SENSITIVITY OPTIMIZATION METHOD FOR 4TH ORDER CBQ STRUCTURE LP FILTER

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Abstract:

This paper describes the design and optimization method for a 4th order low-pass filter designed as a cascade of biquarts and approximated by Butterworth. Schoeffler sensitivity, multiparameter sensitivity measurement, and Monte Carlo analysis were used for sensitivity measurement, filter tuning, and optimization confirmation. The constraints on the filter characteristics due to the Butterworth approximation are also presented, together with the constraints on optimization to obtain a narrow range of values for the filter elements. Three different types of optimization parameters were used to confirm the optimization method. Finally, a 4th order low pass filter with a cascade structure of biquarts was compared to a 4th order filter without feedback and confirmed to have lower sensitivity.

1 Introduction

Analog filter structures are widely used in signal processing devices such as biomedical sensor applications [1,2], high speed machines using active magnetic bearing system [3], high-precision vacuum microelectronic accelerometer [4], and more others (e.g. [5-7]). Moreover, these filters can be fabricated in CMOS technology [8-10], which gives them a compact design. However, due to the tolerances of the elements, the aging process, and various environmental factors, the filter structures may change their specifications. Such behaviour can lead to erroneous readings in signal processing devices or even make the system unstable [11]. To prevent the above effects, Schoeffler sensitivity is introduced as a reference measure for filter sensitivity optimization [12,13]. Such optimization can be used in signal processing devices [1-4] to make them more reliable in a demanding environment.

In this paper an optimization method will be applied on 4^{th} order low-pass (LP) filter structure designed as a cascade of biquarts (CBQ). Such design can be seen in Figure 1.

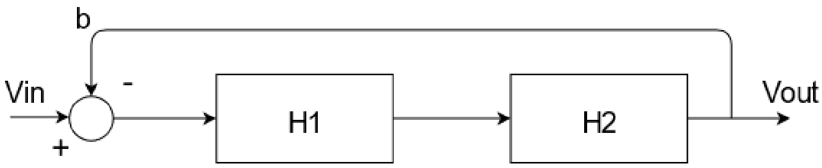

Figure 1. 4^{th} Order LP filter- block diagram of CBQ.

Compared to other existing optimization methods [12,13], described one is not much improved. Reason behind is more complex filter structure with closed loop system which seeks for different kind of filter tuning and higher computation power. Having that in mind, this article is oriented finding a way how to implement known method on new, more complex filter structure. However, in order to give directions how to update this method, new optimization parameter will be partially introduced. This parameter refers to minimum and

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maximum filter element value ratio, and it's briefly described later. The optimization method will consist of several steps. First, the Schoeffler sensitivity will be represented by a multiparameter sensitivity measure (M-parameter) [12,13]. Second, a diagram of the ratio between the minimum and maximum values of the filter elements will be shown, and finally, a Monte Carlo analysis will be performed to confirm the optimization method.

2 Mathematical model

2.1 Filter structure and its transfer function

Detail description of 4^{th} order LP filter follows. Filter consists of two 2^{nd} order LP filter with Sallen and Key (SAK) architecture forming a cascade of biquart structure- see Figure 2. Furthermore, filter is approximated according to Butterworth [11], so procedure of getting the transfer function of the filter can be seen in the equations below.

