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Decentralized robust interval type-2 fuzzy model predictive control for Takagi–Sugeno large-scale systems

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ABSTRACT

Here, decentralized robust interval type-2 (IT2) fuzzy model predictive control (MPC) for Takagi–Sugeno (T-S) large-scale systems is studied. The large-scale system consists of many IT2 fuzzy T–S subsystems. Important necessities that limit the practical application of MPC are the online computational cost and burden of the frameworks. For MPC of T–S fuzzy large-scale systems, the online computational burden is even worse, and in some cases, they cannot be solved timely. Especially for severe, large-scale systems with disturbances, the MPC of T–S fuzzy large-scale systems usually give a conservative solution. So, researchers have many challenges and in finding a reasonable solution in a short time. Although more comfortable results can be achieved by the proposed fuzzy MPC approach, which adopts T–S large-scale systems with nonlinear subsystems, many restrictions are not considered. In this paper, challenges are solved, and the MPC is designed for a nonlinear IT2 fuzzy large-scale system with uncertainties and disturbances. Besides, the online optimization problem is solved, and results are proposed. Consequently, the online computational cost of the optimization problem is reduced considerably. Finally, the effectiveness of the proposed algorithm is illustrated with two practical examples.

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Large-scale systems; Interval type-2 fuzzy Takagi–Sugeno systems; Model predictive control

1. Introduction

Several years ago, the main task in engineering was to control processes such as mechanical engineering, electrical engineering, chemical engineering etc. Researchers have proposed many algorithms and approaches to solve the instability of systems. Systems had been turned very large in scope and dynamic. Consequently, the control of the process has become an essential task. Various controllers have been designed to encounter the instability of systems in industry and academics such as adaptive and fuzzy control [12]. To design an effective and authentic controller, it is an important step to identify the dynamics of the system precisely. In large-scale systems, it is almost impossible to determine the dynamics of the system precisely. Hence, the well-known fuzzy logic method is used. A useful controller for large-scale systems that can control the process is model predictive control (MPC). Because MPC uses a cost function to compute the input vector, it has been practical and popular for many years.

Today, almost all real systems investigated in industry and academic are large scale and have a distinct appeal for decades [34]. Besides, many control systems have become prominent and complex in computation [5]. These systems are constructed by several independent subsystems that work independently with some known and unknown interactions. Lots of research studies conducted in large-scale systems have received much attention [67]. Recently, fuzzy systems with IF–THEN rules have become more prevalent, and most of the nonlinear and complex systems are modelled by fuzzy logic method [89]. One of the most powerful approaches to fill the gap between linear and severe nonlinear systems is the well-known Takagi–Sugeno fuzzy model. Many investigations have been done based on the T–S fuzzy model [1011]. Zamani and Zarif [12] and Bahrami et al. [13] propose a fuzzy inference system popular Takagi–Sugeno fuzzy model. In [14], Takagi–Sugeno fuzzy model represents the dynamic of the unknown nonlinear system. But in the previous research studies, uncertainties were not involved in membership functions for the type-1 fuzzy set. So the control problems for nonlinear plants subject to uncertainties are not handled successfully. If uncertainties of nonlinear plants result in grades of membership uncertainties functions, conditions based on Takagi–Sugeno type-1 might be directed to the conservativeness. Here, the interval type-2 (IT2) fuzzy Takagi–Sugeno is used to fix uncertainties captured by IT2 membership functions [2,15]. In fact, IT2 has many merits in handling the grades of membership uncertainties over type-1. Applicable IT2 has been used widely in control algorithms such as [1617], in which a nonlinear network control system has been modelled

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by IT2 for the synthesizing approach of dynamic output feedback MPC. In [18], the adaptive sliding mode control problem is introduced for the uncertain nonlinear system modelled by IT2 Takagi–Sugeno fuzzy model.

Recently, MPC has become popular as a reliable control approach. By properly using the system model to predict the output response, MPC methods allow us to choose the optimal control action that minimizes the desired cost function. In many applications, MPC has been a viable alternative to many other classic schemes. For such applications, the advantages of MPC are higher dynamic performance, a multivariable controller design, the constraints on input and output variables, and the possibility of including nonlinearities in the model and constraints. Although the need for an accurate model may be a drawback in some applications, it is wondered how reliable models of systems are usually available for control design [1920]. The gist of the MPC is an optimal control sequence that is computed by minimizing a finite horizon cost function at each sampling time. Although, lots of research studies have been done for many years without considering the uncertainties and disturbances [21], both uncertainties and disturbances in real systems may lead to inconstancy. So, using the robust MPC and the robust H_∞ strategy, we try to diminish the effect of disturbances and uncertainties [1922]. Many works have been studied in a nonlinear robust MPC, e.g. [2324]. In [25], both online and offline robust fuzzy MPCs with structured uncertainties and persistent disturbances are investigated for usual systems. In [19], robust fuzzy MPC with nonlinear local models is introduced for which separate controllers are proposed for nonlinear and linear states of the system. The integration of photovoltaics into distribution power systems with grid fault ride-through capability is investigated by proposing a robust MPC scheme in [26]. After that, many studies have been improved as found in [27]. An important avenue to MPC is based on the linear matrix inequalities (LMIs) technique is proposed in [28] for linear parameter varying (LPV) systems. In [29], a new MPC is made for polytypic LPV systems and a paradigm is adopted in gain scheduling.

Obviously, it seems that MPC for IT2 fuzzy large-scale systems with persistent disturbances have not been studied, yet several problems remained unsolved. Designing the MPC for the fuzzy large-scale systems with nonlinear and complex dynamic, considering the uncertainties and disturbances, has become a real challenge and almost in all systems, it is impossible to obtain a non-conservative solution for solving the online optimization problem. Consequently, by the proposed method, this challenge is solved, and the MPC is designed for a nonlinear fuzzy large-scale system with uncertainties and disturbances. Besides, the online optimization problem is solved, and the results are illustrated. So, this paper contributes the following: (1) an optimal control law is obtained at each sampling instant solving an online optimization problem, (2) proposing the IT2 fuzzy MPC for the discrete nonlinear large-scale system, (3) more relaxed and robust results are possible by the proposed method, (4) using the IT2 fuzzy Takagi–Sugeno model for large-scale systems, (5) using H_∞ performance to encounter the disturbance.

The rest of this paper is constructed as follows: in Section 2 some preliminay information about robust IT2 fuzzy MPC of Takagi–Sugeno systems is introduced. In Section 3, robust positive invariance (RPI) and the computation of terminal constraint set for nonlinear model-based fuzzy systems are provided. In Section 4, a numerical example is offered. Finally, the concluding remarks are given in Section 5.

2. Preliminaries

First, the input-to-state stability (ISS) is presented. Second, the system description, IT2 fuzzy Takagi–Sugeno large-scale system and the MPC are proposed. Furthermore, the prediction model, cost function and RPI set are introduced.

2.1. Useful definitions and lemmas

ISS [30] is a stability notion broadly used to assess the stability of nonlinear control systems with external inputs. In the following, some practical definitions and lemmas are provided.

