

# PRORAČUN ELEKTRIČNIH MREŽA S NESINUSNIM PERIODIČKIM VALNIM OBLICIMA COMPUTATION OF ELECTRIC NETWORKS WITH NONSINUSOIDAL PERIODIC WAVEFORMS

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Dan je opis višefaznih električnih sustava i njihove klasifikacije, s posebnim osvrtom na trofazni sustav. Opisan je proračun trofaznog simetričnog sustava, s posebnim osvrtom na snagu simetričnog sustava. U trofaznim mrežama se događaju izobličenja sinusnog oblika, što bitno mijenja karakteristike te mreže. Pokazano je kako se računaju fazne i linijske veličine u mrežama s izobličenim sinusoidama. Na kraju je dan jedan primjer proračuna viših harmonika u određenoj konfiguraciji elektroenergetske mreže primjenom softverske simulacije, te jedan primjer iz prakse, gdje su prikazani rezultati mjerenja viših harmonika u distributivnoj 10 kV elektroenergetskoj mreži.

A description is given of polyphase electric systems and their classification, with special reference to the three-phase system. The computation of a three-phase symmetrical system is described, with special reference to the power of the symmetrical system. Distortions of the sinusoidal form occurring in three-phase networks essentially change the characteristics of the network. It is shown how the phase and line values in networks with distorted sinusoids are computed. At the end, an example is given of how higher harmonics in a certain power supply network are computed by using software simulation, and another example from practice where the results of measuring higher harmonics in a 10 kV power distribution network are shown..

**Ključne riječi: fazne i linijske veličine struje i napona, nesinusni valni periodički oblik, trofazni sustav, višefazni sustav, viši harmonici**  
**Keywords: higher harmonics, nonsinusoidal periodic waveform, phase and line current and voltage values, polyphase system, three-phase system**



## 1 UVOD

Teslino otkriće principa stvaranja rotirajućeg magnetskog polja izmjeničnim strujama dalo je osnovu za rješenje mnogih problema koji su se javljali na području elektroenergetike u to vrijeme, posebno pri prijenosu električne energije. U današnje vrijeme, postavljaju se veliki zahtjevi na trofazni sustav, naročito kad su u pitanju izobličenja sinusnog valnog oblika struje i napona. Pored osnovnih proračuna za određene konfiguracije sustava, ovim radom je posebno i analizirano ponašanje viših harmonika u trofaznim sustavima.

Tesla je svoj motor izmjenične struje i višefazni sustav razradio u Americi i zaštitio u 36 patentata prijavljenih u periodu od 1887-10-12 do 1891-07-13. U tim patentima je razradio postupak konstruiranja višefaznih generatora, odnosno višefaznih električnih sustava.

## 2 STVARANJE VIŠEFAZNIH SUSTAVA

Za prijenos električne energije od izvora prema trošilu potrebna su dva vodiča, odvodni i povratni vodič. Najjednostavniji sustav prijenosa električne energije sastoji se od izvora sinusnoga napona, dvožičnoga voda i trošila.

Spajanjem nekoliko jednofaznih sustava, čije su struje iste frekvencije i međusobno fazno pomaknute za određeni kut, može se postići da je suma struja u povratnim vodičima jednaka nuli. Ako se spoje zajedno svi povratni vodiči takvoga sustava, tada kroz njih ne teče struja pa ih se može ukloniti ili bitno smanjiti presjek žice.

Za razliku od jednofaznih generatora koji imaju dvije izlazne priključnice, kod  $m$ -faznih generatora na izlazu postoje najmanje  $(m + 1)$  priključnice.

Samo jedna priključnica predviđena je za povratni ili nulvodič, a ostale  $(m)$  su fazne priključnice.

Skup sinusnih elektromotornih sila (EMS) koje djeluju u pojedinim fazama čine višefazni sustav EMS, a skup napona i struja koje teku u tom sustavu čine višefazni sustav napona, odnosno višefazni sustav struja [1], [2] i [3].

Od višefaznih sustava susreću se dvofazni, trofazni, šesterofazni, dvanestofazni, a najčešće su u uporabi trofazni sustavi. Sustavi sa 6 ili 12

## 1 INTRODUCTION

Tesla's discovery of the creation of the rotating magnetic field by means of alternating currents has provided a basis for solving many problems encountered in electrical engineering of that time, especially in power transmission. Today considerable demands are made on three-phase systems, especially when it comes to the distortions of the sinusoidal current and voltage waveform. In addition to the basic computations for certain configurations of the system, this work also deals with the behaviour of higher harmonics in three-phase systems.

His alternating-current motor and polyphase system Tesla elaborated in USA and protected through 36 patents registered over the period from 12 October 1887 to 13 July 1891. In these patents he developed the method of designing polyphase generators or polyphase electric systems.

## 2 CREATION OF POLYPHASE SYSTEMS

The transmission of electricity from the source to the consumer requires two conductors, output conductor and return conductor. The simplest electricity transmission system consists of a sinusoidal voltage source, a two-wire line and a consumer.

By connecting several single-phase systems, the currents of which are of the same frequency and mutually phase-shifted, the sum of currents in the return conductors can be made to equal zero. If all return conductors of such a system are connected together, no current will flow through them, so that they can be removed or the wire cross-section can be greatly reduced.

Unlike the single-phase generators which have two output clamps, with  $m$ -phase generators there are at least  $(m + 1)$  clamps at the output.

Only one clamp is envisaged for the return conductor or zero-conductor, the rest  $(m)$  are phase clamps.

A set of sinusoidal electromotor forces (EMF) operating in certain phases constitutes the EMF polyphase system, whereas a set of voltages and currents flowing in that system constitutes the polyphase voltage system and the polyphase current system respectively [1], [2] and [3].

Of the polyphase systems, two-phase, three-phase, six-phase, twelve-phase systems can be found, but

faza pojavljuju se u nekim uređajima, i to obično ispravljačkim.

EMS koje čine višefazni sustavi mogu se proizvesti s više jednofaznih generatora, ali se one proizvode isključivo pomoću jednoga višefaznoga generatora, koji ima više nezavisnih namota međusobno prostorno pomaknutih, tako da se pri radu generatora u namotima induciraju EMS iste frekvencije, ali različitih faznih kutova.

### 3 KLASIFIKACIJA VIŠEFAZNOG SUSTAVA

Razmotrit će se neke klasifikacije višefaznih sustava EMS. Tako npr. postoje simetrični i nesimetrični sustavi.

Simetrični  $m$ -fazni sustav takav je višefazni sustav kod kojeg su efektivne vrijednosti napona (struja) jednake, a fazni pomak između bilo koje dvije susjedne faze jednak je  $\frac{2\pi}{m}$ .

Elektromotorna sila  $k$ -te faze nekog  $m$ -faznog sustava može se napisati u vremenskom obliku ovako:

typically in use are the three-phase systems. Systems with 6 or 12 phases occur in some devices, usually rectifiers.

EMFs constituting the polyphase systems can be made with more single-phase generators, but they are exclusively made by means of one polyphase generator which has several independent mutually spaced windings, so that in the generator's operation EMFs of the same frequency, but of different phase angles, are induced.

### 3 CLASSIFICATION OF POLY-PHASE SYSTEMS

Some classifications of the EMF polyphase systems will be considered below. Thus, for example, there are symmetrical and non-symmetrical systems.

A symmetrical  $m$ -phase system is a polyphase system with which the effective voltage (current) values are equal and the phase shift between any two adjacent phases equals  $\frac{2\pi}{m}$ .

The electromotor force of the  $k$ -th phase of an  $m$ -phase system may be written in the time form as follows:

$$e_k = \sqrt{2} E \sin \left[ \omega t - (k-1) \frac{2\pi}{m} \right] \quad (1)$$

čiji je fazor:

the phasor of which is:

$$\dot{E} = E_k e^{-j(k-1) \frac{2\pi}{m}} \quad (2)$$

Struje u takvom simetričnom sustavu zaostaju iza svojih faza za kut  $\varphi$ , pa je za  $k$ -tu fazu:

Currents in such a symmetrical system lag behind their phases by angle  $\varphi$ , so for the  $k$ -th phase it is:

$$i_k = \sqrt{2} I \sin \left[ \omega t - (k-1) \frac{2\pi}{m} - \varphi \right] \quad (3)$$

čiji je fazor:

the phasor of which is

$$\dot{I} = I_k e^{-j \left[ (k-1) \frac{2\pi}{m} - \varphi \right]} \quad (4)$$

Fazorski prikaz napona (struja) predstavlja pravilnu zvijezdu od  $m$  jednako dugih krakova, koji su razmaknuti za  $\frac{2\pi}{m}$  radijana.

Treba reći da postoji simetrični sustav EMS (napona) i simetrični sustav struja. Karakteristično je za takav simetrični sustav da je:

$$\sum_{n=1}^m e_k = 0 \text{ ili / or } \sum_{n=1}^m \dot{E}_k = 0 \quad (5)$$

$$\sum_{n=1}^m i_k = 0 \text{ ili / or } \sum_{n=1}^m \dot{I}_k = 0 \quad (6)$$

Ako svaka iduća EMS  $m$ -faznog sustava faza zaostaje za prethodnom tada kažemo da je takav sustav direktni, a ako prethodi onda je takav sustav inverzni.

Postoji još i tzv. nulfazni, višefazni sustav kod kojega su sve EMS u fazi, a mogu imati potpuno jednake efektivne vrijednosti EMS. Za takav sustav ne vrijedi da je:

$$\sum_{n=1}^0 \dot{E}_k = 0 \quad (7)$$

Višefazni sustavi direktni i inverzni mogu biti uravnoteženi i neuravnoteženi.

Uravnotežen naziva se višefazni sustav kod kojeg trenutačna vrijednost snage ne ovisi o vremenu nego je konstanta. Neuravnoteženi sustav je takav višefazni sustav kod kojeg je trenutačna vrijednost snage funkcija vremena.