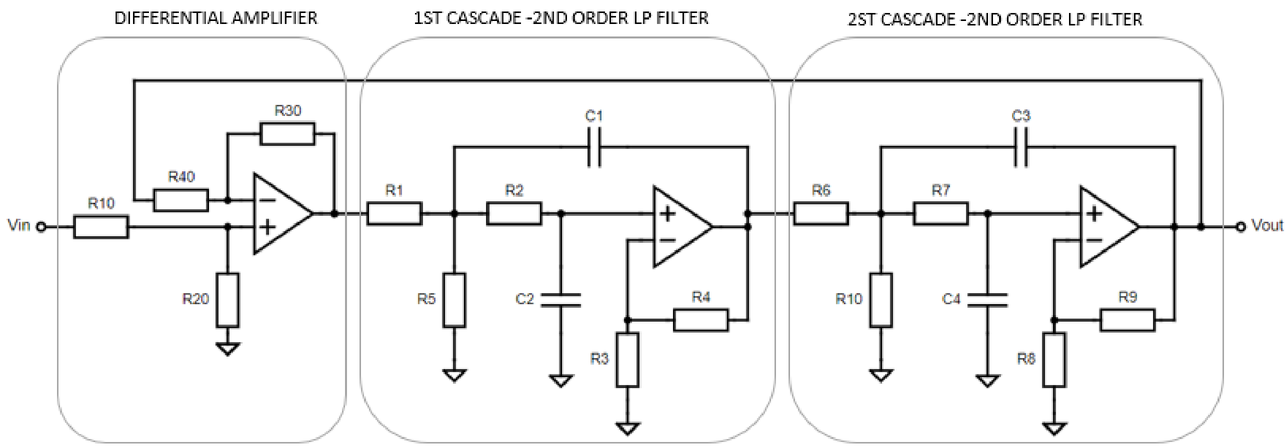

Figure 2. 4^{th} order LP filter showing CBQ structure and SAK architecture *of 2^{nd} order LP filters.*

$$H_{CL}(s) = \frac{\alpha \cdot H_{OL}(s)}{1 + \beta \cdot H_{OL}(s)} \tag{1}$$

$$H_{CL}(s) = \frac{1}{s^2 + 1.847759s + 1} \cdot \frac{1}{s^2 + 0.7653668s + 1} \tag{2}$$

$$H_{CL}(s) = \frac{1}{s^4 + 2{,}6131258s^3 + 3{,}414213s^2 + 2{,}6136258s + 1} \tag{3}$$

$$H_{OL}(s) = H_1(s) \cdot H_2(s) \tag{4}$$

$$H_1(s) = K_1 \frac{{\omega_1}^2}{s^2 + \frac{\omega_1}{Q_1}s + {\omega_1}^2} \tag{5}$$

$$H_2(s) = K_2 \frac{{\omega_2}^2}{s^2 + \frac{\omega_2}{Q_2}s + {\omega_2}^2} \tag{6}$$

where:

H_{CL} = transfer function of closed loop system
H_{OL} = transfer function of open loop system
α, β = closed loop gain
H_1, H_2 = transfer function of 1^{st} and 2^{nd} cascade (2^{nd}order LP filters)
ω_1, ω_2 = filter poles frequencyes
K_1, K_2 = gain of 1^{st} and 2^{nd} cascade (2^{nd}order LP filters)
Q_1, Q_2 = filter poles quality factors
s = transfer function operator

In (1), equations (4)-(6) are inserted:

$$H_{CL}(s) = \frac{a}{s^4 + b\,s^3 + c\,s^2 + d\,s + e} \tag{7}$$

where:

$$a = \alpha K_1 K_2 \omega_1 \omega_2 \tag{8}$$

$$b = \frac{\omega_2}{Q_2} + \frac{\omega_1}{Q_1} \tag{9}$$

$$c = {\omega_2}^2 + \frac{\omega_1 \omega_2}{Q_1 Q_2} + {\omega_1}^2 \tag{10}$$

$$d = \frac{\omega_1 {\omega_2}^2}{Q_1} + \frac{{\omega_1}^2 \omega_2}{Q_2} \tag{11}$$

$$e = {\omega_1}^2 {\omega_2}^2 + \beta K_1 K_2 {\omega_1}^2 {\omega_2}^2 \tag{12}$$

2.2 Filter element values calculations

After (7) is compared to (3), system of three equation with four variables is lined,

$$\frac{\omega_2}{Q_2} + \frac{\omega_1}{Q_1} = 2{,}6131258 \tag{13}$$

$${\omega_2}^2 + \frac{\omega_1 \cdot \omega_2}{Q_1 \cdot Q_2} + {\omega_1}^2 = 3{,}414213 \tag{14}$$

$$\frac{\omega_1 \cdot {\omega_2}^2}{Q_1} + \frac{{\omega_1}^2 \cdot \omega_2}{Q_2} = 2{,}6131258 \tag{15}$$

thus, enabling one variable to be defined as an arbitrary value. Usually, Q_2 is set to be equal to Q_1 in order to make easier calculations, but in this case this is not possible since there are no real solutions for that condition. Hence, variable Q_2 is chosen to be 0,5, so the solution returns all variables as positive real values.

