

# Life Assessment of Railway Tunnel Lining Structure Based on Reliability Theory

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**Abstract:** The reliability of the tunnel lining during its service life has significance for tunnel safety management. To capture the performance of the lining under the effect of deterioration factors, the time-varying reliability theory was applied to predict the service life of the lining. The failure process of the lining structure under an erosion environment was analyzed. The limit state equations of the lining structure were established based on the durability criterion and the bearing capacity criterion, respectively. The time-varying reliability of the tunnel was calculated using the Monte-Carlo method with an engineering example, and the service life of the tunnel under different criteria was predicted based on the target reliability. The results show that the predicted service life of the tunnel is 77.5 years under the durability criterion and 95 years under the bearing capacity criterion, assuming that the tunnel structure is in an erosive environment at the beginning of construction and that no protective measures are taken under the most unfavourable conditions. The durability meets the structural applicability, and the bearing capacity meets the structural safety, which is in line with the actual needs of the project. The study results can provide a basis and reference for the future durability design, life prediction, and maintenance management of similar service tunnels.

**Keywords:** bearing capacity; durability; service life; tunnel engineering; time-varying reliability

## 1 INTRODUCTION

The reliability of an engineering structure includes its durability in addition to its safety and serviceability [1]. Similar to other reinforced concrete structures, tunnel linings face various environmental conditions during service and need to resist the continuous action of destructive factors as well [2-3]. With increasing service time, the performance of the lining deteriorates under the influence of external conditions, resulting in changes in structural dimensions, material strength, etc. The loss of structural bearing capacity is mainly due to the lining's resistance decay and crack expansion. Compared with other structures, tunnel projects are characterized by high life-cycle costs, long service cycles, and high damage hazards. Therefore, the system should be required to have a long enough service life to resist the deterioration of structural materials so that no rework and significant repairs are needed during the service life cycle. This calls for the need to ensure the safety and durability of the lining during its service life in terms of concrete aging and reinforcement corrosion.

Considering the deterioration of structural performance from the perspective of full life has become the focus of scholars. Liu et al. [4] analyzed the progressive erosion degradation law of subsea tunnel segment joints during the whole life cycle and established an analysis model of shield duct joint erosion degradation considering the coupling of seawater pressure infiltration and chloride ion erosion. Based on Fick's second law, Zhang [5] investigated the crack propagation behavior and structural bearing capacity of the reinforced concrete bending members degraded by corrosion. Visser [6] obtained an analytical model of carbonation resistance by studying the parameters of concrete materials and the concentration of CO<sub>2</sub>. Saetta [7-8] established a mechanical model of reinforced concrete beams under carbonation by theoretical analysis and experimental verification. Almusallam [9] and Mak [10] investigated the bearing capacity of rusted reinforced concrete beams. The mentioned study provides an in-depth analysis of the mechanical behavior of the lining under an erosion environment, which has significant value in guiding the

service performance analysis of the lining structure during the service period. However, most of the current researches are directed to a specific factor. Considering the influence of the complex erosion environment [11] and variable loading characteristics [12] of underground tunneling projects, the time-varying bearing capacity calculation model and life prediction model specifically for the life cycle of the lining need to be further studied.

In addition to the time-dependent deterioration, substantial uncertainties exist in tunneling. First, the performance parameters of geotechnical materials possess great uncertainty, which significantly impacts the related mechanical analysis. Secondly, Wang et al. [13] also showed that there will still be uncertainties in the dimensions and material performance parameters of the tunnel lining. In addition, the uncertainty of the random acts of environmental variables is also a potential threat to the structural safety of the tunnel. Considering the impact of such uncertainties on the safety of tunnel structures, Mikaeil et al. [14] evaluated the safety of tunnel structures through reliability based on a probabilistic approach. However, the connotation of lining reliability is abundant, including safety, applicability, durability, etc., all need to meet the requirements of the underground building structure, thus leading to lining failure for various reasons.

The present study proposes to predict the service life of the tunnel lining structure based on the durability criterion and the ultimate bearing capacity criterion. The lining failure process is described by defining the service life of the lining at different failure stages. To establish the lining reliability analysis model during the service period by combining the force characteristics of the lining, develop lining failure probability and reliability calculation method based on probabilistic statistics principle. It aims to provide theoretical support for the life prediction, durability assessment, and scientific maintenance of in-service tunnel lining structures by describing and displaying the deterioration process of lining performance during the service period.

## 2 SERVICE LIFE OF TUNNEL STRUCTURE

### 2.1 Definition

Considering the structure's service life from the view of durability, it usually refers to the time when the system satisfies the intended function under regular maintenance and service conditions. Service life can be divided into technical, functional, and economical.

