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ANALYSIS OF DIFFERENT MODELING APPROACH AT DETERMINING OF BACKWARD EXTRUSION FORCE ON AlCu5PbBi MATERIAL

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The goal of the paper is to present an outline of different modeling approach at determining of backward extrusion force on AlCu5PbBi material and to compare them with experimental obtained results. Stochastic modeling in the paper is based on the statistic processing of central composite experimental design i.e. in this investigations central composite circumscribed (CCC) design. The numerical modeling is based on the finite element method (FEM) using ABAQUS 6.4.1. Explicit software.

Key words: stochastic modeling, numerical modeling, backward extrusion, experimental design

Analiza različitih pristupa modeliranja kod određivanja sile protusmjernog istiskivanja na AlCu5PbBi materijalu. Cilj rada je prezentirati prikaz različitih pristupa modeliranja kod određivanja sile protusmjernog istiskivanja na AlCu5PbBi materijalu i usporediti ih s eksperimentalno dobivenim vrijednostima. Stohastičko modeliranje u radu je zasnovano na statističkoj obradi centralnog kompozicijskog eksperimentalnog plana tj. u ovom istraživanju centralnoga kompozicijskoga opisanog plana. Numeričko modeliranje je zasnovano na metodi konačnih elemenata (MKE) koristeći ABAQUS 6.4.1. Explicit program.

Ključne riječi: stohastičko modeliranje, numeričko modeliranje, protusmjerno istiskivanje, eksperimentalni plan

INTRODUCTION

Backward extrusion process is a widely used cold forming process for the manufacturing of hollow-shape symmetric, cylindrical products and it is usually performed on a high-speed and accurate mechanical presses. The punch descends at a high speed and strikes the blank, extruding it upwards by means of high pressure. The thickness of the extruded tubular section is a function of the clearance between the punch and the die [1]. Schematic outline of backward extrusion process is presented in Figure 1. Backward extrusion is restricted to softer metals and their alloys, according to the much quoted theory on backward extrusion force i.e.

Dipper's theory of double upsetting [2], based on a grid pattern deforming as well as a grid element moving, the analytic expression for fast calculating of backward extrusion force has a form:

$$F_{be} = \left[k_{f\,1} \cdot \left(1 + \frac{1}{3} \cdot \mu \cdot \frac{d_1}{h_1} \right) + k_{f\,2} \left[1 + \frac{h_1}{s} \left(\frac{\mu}{2} + 0.25 \right) \right] \right] \cdot \cdot \frac{\pi d_1^2}{4} \tag{1}$$

where: μ - coefficient of friction between tool, die and workpiece, d_1 - diameter of punch, s -wall thickness, h_1 - bottom thickness. For calculating $k_{f\,1}$, $k_{f\,2}$ true stresses versus the logarithmic plastic strain φ has to be included in the analysis i.e. it means that for φ_1 from flow stress curve the value of kf_1 has been obtained and for φ_2 from flow stress curve the value of kf_2 has been obtained, where:

$$\varphi_1 = \ln \frac{h_1}{h_0}, \tag{2}$$

$$\varphi_2 = \left(\ln \frac{h_1}{h_0} + \frac{d_1}{8s} \ln \frac{h_1}{h_0} \right). \tag{3}$$

Flow stress curve is described in the form of hardening law Hollomon-Ludwik's power law as [3, 4, 5]:

$$k_f = C \cdot \varphi^n, \tag{4}$$

where: k_f - true stress, C - strength coefficient, n - work hardening exponent, φ - true strain.

Because of a grid element twisting the total strain φ_u is derived as addition of axial φ_1 (in the range beyond punch) and radial φ_2 (in the range between wall of die and punch) strain:

$$\varphi_u = |\varphi_1| + |\varphi_2| = \left| \ln \frac{h_1}{h_0} + \frac{d_1}{8s} \ln \frac{h_1}{h_0} \right| = \varphi_1 \cdot \left(1 + \frac{d_1}{8s} \right) \tag{5}$$

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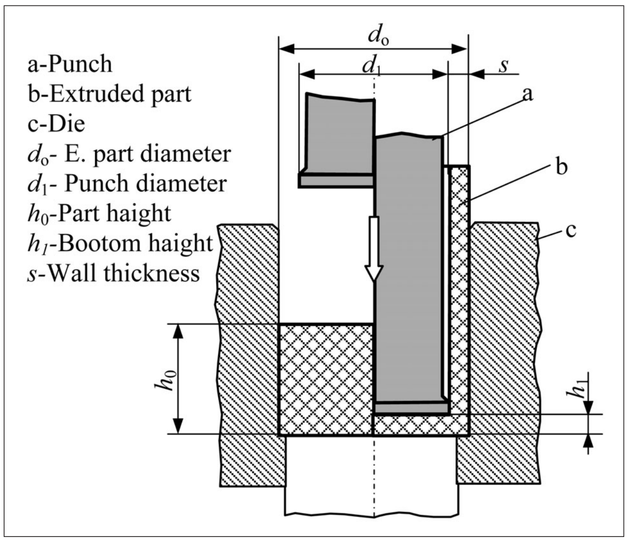