Q_2=0,5- arbitrary
ω_1=1,06706
ω_2=0,85749
Q_1=1,18809

Since sections ‘a’ and ‘e’ in (7) are both equal to ‘1’ if compared to (3), following equation can be written:

$$\omega_1{}^2\omega_2{}^2 + \beta K_1 K_2 \omega_1{}^2\omega_2{}^2 = \alpha K_1 K_2 \omega_1{}^2\omega_2{}^2 \tag{16}$$

If $K_1 = K_2$:

$$\alpha = \frac{\beta}{1 - \omega_1{}^2 \cdot \omega_2{}^2} \tag{17}$$

$$K_1 = K_2 = \sqrt{\frac{1}{\alpha - \beta}}, \tag{18}$$

$$\alpha > \beta \tag{19}$$

Values of parameters K_1, K_2, α and β will be defined later, since there are some limitations. After defining equations for calculation variables $\omega_1, \omega_2, Q_1, Q_2, K_1, K_2, \alpha, \beta$, transfer function with electrical elements can be introduced in order to find their values. According to Figure 2 following equations are written, starting with 1st cascade:

$$H_1 = \frac{\left(1 + \frac{G_3}{G_4}\right)\frac{G_1 G_2}{C_1 C_2}}{s^2 + s\left(\frac{G_{11}}{C_1} + \frac{G_2}{C_1} - \frac{G_2 G_3}{C_2 G_4}\right) + \frac{G_{11} G_2}{C_1 C_2}} \tag{20}$$

where

$$G_{11} = G_1 + G_5 \tag{21}$$

and for 2nd cascade:

$$H_2 = \frac{\left(1 + \frac{G_8}{G_9}\right)\frac{G_6 G_7}{C_3 C_4}}{s^2 + s\left(\frac{G_{22}}{C_3} + \frac{G_7}{C_3} - \frac{G_7 G_8}{C_4 G_9}\right) + \frac{G_{22} G_7}{C_3 C_4}} \tag{22}$$

where

$$G_{22} = G_6 + G_{10} \tag{23}$$

and C_{xx} and G_{xx} are capacitances and conductances from Figure 2. Differential amplifier equation follows:

$$\alpha = \frac{R_{20}}{R_{10} + R_{20}} \cdot \left(1 + \frac{R_{30}}{R_{40}}\right) \tag{24}$$

$$\beta = \frac{R_{30}}{R_{40}} \tag{25}$$

with elements R_{xx} as resistances from Figure 2. Equations (20) and (22) are then compared to (5) and (6) and solved using (17) and (18), which is giving following results:

$$R_{20} = 1 - \text{arbitrary} \tag{26}$$

$$R_{10} = R_{20}\left(\frac{1+\beta}{\alpha} - 1\right) \tag{27}$$

$$R_{40} = 1 - \text{arbitrary} \tag{28}$$

$$R_{30} = \beta \cdot R_{40} \tag{29}$$

Where, in order to have positive value of R_{10}:

$$\frac{1+\beta}{\alpha} - 1 > 0 \tag{30}$$

Follows:

$$\beta < \frac{1}{{\omega_1}^2{\omega_2}^2} - 1 \tag{31}$$

Now, values of parameters K_1, K_2, α and β can be defined according to (17) and (18).