Višefazni sustav bit će uravnotežen ako ima simetričan sustav EMS i ako su sve faze jednako opterećene, pa fazne struje također čine simetrični sustav. Međutim, dvofazni sustav s tri vodiča nesimetričan je sustav EMS, ali kad je s jednakim opterećenjem po fazama također je uravnotežen sustav. Svojstvo višefaznih sustava da pri jednakim opterećenjima daju vremenski konstantnu ukupnu snagu od velike je važnosti, jer se za takve sustave mogu izgraditi motori koji proizvode konstantan moment vrtnje.

Višefazni sustav simetričnih EMS i simetričnih struja uravnotežen je ako je broj faza  $m > 2$ . Trenutačna snaga u  $k$ -toj fazi takvoga simetričnog sustava je:

The phasor presentation of voltages (currents) looks like a regular star consisting of  $m$  points equal in length, spaced out by  $\frac{2\pi}{m}$  radian.

It should be noted that there is a symmetrical EMF (voltage) system and a symmetrical current system. Characteristic of such a symmetrical system is that:

If every next EMF of an  $m$ -phase system lags behind the preceding one, such a system is said to be direct, and if it runs ahead, it is said to be inverse.

There is also a zero-phase, polyphase system in which all EMFs are in phase and may have completely equal effective EMF values. To such a system:

does not apply.

The polyphase systems, direct and inverse, can be balanced and unbalanced.

As balanced is described a polyphase system with which the instantaneous power value does not depend on time but is constant. An unbalanced system is such a polyphase system with which the instantaneous power value is the function of time.

A polyphase system will be balanced if it has a symmetrical EMF system and if all phases are equally loaded, so that the phase currents also make a symmetrical system. However, a two-phase system with three conductors is a non-symmetrical EMF system, but when equally loaded across phases, it is also a balanced system. The property of polyphase systems that at equal loadings they provide a time-constant total power is of great importance, because for such systems motors can be made which produce a constant rotation moment.

The polyphase system of symmetrical EMFs and symmetrical currents is balanced if the number of phases  $m > 2$ . The instantaneous power in the  $k$ -th phase of such a symmetrical system is:

$$\begin{aligned}
 p_k &= e_k \cdot i_k = \sqrt{2} E \sin \left[ \omega t - (k-1) \frac{2\pi}{m} \right] \cdot \sqrt{2} I \sin \left[ \omega t - (k-1) \frac{2\pi}{m} - \varphi \right] \\
 &= E I \cos \varphi - I E \cos \left[ 2\omega t - 2(k-1) \frac{2\pi}{m} - \varphi \right].
 \end{aligned}
 \tag{8}$$

Suma trenutanih vrijednosti snage u svim fazama je:

The sum of instantaneous power values in all phases is:

$$p = \sum_{k=1}^m p_k = m E I \cos \varphi = P = \text{konst.}
 \tag{9}$$

Dakle, suma trenutanih snaga simetričnog sustava ( $m > 2$ ) konstanta je i ne ovisi o vremenu. Može se pokazati da je dvofazni sustav simetrično opterećeni sustav također uravnotežen.

Therefore, the sum of instantaneous power values of a symmetrical system ( $m > 2$ ) is constant and not time-dependent. It can be shown that a symmetrically loaded two-phase system is also balanced.

### 3.1 Trofazni sustav

EMS trofaznog direktnog simetričnog sustava su međusobno fazno pomaknute za kut  $\frac{2\pi}{3}$ , a efektivne vrijednosti su jednake pa vrijedi da je:

### 3.1 Three-phase system

The EMFs of a three-phase direct symmetrical system are mutually phase-shifted by angle  $\frac{2\pi}{3}$ , whereas the effective values are equal, so that:

$$e_1 = \sqrt{2} E \sin \omega t
 \tag{10}$$

$$e_2 = \sqrt{2} E \sin \left( \omega t - \frac{2\pi}{3} \right)
 \tag{11}$$

$$e_3 = \sqrt{2} E \sin \left( \omega t - \frac{4\pi}{3} \right)
 \tag{12}$$

## 4 PRORAČUN MREŽA S NESINUSNIM PERIODIČKIM VALNIM OBLICIMA

## 4 COMPUTATION OF NETWORKS WITH NONSINUSOIDAL PERIODIC WAVEFORMS

### 4.1 Nastanak nesinusnih periodičkih valnih oblika

Kada se na energetska električna mrežu priključe nelinearni elementi, tada dolazi do izobličenja sinusnoga valnog oblika napona i struje. Tako npr. ispravljači s raznim poluvodičkim elementima i s feromagnetskim prigušnicama i transformatorima, čije su karakteristike također nelinearne, izobličuju sinusni napon energetske mreže. Ta izobličenja stvaraju niz nepovoljnih efekata kao što su: smetnje u radiokomunikacijskim

### 4.1 Formation of nonsinusoidal periodic waveforms

When non-linear elements are connected to the electric power network, distortions of sinusoidal voltage and current waveforms appear. Thus the rectifiers with various semi-conductor elements and with ferromagnetic inductors and transformers, whose characteristics are also non-linear, distort the sinusoidal voltage of the electric power network. These distortions produce a series of adverse effects, such as: interferences in radio

uređajima, povećanje gubitaka u električnoj mreži, mogućnost nepoželjne rezonancije, što može izazvati naponska i strujna preopterećenja. Zbog toga u mnogim zemljama postoje vrlo strogi propisi kojima se zabranjuje uporaba uređaja koji izazivaju izobličenja napona u električnoj mreži. Izobličenja također mogu nastati u samim generatorima kao i transformatorima zbog petlje histereze. Karakteristika je tako nastalih izobličenih valnih oblika da imaju tzv. više harmonike, koji se mogu izmjeriti spektralnim analizatorima ili izračunati. Krivulja magnetiziranja ima oblik neparne funkcije, pa zato u uređajima s feromagnetskom jezgrom izobličenja imaju uglavnom neparne harmonike, dok kod poluvodičkih uređaja izobličenja imaju parne i neparne harmonike. Stupanj izobličenja uređaja mjeri se faktorom izobličenja [3] i [4].

Svaki periodički valni oblik  $f(t)$ , kako unipolarni, tako i bipolarni, može se rastaviti na harmonike pomoću Fourierova reda, tj.:

$$f(t) = f(t \pm nT) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(n\omega t) + b_n \sin(n\omega t)] \quad (13)$$

odnosno:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} c_n \sin(n\omega t + \varphi_n) \quad (14)$$

gdje je:

$$c_n = \sqrt{a_n^2 + b_n^2} \quad (15a)$$

$$\varphi = \arctg\left(\frac{a_n}{b_n}\right) \quad (15b)$$

$n$  – cijeli broj ( $n = 1, 2, 3, \dots, \infty$ ),  
 $\omega = \frac{2\pi}{T}$  – kružna frekvencija prvog harmonika,  
 $T$  – perioda periodičke funkcije.

Koeficijenti Fourierova reda računaju se pomoću relacija:

$$\frac{a_0}{2} = \frac{1}{T} \int_0^T f(t) dt \quad (16)$$

communication devices, increased losses in the electric power network, possibility of undesired resonance, which in turn may cause voltage and current overloads. For that reason, in many countries very strict regulations are in force prohibiting the use of devices which cause voltage distortions in the electric power network. Distortions may also occur in generators and transformers due to the hysteresis loop. Characteristic of the thus created distorted waveforms is that they have the so-called higher harmonics which can be measured by spectral analysers or computed. The magnetization curve has the form of an odd function, so that distortions in devices with the ferromagnetic core largely have odd harmonics, whereas with semi-conductor devices they have both even and odd harmonics. The degree of distortion is measured by the distortion factor [3] and [4].

Each periodic waveform  $f(t)$ , unipolar and bipolar alike, can be decomposed into harmonics by means of the Fourier series, viz.:

or:

where:

$n$  – whole number ( $n = 1, 2, 3, \dots, \infty$ ),  
 $\omega = \frac{2\pi}{T}$  – circular frequency of the first harmonic,  
 $T$  – circular frequency of the first harmonic,

The Fourier series coefficients are computed by means of the following relations:

$$a_n = \frac{2}{T} \int_0^T f(t) \cos(n\omega t) dt \quad (17)$$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin(n\omega t) dt \quad (18)$$

Nulti član  $\frac{a_0}{2}$  predstavlja srednju vrijednost periodičke funkcije, a koeficijenti  $a_n$  i  $b_n$  amplitude  $n$ -harmonika ( $n=1, 2, 3, \dots, \infty$ ) pomoću koje se dobije  $c_n$  i  $\Phi_n$ .

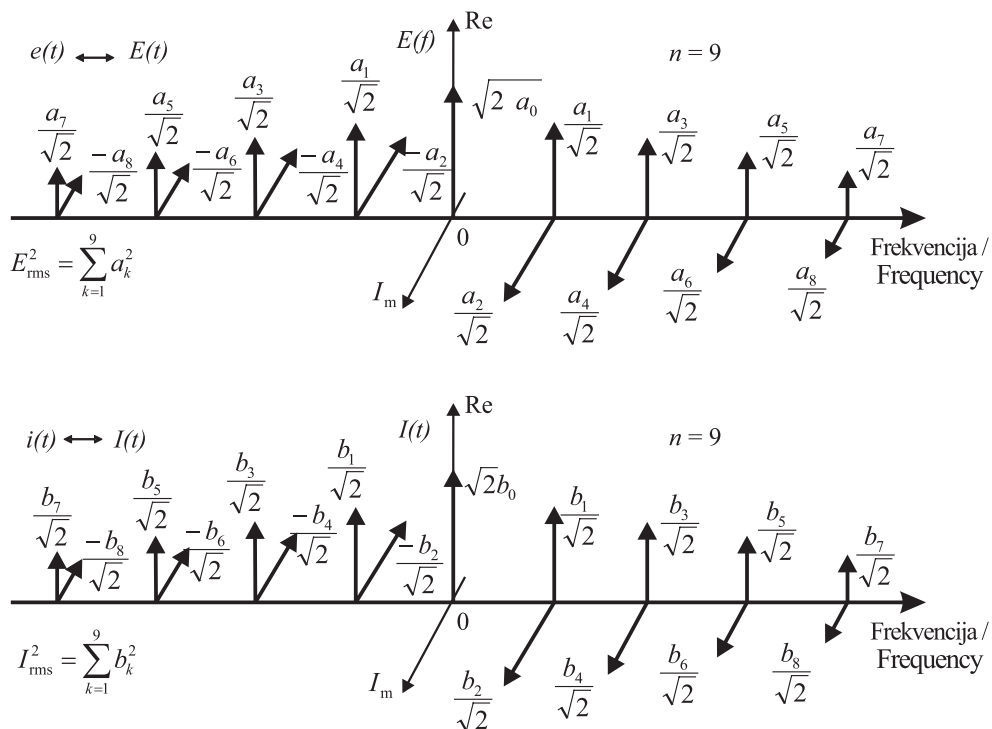
Bipolarni valni oblici parne simetrične funkcije [ $f(t)=f(-t)$ ] imaju samo kosinusne harmonike, a unipolarni valni oblici uvijek imaju nulti član  $\frac{a_0}{2}$ , tj. istosmjernu komponentu. Ako je funkcija neparna  $f(t)=f(-t)$ , tada se javljaju samo sinusni harmonici. Svojstva periodičkih funkcija i razvoj u Fourierovu redu razrađeni su detaljno u matematičkim udžbenicima.