Figure 1. **Backward extrusion process**

EXPERIMENTAL DESIGN

Central composite design has been used in these investigations. It contains an imbedded factorial or fractional factorial design with center points that is augmented with a set of symmetric points that allow estimation of curvature. Symmetric points represent extreme values for each factor in the design. If the length from the center of the design space to a factorial point is ±1 unit for each factor, the distance from the center of the design space to a symmetric point is $\pm\alpha$ with $1|\alpha| \geq 1$. There are three varieties of central composite designs: central composite circumscribed (CCC) designs, central composite inscribed (CCI) designs and central composite face centered (CCF) designs [6]. In this investigation CCC design has been used as an experimental design (Figure 2).

In the CCC design, design points describe a circle circumscribed around the factorial square and symmetric points are at some distance from the center based on the properties desired for the design and the number of factors in the design [6]. CCC design provides a maximum of information on process model effects achieving by point arrangement in the experimental hyperspace based on optimality criterion. All factors are altered simultaneously, therefore every model parameter is determined on the basis of all N experimental results, the consequence of which is that the dispersion $\sigma^2(b_i)$ of any b_i model coefficient is N times smaller than experimental error. The dispersion $\sigma^2(b_i)$ of model parameters is minimal, which leads towards greater precision and reliability of the required function y of the process description [7].

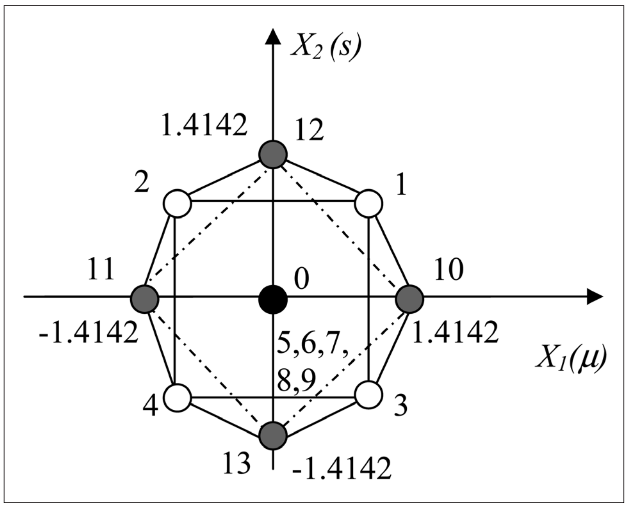

Figure 2. **Schematic outline of CCC design for two variables $X_1(\mu)$ and X_2 (s)**

EXPERIMENTAL SET UP

The basic system for the experimental determining of backward extrusion force and compression test (specimen ϕ20mm × 20 mm) on AlCu5PbBi consists: a strain ring attached to a load cell system with a sensor, a displacement transducer, measuring converter, data processing system, and press KP 200 (Knuth -Germany, maximum force: 2000 kN, maximum stroke: 500 mm.).

After compression test was performed the result in the form of Ludwik– Hollomon's power law is obtained as:

$$k_f = 334{,}33 \cdot \varphi^{0{,}192} \qquad (5)$$

Other experimentally obtained data for material AlCu5PbBi are: yield strength k_0 = 150 N/mm^2, ultimate tensile strength R_m = 280 N/mm^2, state: soft annealed. Bacis values of friction coefficients through Male-Cocroft experimental test [8] were obtained and their values are: hydrolubric μ = 0,17, cold forming lubricant μ = 0,15, Zn stearat μ = 0,1, liqui molly μ = 0,05, paraffinol phenolic mixature μ = 0,03.

The experimental design matrix with varying variables (μ and s), designed by means of CCC design are shown in Table 1. The diameter of backward extrusion specimens used in these experiments has been predetermined on 32 mm, and both coefficient of friction and wall thickness of parts has been varied according to experimental design.