$$\beta = 0.184415 - arbitrary$$
$$\alpha = 1.133$$
$$K_1 = K_2 = 1{,}026754$$

Finally, differential amplifier elements values can be seen in Table 1. While on the way to calculate all elements values, new parameters are introduced, and it will serve to optimize the filter. Those parameters k_1 (and k_2) will define ratio between capacitors C_1 and C_2 values (and between C_{12} and C_{22}). Now, 1st section and 2nd section elements values can be defined, and written altogether in Table 1. Additionally, when setting values of k_1 and k_2, limitations from Table 1 need to be taken into account:

$$R_1 > R_2 \tag{32}$$

$$2 \cdot k_1 > \frac{\sqrt{k_1}}{Q_1} \tag{33}$$

$$R_6 > R_7 \tag{34}$$

$$2 \cdot k_2 > \frac{\sqrt{k_2}}{Q_2} \tag{35}$$

Which results with:

$$k_1 > 0.2029$$

$$k_2 > 1.0265$$

Table 1. Elements values of simple design filter.

Element	Value (per unit)	Element	Value (per unit)
R_{10}	0.04539	R_1	$=\frac{R_2}{K_1}\left(1+\frac{R_4}{R_3}\right)$
R_{20}	0.18441	R_5	$\frac{R_1 R_2}{R_1-R_2}$
R_{30}	1	R_9	1
R_{40}	1	C_3	1
R_4	1	C_4	$=k_2\cdot C_3$
C_1	1	R_7	$=\frac{1}{\omega_2 C_3\sqrt{k_2}}$
C_2	$=k_1\cdot C_1$	R_8	$=\frac{R_9}{2k_2-\frac{\sqrt{k_2}}{Q_2}}$
R_2	$=\frac{1}{\omega_1 C_1\sqrt{k_1}}$	R_6	$=\frac{R_7}{K_2}\left(1+\frac{R_9}{R_8}\right)$
R_3	$=\frac{R_4}{2k_1-\frac{\sqrt{k_1}}{Q_1}}$	R_{10}	$\frac{R_6 R_7}{R_6-R_7}$

3 Simulations

3.1 Simple design

Simulations of 4th order low pass filters are done in Matlab, along with all other needed calculations. As a confirmation that filter parameters have been chosen properly, a transfer function of closed loop system is shown on Figure 3. Besides that, transfer functions of cascades and open loop system is also printed on figure 3, defined with legend in upper right corner. Additional, one of main parameter of transfer function is cut-off frequency, and it’s depicted on figure 4. It can be seen that -3dB gain corresponds to cut-off frequency of 3 kHz.

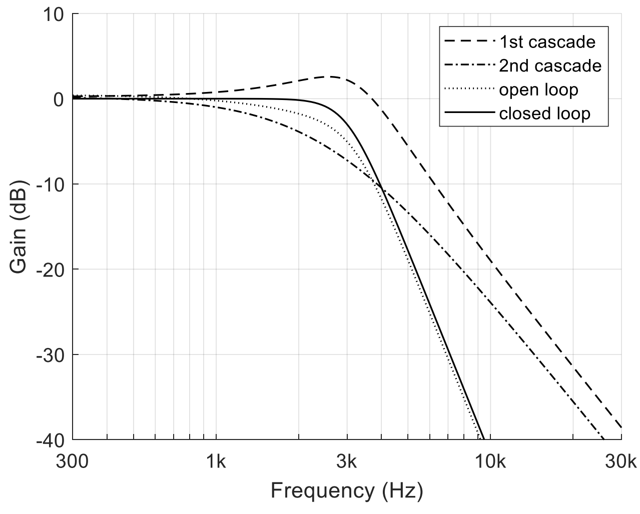

Figure 3. Transfer function of 4th order LP filter.