Na slici 1 dan je fazorski dijagram Fourierovog reda [3].

The zero member  $\frac{a_0}{2}$  represents the mean value of a periodic function, whereas the coefficients  $a_n$  and  $b_n$  represent the amplitudes of  $n$ -harmonics ( $n=1, 2, 3, \dots, \infty$ ) by means of which  $c_n$  and  $\Phi_n$  are obtained.

The bipolar waveforms of an even symmetrical function [ $f(t)=f(-t)$ ] have only cosine harmonics, whereas the unipolar waveforms always have the zero-member  $\frac{a_0}{2}$ , i.e. a dc component. If a function is odd  $f(t)=f(-t)$ , then only the sinusoidal harmonics appear. The properties of the periodic functions and development in the Fourier series are described in detail in math textbooks.

Figure 1 shows a phasor diagram of the Fourier series [3].



Slika 1 – Fazorski dijagram Fourierovog reda  
Figure 1 – Phasor diagram of the Fourier series



#### 4.2 Računanje efektivnih vrijednosti napona i struje, te snage u mrežama koje sadrže više harmonike

U linearnim mrežama koje sadrže više harmonike proračun se provodi pomoću fazorskog računa za svaki harmonik zasebno.

Efektivna vrijednost struje, odnosno napona, koji sadrži više harmonike računa se pomoću sljedeće relacije:

$$I = \sqrt{I^2(0) + \sum_{n=1}^{\infty} I^2(n)} \quad (19a)$$

odnosno:

$$U = \sqrt{U^2(0) + \sum_{n=1}^{\infty} U^2(n)} \quad (19a)$$

gdje je:

- $I(0)$  – istosmjerna komponenta struje,
- $U(0)$  – istosmjerna komponenta napona,
- $I(n)$  – efektivna vrijednost struje  $n$ -tog harmonika,
- $U(n)$  – efektivna vrijednost napona  $n$ -tog harmonika.

Radna snaga u mrežama s višim harmonicima računa se pomoću sljedeće relacije:

$$P = U(0) I(0) + \sum_{n=1}^{\infty} U(n) I(n) \cos \varphi(n) = P(0) + \sum_{n=1}^{\infty} P(n) \quad (20)$$

gdje je radna snaga  $n$ -tog harmonika:

$$P(n) = U(n) I(n) \cos \varphi(n) \quad (21)$$

Može se reći da je radna snaga nesinusne periodičke struje i napona jednaka sumi radne snage svih harmonika dodajući snagu istosmjerne komponente struje i napona. Radna snaga  $i$  u ovom slučaju može se izraziti pomoću efektivne vrijednosti nesinusne periodičke struje  $I$  i napona  $U$ :

#### 4.2 Computation of effective voltage and current values and of power in networks containing higher harmonics

In linear networks containing higher harmonics computation is performed by means of the phasor calculus for each harmonic separately.

The effective current or voltage value containing higher harmonics is computed by means of the following relation:

or: (14)

where:

- $I(0)$  – dc current component
- $U(0)$  – dc voltage component
- $I(n)$  – effective current value of the  $n$ -th harmonic
- $U(n)$  – effective voltage value of the  $n$ -th harmonic

Active power in networks with higher harmonics is computed by means of the following relation:

where the active power of the  $n$ -th harmonic is:

The active power of the nonsinusoidal periodic current and voltage can be said to equal the sum of the active power of all harmonics adding the power of dc current and voltage component. The active power can also in this case be expressed by means of the effective value of nonsinusoidal periodic power  $I$  and voltage  $U$ :

$$P = k U I \quad (22)$$

gdje je  $k$  faktor snage čija je vrijednost između 0 i 1 ( $0 \leq k \leq 1$ ). Što su izraženiji viši harmonici, izobličenja su veća, a faktor snage sve manji.

Viši harmonici stvaraju razne neprilike, tako da npr. ometaju radio i telefonske veze, zatim mogu izazvati dodatne gubitke u generatorima i prijenosnim vodovima i uređajima. Zbog toga se traži da generatori proizvode sinusne EMS.

Po analogiji s definicijom reaktivne snage sinusne struje i napona definira se reaktivna snaga mreže s nesinusnom periodičkom strujom i naponom na sljedeći način:

where  $k$  is the power factor with value varying between 0 and 1 ( $0 \leq k \leq 1$ ). The more marked the higher harmonics, the more marked the distortions, with the power factor becoming smaller and smaller.

The higher harmonics are a cause of all kinds of troubles, for example, they jam radio and telephone lines, or they may cause additional losses in generators, transmission lines and devices. That is why generators are required to produce sinusoidal EMFs.

By analogy with the definition of the reactive power of the sinusoidal current and voltage, the reactive power of a network with nonsinusoidal periodic current and voltage is defined as follows:

$$Q = \sum_{n=1}^{\infty} Q(n) = \sum_{n=1}^{\infty} U(n) I(n) \cos \varphi(n) \quad (23)$$

Analogno, prividna snaga je:

Analogno, prividna snaga je:

$$S = UI \quad (24)$$

U ovom slučaju je  $S^2 \neq P^2 + Q^2$ , pa se uvodi tzv. snaga izobličenja  $D$ , tako da je:

In this case,  $S^2 \neq P^2 + Q^2$ , so the so-called distortion power  $D$  is introduced, therefore:

$$D = \sqrt{S^2 - P^2 - Q^2} \quad (25)$$

Snaga izobličenja  $D$  karakterizira stupanj različitosti oblika napona i struje. Za mrežu od otpora vrijedi da je  $D = Q = 0$ . Kod nesinusnih periodičkih valnih oblika napona i struje kompenzacija faktora snage nije moguća, pa je i to još jedan razlog za očuvanje čistoga sinusnoga napona i struje u energetske mrežama [3].

The distortion power  $D$  characterises the degree of the variability of voltage and current forms. For a resistance network it applies that  $D = Q = 0$ . With the nonsinusoidal periodic voltage and current waveforms the power factor compensation is not possible, which is one more reason for preserving a clean sinusoidal voltage and current in electric power networks [3].

#### 4.3 Viši harmonici u trofaznim mrežama

Neke su faze EMS trofazne mreže simetrične i izobličene, pa se mogu napisati na sljedeći način:

#### 4.3 Higher harmonics in three-phase networks

Some phase EMFs of a three-phase network are symmetrical and distorted, which is why they can be written as follows:

$$e_1 = \sum_{n=1}^{\infty} \sqrt{2} E(n) \sin[n\omega t + \Psi(n)] \quad (26)$$

$$e_2 = \sum_{n=1}^{\infty} \sqrt{2} E(n) \sin\left[n\omega\left(t - \frac{T}{3}\right) + \Psi(n)\right] \quad (27)$$

$$e_3 = \sum_{n=1}^{\infty} \sqrt{2} E(n) \sin\left[n\omega\left(t + \frac{T}{3}\right) + \Psi(n)\right] \quad (28)$$

U ovom slučaju postoje tri karakteristične skupine broja  $n$ .

Prva skupina karakterizirana je brojevima  $n = 3k$  ( $k = 1, 2, 3$ ), pa je odgovarajući niz brojeva  $n = 3, 6, 9, 12$ , itd. U tom slučaju svi su naponi u sve tri faze jednaki, jer je:

In this case there are three characteristic groups of number  $n$ .

The first group is characterised by the numbers  $n = 3k$  ( $k = 1, 2, 3$ ), so the corresponding series of numbers is  $n = 3, 6, 9, 12$ , etc. In that case, all voltages in all three phases are equal, because:

$$\begin{aligned} \sin\left[n\omega\left(t - \frac{T}{3}\right) + \Psi(n)\right] &= \sin\left[n\omega\left(t + \frac{T}{3}\right) + \Psi(n)\right] \\ &= \sin[n\omega T + \Psi(n)], \end{aligned} \quad (29)$$

pa ovi harmonici čine tzv. simetrični nulti sustav, tj.  $n$ -ti harmonik u sve tri faze ima isti početni fazni kut i jednake tjemene vrijednosti [3]. Drugu skupinu karakteriziraju brojevi  $n = 3k + 1$  ( $k = 0, 1, 2, 3, \dots$ ), pa je odgovarajući niz brojeva  $n = 1, 4, 7, 10, 13$  itd. U ovom slučaju sve tri faze EMS  $n$ -tog harmonika čine direktni simetrični sustav, jer je pri tome:

so these harmonics constitute the so-called symmetrical zero system, i.e., the  $n$ -th harmonic in all three phases has the same initial phase angle and the same peak values [3]. The second group is characterised by the numbers  $n = 3k + 1$  ( $k = 0, 1, 2, 3, \dots$ ), so the corresponding series of numbers is  $n = 1, 4, 7, 10, 13$ , etc. In that case all three EMF phases of the  $n$ -th harmonic constitute a direct symmetrical system, because:

$$\sin\left[n\omega\left(t - \frac{T}{3}\right) + \Psi(n)\right] = \sin\left[n\omega t - \frac{2\pi}{3} + \Psi(n)\right], \quad (30)$$

$$\sin\left[n\omega\left(t + \frac{T}{3}\right) + \Psi(n)\right] = \sin\left[n\omega t + \frac{2\pi}{3} + \Psi(n)\right] \quad (31)$$

tj.  $n$ -ti harmonik faze (2) zaostaje iza faze (1) za  $120^\circ$ , a faza (3) prethodi za  $120^\circ$  fazi (1).