In this paper, the technical service life is used to reflect the service life of the railway tunnel lining structure. Given the general service environment of a railway tunnel in an atmospheric setting, it is pointed out that the durability of the lining structure is affected by concrete carbonation, or the ultimate bearing capacity of the structure cannot meet the design requirements. Some or all of the functions of the structure cannot meet the expected time of use.

### 2.2 Analysis of Deterioration Process of Lining Structure

The research on the mechanism of structural deterioration is the key to the service life assessment. For the ordinary railway tunnel, the lining structure is exposed to the general atmosphere, and concrete carbonization is the leading cause of the deterioration of the structure. Carbonation leads to the failure of the protective layer, and corrosion of steel bars leads to the cracking of the protective layer, which is the leading cause of structural failure. With the increase in service time, the process of structural deterioration can be represented by Fig. 1.

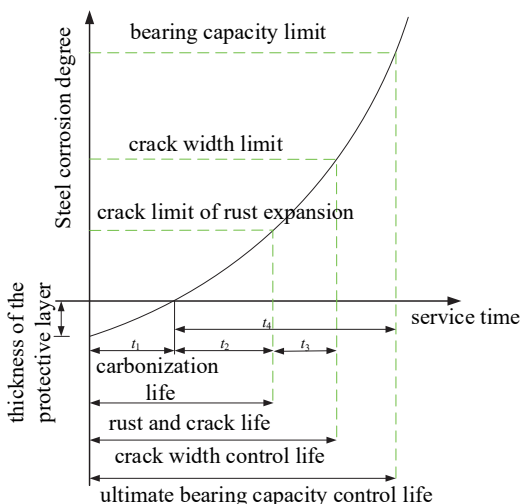


Figure 1 Process of deterioration of lining performance with time

(1) Carbonation life refers to the time from the start of the tunnel lining until the depth of concrete carbonation reaches the surface of the reinforcing steel. The concrete protective layer fails in this process, and the reinforcement rusts.

(2) Corrosion cracking life refers to the time between the start of corrosion and expansion of the reinforcing steel and the cracking of the concrete cover.

(3) The crack width control life refers to the time when the crack width reaches the limit value.

(4) The ultimate bearing capacity control life means that with the continuous increase of service time, the progressive development of cracks leads to the constant reduction of the structure's bearing capacity. The structure

cannot guarantee the standard service time after the crack reaches the limit value, so the structure's reliability level reaches the lower limit of the code.

Fig. 1 shows several essential stages in the service period of the structure, namely, the time when carbonation causes the steel bar to begin to rust, the time when the concrete cracking reaches the limit value, and the time when the bearing capacity of the structure reaches the lower limit value. These time points are virtual nodes for predicting the service life of structures.

### 2.3 Life Prediction Method based on Time-Varying Reliability

From the definition of structural service life, it is clear that defining the function of structural use is the prerequisite for structural service life prediction; the structural function failure criterion is the key to predicting structural service life. The following criteria are summarized: crack width and corrosion limit life criterion, carbonation or chloride erosion life criterion, bearing capacity life criterion, rust expansion cracking life criterion and so on. Based on the understanding of the above criteria, the service life of the lining structure is predicted by using the limit life criterion of crack width and steel corrosion amount and bearing capacity life criterion, respectively. The smaller value between them is taken as the service life of the tunnel lining structure.

A railroad passenger line tunnel is used as the engineering background. The length of the tunnel is 900.76 m, which is a single-hole and two-lane tunnel; the maximum buried depth in the tunnel is 127.4 m. The design speed target value is 300 ~ 350 km/h, and the service life of the structural design is 100 years. The thickness of the lining is 45 cm, the second lining is C35 reinforced concrete, and the thickness of the protective layer is 5 cm. The annual average temperature of the engineering area is 15.7 °C, the annual average relative humidity is 0.8, CO<sub>2</sub> concentration in the tunnel is approximately 0.2%. According to the research results of the structure life obtained at present, the service life of the tunnel lining structure is studied and predicted using the reliability theory. The service life of the lining structure is obtained.

## 3 SERVICE LIFE PREDICTION OF STRUCTURES BASED ON DURABILITY CRITERION

The life prediction based on the durability criterion is that the lining structure will be carbonized, rusted, and cracked until the structure's durability does not meet the requirements. According to Fig. 1, the formula for calculating the lining life controlled by the durability criteria is:

$$T_d = t_1 + t_2 + t_3 \quad (1)$$

where,  $t_1$  indicates the carbonization life of the concrete cover, the  $t_2$  indicates the time taken for the steel bar to rust until the protective layer cracks, and the  $t_3$  represents the time taken when the cover cracks to the maximum crack width.