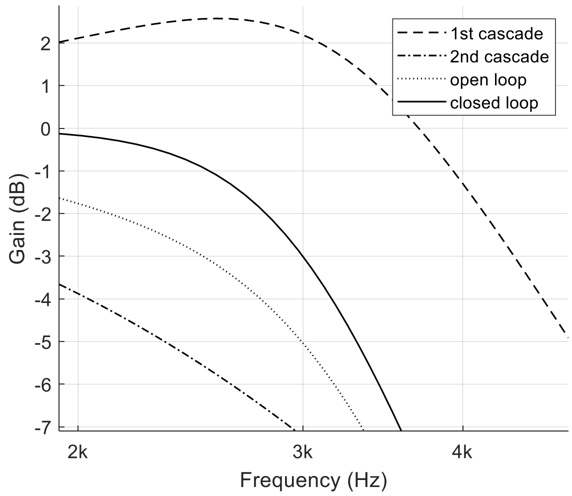

Figure 4. Detail preview of 4th order LP filter cut-off frequency.

As mentioned in the introduction, Schoeffler sensitivity is used as a measure of filter sensitivity. Moreover, reference value for optimization method is chosen to be a simple design filter [11], where $k_1 = 1$ and $k_2 = 2$, shown in Figure 5.

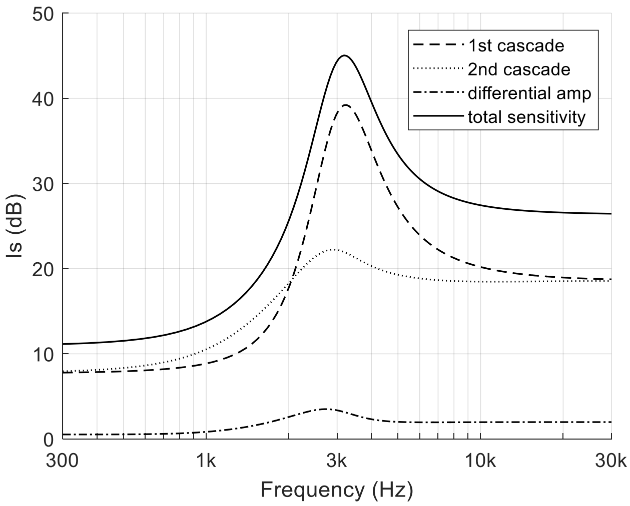

Figure 5. Schoeffler sensitivity of simple design filter.

3.2 Optimization method

Along with Schoeffler sensitivity, multi-parameter sensitivity measure is adopted in order to enable the optimization method used for reducing filter sensitivity. Basically, optimization method is an iteration process where, for different values of parameters k_1 and k_2, multi-parameter is recorded [12,13]. Result of such a process can be seen on Figure 6.

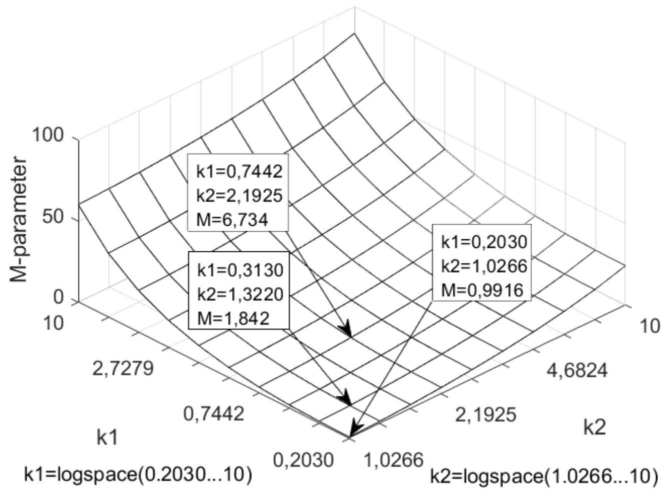

Figure 6. M-parameter results.