Treću skupinu karakteriziraju brojevi  $n = k - 1$  ( $k = 1, 2, 3, \dots$ ), pa je odgovarajući niz brojeva  $n = 2, 5, 8, 11, 14$  itd. U tom slučaju sve tri faze EMS  $n$ -tog harmonika čine inverzni simetrični sustav, jer je pri tome:

i.e., the  $n$ -th harmonic of phase (2) lags behind phase (1) by  $120^\circ$ , and phase (3) precedes phase (1) by  $120^\circ$ .

The third group is characterised by the numbers  $n = k - 1$  ( $k = 1, 2, 3, \dots$ ), so the corresponding series of numbers is  $n = 2, 5, 8, 11, 14$  etc. In that case, all three EMF phases of the  $n$ -th harmonic constitute an inverse symmetrical system, because:

$$\sin\left[n\omega\left(t - \frac{T}{3}\right) + \Psi(n)\right] = \sin\left[n\omega t + \frac{2\pi}{3} + \Psi(n)\right] \quad (32)$$

$$\sin\left[n\omega\left(t + \frac{T}{3}\right) + \Psi(n)\right] = \sin\left[n\omega t - \frac{2\pi}{3} + \Psi(n)\right] \quad (33)$$

tj.  $n$ -ti harmonik faze (2) prethodi fazi (1) za  $120^\circ$ , a faza (3) zaostaje iza faze (1) za  $120^\circ$ .

Izobličenja u trofaznim mrežama nastaju zbog petlje histereze feromagnetskih materijala koji su ugrađeni u generatore, prigušnice i transformatore. Pri analizi nesinusoidnih trofaznih mreža treba se ograničiti zapravo na najvažniji slučaj, a to je da fazne EMS imaju neparne harmoničke članove, dok su parni članovi kao i istosmjerni član zanemarivi. Kod spoja EMS i simetričnog trošila u zvijezdu s nulvodičem, struja kroz nulvodič u ovom slučaju nije nula zbog harmonika koji čine nulti sustav (prva skupina) [3], pa je:

i.e., the  $n$ -th harmonic of phase (2) precedes phase (1) by  $120^\circ$ , and phase (3) lags behind phase (1) by  $120^\circ$ .

Distortions in three-phase networks occur due to the hysteresis loop of ferromagnetic materials built into generators, inductors and transformers. In analysing nonsinusoidal three-phase networks one should actually confine oneself to the most relevant fact that the phase EMFs have odd harmonic members, whereas the even members and the dc member are negligible. If EMF and a symmetrical consumer are star-connected with the zero conductor, the current through the zero conductor in that case is not zero due to the harmonics making up the zero system (the first group) [3], therefore:

$$i_0 = i_1 + i_2 + i_3 = 3\sqrt{2} I(3) \sin[3\omega t - \varphi(3)] + 3\sqrt{2} I(9) \sin[9\omega t - \varphi(9)] + \dots, \quad (34)$$

a efektivna vrijednost struje je:

and the effective current value is:

$$I_0 = \sqrt{I^2(3) + I^2(9) + I^2(15) + \dots} \quad (35)$$

Da bi se izbjegli harmonici prve skupine ( $n = 3k$ ), dovoljno je isključiti nul-vodič. Tada se ti harmonici ne pojavljuju ni u faznim strujama. Isto tako linijski naponi, koji su jednaki razlici faznih napona, ne sadrže harmonike prve skupine ( $n = 3k$ ). Zbog toga je  $U_l < \sqrt{3} U_f$ , jer je:

To avoid the first group harmonics ( $n = 3k$ ), it suffices to turn off the zero conductor. Then these harmonics will not appear in the phase currents either. Likewise, the line voltages, which equal the difference of the phase voltages, do not contain the first group harmonics ( $n = 3k$ ). Hence  $U_l < \sqrt{3} U_f$ , because:

$$U_l = \sqrt{3} \cdot \sqrt{U^2(1) + U^2(3) + U^2(5) + U^2(7) + \dots} \quad (36)$$

$$U_f = \sqrt{U^2(1) + U^2(3) + U^2(5) + U^2(7) + \dots} \quad (37)$$

Kod trokutnog spoja faza generatora s nesinusnim EMS, suma EMS neće biti jednaka nuli, već će biti jednaka trostrukoj sumi harmonika prve skupine ( $n = 3k$ ), tj.:

$$e_1 + e_2 + e_3 = 3 \sqrt{2} E(3) \sin[3\omega t + \Psi(3)] + 3 \cdot \sqrt{2} E(9) \sin[9\omega t + \Psi(9)] + \dots, \quad (38)$$

In the case of a delta-connection of the generator's phases with nonsinusoidal EMFs, the sum of EMFs will not equal zero, it will equal a three-fold sum of the first group harmonics ( $n = 3k$ ), viz.:

a efektivna vrijednost te EMS je:

while the effective value of that EMF is:

$$E = 3 \cdot \sqrt{U^2(3) + U^2(9) + \dots} \quad (35)$$

Ta EMS uzrokuje da u trokutu teku tzv. struje izjednačenja bez obzira na trošilo, te stvara dodatne gubitke i dodatno zagrijavanje generatora. To je jedan od razloga da se trofazni generator obično ne spaja u trokut, nego u zvijezdu [3].

This EMF causes that in the delta the so-called equalising currents flow regardless of the consumer, leading to additional losses and additional generator heating. That is one of the reasons why three-phase generators are usually star-connected rather than delta-connected [3].

Treća skupina harmonika ( $n = k + 1$ ) stvara obrnuto rotirajuće polje, što izaziva kočenje. Uz sve te nedostatke izobličene sinusoide, postoji još jedan veliki, a to su povećani gubici u vodovima zbog viših harmonika [4].

The third group of harmonics ( $n = k + 1$ ) produces a reverse rotating field, causing motor stalling. In addition to all these shortcomings of a distorted sinusoid, there is another big disadvantage: increased line losses due to the higher harmonics [4].

Vidi se da su viši harmonici matematički opis valnog oblika struje i napona, odnosno snage u električnom sustavu, a definirani su međunarodnim standardima.

The higher harmonics, as seen, are a mathematical description of the waveform of current and voltage, or of power in an electricity supply system, and are defined by international standards.

Standard IEEE 519-1981 definirao je totalno harmonično izobličenje u iznosu od 5 %, kao ograničenje izobličenja napona u elektroenergetskom sustavu. Ovo ograničenje odnosilo se na mjesto priključka između dobavljača električne energije i potrošača [5].

The IEEE 519-1981 standard defined the total harmonic distortion in the amount of 5 % as the limit of voltage distortion in an electric power system. This limit applied to the connection point between the electricity supplier and the consumer [5].

Poslije ovog, uveden je IEEE Standard 519-992, pod nazivom *IEEE Recommended Practices and Requirements for Harmonics Control in Electric Power System*, koji je bitno izmijenio prethodni Standard, jer se njime uvodi pored naponskog izobličenja i valno izobličenje struje. Uvodi se pet različitih kategorija valnog izobličenja struje, koje se odnose na različite kategorije potrošača. Jednu ili dvije skupine čine kategorije velikih potrošača, a ostale skupine pripadaju srednjim i malim potrošačima. Osnova za ovu podjelu bio je omjer između struje kratkog spoja i maksimalno zahtijevane struje na mjestu primopredaje električne energije. Najveći industrijski potrošači trebaju biti s ograničenjem valnog izobličenja struje ispod 5 % [5].

Then the IEEE 519-992 standard was introduced under the title *IEEE Recommended Practices and Requirements for Harmonics Control in Electric Power System*, which essentially changed the previous standard, because in addition to the voltage distortion it introduces the current wave distortion. Five different categories of current wave distortions are introduced applying to different consumer categories. One or two groups are large consumers, other are medium and small consumers. The basis of this categorisation was the ratio between the short-circuit current and the maximum current demand at the place of power handover. The largest industrial consumers should have the current wave distortion limit below 5 % [5].

Za proračun viših harmonika koriste se za to pripremljeni softveri, pomoću kojih se mogu unaprijed predvidjeti ukupna izobličenja valnog oblika struje i napona za svaku od sabirnica ili grana sustava. Proračun je obično kreiran kao sustav od  $n$  nepoznanica primjenom Ohmovog i Kirchoffovih zakona. Primjenom matricnog računa pronađu se nepoznate vrijednosti, struje i naponi. Naponska i strujna izobličenja tada mogu biti uspoređena sa Standardima. Softver bi trebao imati mogućnost za proračun iznosa pojedinih harmonika struje i napona [4] i [5].

Analizom pojedinih harmonika moguće je unaprijed predvidjeti rezonantnu frekvenciju. Efekt paralelne rezonancije pojačava sve više one harmonike struje koji su u blizini prirodne frekvencije sustava. Isto tako serijska rezonancija sustava može imati za posljedicu pojačanje harmonika napona. Iznos maksimalne vrijednosti pozitivne ili negativne, ukazuje na očekivano pojačanje odgovarajućeg harmonika, a to upućuje na rezonantnu frekvenciju. Omska vrijednost impendancije u sustavu određivat će iznos pojačanja i rezonantne uvjete [5].