### 3.1 Calculation of Carbonation Life

The failure of the passivation film caused by the carbonization's depth reaching the steel bar's surface is the premise of steel corrosion. The limit state equation of concrete carbonization durability is:

$$Z_1(t) = c - X(t) \tag{2}$$

where,  $c$  is the concrete cover thickness (mm),  $X(t)$  is the carbonation depth (mm),  $t$  is the service life of the structure.

Based on the empirical model of the compressive strength of concrete, Niu [15] considers that the carbonation depth of concrete is related to time, material properties, and environmental factors. The prediction formula for carbonation depth can be expressed as follows:

$$X(t) = k\sqrt{t} \tag{3}$$

where,  $k$  is the carbonation development condition coefficient and its formula is:

$$k = 3k_j k_{CO_2} k_p k_s \sqrt[4]{T(1-RH)RH} \cdot \left( \frac{57.94}{f_{cuk}} - 0.76 \right) \tag{4}$$

where,  $k_j$  is the angle correction factor (1.4 for angle and 1.0 for non-angle),  $k_{CO_2}$  is the influence coefficient of carbon dioxide concentration,  $k_p$  is the casting surface correction factor (1.3 for pouring surface);  $k_s$  is the influence coefficient of working stress (take 1.0 under pressure and 1.2 under tension),  $f_{cuk}$  is the standard value of compressive strength of concrete cube (MPa),  $T$  is the average annual temperature of the environment (°C).

Based on reliability theory, the probability of failure of concrete cover and corrosion of steel bar is as follows:

$$P_{f1}(t) = P\{c - x_0 - X(t) < 0\} = \Phi(-\beta_1) \tag{5}$$

where,  $\Phi^{-1}(\cdot)$  is the probability distribution function of the standard normal distribution,  $\beta$  is the reliability index.

The Monte Carlo method is used to calculate reliability indexes. The basic principle is that the probability of an event can be estimated by the frequency of occurrence in a large number of trials. A large number of random samples of random variables are first taken. Then these sample values are substituted into the functional function equation to determine whether the structure fails or not. Finally, the probability of failure of the system is derived from it. Therefore, when the sample size is large enough, it can be considered that the frequency of the event is its probability. The parameter values of random variables in the calculation process, as shown in Tab. 1, the simulation times are 100000 times To ensure calculation accuracy. The calculation results of the reliability index and failure probability under the carbonation depth criterion are shown in Tab. 2 and Fig. 2, Fig. 3.

According to the reference [16], it is considered that when the failure probability of components reaches 0.05, or the reliability index is 1.64, the structure will fail.

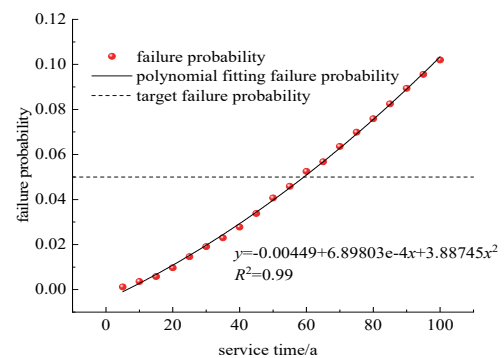
According to the calculation results of the program combined with Fig. 2 and Fig. 3, the corresponding time  $T = t_1 = 60$  years, that is, when the railway tunnel lining structure is in service for 60 years, the carbonization depth will reach the surface of steel bars, and the steel bars begin to rust.

**Table 1** Statistical characteristics of stochastic parameters under carbonization life control

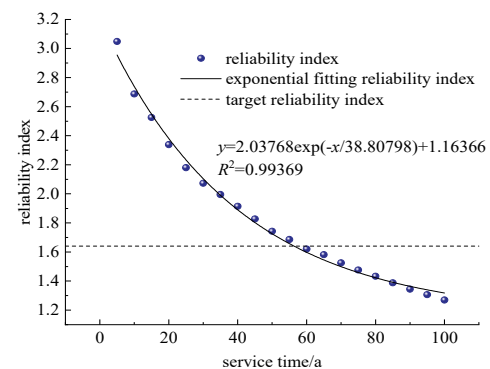
| Random parameter       | Mean value | Coefficient of variation | Distribution |
|------------------------|------------|--------------------------|--------------|
| $c / \text{mm}$        | 50         | 0.25                     | normal       |
| $T / ^\circ\text{C}$   | 15.7       | 0.5                      | normal       |
| $RH$                   | 0.8        | 0.15                     | normal       |
| $f_{cuk} / \text{MPa}$ | 35         | 0.25                     | normal       |