Definition 1: [19] *R, R_+, Z and Z_+ illustrate sets of real numbers, positive real numbers, integers and positive integers, respectively. $Z_{[m,n]}$ is the symbol of set of integers in the interval $[m, n]$ for convenience. x depicts the norm of vector $x_i \in R^c$. A real-valued scalar function $\kappa : R_+ \to R_+$ is an* M*-function ($\kappa \in$ M), if it is continuous, rigidly increasing and $\kappa(0) = 0$. So we can say, $\kappa \in \mathrm{M}_\infty$ if $\partial \in$ M and $\lim_{s\to\infty} \kappa(s) = \infty$. A function $\beta : R_+ \times R_+ \to R_+$ is an* ML*-function ($\beta \in$ ML), if for each fixed $s > 0, \beta(., s)$ is a* M*-function, and for each fixed $r > 0$, $\beta(r, .)$ rigidly decreases and $\beta(r, s) \to 0$ as $s \to \infty$.*

Definition 2: (*Input-to-state stability) A discrete-time nonlinear system $x(k + 1) = f(x(k), d(k))$ where d is the symbol of the disturbance vector, is input-to-state stable (ISS) if there are an* ML*-function β and an* M*-function γ such that for each input d, $\|x(k)\| \le \beta(x(0), k) + \gamma(\| d \|)$, where $x(0)$ is the initial state vector, d is the disturbance sequence $\{d(0), d(1), \ldots, d(k-1)\}$ and $k \in Z_+$.*

Definition 3: (*ISS-Lyapunov Function* [25]) *A continuous positive definite function $V(x(k))$ is called an ISS-Lyapunov function for the system $x(k + 1) = f(x(k), d(k))$, if there are M_∞–function $\kappa_1, \kappa_2, \kappa_3$, and* M*-function ρ such that*

$$\kappa_1(\| x(k) \|) \le V(x(k)) \le \kappa_2(\| x(k) \|)$$

$$V(x(k+1)) - V(x(k)) \le -\kappa_3(\| x(k) \|)+\rho(\| d(k) \|)$$

Lemma 1: *[30] If the system $x(k+1) = f(x(k), d(k))$ is acknowledged an ISS-Lyapunov function, then it is ISS.*

2.2. System description

The IT2 fuzzy large-scale system, which consists of N subsystems, is assumed. Here, each subsystem S_i can be designated by an IT2 fuzzy Takagi–Sugeno model as shown below:

$$S_i^l : \begin{cases} \text{IF } z_{i1} \text{ is } \tilde{F}_{i1}^l \text{ and } \dots \text{ and } z_{ig} \text{ is } \tilde{F}_{ig}^l \\ \text{THEN } x_i(k+1) = A_i^l x_i(k) + B_i^l u_i(k) \\ \quad + E_i^l d_i(k) + \sum\limits_{\substack{j=1 \\ i \ne j}}^{N} g_{ij} x_j(k) \end{cases}$$

where $i = 1, 2, \dots, N; l = 1, 2, \dots, r_i$, A_i^l, B_i^l and E_i^l are the system matrices and disturbances of rule-l in the subsystem S_i; $x_i(k) \in R^c$ the state vector, $u_i(k) \in R^n$ the input vector, $d_i(k) \in R^m$ the addition disturbance. r_i, g_{ij} and $F_{iq}^l (q = 1, 2, \dots, g)$ introduce a number of the fuzzy rules in the subsystem S_i, the interconnection between subsystems and the linguistic IT2 fuzzy sets of the rule l according to the function $z_i(k)$, respectively. $z_i(k) = [z_{i1}, z_{i2}, \dots, z_{ig}]$ are some measurable premise variables for the subsystem S_i. By using singleton fuzzifier, product fuzzy inference and centre-average defuzzifier, the IT2 fuzzy Takagi–Sugeno large-scale system (3) is

$$x_i^+ = \tilde{A}_{i\mu} x_i + \tilde{B}_{i\mu} u_i + E_{i\mu} d_i + \sum_{\substack{j=1 \\ i \ne j}}^{N} g_{ij} x_j, \ i = 1, 2, \dots, N \tag{4}$$

$$\tilde{A}_{i\mu} = \sum_{l=1}^{r_i} w_i^l(z_{iq}) A_i^l \, ; \ \tilde{B}_{i\mu} = \sum_{l=1}^{r_i} w_i^l(z_{iq}) B_i^l$$
$$E_{i\mu} = \sum_{l=1}^{r_i} w_i^l(z_{iq}) E_i^l \, ; \ g_{ij} = \sum_{l=1}^{r_i} w_i^l(z_{iq}) g_{ij} \tag{5}$$

in which $\underline{w}_i^l(z_{iq}(k)) = \prod_{q=1}^{g} \underline{\upsilon}_{\tilde{F}_{i\theta}^l}(z_{iq}(k)) \ge 0$ and $\bar{w}_i^l(z_{iq}(k)) = \prod_{q=1}^{g} \bar{\upsilon}_{\tilde{F}_{i\theta}^l}(z_{iq}(k)) \ge 0$ illustrate the lower and upper grades of membership functions, respectively. It is obvious that $\underline{\upsilon}_{\tilde{F}_{i\theta}^l}(z_{iq}(k)) \in [0, 1]$ and $\bar{\upsilon}_{\tilde{F}_{i\theta}^l}(z_{iq}(k)) \in [0, 1]$ are the lower and upper membership functions, respectively. And here, needless to say that, $\bar{\upsilon}_{\tilde{F}_{i\theta}^l}(z_{iq}(k)) \ge \underline{\upsilon}_{\tilde{F}_{i\theta}^l}(z_{iq}(k))$, therefore, $\bar{w}_i^l(z_{iq}(k)) \ge \underline{w}_i^l(z_{iq}(k))$.

Here, x_i^+ illustrates the ith system state in the next instant for simplicity. The following interval sets are the firing strengths of the rule:

$$W_i^l = [\bar{w}_i^l, \underline{w}_i^l]$$
$$w_i^l(z_{iq}) = \bar{w}_i^l(z_{iq}(k)) \bar{\rho}_i^l(x(k)) + \underline{w}_i^l(z_{iq}(k)) \underline{\rho}_i^l(x(k)) \tag{6}$$

The parameter uncertainties existing in the nonlinear plant can result in uncertainties of the membership functions and determine the lower and upper membership functions. We chose the lower and upper functions as nonlinear functions related to the state variables.

Where $\bar{\rho}_i^l(x(k)) \in [0, 1]$ and $\underline{\rho}_i^l(x(k)) \in [0, 1]$ are nonlinear functions and satisfy $\bar{\rho}_i^l(x(k)) + \underline{\rho}_i^l(x(k)) = 1$.

Remark 1 It is evident that the parameter uncertainties are in almost all nonlinear plants, and they can result in uncertainties in the membership functions and determine the lower and upper membership functions. In some investigations, $\bar{\rho}_i^l(x(k))$ and $\underline{\rho}_i^l(x(k))$ are the known and constant parameters, respectively [31]. But, in this paper the lower and upper functions are chosen as nonlinear functions, related to the state variables. Thus, for the stability analysis and design of the IT2, the lower and upper membership functions can be exerted.

The nonlinear fuzzy MPC with the schematic as shown in Figure 1, is as follows:

$$C_i^l : \begin{cases} \text{IF } z_{il} \text{ is } \tilde{G}_{i1}^l \text{ and } \dots \text{ and } z_{ig} \text{ is } \tilde{G}_{ig}^l \\ \text{THEN } u_i(k) = k_i^l x_i(k) \end{cases} \tag{7}$$

where $i = 1, 2, \dots, N; l = 1, 2, \dots, r_i$. For convenience, we use the same weight notation $w_i^l(z_{iq})$ as in the subsystem S_i. Analogous to (7), the final output of the controller for the corresponding subsystem S_i is

$$u_i(k) = \sum_{l=1}^{r_i} h_i^l(z_{iq}) k_i^l x_i(k) \tag{8}$$

in which $\underline{h}_i^l(z_{iq}(k)) = \prod_{q=1}^{g} \underline{\sigma}_{\tilde{G}_{i\theta}^l}(z_{iq}(k)) \ge 0$ and $\bar{h}_i^l(z_{iq}(k)) = \prod_{q=1}^{g} \bar{\sigma}_{\tilde{G}_{i\theta}^l}(z_{iq}(k)) \ge 0$ illustrate the lower and upper grades of membership functions, respectively. It is obvious that $\underline{\sigma}_{\tilde{G}_{i\theta}^l}(z_{iq}(k)) \in [0, 1]$ and $\bar{\sigma}_{\tilde{G}_{i\theta}^l}(z_{iq}(k)) \in [0, 1]$ are the lower and upper membership functions, respectively. And here, it is needless to say that $\bar{\sigma}_{\tilde{G}_{i\theta}^l}(z_{iq}(k)) \ge \underline{\sigma}_{\tilde{G}_{i\theta}^l}(z_{iq}(k))$, therefore, $\bar{h}_i^l(z_{iq}(k)) \ge \underline{h}_i^l(z_{iq}(k))$. The following interval sets are the firing strengths of the rule:

$$h_i^l(z_{iq}) = \frac{\bar{\mu}_i^l(z_{iq}(k)) \bar{h}_i^l(x(k)) + \underline{\mu}_i^l(z_{iq}(k)) \underline{h}_i^l(x(k))}{\sum_{l=1}^{r_i} \bar{\mu}_i^l(z_{iq}(k)) h_i^l(x(k)) + \underline{\mu}_i^l(z_{iq}(k)) \underline{h}_i^l(x(k))},$$
$$H_i^l = [\bar{h}_i^l, \underline{h}_i^l] \tag{9}$$

where $\bar{\mu}_i^l(x(k)) \in [0, 1]$ and $\underline{\mu}_i^l(x(k)) \in [0, 1]$ are nonlinear functions and satisfy $\bar{\mu}_i^l(x(k)) + \underline{\mu}_i^l(x(k)) = 1$.