STOCHASTIC MODELLING

Defining of stochastic model starts with identification of set of all process or system parameters. Working

Table 1. **The experimental design matrix**

Design point	Workpi. diameter d_0 / mm	Wall thickness s/ mm	Workpi. height h_i / mm	Punchdiam. d_1 /mm	Frict. coeffic. μ
1.	32	4,5	16,057	23	0,15
2.	32	4,5	16,057	23	0,05
3.	32	2,5	8,04	27	0,15
4.	32	2,5	8,04	27	0,05
5-9.	32	3,5	12,28	25	0,1
10.	32	3,5	12,28	25	0,17
11.	32	3,5	12,28	25	0,03
12.	32	4,9	17,46	22,2	0,1
13.	32	2,1	6,2	27,8	0,1

out stochastic model is based on the statistic processing CCC experimental design. In CCC experimental design all conditions are predetermined according to the mathematical theory of experimental design. For modeling of backward extrusion force the second order model has been introduced:

$$Y = b_0 X_0 + b_1 X_1 + b_2 X_2 + b_{12} X_{12} + b_{11} X_1^2 + b_{22} X_2^2 \tag{6}$$

According to introduced model the Table 2 of two-factorial design with interaction has been formed.

Table 2. **Matrix of coded variables and measuring results of extrusion force**

Exp.	Coded values						Backw. extr. force F_{be} / kN
	X_0	X_1	X_2	X_{12}	X_1^2	X_2^2	
1	1	+1	+1	1	1	1	495
2	1	-1	+1	1	1	1	486
3	1	+1	-1	-1	1	1	870
4	1	-1	-1	-1	1	1	750
5	1	0	0	0	0	0	590
6	1	0	0	0	0	0	585
7	1	0	0	0	0	0	595
8	1	0	0	0	0	0	593
9	1	0	0	0	0	0	586
10	1	1.414	0	0	2	0	660
11	1	-1.414	0	0	2	0	560
12	1	0	1.414	0	0	2	390
13	1	0	-1.414	0	0	2	1050

Dispersion homogeneity evaluation of backward extrusion force experimental results was performed according to Cochran′s criterion for level of reliability P=0,95:

$$K_h = \frac{\max S_j^2}{\sum_{j=1}^{N} S_j^2} \tag{7}$$

where: K_t – value according to Cochran′s criterion for degrees of freedom f_j and N [9], f_j – degree of freedom (f_j=n_j-1), n_j – repetition number on design, $\sum_{j=5}^{9} S_j^2 = S_0^2$ – variance of central points of CCC design, S_j^2 – maximal variance at central design.

After checking of dispersion and the checking of significance according to Student′s criterion has been performed according to expressions:

$$\left| b_{ii} \right| \geq \Delta b_i = \pm t_{t(f_0,\, \alpha)} \sqrt{a_{ij}}\, S, \tag{8}$$

$$\left| b_{im} \right| \geq \Delta b_{im} = \pm t_{t(f_0,\, \alpha)} \sqrt{a_{13}}\, S, \tag{9}$$

$$\left| b_{ii} \right| \geq \Delta b_{ii} = \pm t_{t(f_0,\, \alpha)} \sqrt{a_{14}}\, S, \tag{10}$$

$$b_0 = a_1 \sum_{j=1}^{N} Y_j + a_2 \sum_{l=1}^{k} \sum_{j=1}^{N} X_{ij}^2 Y L_j , \tag{11}$$

$$b_i = a_3 \sum_{j=1}^{N} X_{ij} Y_j , \tag{12}$$

$$b_{im} = a_4 \sum_{j=1}^{N} X_{ij} X_{nj} Y_j , \tag{13}$$

$$b_{ii} = a_5 \sum_{j=1}^{N} X_{ij}^2 Y_j + a_6 \sum_{j=1}^{k} \sum_{j=1}^{N} X_{ij}^2 Y_j + a_7 \sum_{j=1}^{N} Y_j , \tag{14}$$

where: a_1 = 0,2, a_2 = −0,1, a_3 = 0,125, a_4 = 0,25, a_5 = 0,125, a_6 = 0,01875, a_7 = − 0,1 (for $k = 2$, n_0 = 5 and N = 13 [9]), $t_{t(f_j,\, \alpha)} = t_{t(4;0,05)}$ = 2,13-table value Student’s t criterion, α-experimental value error approximation,- level of model reliability, a_{11} = 0,2, a_{12} = 0,125, a_{13} = 0,25, a_{14} = 0,14375 – correlation matrix elements [9].