**Table 2** Calculation results under carbonation depth criterion

| Service time, a | Failure probability | Reliability index |
|-----------------|---------------------|-------------------|
| 5               | 0.001 2             | 3.048 5           |
| 10              | 0.003 6             | 2.687 4           |
| 15              | 0.005 8             | 2.525 3           |
| 20              | 0.009 7             | 2.338 9           |
| 25              | 0.014 6             | 2.180 0           |
| 30              | 0.019 1             | 2.073 1           |
| 35              | 0.023 0             | 1.995 2           |
| 40              | 0.027 8             | 1.914 8           |
| 45              | 0.033 8             | 1.827 4           |
| 50              | 0.040 7             | 1.743 0           |
| 55              | 0.045 9             | 1.685 7           |
| 60              | 0.052 5             | 1.620 6           |
| 65              | 0.056 8             | 1.581 9           |
| 70              | 0.063 6             | 1.525 2           |
| 75              | 0.069 9             | 1.476 2           |
| 80              | 0.075 9             | 1.433 3           |
| 85              | 0.082 5             | 1.388 2           |
| 90              | 0.089 4             | 1.344 6           |
| 95              | 0.095 6             | 1.307 0           |
| 100             | 0.102 0             | 1.270 3           |



**Figure 2** The change curve of structural failure probability under the criterion of carbonization life



**Figure 3** Variation curve of structural reliability index under carbonation life criterion

### 3.2 Calculation of Corrosion Expansion and Cracking Life

When the corrosion depth of steel bars in a concrete structure reaches critical corrosion depth, the cracking phenomenon will occur in the concrete protective layer of the structure. According to this phenomenon, the limit state equation is established as follows:

$$Z_2(t) = \delta_{cr} - \delta(t) \tag{6}$$

where  $\delta_{cr}$  is the limit value of the corrosion depth of steel bars at the time of crack of a protective layer,  $\delta(t)$  is the corrosion depth of rebar at  $t$  moment, and  $t$  is the service time of the structure after the steel bar begins to rust.

$$\delta_{cr} = k_{crs} \left( 0.008 \frac{c}{d} + 0.00055 f_{cuk} + 0.022 \right) \tag{7}$$

where  $d$  is the steel bar diameter, and  $k_{crs}$  is the influence factor of reinforcement position (take 1.0 at the corner and 1.35 at the non-corner).

$$\begin{cases} \delta(t) = \lambda_{e1}(t) \\ \lambda_{e1} = 46k_{cr}k_{ce}e^{0.04T} (RH - 0.45)^{\frac{2}{3}} c^{-1.36} f_{cuk}^{-1.83} \end{cases} \tag{8}$$

where  $\lambda_{e1}$  is the corrosion rate before rust bulging and cracking;  $k_{cr}$  is the rebar position correction factor (take 1.6 at the corner and 1.0 at the non-corner),  $k_{ce}$  is the correction factor for secondary environmental conditions, which is 3.0 ~ 4.0 for outdoors and 1.0 ~ 1.5 for indoors under wet conditions, 2.5 ~ 3.5 for outdoors and 1.0 for indoors under dry conditions.

The probability of rust bulging and cracking of the protective layer, that is, the failure probability, is as follows:

$$P_{f2}(t) = P\{\delta_{cr} - \delta(t) < 0\} = \Phi(-\beta_2) \tag{9}$$

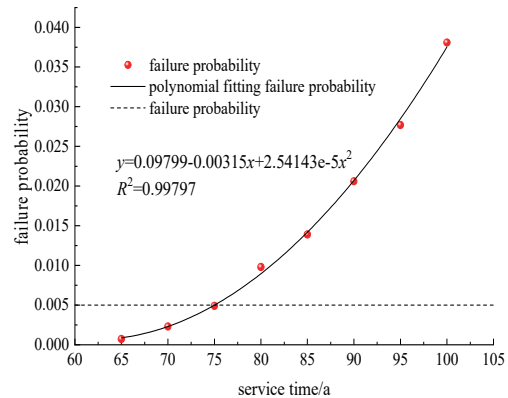
The mean diameter of the reinforcing bar is  $\mu_d = 22$  mm, and the coefficient of variation is  $V_d = 0.01$ , which obeys the normal distribution. The parameters of other random variables are shown in Tab. 1. The reliability index and failure probability under the criterion of dilatancy and cracking of lining structure are calculated as shown in Tab. 3 and Fig. 4, and Fig. 5, respectively.