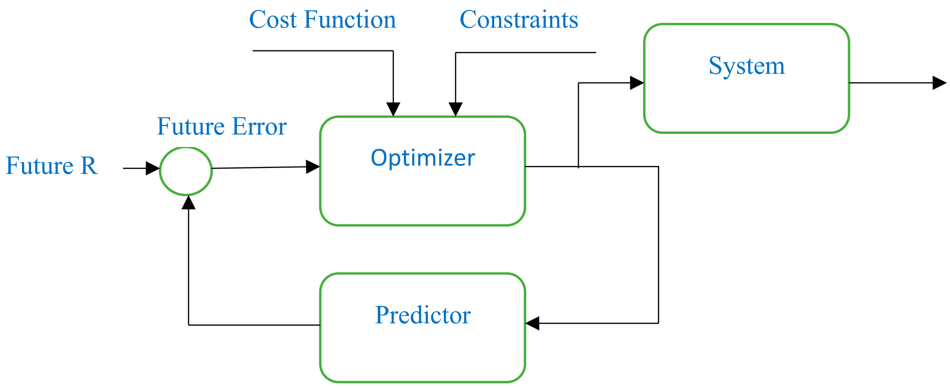


Figure 1. Schematic diagram of the control.

Remark 2 To diminish the conservativeness in designing the controller, it is better to choose the rational values of the weighting coefficients [32]. So, $\bar{\mu}_i^l(x(k))$ and $\underline{\mu}_i^l(x(k))$ are chosen as the nonlinear functions related to the state variables rather than the constant ones to diminish the conservativeness.

Here, combining (4) and (8), the closed-loop IT2 fuzzy subsystem will be

$$x_i(k+1) = \sum_{l=1}^{r_i}\sum_{m=1}^{r_i} w_i^l(z_{iq})h_i^m(z_{iq})([\tilde{A}_i^l + \tilde{B}_i^l k_i^m]x_i(k) + E_i^l d_i(k)) + \sum_{l=1}^{r_i}\sum_{\substack{j=1\\ i\neq j}}^{N} w_i^l(z_{iq})g_{ij}x_j(k) \tag{10}$$

2.3. Model predictive control

In this section, the prediction model of the system and a specified finite horizon cost function are introduced.

$$x_i(k+m+1|k) = \tilde{A}_{i\mu}x_i(k+m|k) + \tilde{B}_{i\mu}u_i(k+m|k) + E_{i\mu}d_i(k+m|k) + \sum_{\substack{j=1\\ i\neq j}}^{N} g_{ij}x_j(k+m|k) \tag{11}$$

and finite horizon cost function that must be minimized is

$$J(k) = \sum_{i=1}^{N} j_i(k) = \sum_{i=1}^{N}\left(\psi_i(K) + \sum_{m=0}^{T-1}\psi_i(k+m|k) + V_{in}(x_i(k+m|k))\right) \tag{12}$$

where $\psi_i(k+m|k)$ is the stage cost at the predicted time. $V_{in}(x_i(k+T|k))$ is called the terminal cost, with $V_{in}(\cdot)$ being a positive definite function [33], and the stage cost is selected as

$$\psi(k) = \sum_{i=1}^{N}\psi_i(k) == \sum_{i=1}^{N}(x_i^T(k+m|k)Qx_i(k+m|k) + u_i^T(k+m|k)Ru_i(k+m|k) - \tau_i d_i^T(k+m|k)d_i(k+m|k)) \tag{13}$$

where R and Q are fixed real matrices. By choosing rational values of the weighs, we can reduce the conservativeness. So, different values of weights are applied iteratively to get the best result, and τ_i is a positive scalar. As it is evident, the cost function entails the disturbance, and obviously, the cost function is impressed by the well-known H_∞ control [34]. Thus, it is not suitable to optimize the cost function directly, while the disturbance is involved. So, in that case, a min–max approach is chosen that minimizes the worst-case cost function [35]. At the end of the prediction, the states enter a terminal constraint set to be asymptotic stability. $x_i(k+T|k) \in \Omega_{iw}$ is the terminal constraint set. The online optimization problem can be illustrated as follows:

$$\min_{u_i(k+m|k)}\ \max_{d_i(k+m|k)} J_i(k), \tag{14}$$
$$\text{s.t. } u_i(k+m|k) \in U_i$$
$$d_i(k+m|k) \in D_i$$
$$x_i(k+m|k) \in \Omega_{iw}$$

and $d_i \in D_i := \{d_i | d_i^T d_i \le \eta_i{}^2\}$, $u_i \in U_i := \{u_i||u_{is}| \le u_{is.max}\}$ should be satisfied. Where $\eta_i{}^2$ is a positive scalar, u_{is} is the sth element of the inputs , $s \in Z_{[1,m]}$.

3. Main results

3.1. Robust positively invariant set for the interval type-2 fuzzy T–S large-scale systems

In this section, the robust positively invariant and constraint sets are defined. The trajectories of the system are bounded robustly and this feature is guaranteed by the controller, as mentioned previously. The RPI set is shown by Ω_{iw}, and $u_i(k)$ is the corresponding control law. The RPI set property is provided as $\forall x_i \in \Omega_{iw}, x_i^+ \in \Omega_{iw}$ for all permissible uncertainties and disturbances. Ω_{iw} is defined for the IT2 fuzzy T–S large-scale system (3) as follows.

$$\Omega_{iw} := \left\{ x_i \middle| \sum_{l=1}^{r_i} w_i^l(z_{iq}) x_i^{\mathrm{T}} P_{i\mu} x_i \le \xi_i \right\} \tag{15}$$

where $P_{i\mu} = \sum_{l=1}^{r_i} w_i^l(z_{iq})P_i$, and ξ_i are positive scalars and P_i is a positive constant matrix. The controller for IT2 fuzzy Takagi–Sugeno large-scale system is $u_i(k) = \sum_{l=1}^{r_i} h_i^l(z_{iq}) h_i^l x_i(k)$.

Lemma 2: *[25] The set Ω_{iw} is an RPI set if there is a positive scalar λ_i, $(0 < \lambda_i < 1)$, such that*

$$\sum_{i=1}^{N} \left\{ \left(\frac{1}{\xi_i} {x_i^+}^T P_{i\mu}^+ x_i^+ - \frac{1}{\xi_i} x_i^T P_{i\mu} x_i \right) - \lambda_i \left(\frac{1}{{\eta_i}^2} d_i^T d_i - \frac{1}{\xi_i} x_i^T P_{i\mu} x_i \right) \right\} \le 0 \tag{16}$$

with $P_{i\mu}^+ = \sum_{l=1}^{r_i} w_i^l(x_i^+)P_i$, *for all* $x_i^+ \in \tilde{A}_{i\mu}x_i + \tilde{B}_{i\mu}u_i + E_{i\mu}d_i + \sum_{\substack{j=1 \\ i \ne j}}^{N} g_{ij}x_j$, $u_i \in U_i$, *and* $d_i \in D_i$.

Based on Lemma 2 and the concept of Ω_{iw}, Theorem 1 is introduced to assure the trajectories of the system in the Ω_{iw} set.