Viši harmonici nepovoljno djeluju na prienosnu moć elektroenergetskih vodova, zbog povećanih gubitaka u vodovima. Zatim mogu izazvati smetnje na uređajima koji se priključuju na električnu mrežu, a posebno na uređajima za upravljanje. Viši harmonici također mogu izazvati kočenje električnih motora. Kod javne rasvjete lampe punjene plinom mogu imati manji faktor iskoristivosti zbog viših harmonika. Sve su ovo razlozi zbog kojih se potrebno zalagati za veću čistoću sinusnog valnog oblika struje i napona.

U tablici 1 dane su europske norme koje se odnose na više harmonike te koje su izdane unatrag 20 godina [6].

Used for the computation of higher harmonics is software which is created for the purpose and with which total current and voltage waveform distortions for each of the bus or the branches of the system can be anticipated. Computation is usually created as a system of  $n$  unknowns by using Ohm's and Kirchoff's Laws. The unknown current and voltage values are found by applying the matrix calculus. Then the voltage and current distortions can be compared with the applicable standards. The software should be able to work out the values of particular current and voltage harmonics [4] and [5].

Through an analysis of individual harmonics it is possible to anticipate the resonant frequency. The effect of the parallel resonance increasingly amplifies those current harmonics which are close to the natural frequencies of the system. Likewise, the series resonance of the system may have an effect of amplified voltage harmonics. The amount of maximum value, positive or negative, indicates the expected amplification of a harmonic, and that in turn indicates the existence of the resonant frequency. The ohmic impedance value in the system will determine the amount of amplification and the resonance conditions [5].

Higher harmonics adversely affect the transmissibility of electrical power lines due to higher losses in the lines. Besides, they can cause disturbances in devices connected to the power supply network, especially in the control devices. Higher harmonics may also cause motor stalling. In public lighting the gas-filled lamps may have a lower efficiency factor due to higher harmonics. These are all reasons in favour of the need to ensure greater purity of the sinusoidal current and voltage waveform.

Table 1 specifies the European standards relating to higher harmonics and issued over the last 20 years [6].

Tablica 1 — Pregled europskih normi koje se odnose na više harmonike  
Table 1 — Overview of the European standards relating to higher harmonics

Redni broj / No	Dokument Norma – Standard / Document – Standard	Naziv / Title	Vrijeme izdavanja (godina-mjesec) / Time of issue (year-month)
1	EN 60555-1	Disturbances in supply systems caused by household appliances and similar electrical equipment. Part 1: Definitions	1987-04
2	IEC/TR3 61000-2-1 CEI/TR3 61000-2-1	Electromagnetic compatibility (EMC); part 2; environment; section 1: description of the environment; electromagnetic environment for low-frequency conducted.	1990-05
3	IEC/TR3 61000-3-6 CEI/TR3 61000-3-6	Electromagnetic compatibility (EMC)-Part 3-Limits-Section 6; Assessment of emission limits for distorting loads in Mv and HV power systems.	1996-10
4	EN 61642	Industrial ac networks affected by harmonics – Application of filters and shunt capacitors (IEC 61642:1997)	1997-10
5	IEC 77A/262/CDV CEI	IEC Amendment related to subclause 6.1 and to professional equipment < > 1 kW to IEC 61000 – 3 - 2; 1995	1998-10
6	IEC 60050-551-20 CEI 60050-551-20	International Electrotechnical Vocabulary – Part 551 – 20 ; Power electronics; Harmonic analysis	2001-07
7	IEC 61000 -4-13 CEI 61000-4-13	Electromagnetic compatibility (EMC) – Part 4-13 ; Testing and measurement techniques; Harmonics and interharmonics including mains signaling.	2002-03
8	EN 61000- 4 – 3	Electromagnetic compatibility (EMC) – Part 4-13; Testing and measurement techniques; Harmonics and interharmonics including mains signaling.	2002-06
9	IEC 61000 -4- 7 CEI 61000 -4- 7	Electromagnetic compatibility (EMC) – Part 4- 7 Testing and measurement techniques; General guide on harmonics and interharmonics measurements and instrumentation, for power supply systems and equipment.	2002-08
10	EN 61000 2-4	Electromagnetic compatibility (EMC) – Part 2- 4; Environment ; Compatibility Levels in industrial plants for low – frequency conducted disturbances	2002-09
11	UIC 737 - 4	Measures for limiting the disturbance of light current installations by electric traction ( in particular thyristor apparatus)	2003-02
12	IEC 61000 -4- 7	Corrigendum 1 - Electromagnetic compatibility (EMC) – Part 4- 7	2004-07
13	IEC/ PAS 62001 CEI/ PAS 62001	Guide to the specification and design evaluation of ac filters for HVDC system	2004-07
14	IEC/TR 60919-1 CEI/TR 60919-1	Performance of high – voltage direct current (HVDC) systems with line- commutated converters – Part 1; Steady –state conditions.	2005-03
15	IEC/TR 61000-1-4 CEI/TR 61000-1-4	Electromagnetic compatibility (EMC) – Part 1- 4; General – Historical rationale for the limitation of power – frequency conducted harmonics from equipment.	2005-05
16	IEC 61000-3-2 CEI 61000-3-2	Electromagnetic compatibility (EMC) – Part 3-2; Part – Limits for harmonics current emissions (equipment input current < > 16 A per phase.	2005-11
17	CLC/TR 60919-1	Performance of high – voltage direct current (HVDC) systems with line- commutated converters – Part 1; Steady-state conditions.	2005-12
18	IEC 77A/531/CDV CEI 77A/531/CDV	Amendment to IEC 61000-3-2; Electromagnetic compatibility (EMC) – Part 3-2; Part – Limits for harmonics current emissions (equipment input current < > 16 A per phase.	2006-03
19	prEN 61557-12	IEC 61557-12 Ed. 1: Electrical safety in low voltage distribution systems up to 1000 V a.c. and 1500 V d.c.- Equipment for testing, measuring or monitoring.	2006-05
20	EN 61000-4-7/ prA1	Amendment to IEC 61000-4-7, Ed.2; Electromagnetic compatibility (EMC) – Testing and measurement techniques – General guide on harmonics measurement and instrumentation for power supply system	2006-12
21	IEC 77A/575DTR CEI 77A/575DTR	IEC 61000-3-6,Ed.2: Ed.2; Electromagnetic compatibility (EMC) – Part 3-6 Assessment of harmonics emission limits for the connection of distorting installations to MV, Hv and EHV power systems	2007-03

Zemlje jugoistočne Europe su u procesu otvaranja regionalnih tržišta električnom energijom. U zadnjih dvadeset godina u zemljama EU, koristili su se zakoni u skladu s direktivama: *Directive of European Community Council 85/374/EEC* od 1985. godine. Jedan od najvažnijih zahtjeva na dobavu električne energije, odnosno napon jest: frekvencija, amplituda, oblik i valna simetričnost, što je dano normom EN 50160 (*Characteristics of supply voltage in public distributions systems*) i IEC normama serije 61000 (*Elektromagnetic compatibility*).

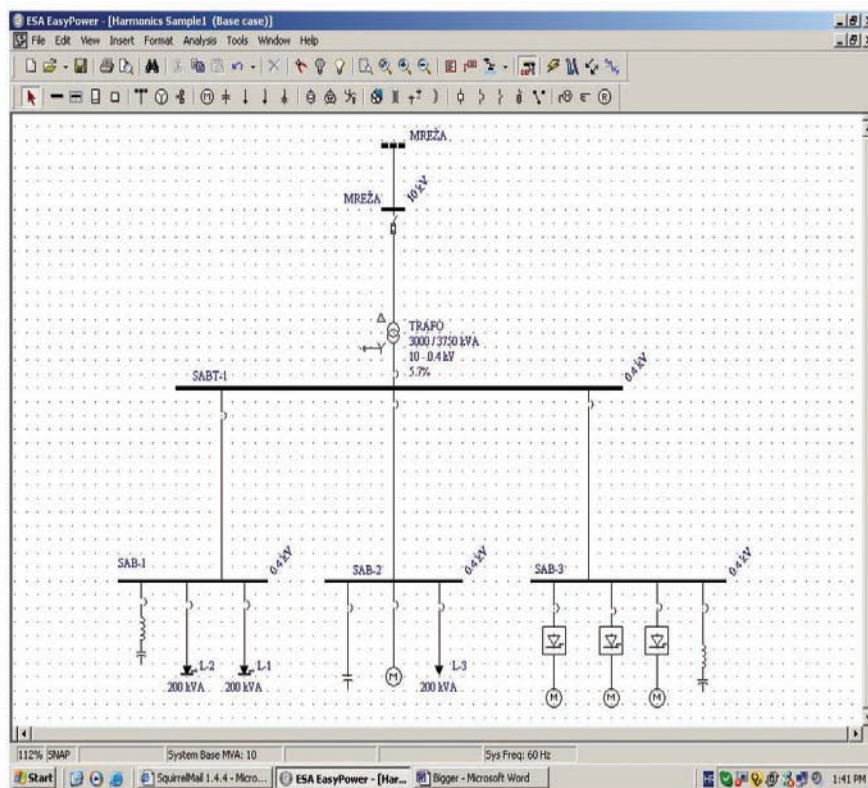
U državama diljem svijeta definiran je napon pri opskrbi električnom energijom prema IEC 38, kao dio procesa harmonizacije. Ove norme su prilagođene okolnostima u pojedinim zemljama (npr. HD472.1 amendment od 2003. godine), u kojima se naponska razina 220/380 V i 240/415 V zamjenjuje sa standardnom vrijednostima 240/400 V. Dozvoljena tolerancija  $\pm 10\%$  preporučena je u okviru novih normi EN 50160. Hrvatska trenutačno ima naponsku razinu 220/380 V,  $\pm 5\%$ .

U nastavku dan je jedan primjer proračuna, gdje je korišten softverski paket *Easy Power Spectrum*, a prikazan je na slikama 2 do 10 i u tablicama 2 do 20.