**Table 3** Calculation results of rust expansion cracking criterion

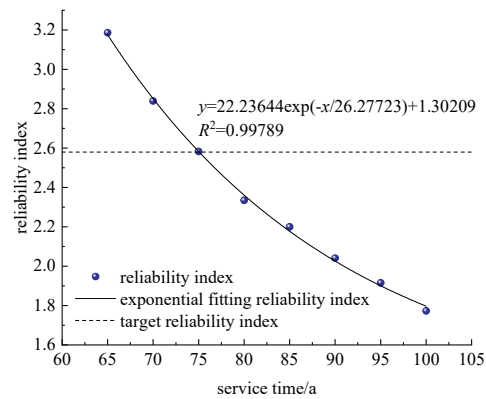
| Service time, a | Failure probability     | Reliability index |
|-----------------|-------------------------|-------------------|
| 65              | $7.2000 \times 10^{-4}$ | 3.1865            |
| 70              | 0.0023                  | 2.8394            |
| 75              | 0.0049                  | 2.5828            |
| 80              | 0.0098                  | 2.3347            |
| 85              | 0.0139                  | 2.1995            |
| 90              | 0.0206                  | 2.0411            |
| 95              | 0.0277                  | 1.9154            |
| 100             | 0.0381                  | 1.7728            |

According to the code for the design of concrete structures [17], the failure probability is 0.005, that is, the reliability index is 2.58, for the stage of corrosion expansion and cracking of members. Based on the

calculated results of the program, combined with Fig. 4 and Fig. 5, corresponding time  $T = t_1 + t_2 = 60 + t_2 = 75$  years,  $t_2 = 15$  years. That is, the crack time of the concrete surface caused by corrosion of steel bar in the service of this railway tunnel lining structure for 75 years.



**Figure 4** The change curve of structural failure probability under the criterion of the rust expansion and cracking



**Figure 5** The change curve of structural reliability index under the criterion of the rust expansion and cracking

### 3.3 Calculation of Crack Width Controlled Life

After the protection layer is cracked, the corrosion rate of the reinforcement accelerates, and the crack width continues to expand. Take the maximum crack width of concrete exceeding the allowed value as the failure criterion; the limit state equation is as follows:

$$Z_3(t) = [\omega] - \omega(t) \tag{10}$$

where  $[\omega]$  is the maximum allowed value of crack width, 0.2 mm is selected [18],  $\omega(t)$  is the width of the crack at time  $t$ .

After cracks appear in the surface concrete, the corrosion rate of reinforcement increases and the time-varying formula for calculating the crack width of the reinforced concrete lining can be expressed as follows:

$$\omega(t) = \lambda_{e1}t^2 + 2t\sqrt{\lambda_{e1}(0.0229f_{cuk} - 0.051)} \tag{11}$$

where  $t$  is duration of continuous service of structure after cracking of protective layer.

When the crack reaches the limit value, the limit state equation is as follows:

$$Z_3(t) = 0.2 - \frac{46k_{cr}k_{ce}e^{0.04T}(RH - 0.45)^{\frac{2}{3}}t^2}{c^{1.36}f_{cuk}^{1.83}} \quad (12)$$

$$- 2t\sqrt{\frac{46k_{cr}k_{ce}e^{0.04T}(RH - 0.45)^{\frac{2}{3}}(0.0229f_{cuk} - 0.051)}{c^{1.36}f_{cuk}^{1.83}}}$$

The probability that the crack width reaches the limit value, that is, the failure probability is:

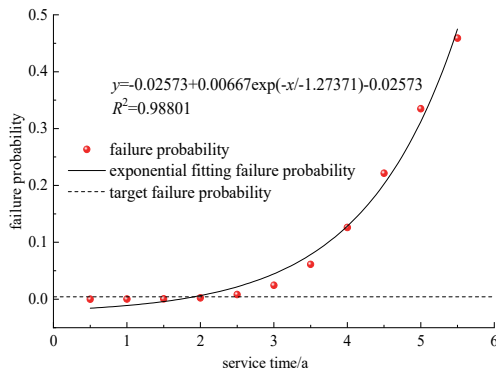
$$P_{f3}(t) = P\{\omega_{max} - \omega(t) < 0\} = \Phi(-\beta_3) \quad (13)$$

The calculated results are as shown in Tab. 4 and Fig. 6, Fig. 7.

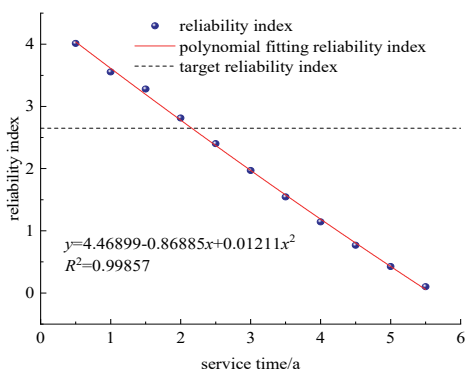
**Table 4** Calculation results under crack width criterion

| Service time, a | Failure probability | Reliability index |
|-----------------|---------------------|-------------------|
| 0.5             | 3.000 0 e -05       | 4.0128            |
| 1               | 1.900 0 e -04       | 3.5536            |
| 1.5             | 5.200 0 e -04       | 3.2795            |
| 2               | 0.0024              | 2.8149            |
| 2.5             | 0.0082              | 2.4017            |
| 3               | 0.0245              | 1.9695            |
| 3.5             | 0.0612              | 1.5449            |
| 4               | 0.1261              | 1.1449            |
| 4.5             | 0.2214              | 0.7675            |
| 5               | 0.3349              | 0.4263            |
| 5.5             | 0.4589              | 0.1031            |

According to the code for the design of concrete structures, the target reliability index for this stage is 2.65, and the corresponding failure probability is 0.004. According to the calculation results of the program, it is obtained that the crack reaches the limit requirement after 2.5 years of continuous use after cracking. That is, the maximum crack width control life is  $t_3 = 2.5$  a.