Remark 3 Here, by proposing Theorem 1, if two LMIs are solved, two results are achieved. (1) It is proved that the considered large-scale fuzzy system is stable in terms of Lyapunov. (2) Controller gains are computed optimally in the restricted bound.

Theorem 1: *Consider the fuzzy system (3), if there are positive definite matrices X_i X_j and $X_{i\mu}$, and λ_i such that the following matrix inequalities are feasible*

$$\begin{bmatrix} E_{i\mu}^T X_i E_{i\mu} - \xi_i \lambda_i \mathrm{N}_i & \star & \star & \cdots & \star & \star \\ \tilde{\Theta}_i^T X_i E_{i\mu} & N\sqrt{a} \sum_{\substack{j=1 \\ i \ne j}}^{N} g_{ij}^T X_j g_{ij} - (-\lambda_i + 1) X_{i\mu} & \star & \cdots & \star & \star \\ g_{ij}^T X_i E_{i\mu} & \left(1 - \sqrt{\alpha}\right) g_{ij}^T X_i \tilde{\Theta}_i & -(\alpha - 1) g_{ij}^T X_i g_{ij} & \cdots & \star & \star \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ g_{iN}^T X_i E_{i\mu} & \left(1 - \sqrt{\alpha}\right) g_{iN}^T X_i \tilde{\Theta}_i & -(\alpha - 1) g_{iN}^T X_i g_{ij} & \cdots & -(\alpha - 1) g_{iN}^T X_i g_{iN} & \star \\ 0 & X_i \tilde{\Theta}_i & 0 & \cdots & 0 & -N^{-1} X_i^T \end{bmatrix} \le 0 \tag{17}$$

$$\begin{bmatrix} Z_i & k_{i\mu}^T \\ k_{i\mu} & 1 \end{bmatrix} \ge 0 \; Z_{iss} \le u_{is,max}^2, \; s \in Z_{[1,m]} \tag{18}$$

then the set $\Omega_{iw} = \{x_i | x_i^T P_{i\mu} x_i \le \xi_i\}$ is an RPI set for the IT2 fuzzy system (3) corresponding to the feedback control law $u_i(k) = \sum_{l=1}^{r_i} h_i^l(z_{iq}) k_i^l x_i(k)$. $P_{i\mu} = \sum_{l=1}^{r_i} w_i^l(z_{iq})P_i$.

Z_{iss} is the sth diagonal element of matrix Z_i, N represents the number of subsystems, $N_i = \frac{\xi_i}{{\eta_i}^2}$, $\alpha \ge 2$ and $i, j, l, N, \alpha \in R_+$, and $\tilde{\Theta}_i = A_{i\mu} + B_{i\mu} k_{i\mu}$.

Proof: See Appendix A. ■

3.2. The terminal constraint set

In this section, it will be proved by an LMI that the terminal constraint set Ω_{iw} be the RPI set. The sub-cost function is

$$\psi_i(.) = \sum_{k=0}^{K-1} \{ x_i^{\mathrm{T}}(k) Q x_i(k) + u_i^{\mathrm{T}}(k) R u_i(k) - \tau_i d_i^{\mathrm{T}}(k) d(k) \} \tag{19}$$

a positive definite function (terminal cost function) V_i (x) is such that $\forall x_i \in \Omega_{iw}$

$$\gamma_3(\| x_i \|) \le V(x_i) \le \gamma_4(\| x_i \|) \tag{20}$$

$$\sum_{i=1}^{N} V(x_i^+) - V(x_i) < -\sum_{i=1}^{N} \psi_i(\cdot) \tag{21}$$

where ς_3 and ς_4 are H_∞ functions, $V(x)$ is given as follows:

$$V(x) = \sum_{i=1}^{N} V_i(x) = \sum_{i=1}^{N} \sum_{l=1}^{r_i} w_i^l(z_{iq}) x_i^{\mathrm{T}} P_i x_i \tag{22}$$

The main and core of every proposed control algorithm is the definition of a positive function to prove the stability and effectiveness of the proposed algorithm. In the following, an LMI proves that the Ω_{iw} is a terminal constraint set corresponding to the terminal cost.

Remark 4 In this section, by proposing Theorem 2, the concept of the robust positively invariant and constraint set is achieved by solving an LMI, and it will be ensured that trajectories of the large-scale systems are stable robustly.

Theorem 2: *Consider the IT2 fuzzy Takagi–Sugeno system* (3), *if* (17), (18) *and the following matrix inequality are feasible*

$$\begin{bmatrix} E_{i\mu}^T X_i E_{i\mu} - \xi_i \tau_i & \star & \star & \cdots & \star & \star & \star \\ \tilde{\Theta}_i^T X_i E_{i\mu} & \zeta_i & \star & \cdots & \star & \star & \star \\ g_{ij}^T X_i E_{i\mu} & (1-\sqrt{\alpha})\, g_{ij}^T X_i \tilde{\Theta}_i & -(a-1) g_{ij}^T X_i g_{ij} & \cdots & \star & \star & \star \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ g_{iN}^T X_i E_{i\mu} & (1-\sqrt{\alpha})\, g_{iN}^T X_i \tilde{\Theta}_i & -(a-1) g_{iN}^T X_i g_{ij} & \cdots & -(a-1) g_{iN}^T X_i g_{iN} & \star & \star \\ 0 & \mathrm{M}_i k_{i\mu} & 0 & \cdots & 0 & -\mathrm{M}_i & \star \\ 0 & X\tilde{\Theta}_i & 0 & \cdots & 0 & 0 & -N^{-1} X_i^T \end{bmatrix} < 0 \quad (23)$$

then Ω_{iw} *is a terminal constraint set corresponding to the terminal cost function* $V_i(x)$. *Where* N *represents the number of subsystems,* $\zeta_i = N\sqrt{a}\sum_{\substack{j=1 \\ i \neq j}}^{N} g_{ij}^T X_j g_{ij} - X_{i\mu} + \xi_i Q$, $\tilde{\Theta}_i = A_{i\mu} + B_{i\mu} k_{i\mu}$, $\mathrm{M}_i = \xi_i R$, $\alpha \geq 2$, $X_i = \xi_i P_i$, $X_{i\mu} = \xi_i P_{i\mu}$, $X_j = \xi_i P_j$ *and* $i, j, l, N, \alpha \in R_+$.

Proof: See Appendix B. ∎

3.3. Control algorithm

According to the aforementioned results and to accomplish the control design, the online control algorithm is studied in this section. So, the terminal constraint set $V(x(k))$, see (21), should satisfy the following condition

$$V(x(k)) = \sum_{i=1}^{N} V_i(x(k)) = \sum_{i=1}^{N} \sum_{l=1}^{r_i} w_i^l(z_{iq}) x_i^{\mathrm{T}} P_i x_i \leq \xi_i \quad (24)$$

the following optimization problem, thus, is discussed to minimize ξ_i :

$$\min \xi_i \text{ subject to} V_i(x(k)) \leq \xi_i \quad (25)$$

in addition, a sufficient condition for $V_i(x(k)) \leq \xi_i$ is $x_i^{\mathrm{T}}(k) P_i x_i(k) \leq \xi_i$, which is $\xi_i - x_i^{\mathrm{T}}(k) P_i x_i(k) \geq 0$ and equal to $\xi_i - x_i^{\mathrm{T}}(k) \frac{\xi_i P_i}{\xi_i} x_i(k) \geq 0$.

By defining symmetrical matrix $X_i = \xi_i P_i$, is guaranteed by the following LMIs

$$\begin{bmatrix} \xi_i & x_i^{\mathrm{T}}(k) \\ x_i(k) X_i^{-1} \xi_i \end{bmatrix} \geq 0 \quad (26)$$

Note: X_i is a fixed value and can appear in a negative form in the LMI.

Remark 5 The principal part of the MPC is optimizing a cost function. In this paper, the considered system is the type of large-scale one, and the system is stabilized based on the minimized values of ξ_i. In previous *theorems,* the bilinear matrix inequalities have been proposed. Because the significant part of this paper is minimized. To reduce the computational burden, first try different values of λ_i, $X_{i\mu}$, X_i and $X_{i\mu}$, and then try to obtain the minimum amount of ξ_i.