After checking of significance according to Student′s criterion, mathematical model has a form:

$$\begin{aligned} Y &= 589{,}8 + 33{,}8 X_1 - 196{,}53 X_2 - 27{,}75 X_{12} + \\ &+ 6{,}4125 X_1^2 + 61{,}425 X_2^2 \end{aligned} \tag{15}$$

Checking of adequacy according to F-criterion was examined. After model decoding, the stochastic model of backward extrusion force in physical form was obtained:

$$\begin{aligned} Y &= 2565\mu^2 + 61{,}425 s^2 - 571{,}005 s + \\ &+ 2105{,}5\mu - 5555\mu \cdot s + 1793{,}91 \end{aligned} \tag{16}$$

Comparison of experimental and stochastic results has been presented in Table 3.

Table 3. **Comparison of experimental and stochastic obtained results**

CCC desig npoints	Stochastic obtained force / kN	Experimental obtained force / kN	Diference / kN
1.	467,157	495	27,843
2.	455,058	486	30,942
3.	915,717	870	45,717
4.	792,618	750	42,618
5-9.	589,8	590	0,2
10.	649,689	660	10,311
11.	555,048	560	4,952
12.	435,051	390	45,051
13.	985,335	1050	64,665

NUMERICAL MODELLING

Defining of numerical model starts with ABAQUS 6.4.1. Explicit model definition. The workpiece model consisted of 750 axisymmetric quadrilateral elements with four node of reduced integration (TYPE=CAX4R), because of large plastic deformation that take place [10]. The number of elements, their size and nodes are very important for correct description of simulation. When using under integrated elements in simulations governed by large hydrostatic pressure, like extrusion processes, hourglassing effects may corrupt the simulation results. In this forming process during the material flow the element undergo a mesh distortion and that is the most important problem during the numerical analysis in a sever software. It was found that deformation is different with other type of element. CAX4R elements overcome the problem by means of its interpolation functions. Used functions in model discretization were: NODE, NSET, NGEN, NFILL, ELEMENT, ELGEN, ELSET, SURFACE DEFINITION, and RIGID SURFACE.

One half of workpiece and tools can be taken into consideration because of symmetry of the process. AlCu5PbBi material is assumed as rigid-plastic and it obeys the von Mises yield criterion and its elastic characteristic is governed by Poisson's ratio and modulus of elasticity. Used functions in material definition were: MATERIAL, ELASTIC and PLASTIC (it starts from $\varphi = 0$).

Die and punch were interpreted as rigid unmovable body by means of lines and curves. They are fixed with a determined point on them. Punch and die are in contact with the workpiece which is a deformable body. At the contact between a workpiece and tool the nodes do not penetrate the tools. Used functions in tools modeling were: START, LINE, CIRCLE, REF NODE, TYPE=SEGMENTS and FILLET RADIUS.

Boundary conditions assure the full symmetry of process. The moving in X and Y directions is constrained by means of S instructions. In this software there is automatic remeshing procedure but mesh was created by hand (elements and nodes are set by means of increments).

Some interactions can be modeled with the general contact algorithm, and others with the contact pair algorithm. The general contact algorithm uses a penalty method to enforce the contact constraints. At modeling contact interactions with contact pairs in the paper the contact pair algorithm was used. Other used functions in process simulation were: CONTACT PAIR, FRICTION, STEP, INC, BOUNDARY and NODE PRINT (U, RF).

On the basis of subroutine in ABAQUS 6.4.1. Explicit software that calculates the backward extrusion force in process, the backward extrusion force curve has been outcome in all points of CCC experimental design. Comparison of experimental and numerical results has been presented in Table 4.

Table 4. **Comparison of experimental and numerical obtained results**

CCC design points	Numerical obtained force / kN	Experimental obtained force / kN	Diference / kN
1.	470	495	25
2.	460	486	26
3.	895	870	25
4.	780	750	30
5-9.	567	590	23
10.	640	660	20
11.	565	560	5
12.	368	390	22
13.	1020	1050	30

CONCLUSION

In the paper the outcomes obtained from stochastic modeling (with CCC multi factorial experimental design) and experimental research show that the results of backward extrusion force founded on stochastic analysis are very close to experimental ones (especially according to results in 5-9, 10 and 11 point of CCC design). The outcomes obtained from numerical modeling (with the same multi factorial experimental design) and experimental comparison show that the results of backward extrusion force founded on numerical analysis are closer to experimental ones (especially according to results in 1-4, 12 and 13 point of CCC design. As final conclusion, it may be concluded that stochastic and numerical modeling of backward extrusion can be used to determine forming force very successful because the calculated results of modeling agree well with the experimental investigations. Generally regarding the time invested in modeling, stochastic modeling process is faster than numerical modeling process but disadvantage of this type of modeling is an expensive experiment.

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