**Figure 6** The change curve of failure probability under the crack width criterion



**Figure 7** Reliability Index Change Curve under the Crack Width Criterion

In summary, the railway tunnel lining structure's service life is calculated using the time-varying reliability theory based on the durability criterion. The theoretical value of the durability service life of the tunnel lining structure is as follows:

$$T_d = t_1 + t_2 + t_3 = 60 + 15 + 2.5 = 77.5 \text{ years}$$

#### 4 PREDICTION OF SERVICE LIFE BASED ON LIMIT CRITERION OF BEARING CAPACITY

##### 4.1 Life Composition of Lining Structure under Bearing Capacity Criterion

Due to environmental erosion, the concrete and steel of the tunnel structure may degrade, the strength of the material itself gradually decreases, and there is a continuous reduction in the ultimate bearing capacity of the structure. When the ultimate bearing capacity is reduced to the ultimate value, it is possible to judge that the structure has entered the ultimate life state of the bearing capacity by this point. This process is the service life  $t_d$  of the structure under the condition of bearing capacity life criterion;  $t_d$  can be roughly divided into two phases:

(1) The time when the depth of carbonization reaches the critical value, e.g., 3.1, expressed as  $t_1$ .

(2) Corrosion of rebar leads to a continuous decrease of ultimate bearing capacity until the lower limit value is reached, expressed as  $t_4$ . Under this criterion, the service life of lining structure  $t_d$  can be expressed as:

$$T_d = t_1 + t_4 \quad (14)$$

where  $t_4$  indicates the attenuation life of the reinforcement bearing capacity.

##### 4.2 Calculation of the Ultimate Bearing Capacity Control Life

###### 4.2.1 Analysis of Resistance R

The time-varying model of structural resistance is often expressed as the product of initial resistance and attenuation function:

$$R(t) = R_0\varphi(t) \quad (15)$$

where  $R_0$  is the initial value of resistance,  $\varphi(t)$  is deterministic attenuation function.

$$R_N(t) = \left( R_a b - \frac{a_s(t)a_c(t)BA_g}{h_{0c}} \right)$$

$$\left\{ \frac{\left[ R_a b(e - h_{0c}) - \frac{Ba_s(t)a_c(t)A_g e}{h_{0c}} \right]^2 + \sqrt{2R_a b(a_s(t)a_c(t)A_g e C + a_s(t)R_{g1}A_{g1}e_1)}}{R_a b} \right\} \quad (16)$$

$$\left\{ \frac{\left[ R_a b(e - h_{0c}) - \frac{Ba_s(t)a_c(t)A_g e}{h_{0c}} \right]}{R_a b} \right\}$$

$$+ a_s(t)R_{g1}A_{g1} - a_s(t)a_c(t)CA_g$$

where,  $a_s(t)$  is the reinforcement strength and sectional area reduction factor,  $a_c(t)$  is the coefficient of cooperation between reinforcement and concrete,  $R_a$  is the ultimate compressive strength of concrete,  $h_{0c}$  is the effective height of concrete cross section after damage,  $A_g$  and  $A_{g1}$  is the cross-section area of the tensile steel bar and the compressed steel bar respectively,  $e$  and  $e_1$  are the distance from the point of axial force to the point of tension and compression of steel bar,  $R_{g1}$  is the calculated compressive strength of steel bars. Under the action of surrounding rock pressure, the tunnel lining structure mainly displays the state of slight eccentricity compression. The damage coefficient is introduced to reflect the effects of steel bar section loss and strength reduction, concrete cross-section damage, and decreased bond force between reinforcement and concrete caused by steel corrosion. According to the literature [19], the expression for structural resistance is:

The above parameters are assumed to be the normal distribution.  $a_s(t)$  and  $a_c(t)$  are time-dependent random processes, the specific parameters and expressions are presented in the literature [20]. The other three parameters are determinative values, which are  $b = 1000$  mm,  $B = -1309$  MPa,  $C = 1047$  MPa.