The SEE countries are undergoing the process of opening their regional electricity markets. Over the last 20 years the relevant laws of the EU Member States have been implemented in compliance *Directive of European Community Council 85/374/EEC* of 25 July 1985. The most important power supply or voltage requirements include: frequency, amplitude, wave form and symmetry, as laid down in the power quality standard EN 50160 (*Characteristics of supply voltage in public distributions systems*) and IEC Series (*Electromagnetic compatibility*).

In countries worldwide the power supply voltage is defined according to IEC 38, as part of the approximation process. The standards are adapted to country-specific circumstances (e.g., HD472.1 amendment of 2003), where the voltage levels 220/380 V and 240/415 V are replaced by standard values 240/400 V. The permissible limit of  $\pm 10\%$  is recommended within the framework of the new EN 50160 standards. At present Croatia has the voltage level of 220/380 V,  $\pm 5\%$ .

Given below is a computation example, where the software package *Easy Power Spectrum* is used, and is shown in Figures 2 to 10 and Tables 2 to 20.



Slika 2 — Shema primjera proračuna harmonične analize  
Figure 2 — Example of computational harmonic analysis



Tablica 2 – Ulazni podaci  
Table 2 – Input data

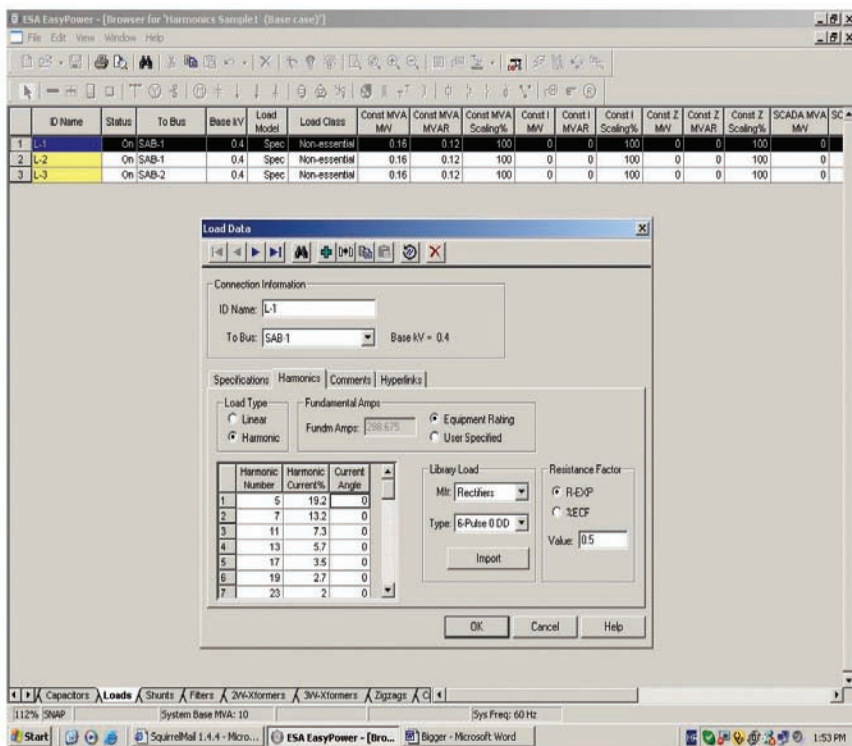
Ulazni podaci / Input data			(kV)	(MVA)	(Mvar)
1	Mreža /	Uključeno / On	10		
2	SAB – 1	Uključeno / On	0,4		
3	SAB – 2	Uključeno / On	0,4		
4	SAB – 3	Uključeno / On	0,4		
5	SABT – 1	Uključeno / On	0,4		
Opterećenja / Loads					
L – 1	Uključeno / On	SAB – 1	0,4	0,16	0,12
L – 2	Uključeno / On	SAB – 2	0,4	0,16	0,12
L – 3	Uključeno / On	SAB – 3	0,4	0,16	0,12

Tablica 3 – Harmonični spektar opterećenje L1  
Table 3 – Harmonic load spectrum L1

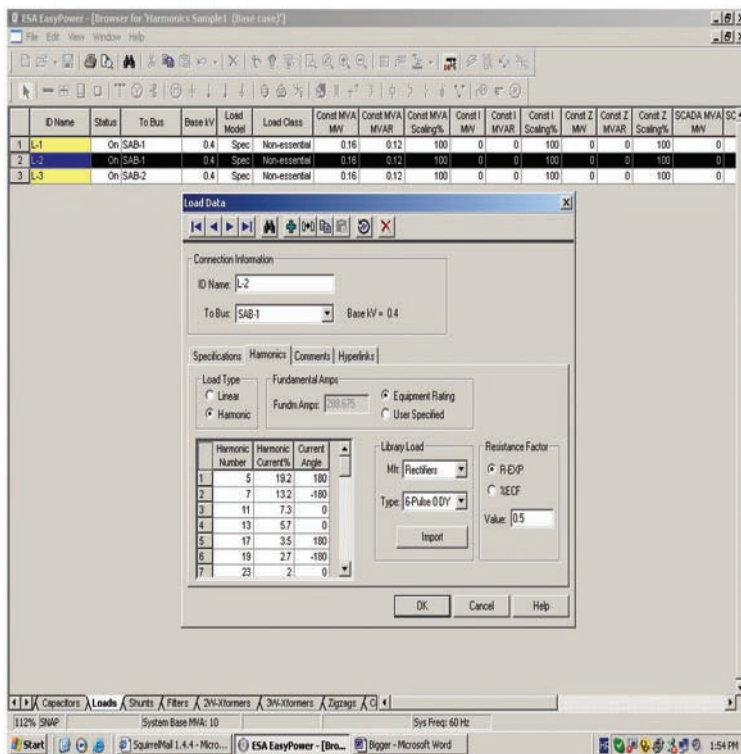
Red / Order	Postotak / Percentage [%]	Kut / Angle
5	19,2	0
7	13,2	0
11	7,3	0
13	5,7	0
17	3,5	0
19	2,7	0
23	2	0
25	1,6	0
29	1,4	0
31	1,2	0
35	1,1	0
37	1	0
41	0,9	0
43	0,8	0
47	0,8	0
49	0,7	0

Tablica 4 – Harmonijski spektar opterećenja L2  
Table 4 – Harmonic load spectrum L2

Red / Harmonic No	Postotak / Percentage [%]	Kut / Angle
5	19,2	180
7	13,2	-180
11	7,3	0
13	5,7	0
17	3,5	180
19	2,7	-180
23	2,0	0
25	1,6	0
29	1,4	180
31	1,2	-180
35	1,1	0
37	1,0	0
41	0,9	180
43	0,8	-180
47	0,8	0
49	0,7	0



Slika 3 – Harmonični spektar opterećenja L1  
 Figura 3 – Harmonic load spectrum L1



Slika 4 – Harmonični spektar opterećenja L2  
 Figure 4 – Harmonic load spectrum L2

Tablica 5 – Proračun osnovnog harmonika tokova snaga  
Table 5 – Computation of the basic power flow harmonic

Broj iteracija / No of iterations	Odstupanje radnih snaga / Active power deviation (MW)		Odstupanje jalovih snaga / Reactive power deviation (Mvar)	
Broj / No	Ime sabirnice / Bus ID name	Jedinična vrijednost (jv) / Unit value (uv)	Ime sabirnice / Bus ID name	Jedinična vrijednost (jv) / Unit value (uv)
0	SAB-2	0,002 22	SAB-2	0,000 98
1	SAB-2	0,000 08	SAB-2	0,000 04
2	SAB-3	0,000 00	SAB-2	0,000 00
3	SAB-3	0,000 00	SAB-2	0,000 00

Tablica 6 – Sumarno izvješće o generatorima  
Table 6 – Summary report on generators

Generator / Generator			Planirano / Planned			Granice / Limits		Rješenje / Solution							
Ime / ID name	Tip / Type	Na-zivna snaga / Rated power (MVA)	[MW]	[Mvar]	Vjv	Min [Mvar]	Max [Mvar]	[MW]	[Mvar]	[MVA]	Cos $\phi$	Vjv	Stu-panj / De-gree	Napon iza prijelazne reaktancije u poprečnoj osi / Voltage behind transition reactance in transverse axis $E_q'jv$	Stu-panj / De-gree
MREA / Network	Sw				1,000			1,684	0,199	1,696	0,993	1,000	0,00	1,003	0,44

Tablica 7 – Sumarno izvješće o opterećenjima  
Table 7 – Summary report on loads

Sabirnica / Bus		Rješenje / Solution							
Ime / ID name	Bazni napon / Base voltage	(kV)	(kV)	Vjv (MW)	(Mvar)	(MVA)	Stupanj / Degree	cos $\phi$	
MREA / Network	10,000	10,000	1,000	0,000	0,000	0,000	0,00	0,000	
SAB-1	0,400	0,400	1,000	0,318	-0,276	0,422	-2,79	-0,755	
SAB-2	0,400	0,387	0,968	0,545	0,419	0,687	-2,77	0,793	
SAB-3	0,400	0,394	0,984	0,789	-0,051	0,791	-3,74	-0,998	
SABT-1	0,400	0,398	0,994	0,000	0,000	0,000	-1,82	0,000	

Tablica 8 – Sumarno izvješće o sustavu  
Table 8 – Summary report on the system

Ukupno / Total	(MW)	(Mvar)	(MVA)	cos $\phi$
Proizvodnja u sustavu / System output	1,684	0,199	1,696	0,993
Opterećenje sustava / System load	1,656	1,305	2,108	0,785
Kondenzatori (prigušnice) u sustavu / Capacitors (inductors) in the system	0,004	-1,214		
Gubici u sustavu / System losses	0,024	0,108		
Provjera balansa / Balance check	0,000	-0,000		

Tablica 9 – Izvješće o prijenosnim omjerima transformatora  
Table 9 – Report on transformer transmission ratios