Algorithm 1: Step 1: Set different values of M_i, N_i, X_i, λ_i. And obtain the system state $x_i(k)$.

Step 2: Solve the following optimization problem

$$\min_{k_{i\mu}, \varepsilon_i, v_i, Z_i} \xi_i, \quad (27)$$

subject to (17), (18), (23), (26), and for each subsystem, find the values of $k_{i\mu}$ and $X_{i\mu}$. Move the time instant from k to $k+1$ and go to Step 1.

One of the principal elements of the MPC is the recursive feasibility. Since the MPC is supposed for discrete-time systems, it is essential to confirm its feasibility for all times. In the following *theorem*, this initial task is examined.

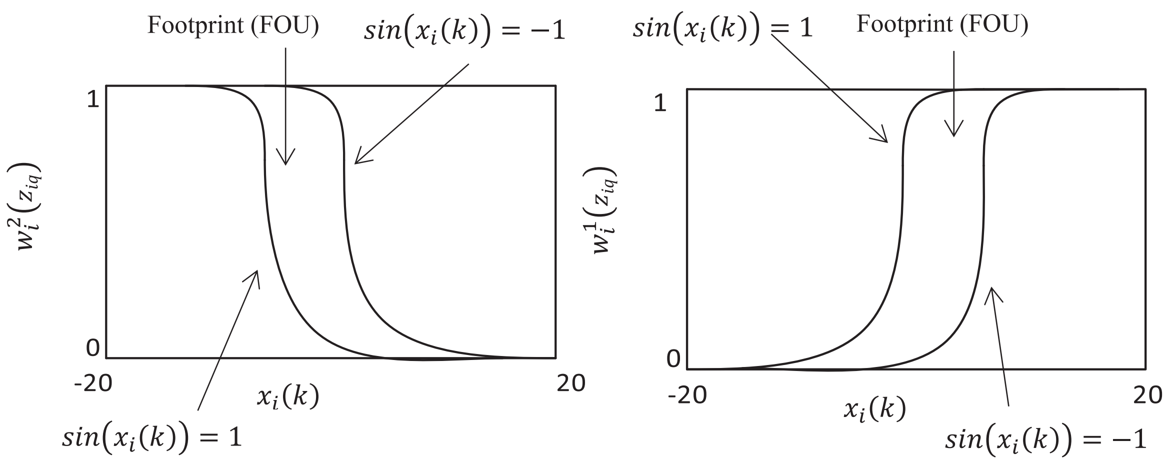
Theorem 3: *In system* (4), *if the solution of the optimization problem can be achieved at time 0, the solution will be obtainable at any time. So, the recursive feasibility was performed.*

Proof: If (27) is feasible at time k, then by the $x_i(k) \in \Omega_{iw}$, and according to Theorem 1, which has been implied that $x_i(k+1) \in \Omega_{iw}$, then it can be concluded that (27) can be solvable at time $k+1$. Besides, the solution that is achieved at the time k, is feasible at time $k+1$ the optimization problem is feasible always. ∎

Theorem 4: *Consider that the optimization problem is feasible at the initial time* 0, *system* (4) *is ISS due to the disturbance d.*

Proof: Assume the Lyapunov function $V(x_i(k)) = x_i^{\mathrm{T}}(k) P_{i\mu} x_i(k)$, in which $P_{i\mu} = \sum_{l=1}^{r_i} w_i^l(z_{iq}) P_i$, here, P_i is defined based on the (22), then it can be achieved

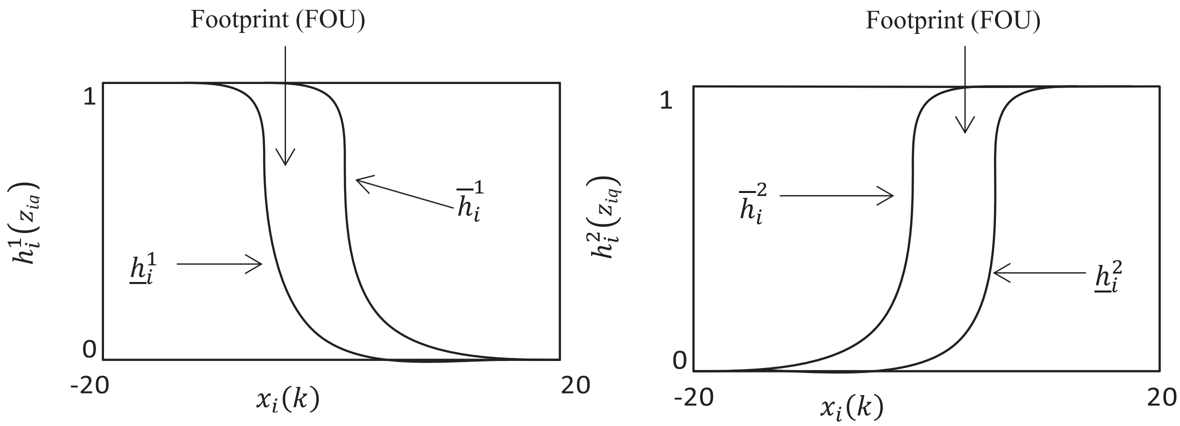
$$\varpi_{\min}^* \|x_i(k)\|^2 \leq V(k, x_i) \leq \varpi_{\max}^* \|x_i(k)\|^2 \quad (28)$$



(a) The membership function of rule 2

(b) The membership function of rule 1

Figure 2. The membership function of the IT2 fuzzy model. (a) The membership function of rule 2 and (b) rule 1.



(a) The membership function of rule 1

(b) The membership function of rule 2

Figure 3. The membership function of the IT2 fuzzy controller. (a) The membership function of rule 1 and (b) rule 2.

in which

$$\varpi_{\max}^* = \max\{\varpi_{\max}(P_i(k))|i \in Z_{[1,L]}, k \in R\}$$

$$\varpi_{\min}^* = \min\{\varpi_{\min}(P_i(k))|i \in Z_{[1,L]}, k \in R\}$$

where $\varpi_{\max}(\cdot)$ and $\varpi_{\min}(\cdot)$ are the maximal and minimal eigenvalues, respectively.

In addition, (22) implies that

$$V_k(x_i(k+1)) - V(x_i(k)) < -(x_i^{\mathrm{T}}(k)Qx_i(k) + u_i^{\mathrm{T}}(k)Ru_i(k) - \tau_i d_i^{\mathrm{T}}(k)d(k)) \tag{29}$$

where $V_k(x_i(k+1)) = x_i^{\mathrm{T}}(k+1)P_{i\mu}x_i(k+1)$. And if

$$V_k(x_i(k+1)) - V(k, x_i) < -x_i^{\mathrm{T}}(k)Qx_i(k) + \tau_i d_i^{\mathrm{T}}(k)d(k) \tag{30}$$

due to (38) at time $(k+1)$:

$$V_{k+1}(x_i(k+1)) \le V_k(x_i(k+1)) \tag{31}$$

ultimately, it achieves that

$$V_{k+1}(x_i(k+1)) - V(k, x_i) < -x_i^{\mathrm{T}}(k)Qx_i(k) + \tau_i d_i^{\mathrm{T}}(k)d(k) \tag{32}$$

resorting to definition (3), (27) and (32) are conducive to the result that $V(x_i(k))$ is an ISS-Lyapunov function. On the other hand, the closed-loop system is ISS due to disturbances. So the proof is completed.

