillations do not exist but only when the time step is small enough can the fully explicit format and Crank-Nicolson format avoid oscillation.

### 3.2 Jump Conditions on the Coupon Date

On coupon dates, the aforementioned non-arbitrage jump conditions should be satisfied. By assuming that the coupon dates were at the time grid points between $z_n$ and $z_{n+1}$, i.e. $z_n \le T - t_k < z_{n+1}$, we got the following finite difference scheme of non-arbitrage jump conditions,

$$V_{i,j}^{n+1} = \widehat{V}_{i,j}^n - \min\left\{C_{\text{cap}} - A_i, \tau(f - sL(t_k, r_j, \tau))^+\right\} \tag{23}$$

where $\widehat{V}_{i,j}^n$ is an approximate value of $V\left(z_n, r_j; A'\right)$. By using the following linear interpolation method, we obtained that,

$$\hat{V}_{i,j}^n = \frac{A_i - A'}{A_i - A_{i-1}}V_{i-1,j}^n + \frac{A' - A_{i-1}}{A_i - A_{i-1}}V_{i,j}^n \tag{24}$$

where

$$A' = A_i + \min\left\{C_{\text{cap}} - A_i, \tau\left(f - sL\left(t_k, r_j, \tau\right)\right)^+\right\}$$

As $A' = 0$ it means that no coupon is paid. At this point, $V_{0,j}^{n+1} = \widehat{V}_{i,j}^n = V_{0,j}^n$.

We assumed that $A_{i-1} < A' < A_i, i = 1, 2, \ldots, i_{\max}$, where $\widehat{V}_{i,j}^{n}$ is the linear interpolation of $a$. The second item of $A'$ is the actual payment of the coupon on the coupon date $t_k$. We take the minimum of $A'$ to make the cumulative sum of coupons the STARN holders have up to $C_{\text{cap}}$ and $A' \in \left[0, C_{\text{cap}}\right]$. Because $A'$ was not exactly falling inthe computing grid, it is necessary to use interpolation methods to obtain approximate values of $\widehat{V}_{i,j}^{n}$.

### 3.3 Early-Excise Conditions of Redemption Dates

On the redemption dates $t_k^m, m = 1, 2, \ldots, M-1$, the holders of STARN can choose the early termination of the contract by applying for part of the face value. By assuming that the redemption date was at the time grid point between $z_n$ and $z_{n+1}$, i.e. $z_n \le T - t_k^m < z_{n+1}$, we got the following finite difference scheme of the early-excise

$$V_{i,j}^{n} = \max\left(V_{i,j}^{n}, \rho\right) \tag{25}$$

Using the Crank-Nicolson format and fully implicit format, it is difficult to solve the above function because solving $V_{i,j}^{n}$ is required to use other computing grid points. We can use an iterative method introduced by Tavella & Randall (2000) [25] for solving linear equations to settle this problem.

## 4 NUMERICAL EXAMPLE

Refering to the contract of STARN issued by HSBC, the contract of STARN for numerical computation is described in the following Tab. 1.

**Table 1** The contract of STARN

| Clause | Content |
|---|---|
| Notional Amount | 1 |
| Maturity | 10 years |
| Target cap rate | 15% |
| First-year fixed coupon rate | 9% |
| Inverse floater formula | max(8.5% − $L$, 0), $L$ = 3 − month LIBOR |
| Coupon payment frequency | Quarter |
| Redemption date | End of non-coupon month |
| Redemption Proportion | $\rho = 0.8$ |

For the CIR model described in Eq. (1), the corresponding parameters are the mean reversion rate $a = 0.5$, the levels of mean reversion $b = 0.02$, the market prices of interest rate $\lambda = 0.01$, and the volatility $\sigma = 0.1$. We follow the direction of short-term interest rate $r$ using 60 grid points, where $r_{\max} = 1.0$. Along the direction of $A$ using 30 grid points, where $A_{\max} = C_{\text{cap}} = 0.15$. In Fig. 1, we compare the calculation accuracy of the finite volume element numerical method of STARN under different time steps. It can be found in Fig. 1 that as $r$ increases, the value of notes $V$ decreased rapidly. So $V$ is a decreasing function of interest rate $r$. Higher interest rates mean receiving coupons at a slower rate and knocking out with a smaller opportunity. With the delay of the expiry date of the ticket, face value will be discounted with a bigger discounting factor and the value of notes becomes slower. However, as $r$ reached a turning point, the rate of decline slowed. That is because as $r$ is running upward to a certain extent, STARN holders can actively choose to get part of the face value of notes to terminate the contract. From Fig. 1, with the decrease in time step, the numerical results obtained from the finite volume method also converge at a very fast speed.