(1)  $a_s(t)$ : Reinforcement strength and section area reduction factor after corrosion:

(2)

$$a_s(t) = 1 - 1.007\eta_s(t) \tag{17}$$

$$\eta_s(t) = 1 - \left(1 - \frac{2\delta(t)}{d}\right)^2 \tag{18}$$

where  $\eta_s(t)$  is the loss rate of rebar section at some point,  $\delta(t)$  is the depth of steel corrosion at some point. Eq. (8) is used before cracking, after cracking:

$$\delta(t) = \delta_{cr} + \lambda_{e1}(3.5 - 200\lambda_{e1})(t - t_1 - t_2) \tag{19}$$

(3)  $a_c(t)$ : Co-working coefficient of steel bar and concrete after corrosion.

(4) When there is no rust expansion crack in the protective layer ( $\delta(t) < \delta_{cr}$ ):

$$ac(t) = 1 \tag{20}$$

when  $\delta_{cr} < \delta(t) \leq 0.3$ :

$$a_c(t) = 1 - 0.85[\delta(t) - \delta_{cr}] \tag{21}$$

when  $\delta(t) > 0.3$ :

$$a_c(t) = 0.745 + 0.7\delta_{cr} \tag{22}$$

#### 4.2.2 Statistical Characteristics of Action Effects S

In the calculation of load effect, the parameter variables, such as the mechanical properties of surrounding rock and supporting material, the geometric size, and the calculation mode of the structure, can be regarded as random variables. The load effect analysis model of tunnel

lining structure is established, and the internal force of the lining structure is analyzed by PDS, a particular module for reliability calculation in Ansys software. Then the mean and variance of cross-section bending moment  $M_0$  and axial force  $N_0$  can be obtained.

#### 4.2.3 Equation of Limit State Considering Resistance Attenuation

According to the analysis of resistance and action effect mentioned above, when the attenuation effect of resistance is considered, the reliability limit state equation of reinforced concrete lining structure can be expressed as follows:

$$Z = R_N(t) - S(T) = \left( R_a b - \frac{a_s(t)a_c(t)BA_g}{h_{0c}} \right) \cdot \left[ \frac{\left[ R_a b(e - h_{0c}) - \frac{Ba_s(t)a_c(t)A_g e}{h_{0c}} \right]^2 + 2R_a b(a_s(t)a_c(t)A_g e C + a_s(t)R_{g1}A_{g1}e_1)}{R_a b} \right]^{1/2} - \left[ \frac{R_a b(e - h_{0c}) - \frac{Ba_s(t)a_c(t)A_g e}{h_{0c}}}{R_a b} \right] + a_s(t)R_{g1}A_{g1} - a_s(t)a_c(t)CA_g - N_0 \tag{23}$$

The probability that the ultimate bearing capacity of the structure reaches the limit value, that is, the failure probability is:

$$P_{f4}(t) = P\{R_N(t) - S(T) < 0\} = \Phi(-\beta_4) \tag{24}$$

The average value of axial force  $N_0$  of the most unfavorable section is  $\mu_{N0} = 7700.25$  kN, and the coefficient of variation is  $V_{N0} = 0.0334$ . The statistical characteristics of random parameters are shown in Tab. 5. The calculation results are shown in Tab. 6, Fig. 8, and Fig. 9.

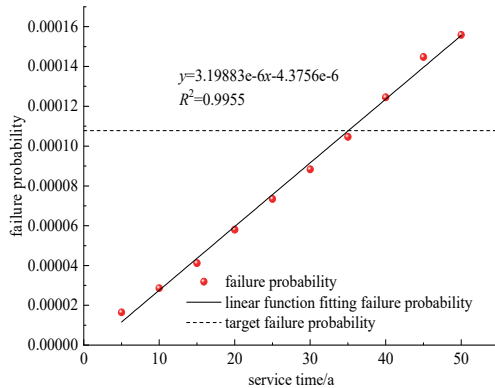
According to the results of Zhao's [21] research on the target reliability of railway tunnels, the failure of tunnel lining is a tensile failure at this stage. The target reliability index is 3.7, and the corresponding failure probability is  $1.078 \times 10^{-4}$ . According to the calculation results of the program, it is found that the ultimate bearing capacity reaches the limit requirement after the steel bar is used for 35 years, and the reliability index is reduced to 3.7. That is, the control life of the ultimate bearing capacity is  $t_4 = 35$  a.