The obvious conclusion from Figure 6 is that the point with k_1=0.2035 and k_2=1.0266 gives the best optimization result (M=0.9916), shown in Figure 7, but sometimes it is not enough to care about only one parameter. When designing a filter, the designer must be aware of the constraints imposed by the real conditions under which the filter must operate. One such constraint is the values of the filter elements. If the values of the filter elements are too large or too small, various problems may occur. Therefore, it is necessary to choose an optimal solution where the element values do not differ too much. To find this solution, the ratio between the maximum and minimum element values of the filter are shown in Figures 8 and 9. This ratio is mostly introduced in order to see that k1 and k2 parameter values have double nature impact on system (reducing sensitivity while increasing discrepancies between element values). In ideal filter where no parasitic elements are included, there is no impact on system. But if system where stray capacitance is included, there could be potential problems with optimization method based exclusively on parameters k1 and k2. Having that in mind, introduced ratio could be potential leading optimization parameter in non ideal LP filter.

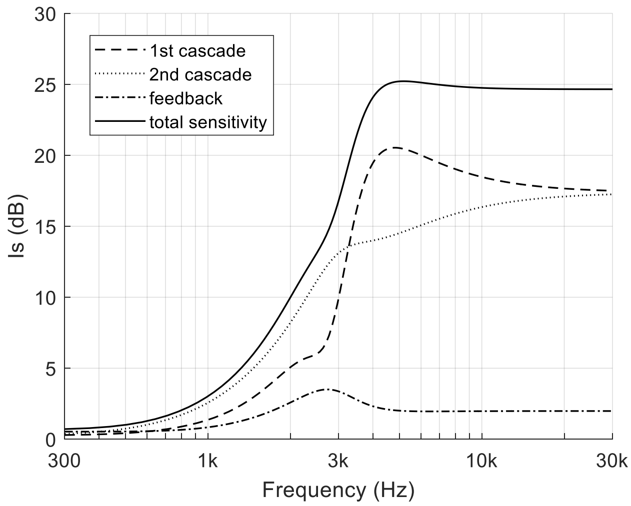

Figure 7. Schoeffler sensitivity- minimum of sensitivity.

Having in mind both Figure 6 and Figure 8 (and 9 respectively), optimal solution is chosen, with parameters $k_1 = 0.3130$, $k_2 = 1.3220$, where M=1.842 and ratio between max. and min. element value is $9 \cdot 10^{11}$. Schoeffler sensitivity for optimized filter is shown in Figure 10, along with denormed elements values in Table 2.

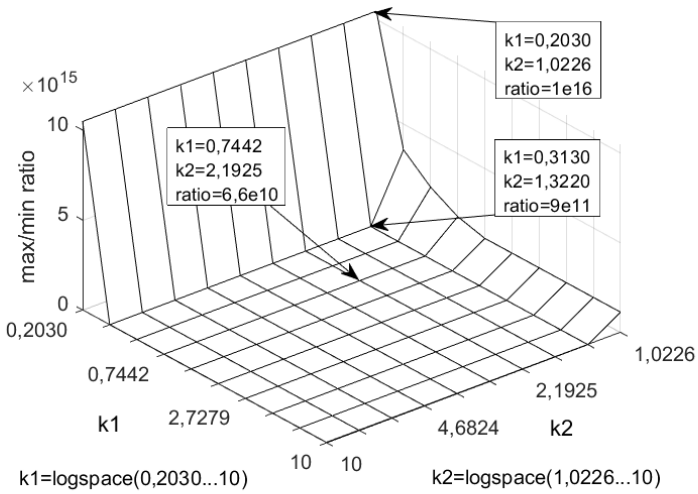

Figure 8. Maximum/minimum element value ratio.

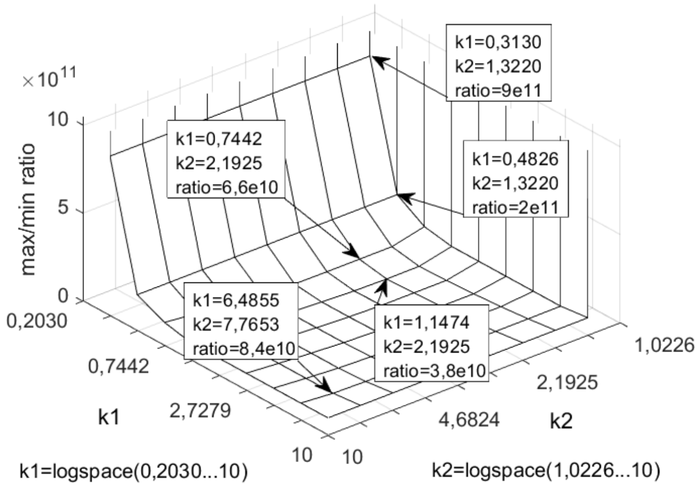

Figure 9. Maximum/minimum element value ratio- detail view.