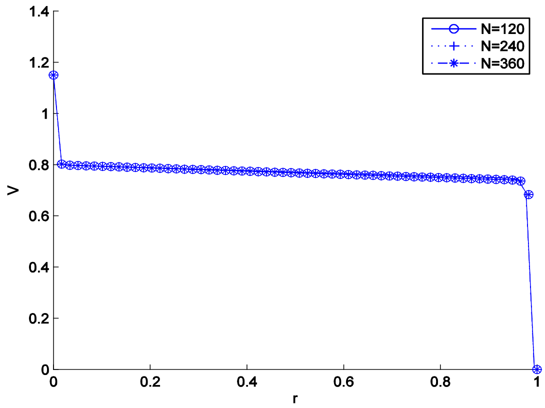


**Figure 1** Comparison of accuracy of numerical valuation of STARN with varying number of time steps

In Fig. 2 and Fig. 3, STARN's sensitivity to interest rate volatility parameters $\sigma$ and the proportion of redemptions $\rho$ are respectively discussed. By observing Fig. 2, it can be found that, when $\sigma$ increases, the value of STARN also increases. Therefore, when the short-term interest rate $r$ reaches a certain critical point, due to the active redemption, the different values $\sigma$ do not affect the values of STARN. Just near the calculation region's boundaries, lager $\sigma$ easily leads to instability of numerical methods and makes the value of STARN at the boundary $r = r_{\max}$ unreliable. It can be found in Fig. 3 that the value of STARN increases when the $\rho$ increases. When the value $\rho$ decreases, the value of STARN also decreases. When $\rho$ = 0 STARN degenerates into an ordinary target redemption note (TARN).

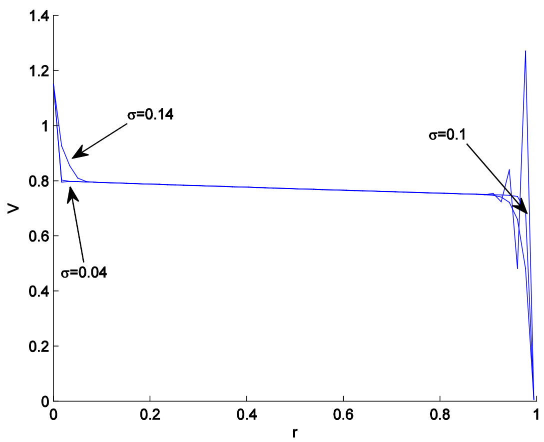


**Figure 2** Plot of note value $V$ against short rate $r$ with varying values of redemption parameter $\rho$

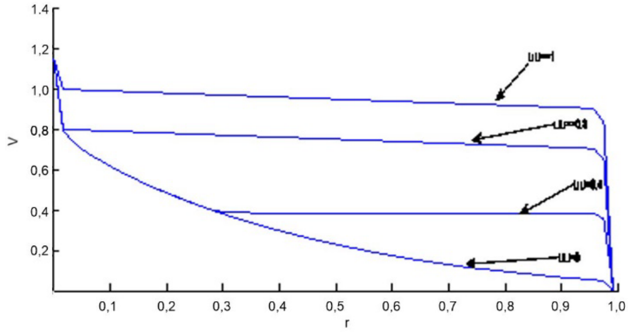


**Figure 3** Plot of note value *V* against short rate *r* with varying values of redemption parameter *ρ*

## 5 CONCLUSION

The complexity of pricing STARN lies in two features of STARN, a premature knock-out and early termination. Its pricing function occurs in discontinuous jump at the points of the coupon dates and has a lower bound on redemption days. All of these make the issue of the pricing of STARN difficult to solve. Therefore,the pricing partial differential equation and its boundary conditions of STARN were proposed. Then, a numerical method for solving the pricing equation was obtained based on the control volume in the theory of finite volume by making use of the upwind weighting scheme to avoid the numerical oscillation phenomenon. Finally, the sensitivity of the model parameters was analyzed. The results show that the STARN value decreases rapidly with the increase in short-term interest rates; furthermore, when short-term interest rates reached a turning point, the rate of decline slowed. When the volatility increases, the value of the Notes is increasing; increasingly as the proportion redeemed gets larger, the STARN value increases.

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