Table 5 Statistical characteristics of random parameters

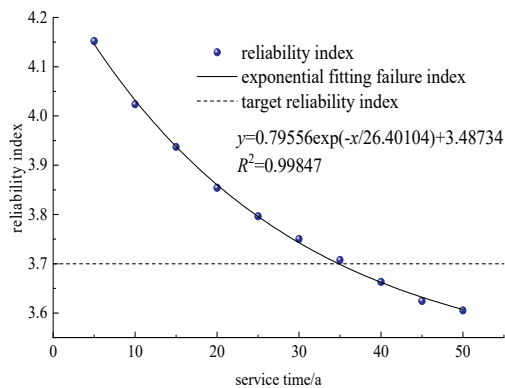
| Parameter              | Mean     | COV    | Distribution |
|------------------------|----------|--------|--------------|
| $A_g / \text{mm}^2$    | 3358     | 0.0030 | normal       |
| $A_{g1} / \text{mm}^2$ | 5859     | 0.0023 | normal       |
| $E / \text{mm}$        | 328.0493 | 0.0078 | normal       |
| $e_1 / \text{mm}$      | 201.9507 | 0.0100 | normal       |
| $R_a / \text{MPa}$     | 34.07    | 0.16   | normal       |
| $R_{g1} / \text{MPa}$  | 351.38   | 0.072  | normal       |
| $h_{0c} / \text{mm}$   | 400      | 0.0295 | normal       |
| $N_0 / \text{kN}$      | 7700.25  | 0.0334 | normal       |

**Table 6** Calculation results under bearing capacity criterion

| Duration of service after corrosion of steel bar, a | Failure probability | Reliability index |
|---|---------------------|-------------------|
| 5   | 1.6458 e -05        | 4.1523            |
| 10  | 2.8645 e -05        | 4.0237            |
| 15  | 4.1219 e -05        | 3.9372            |
| 20  | 5.8031 e -05        | 3.8543            |
| 25  | 7.3436 e -05        | 3.7963            |
| 30  | 8.8312 e -05        | 3.7503            |
| 35  | 1.0462 e -04        | 3.7076            |
| 40  | 1.2454 e -04        | 3.6632            |
| 45  | 1.4476 e -04        | 3.6245            |
| 50  | 1.5590 e -04        | 3.6053            |



**Figure 8** Change curve of structural failure probability under bearing capacity criterion



**Figure 9** Change Curve of Reliability Index under bearing capacity Criterion

In summary, the railway tunnel lining structure's service life is calculated using the time-varying reliability theory based on the ultimate bearing capacity criterion. The theoretical value of the service life of the tunnel lining structure is as follows:

$$T_d = t_1 + t_4 = 60 + 35 = 95 \text{ years}$$

### 5 DISCUSSION

Compare the structure's service life under two different service life criteria. It can be found that the predicted service life under the durability criterion is less than the predicted life under the condition of the bearing capacity limit, and the results accord with the actual engineering situation. For tunnel engineering, the regular service limit state meets the requirements of applicability and durability, and the ultimate state of bearing capacity is to meet the safety requirements. According to engineering experience, under applicable conditions, the bearing capacity can meet the needs of the safe state of the structure. Therefore, the

structure's safety (bearing capacity) service life is longer than its applicable service life (crack limit).

It should be pointed out that the object of this study is completely exposed to the erosion environment, and there are no protective measures, that is, to study according to the most unfavorable conditions. The calculated lifetime of the two structures is shorter than the service life of the structural design (100 years) and cannot meet the requirements in theory. However, in the course of structural service, a series of protective measures are often taken according to environmental conditions to improve the material's durability. Therefore, the tunnel lining structure's actual service life should be longer than this theory predicted.

### 6 CONCLUSION

In this paper, based on a railway passenger dedicated line tunnel, the time-varying reliability theory is used to predict the service life of the tunnel lining. The research conclusions are as follows:

(1) According to the durability criterion, the service life of the tunnel lining structure in the project example is 77.5 years. This life criterion mainly considers the variability of structural parameters and service environment. It is considered that concrete carbonation is the primary cause of reinforcement corrosion, and the structure mainly goes through three stages: carbonation, cracking of reinforcement rust and expansion, and cracks extending further to the limit value.

(2) According to the criterion of ultimate bearing capacity, the service life of the tunnel lining structure is 95 years. The primary purpose of this life criterion is to consider the attenuation of structural resistance after the steel bar begins to rust and to calculate the theoretical life of bearing capacity by the reliability index.

(3) In this study, the time-varying reliability theory is applied to the life prediction of service tunnels, and definite mathematical models are given according to the different failure modes of lining structures. The method sufficiently considers the influence of uncertainties on the service performance of the lining and can reflect the actual condition of the lining more realistically. The study provides a new research method for structural life prediction and can also provide a reference for tunnel management and maintenance.

(4) This study has not yet considered the differences in the distribution of tunnel operating environment characteristics over the length of the tunnel. In addition, factors such as groundwater and train vibration loads may also affect the durability of the lining. Under the influence of multiple factors, the deterioration mechanism of the serving lining will be more complicated. The above issues need to be further refined in the subsequent studies.

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