Transformator / Transformer	Spoj /		Bazni napon / (kV)		Prijenosni omjer /		Opis regulacije prijenosnog omjera / Description of transmission ratio regulation								
	Ime / ID name	Od sabirnice Ime / From bus ID name	Do sabirnice Ime / To bus ID name	Od / From	Na / On	Od / From	Na / On	Tip / Type	Regulacija pod teretom / Under-load regulation	Tip regulacije / Regulation type	Kontrolirana strana / Controlled side	LTC Side	Kontrolirani napon u jv / Controlled voltage in uv	Granica Minimalna / Min. limit (kV)	Granica Maksimalna / Max. limit (kV)
TRAF0 / Transformer	MREA / Network	SABT-1		10,000	0,400	10,000	0,400	2Wnd	Ne / No						

Tablica 10 – Izvješće o preopterećenju grana (Granica preopterećenja = 10,00 %)  
Table 10 – Report on branch overload (Overload limit = 10,00 %)

Vod / Line				Opterećenje / Load			
Od sabirnice / From bus	Do sabirnice / To bus	Ime grane / Branch ID name	Nazivno strujno opterećenje / Rated current load (A)	Stvarno opterećenje / Actual load (A)	Opterećenje / Load (%)	Preopterećenje / Overload (%)	Komentar / Note
SABT-1	SAB-1	C-3	1 520,0	612,0	40,3	-59,7	
SABT-1	SAB-2	C-2	1 520,0	1 024,0	67,4	-32,6	
SABT-1	SAB-3	C-1	1 520,0	1 166,5	76,7	-23,3	

Tablica 11 – Izvješće o opterećenju transformatora  
Table 11 – Report on transformer load

Transformator / Transformer				Opterećenje / Load			
Ime / ID name	Od sabirnice / From bus	Do sabirnice / To bus	Opterećenje / Load (MVA)	Nazivno / Rated (MVA)	Opterećenje / Load (%)	Preopterećenje / Overload (%)	Komentar / Note
TRAF0 / Transformer	MREA / Network	SABT-1	1,696	3,750	45,2	-54,8	

Tablica 12 – Izvješće gubitaka u granama  
Table 12 – Report on losses in branches

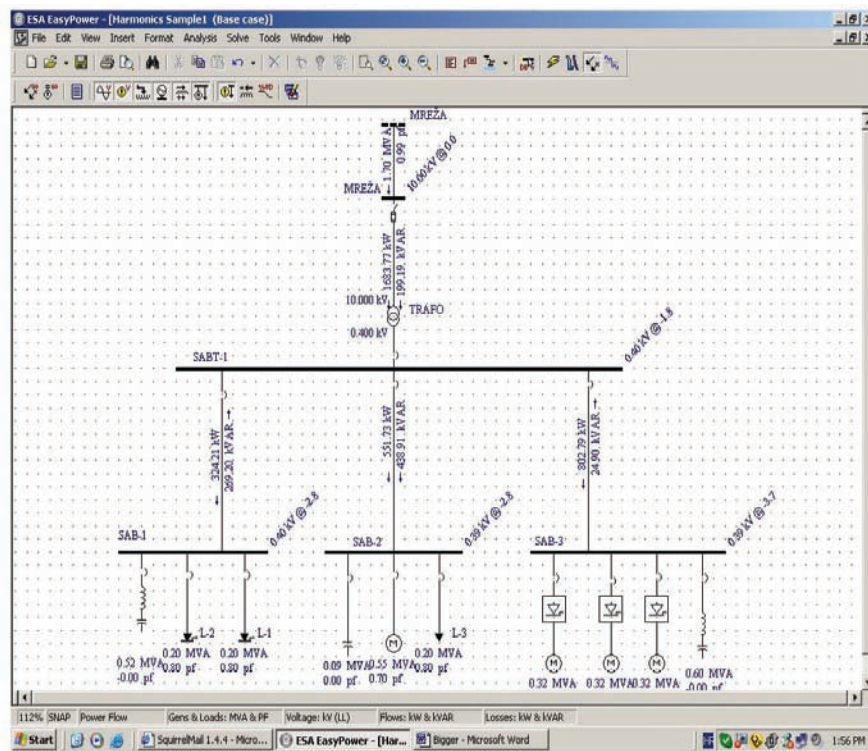
Od sabirnice / From bus		Do sabirnice / To bus		Gubici / Losses	
Ime / ID name	Bazni napon / Base voltage (kV)	Ime / ID name	Bazni napon / Base voltage (kV)	(MW)	(Mvar)
MREA / Network	10,000	SABT-1	0,400	0,005	0,054
SABT-1	0,400	SAB-1	0,400	0,003	0,007
SABT-1	0,400	SAB-2	0,400	0,007	0,020
SABT-1	0,400	SAB-3	0,400	0,009	0,026
Ukupni gubici u sustavu / Total losses in the system				0,024	0,108

Tablica 13 – Izvješće o padovima napona  
Table 13 – Report on voltage drops

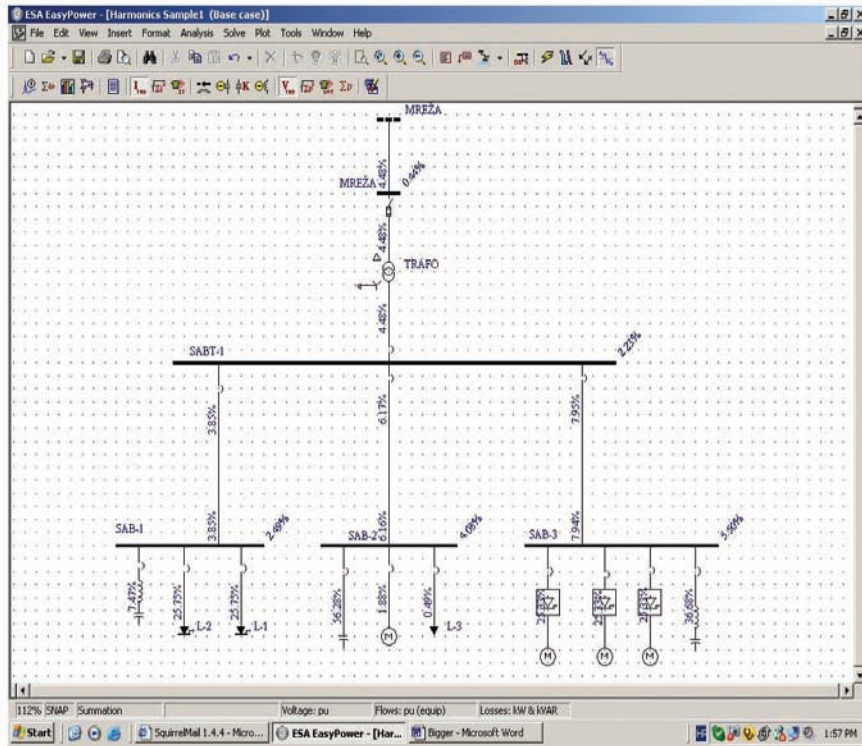
Od sabirnice / From bus		Do sabirnice / To bus		Pad napona / Voltage drop
Ime / ID name	Bazni napon / Base voltage (kV)	Ime / ID name	Bazni napon / Base voltage (kV)	(%)
MREA / Network	10,000	SABT-1	0,400	0,6
SABT-1	0,400	SAB-3	0,400	1,0
SABT-1	0,400	SAB-2	0,400	2,6
SABT-1	0,400	SAB-1	0,400	-0,6

Tablica 14 – Sumarno izvješće s višim harmonicima  
Table 14 – Summary report with higher harmonics

Sabirnica / Bus	Naponi / Voltages	Ukupno harmonično izobličenje / Total harmonic distortion $V_{THD}$ (kV)	Efektivna vrijednost napona / Effective voltage value $V_{RSS}$ (kV)	Naponski telefonski koeficijent / Telephone voltage coefficient $V_{TIF}$ (kV)	Vršna vrijednost napona / Peak voltage value $V_{SUM}$ (kV)
Ime / ID name	Bazni napon / Base voltage (kV)				
MREA / Network	10,000	0,044	10,000	201,563	10,130
SAB-1	0,400	0,010	0,400	54,928	0,429
SAB-2	0,400	0,016	0,388	56,751	0,422
SAB-3	0,400	0,022	0,394	131,528	0,476
SABT-1	0,400	0,009	0,398	40,677	0,424



Slika 5 – Slika tokova snaga bez harmonika  
Figure 5 – Power flows without harmonics



Slika 6 – Proračun harmonične analize  
Figure 6 – Computational load analysis

Tablica 15 – Sumarno izvješće po granama  $t$   
Table 15 – Summary report by branches  $t$

Grama / Branch	Sabirnica / Bus		Struje / Currents			Gubici / Losses	
	Name / ID name	Ime / ID name	Bazni napon / Base voltage (kV)	Ukupno harmonično strujno izobličenje / Total harmonic current distortion $I_{THD}$ (A)	Efektivna vrijednost struje / Effective current value $I_{RMS}$ (A)	IT umnožak / Product of multiplication $IT_{prod}$ (A)	Radne snage / Active power (kW)
C-1	SABT-1	0,400	92,7	1 170,2	316 959,4	9	28
C-2	SABT-1	0,400	63,1	1 025,9	260 406,5	7	21
C-3	SABT-1	0,400	23,6	612,5	60 395,7	3	7
CAP-1	SAB-2	0,400	78,7	160,4		0	-96
FL-1	SAB-1	0,400	55,7	747,2		2	18
FL-1_A	SAB-3	0,400	324,7	943,0		3	42
L-1	SAB-1	0,400	74,3	298,0			
L-2	SAB-1	0,400	74,3	298,0			
L-3	SAB-2	0,400	1,5	298,2		171	128
M-1	SAB-3	0,400	119,5	486,7			
M-2	SAB-3	0,400	119,5	486,7			
M-3	SAB-3	0,400	119,5	486,7			
M-4	SAB-2	0,400	15,4	819,2		6	118
Mreža / Network	MREA / Network	10,000	4,4	98,0	12922,2	3	14
TRAFO / Transformer	MREA / Network	10,000	4,4	98,0	12922,2	5	56