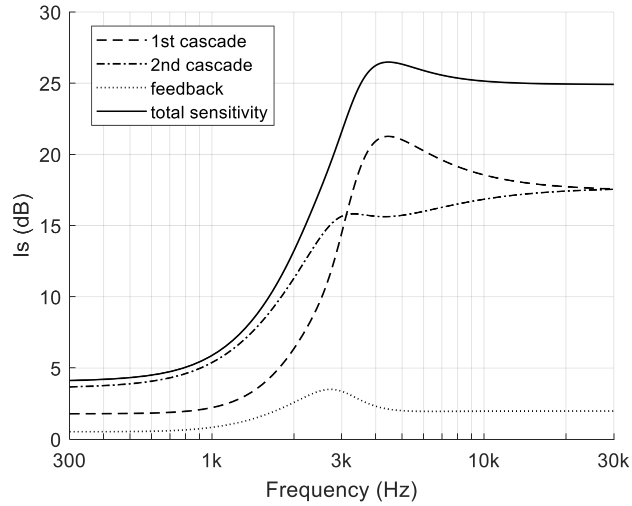

Figure 10. Schoeffler sensitivity of optimized filter.

Before moving forward, additional graph is shown on Figure 11, where multiple Schoeffler sensitivity curves can be seen for better visualization of optimization process. As parameters k_1 and k_2 values are getting higher, filter sensitivity arises (in intermediate amount on low frequencies, but scientifically around cut-off frequency), which is in alignment with Figure 6.

Table 2. Denormalized element values (R_0=1 kΩ, C_0=1 nF) of optimized filter.

Element	Value (denormalized)	Element	Value (denormalized)
R_{10}	45.3989 Ω	R_1	1884.49 Ω
R_{20}	1000 Ω	R_5	15075.03 Ω
R_{30}	184.415 Ω	R_9	1000 Ω
R_{40}	1000 Ω	C_3	53.0516 nF
R_4	1000 Ω	C_4	70.1343 nF
C_1	53.0516 nF	R_7	1014.27 Ω
C_2	16.6051 nF	R_8	2903.30 Ω
R_2	1675.09 Ω	R_6	1328.08 Ω
R_3	6447.18 Ω	R_{10}	4292.41 Ω

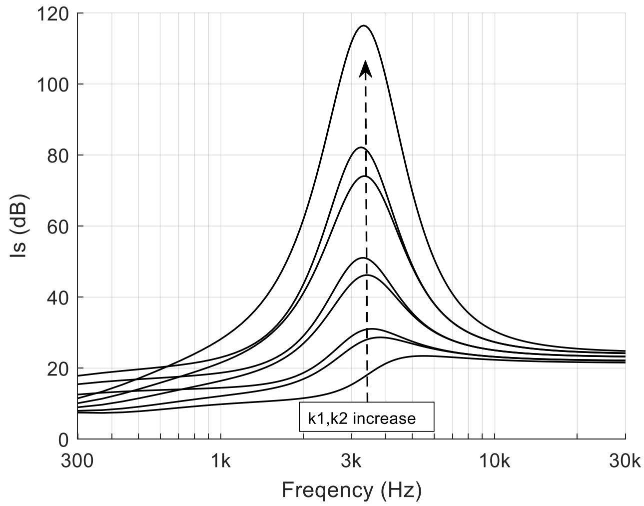

Figure 11. Schoeffler sensitivity for various values of k_1 and k_2 parameters.