Remark 6 In [1925], robust fuzzy MPC for a class of fuzzy systems has been studied. The type of systems studied beforehand was discrete-time systems. But in this paper, the decentralized MPC for a class of IT2 discrete-time large-scale systems is considered. The mentioned controller is applied with the so-called H_∞ performance to reduce the unknown disturbances and the adopted modelling used is IT2 to diminish the uncertainties. ■

4. Numerical example

Example 1 In this section, a numerical example shows the application of the proposed algorithm for IT2 fuzzy T–S large-scale system. The considered fuzzy IT2 large-scale system consists of three subsystems, $S_i, i = 1,2,3 (N = 3)$, and each subsystem involves two rules, $l = 1,2$. In this example, $N_i = 0.5$, $Q = \mathrm{diag}\{1,1\}, M_i = 1, \tau_1 = 1, \tau_2 = 1.5, \tau_3 = 2$. The membership functions are shown in Figures 2 and 3. Here, the sampling time is set as Ts = 0.2 min and the frequency of calculations is 0.8 Hz. The fuzzy model of the plan is

$$S_i^l : \begin{cases} \text{IF } z_{i1} \text{ is } \tilde{F}_{i1}^l \text{and } \dots \text{ and } z_{ig} \textit{ is } \tilde{F}_{ig}^l \\ \textit{THEN } x_i(k+1) = A_i^l x_i(k) + B_i^l u_i(k) + E_i^l d_i(k) \\ + \sum_{\substack{j=1 \\ i \neq j}}^{N} g_{ij} x_j(k) \end{cases}$$

So by applying the controller and the membership function, we have

$$x_i(k+1) = \sum_{l=1}^{r_i} \sum_{m=1}^{r_i} w_i^l(z_{iq}) h_i^m(z_{iq}) ([\tilde{A}_i^l + \tilde{B}_i^l k_i^m] x_i(k) + E_i^l d_i(k)) + \sum_{l=1}^{r_i} \sum_{\substack{j=1 \\ i \neq j}}^{N} w_i^l(z_{iq}) g_{ij} x_j(k)$$

Subsystem $\boldsymbol{S}_1$

$$A_{11} = \begin{bmatrix} 0.55 & 0.05 \\ 0 & 0.42 \end{bmatrix}, \quad B_{11} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$E_{11} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, \quad g_{12} = \begin{bmatrix} 0.08 & 0.05 \\ 0.05 & 0.05 \end{bmatrix},$$

$$g_{13} = \begin{bmatrix} 0.09 & 0.06 \\ 0.06 & 0.09 \end{bmatrix}, \quad \lambda_1 = 0.5,$$

$$X_1 = \begin{bmatrix} 0.015 & 0 \\ 0 & 0.015 \end{bmatrix}$$

$$A_{12} = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.08 \end{bmatrix}, \quad B_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, E_{12} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}.$$

Subsystem $\boldsymbol{S}_2$

$$A_{21} = \begin{bmatrix} 0.325 & 0 \\ 0.4 & 0 \end{bmatrix}, \quad B_{21} = \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$

$$E_{21} = \begin{bmatrix} -0.1 \\ 0 \end{bmatrix}, g_{21} = \begin{bmatrix} 0.1 & 0.1 \\ 0 & 0 \end{bmatrix},$$

$$g_{23} = \begin{bmatrix} 0 & 0 \\ 0.1 & 0.1 \end{bmatrix}, \quad \lambda_2 = 0.488,$$

$$X_2 = \begin{bmatrix} 0.018 & 0 \\ 0 & 0.018 \end{bmatrix}$$

$$A_{22} = \begin{bmatrix} 0.6 & 0.2 \\ 0.1 & 0 \end{bmatrix}, \quad B_{22} = \begin{bmatrix} -1 \\ 1 \end{bmatrix},$$

$$E_{22} = \begin{bmatrix} 0 \\ -0.2 \end{bmatrix}.$$

Subsystem $\boldsymbol{S}_3$

$$A_{31} = \begin{bmatrix} 0.2 & 0.4 \\ 0.2 & 0 \end{bmatrix}, \quad B_{31} = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$E_{31} = \begin{bmatrix} -0.3 \\ 0 \end{bmatrix}, \quad g_{31} = \begin{bmatrix} 0.03 & 0 \\ 0 & 0.02 \end{bmatrix},$$

$$g_{32} = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0 \end{bmatrix}, \quad \lambda_3 = 0.487,$$

$$X_3 = \begin{bmatrix} 0.027 & 0 \\ 0 & 0.027 \end{bmatrix}$$

$$A_{32} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.4 \end{bmatrix}, \quad B_{32} = \begin{bmatrix} -2 \\ 1 \end{bmatrix},$$

$$E_{32} = \begin{bmatrix} 0 \\ -0.4 \end{bmatrix},$$

By considering $\delta(x_i) = \sin(x_i) \in [-1,1]$, the membership functions and parameter uncertainties are

$$\begin{cases} w_i^1(z_{iq}) = 1 - \dfrac{1}{1 + e^{x_i + 4 + \delta(x_i)}} \\ w_i^2(z_{iq}) = 1 - w_i^1(z_{iq}) \end{cases},$$

$$\begin{cases} \underline{h}_i^1(z_{iq}) = 1 - \dfrac{1}{1 + e^{\frac{-x_i - 1.5}{2}}} \\ \bar{h}_i^1(z_{iq}) = 1 - \dfrac{1}{1 + e^{\frac{-x_i + 1.5}{2}}} \\ \underline{h}_i^2(z_{iq}) = 1 - \bar{h}_i^1(z_{iq}) \\ \bar{h}_i^2(z_{iq}) = 1 - \underline{h}_i^1(z_{iq}) \end{cases},$$

$$\begin{cases} \underline{w}_i^1(z_{iq}) = 1 - \dfrac{1}{1 + e^{x_i + 4 - 1}} \\ \bar{w}_i^1(z_{iq}) = 1 - \dfrac{1}{1 + e^{x_i + 4 + 1}} \\ \underline{w}_i^2(z_{iq}) = \dfrac{1}{1 + e^{x_i + 4 + 1}} \\ \bar{w}_i^2(z_{iq}) = \dfrac{1}{1 + e^{x_i + 4 - 1}} \end{cases}$$

The mentioned parameter uncertainty is assumed as $\delta(x_i) = \sin(x_i) \in [-1,1]$:

Subsequently, we can obtain the feedback gains

k11 = [−0.549 −0.222] k12 = [−0.0569 −0.799]

k21 = [4.794e−05 −4.739e−09] k22 = [1.755e−05 1.138e−05]

k31 = [−0.199 −0.111] k32 = [0.073 −0.201]

Remark 7 By exemplifying an instance, it has been shown that the proposed algorithm is entirely practical. Here, in this example, as evident in Figures 4–6, the

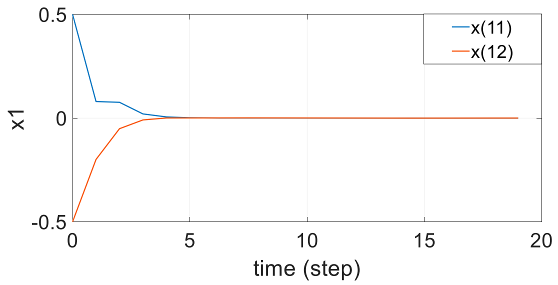


Figure 4. Trajectories of subsystem 1.

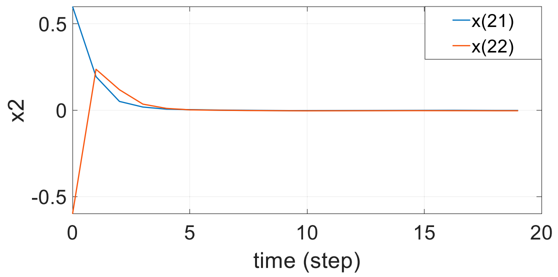


Figure 5. Trajectories of subsystem 2.

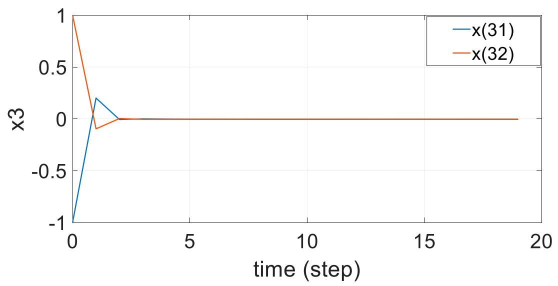


Figure 6. Trajectories of subsystem 3.