Tablica 16 – Izvješće gubitaka u granama  
Table 16 – Report on losses in branches

Grana / Branch	Od sabirnice / From bus		Do sabirnice / To bus		Gubici / Losses	
	Ime / ID name	Bazni napon / Base voltage (kV)	Ime / ID name	Bazni napon / Base voltage (kV)	(kV)	(kvar)
C-1	SABT-1	0,400	SAB-3	0,400	9	28
C-2	SABT-1	0,400	SAB-2	0,400	7	21
C-3	SABT-1	0,400	SAB-1	0,400	3	7
CAP-1	SAB-2	0,400			0	-96
FL-1	SAB-1	0,400			2	18
FL-1_A	SAB-3	0,400			3	42
L-3	SAB-2	0,400			171	128
M-4	SAB-2	0,400			6	118
Mreža / Network	MREA / Network	10,000			3	14
TRAFO / Transformer	MREA / Network	10,000	SABT-1	0,400	5	56

Tablica 17 – Izvješće vršne vrijednosti napona na kondenzatoru VSUM  
Table 17 – Report on capacitor peak voltage value VSUM

Grana / Branch	Sabirnica / Bus		Rezultati / Results		
	Ime / ID name	Bazni napon / Base voltage (kV)	(Mvar)	Nazivni napon / Rated voltage (kV)	Vršna vrijednost / Peak value $V_{SUM}$ (%)
CAP-1	SAB-2	0,400	0,100	0,400	105,4

Tablica 18 – Izvješće o opterećenju transformatora  
Table 18 – Report on transformer load

Grana / Branch	Od sabirnice / From bus		Do sabirnice / To bus		Rezultati / Results	
	Ime / ID name	Bazni napon / Base voltage (kV)	Ime / ID name	Bazni napon / Base voltage (kV)	K-faktor / K-factor	Opteretivost / Loadability (%)
TRAFO / Transformer	MREA / Network	10,000	SABT-1	0,400	1,307	96,2

Tablica 19 – Izvješće o opterećenju vodiča  
Table 19 – Report on conductor load

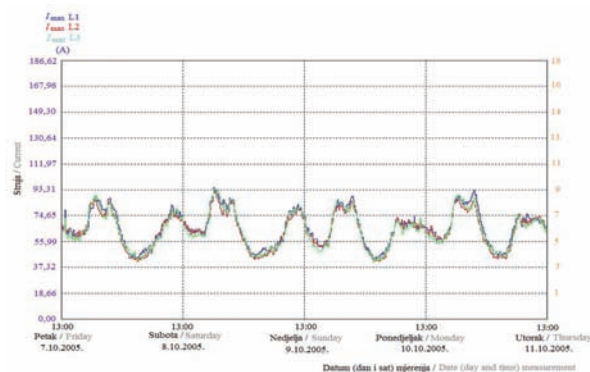
Grana / Branch	Od sabirnice / From bus		Do sabirnice / To bus		Rezultati / Results
	Ime / ID name	Bazni napon / Base voltage (kV)	Ime / ID name	Bazni napon / Base voltage (kV)	
C-1	SABT-1	0,400	SAB-3	0,400	99,4
C-2	SABT-1	0,400	SAB-2	0,400	99,7
C-3	SABT-1	0,400	SAB-1	0,400	100,0

Tablica 20 — Sumarno izvješće o filtrima  
Table 20 — Summary report on filters

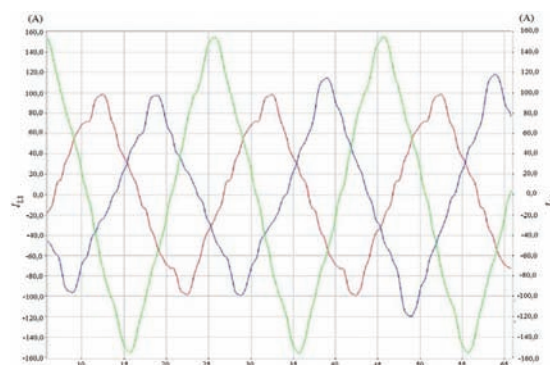
Ime / ID name	Tip / Type	Efektivna vrijednost struje / Effective current value $I_{RMS}$ (A)	Gubici djelatne snage / Active power losses (kW)	Gubici jalove snage / Reactive power losses (kvar)	Vršna vrijednost napona / Peak voltage value $V_{SUM}$ (%)
FL-1	Notch	747,2	2	18	104,9
FL-1_A	Notch	943,0	3	42	112,5

Kao praktičan primjer (nesimetričnog opterećenja) promatrana je kvaliteta električne energije mjerene na kabelskom vodu na 10 kV razini u TS 35/10 kV Vinkovci 1 [7]. Na ovaj kabelski vod priključeno je 12 trafostanica 10/0,4 kV. Svi parametri su mjereni u trajanju od jednog tjedna, prema normi EN50160, koja uključuje veličine na sve tri faze i to: napone, struje, aktivnu, reaktivnu i kompleksnu snagu, struje i napone nesimetrije, harmonike sve do 40-tog, ili potpuno harmonično izobličenje, flikere kratkotrajne i duge, faktor snage u sve tri faze i frekvenciju. Ovdje su predstavljeni neki od dobivenih rezultata (slike 7 do 10).

As an example from practice (of unbalanced load), the quality of electric power measured on the cable line at 10 kV level in TS 35/10 kV Vinkovci 1 [7] was observed. 12 substations 10/0,4 kV are connected to this cable line. All parameters were measured in the duration of one week according to EN50160, which included the values on all three phases, viz.: voltage, current, active, reactive and complex power, unbalanced currents and voltages, harmonics all the way to the 40-th, or complete harmonic distortion, flickers short and long, power factor in all three phases and frequency. Some of the obtained results are presented herein (Figures 7 to 10).

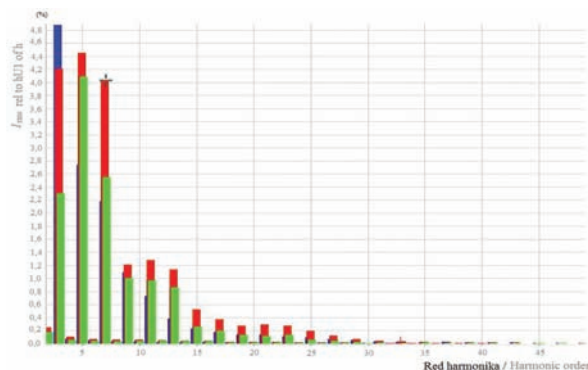


Slika 7 — Nesimetričnost struje na 10 kV kabelskom vodu  
Figure 7 — Current imbalance on 10 kV cable line

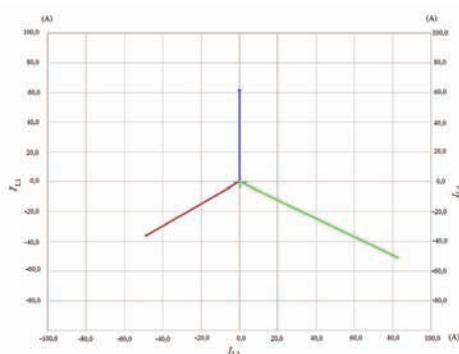


Slika 8 — Valni oblik struja na 10 kV kabelskom vodu  
Figure 8 — Waveform of currents in 10 kV cable line





Slika 9 – Spektar harmonika struje na 10 kV kabelskomvodu  
Figure 9 – Harmonic current spectrum on 10 kV cable line



Slika 10 – Vektorski dijagram asimetričnosti struja na 10 kV kabelskomvodu  
Figure 10 – Vector diagram of unbalanced currents on 10 kV cable line

## 5 ZAKLJUČAK

U trofaznim mrežama viši harmonici čine direktni, inverzni i nulti sustav EMS. Inverzni sustav EMS stvara u strojevima magnetsko polje koje rotira suprotno od direktnog sustava, čime se smanjuje korisnost takvog električnog motora.

Zbog postojanja nul-sustava EMS u nul-vodiču teče struja, a kod spoja generatora u trokut, teče tzv. struja izjednačenja. Sve to stvara dodatne gubitke u vodovima i uređajima. Zbog toga treba spriječiti stvaranje viših harmonika u trofaznim mrežama. Za proračun utjecaja viših harmonika u trofaznim mrežama, potrebno je poznavati osnovne spojeve trofaznih sustava i njihov proračun struja i napona. Pokazano je da se viši harmonici javljaju u direktnom, inverznom i nultom sustavu trofaznih struja i napona. Sve te tri skupine su analizirane u različitim spojevima, te je ukazano na njihove posljedice.

Pregledno su dane europske norme, koje su izdane unatrag 20 godina, a koje se odnose na više harmonike. Na kraju su tabelarno prikazani svi rezultati jednog primjera proračuna elektroenergetske mreže primjenom softverske simulacije.

## 5 CONCLUSION

In three-phase networks higher harmonics constitute a direct, inverse and zero EMF system. The inverse EMF system produces in the machines a magnetic field which rotates counter to the direct system, whereby the electric motor efficiency is diminished.

Due to the existence of EMF zero-system, current flows in the zero-conductor, and when the generator is delta-connected, the so-called equalising current flows. All this leads to additional losses in lines and devices. For that reason the creation of higher harmonics in three-phase networks ought to be prevented. For computing the impact of higher harmonics on three-phase networks it is necessary to know the basic connections of three-phase systems and their current and voltage computation. It has been shown that higher harmonics occur in the direct, inverse and zero systems of three-phase currents and voltages. All these three groups have been analysed in different connections and their effects shown.

An overview of the European standards issued over the last 20 years and relating to higher harmonics has been given. Finally, all the results of

Također je dan i primjer iz prakse gdje su izvedena mjerenja u 10 kV mreži s nesimetričnim opterećenjem, a dobiveni rezultati prikazani su grafički.

an example of electric power network computation by using software simulation are shown in tables. Also given is an example from practice, where measurements were performed in a 10 kV network with unbalanced load, and the obtained results are graphically represented.

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