To conclude the simulations, a Monte Carlo analysis is performed and shown in Figure 12: for the simple design, the optimized filter, and the minimum filter sensitivity. To facilitate comparison, Figure 13 is also shown, which contains Schoeffler sensitivity plots for the simple design, the optimized filter, and the minimum filter sensitivity solution.

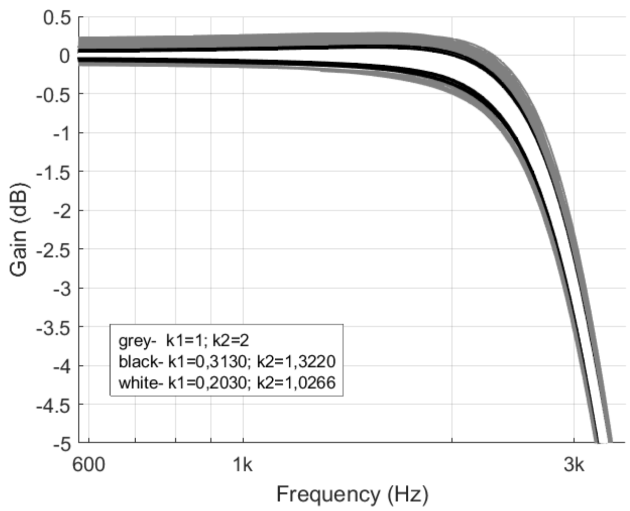

Figure 12. Monte Carlo analysis: element tolerances=1%, 100 repeats for each curve.

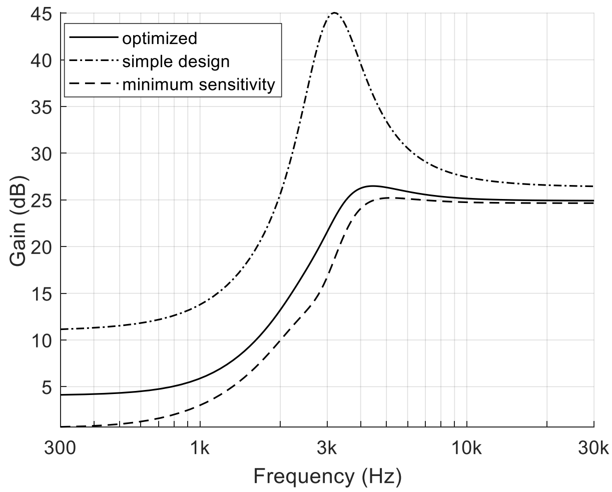

Figure 13. Schoeffler sensitivity of optimized filter, simple design and minimum sensitivity solution.

4 Conclusion

In this paper, the Schoeffler sensitivity optimization method of a 4th order filter LP has been described. At the end of the paper, some conclusions can be drawn, which can be divided into three sections. The first section concerns the low frequency band of the filter. In this frequency range, the optimized filter has a minimum sensitivity, which is important so that the signal does not experience distortion when passing through the filter. As the cutoff frequency band is crossed, the sensitivity can be significantly reduced by optimizing the filter - see Figure 13. In this way, stabilization of the cutoff frequency can be achieved, and the Monte Carlo analysis in Figure 12 confirms this. As can be seen in the same figure, the curve of the optimized filter is thinner than that of the non-optimized filter (simple design), so the transfer function cannot change. Finally, in the high frequency band of the filter, nothing changes dramatically. There is a small decrease in sensitivity when the filter is optimized, but due to the high attenuation of the 4th order filter LP, the sensitivity will not affect the signal propagation. If depicted filter (4^{th} order LP filter with CBQ structure) is compared to filter form [12] (4^{th} order LP filter without feedback), it can be seen that better sensitivity performance is provided with CBQ structure. This is significantly expressed around cut-off frequency in Schoeffler sensitivity curve, with cca. 7dB decrease. Besides results comparison, article introduces new, potentially good variable for filter optimization when stray capacitance is included in filter model. This variable is ratio between minimum and maximum filter element value, and it could be a good opportunity for future work.

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