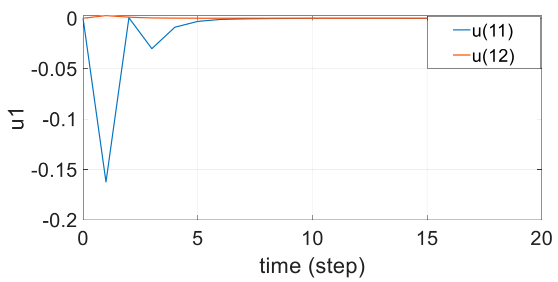


Figure 7. Trajectories of controller 1.

trajectories of the three subsystems are leading to zero. This means that the overall closed-loop system is stable during that time.

Remark 8 In Figures 7–9, an important issue is specified. After a while, the states of the system have been obtained in a boundary. After that, there is no need of the input vector $u_i(k)$, it means a decrease in costs. Since the first example is the mathematical one, no unit is defined for the x-axis. This shows the effectiveness of the proposed approach.

Example 2 To show the effectiveness of the proposed method, a double inverted pendulum is considered due to [36]. For convenience, all configurations

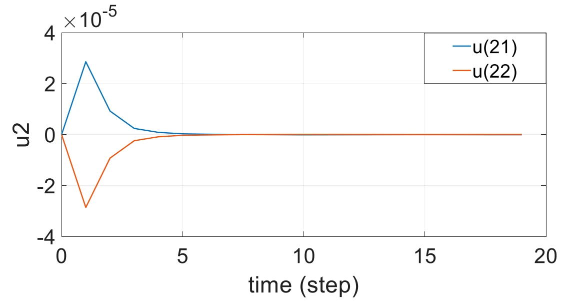


Figure 8. Trajectories of controller 2.

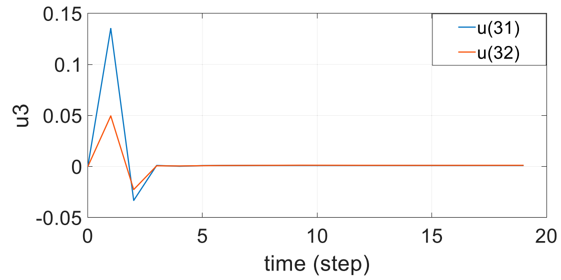


Figure 9. Trajectories of controller 3.

and parameters are chosen the same as [36] and the previous example. In this example, $y_1(k)$ and $y_2(k)$ are assumed as angular position and velocity, respectively. Here, the sampling time is set as Ts = 0.2 min and the frequency of calculation is 0.8 Hz.

Subsystem $\boldsymbol{S}_1$

$$A_{11} = A_{13} = \begin{bmatrix} 1 & 0.005 \\ 0.0262 & 1 \end{bmatrix},$$

$$A_{13} = \begin{bmatrix} 1 & 0.005 \\ 0.0441 & 1 \end{bmatrix},$$

$$B_{11} = B_{12} = B_{13} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$E_{11} = E_{12} = E_{13} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix},$$

$$g_{12} = \begin{bmatrix} 0.08 & 0.05 \\ 0.05 & 0.05 \end{bmatrix}, \ \lambda_1 = 0.5,$$

$$X_1 = \begin{bmatrix} 0.015 & 0 \\ 0 & 0.015 \end{bmatrix}, \ H_1 = [1 \ 0].$$

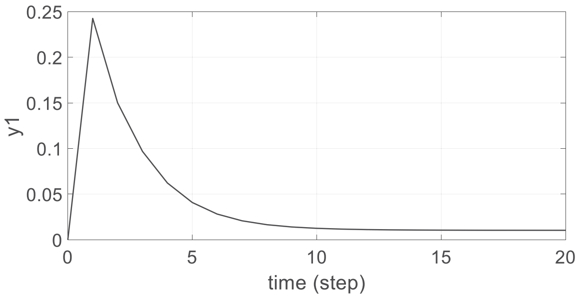
Subsystem $\boldsymbol{S}_2$

$$A_{21} = A_{23} = \begin{bmatrix} 1 & 0.005 \\ 0.0272 & 1 \end{bmatrix},$$

$$A_{23} = \begin{bmatrix} 1 & 0.005 \\ 0.0451 & 1 \end{bmatrix},$$

$$B_{21} = B_{22} = B_{23} = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$E_{11} = E_{22} = E_{23} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix},$$



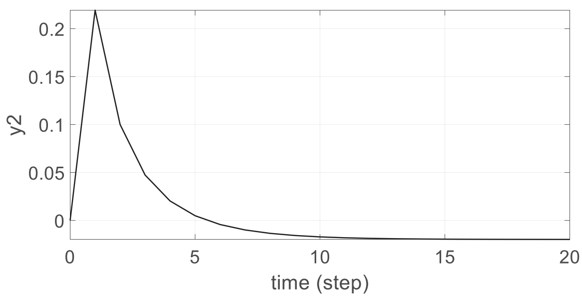


Figure 10. Output responses of the closed-loop system with disturbances.

$$g_{21} = \begin{bmatrix} 0.08 & 0.05 \\ 0.05 & 0.05 \end{bmatrix}, \lambda_2 = 0.448,$$

$$X_2 = \begin{bmatrix} 0.018 & 0 \\ 0 & 0.018 \end{bmatrix}, H_2 = [1\ 0].$$

$$x_i(k+1) = \sum_{l=1}^{r_i} \sum_{m=1}^{r_i} w_i^l(z_{iq}) h_i^m(z_{iq})([\tilde{A}_i^l + \tilde{B}_i^l k_i^m] x_i(k) + E_i^l d_i(k)) + \sum_{l=1}^{r_i} \sum_{\substack{j=1 \\ i \neq j}}^{N} w_i^l(z_{iq}) g_{ij} x_j(k)$$

$$y_i(k) = H_i x_i(k)$$

Subsequently, we can obtain the feedback gains:
k11= [−0.071 −0.024] k12= [−0.0153 −0.219] k13= [−12.15 −9.585]
k21= [21.794 −41.739] k22= [−10.75 −21.138] k23= [−18.255 −32.252]

Remark 9 Figure 10 shows the output responses of the closed-loop discrete-time nonlinear large-scale system with the disturbances. The piecewise controller proposed in this paper based on the fuzzy dynamic model not only stabilizes the original nonlinear large-scale system but also attenuates the disturbances effectively as expected.

Remark 10 To evaluate the effectiveness of the proposed controller, a comparison is made with [37]. In [37], the PI controller is applied to a decentralized large-scale system. In the first set of the experiments, a control scheme using PI controllers currently used in industry is used. Although this scheme is very simple to implement, its performance is often limited. Tuning the P and I gains is a tedious process. In the results of this paper, the proposed controller is used. Experimental results with these control schemes show that the proposed control scheme offers a marked improvement in results.

Remark 11 As it is clear due to gain results, to stabilize the mentioned systems in this paper, computed gains have little value. Specifically, in the first example, it is noticed that by gain values that are < 1, states of the system are stabilized. This means that the proposed method can control the system optimally. In the second example, compared with other methods, it is evident that the double inverted pendulum can be stabilized with a lower cost and this is the efficient proposed approach.

5. Conclusion

In this paper, a decentralized robust IT2 fuzzy MPC for Takagi–Sugeno large-scale systems has been given. Since almost all real systems have complex and severe nonlinearity, they will be hard and impossible to find the dynamic of systems most of the time. Therefore, the IT2 fuzzy model of the large-scale system for deducing uncertainties is considered in this paper. The supposed controller for this system is MPC, and the type of the controller is decentralized. To reduce the effects of disturbance and uncertainties, the H_∞ performance is used. Finally, the control gains and other values are calculated by solving LMIs.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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Appendices

Appendix A

Proof: Using the Schur complement, the inequality (17) is equivalent to

$$\begin{bmatrix} E_{i\mu}^{\mathrm{T}}X_iE_{i\mu}-\xi_i\lambda_i\mathrm{N}_i & \star \\ \tilde{\Theta}_i^{\mathrm{T}}X_iE_{i\mu} & N\sqrt{a}\sum_{\substack{j=1\\ i\neq j}}^{N} g_{ij}^{\mathrm{T}}X_jg_{ij}-(-\lambda_i+1)X_{i\mu} \\ g_{ij}^{\mathrm{T}}X_iE_{i\mu} & \left(1-\sqrt{\alpha}\right)g_{ij}^{\mathrm{T}}X_i\tilde{\Theta}_i \\ \vdots & \vdots \\ g_{iN}^{\mathrm{T}}X_iE_{i\mu} & \left(1-\sqrt{\alpha}\right)g_{iN}^{\mathrm{T}}X_i\tilde{\Theta}_i \end{bmatrix.$$
$$\begin{bmatrix} \star & \cdots & \star \\ \star & \cdots & \star \\ -(\alpha-1)g_{ij}^{\mathrm{T}}X_ig_{ij} & \cdots & \star \\ \vdots & \ddots & \vdots \\ -(\alpha-1)g_{iN}^{\mathrm{T}}X_ig_{ij} & \cdots & -(\alpha-1)g_{iN}^{\mathrm{T}}X_ig_{iN} \end{bmatrix}$$
$$+\begin{bmatrix} 0 \\ \tilde{\Theta}_i^{\mathrm{T}}X_i^{\mathrm{T}} \\ 0 \\ \vdots \\ 0 \end{bmatrix} NX_i^{-\mathrm{T}}\begin{bmatrix}0 & X_i\tilde{\Theta}_i & \cdots & 0 & 0\end{bmatrix}\leq 0 \quad (\mathrm{A.1})$$

where $\tilde{\Theta}_i=\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu}$ and if consider $X_i=\xi_iP_i$, $X_{i\mu}=\xi_iP_{i\mu}$, and $X_j=\xi_iP_j$, according to the Schur complement, we will have

$$\begin{bmatrix} \frac{1}{\xi_i}E_{i\mu}^{T}P_iE_{i\mu}-\frac{\lambda_i}{\eta_i{}^2} & \star \\ \frac{1}{\xi_i}\tilde{\Theta}_i^{T}P_iE_{i\mu} & \Xi_i \\ \frac{1}{\xi_i}g_{ij}^{T}P_iE_{i\mu} & \frac{1}{\xi_i}\left(1-\sqrt{\alpha}\right)g_{ij}^{T}P_i\tilde{\Theta}_i \\ \vdots & \vdots \\ \frac{1}{\xi_i}g_{iN}^{T}P_iE_{i\mu} & \frac{1}{\xi_i}\left(1-\sqrt{\alpha}\right)g_{iN}^{T}P_i\tilde{\Theta}_i \end{bmatrix.$$
$$\begin{bmatrix} \star & \cdots & \star \\ \star & \cdots & \star \\ -\frac{1}{\xi_i}(\alpha-1)g_{ij}^{T}P_ig_{ij} & \cdots & \star \\ \vdots & \ddots & \vdots \\ -\frac{1}{\xi_i}(\alpha-1)g_{iN}^{T}P_ig_{ij} & \cdots & -\frac{1}{\xi_i}(\alpha-1)g_{iN}^{T}P_ig_{iN} \end{bmatrix}\leq 0 \quad (\mathrm{A.2})$$

where $\Xi_i=\frac{1}{\xi_i}N\tilde{\Theta}_i^{\mathrm{T}}P_i\tilde{\Theta}_i+\frac{1}{\xi_i}N\sqrt{a}\sum_{\substack{j=1\\ i\neq j}}^{N} g_{ij}^{\mathrm{T}}P_jg_{ij}-\frac{1}{\xi_i}(-\lambda_i+1)P_{i\mu}$. By multiplying $\left[\begin{matrix} d_i^{\mathrm{T}} & x_i^{\mathrm{T}} & x_j^{\mathrm{T}} & \cdots & x_N^{\mathrm{T}}\end{matrix}\right]$ and its transpose from both sides of the matrix in the inequality (A.2), respectively:

$$\sum_{i=1}^{N} x_i^{T}\frac{1}{\xi_i}\left\{N(\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu})^{T}P_i(\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu})\right.$$
$$+N\sqrt{a}\sum_{\substack{j=1\\ i\neq j}}^{N} g_{ij}^{T}P_jg_{ij}-P_{i\mu}\}\, x_i$$
$$+\sum_{i=1}^{N}\{x_i^{T}\frac{1}{\xi_i}(\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu})^{T}P_iE_{i\mu}d_i$$
$$+d_i^{T}\frac{1}{\xi_i}E_{i\mu}^{T}P_iE_{i\mu}d_i+\frac{1}{\xi_i}\left(\sum_{\substack{j=1\\ i\neq j}}^{N} x_j{}^{T}g_{ij}^{T}\right)P_iE_{i\mu}d_i$$
$$+d_i^{T}\frac{1}{\xi_i}E_{i\mu}^{T}P_i(\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu})x_i$$
$$+d_i^{T}\frac{1}{\xi_i}E_{i\mu}^{T}P_i\left(\sum_{\substack{j=1\\ i\neq j}}^{N} g_{ij}x_j\right)-\frac{\lambda_i}{\eta_i{}^2}d_i^{T}d_i+\frac{\lambda_i}{\xi_i}x_i^{T}P_{i\mu}x_i$$
$$-\frac{1}{\xi_i}(\alpha-1)\left(\sum_{\substack{j=1\\ i\neq j}}^{N} x_j^{T}g_{ij}^{T}\right)P_i\left(\sum_{\substack{j=1\\ i\neq j}}^{N} g_{ij}x_j\right)$$
$$+x_i^{T}\left(1-\sqrt{\alpha}\right)\frac{1}{\xi_i}(\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu})^{T}P_i\left(\sum_{\substack{j=1\\ i\neq j}}^{N} g_{ij}x_j\right)$$
$$+\left(1-\sqrt{\alpha}\right)\frac{1}{\xi_i}\left(\sum_{\substack{j=1\\ i\neq j}}^{N} x_j^{T}g_{ij}^{T}\right)P_i(\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu})x_i\}$$
$$\leq 0 \quad (\mathrm{A.3})$$

by resorting to [38], the inequality (A.3) is

$$\sum_{i=1}^{N}\frac{1}{\xi_i}\left\{\left[(\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu})x_i+\sqrt{\alpha}\sum_{\substack{j=1\\ i\neq j}}^{N} g_{ij}x_j\right]^{T}\right.$$
$$\left.P_i\left[(\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu})x_i+\sqrt{\alpha}\sum_{\substack{j=1\\ i\neq j}}^{N} g_{ij}x_j\right]-x_i^{T}P_{i\mu}x_i\right\}$$
$$+\sum_{i=1}^{N}\left\{x_i^{T}\frac{1}{\xi_i}(\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu})^{T}P_iE_{i\mu}d_i\right.$$
$$+d_i^{T}\frac{1}{\xi_i}E_{i\mu}^{T}P_{il}E_{i\mu}d_i+\frac{1}{\xi_i}\left(\sum_{\substack{j=1\\ i\neq j}}^{N} x_j^{T}g_{ij}^{T}\right)P_iE_{i\mu}d_i$$
$$+d_i^{T}\frac{1}{\xi_i}E_{i\mu}^{T}P_i(\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu})x_i+d_i^{T}\frac{1}{\xi_i}E_{i\mu}^{T}P_i\left(\sum_{\substack{j=1\\ i\neq j}}^{N} f_{ij}x_j\right)$$

$$-\frac{\lambda_i}{{\eta_i}^2}d_i^T d_i + \frac{\lambda_i}{\xi_i}x_i^T P_{i\mu}x_i$$

$$-\frac{1}{\xi_i}(\alpha-1)\left(\sum_{\substack{j=1\\ i\neq j}}^{N} x_j^T g_{ij}^T\right)P_i\left(\sum_{\substack{j=1\\ i\neq j}}^{N} g_{ij}x_j\right)$$

$$+\left(1-\sqrt{\alpha}\right)\frac{1}{\xi_i}\left(\sum_{\substack{j=1\\ i\neq j}}^{N} x_j^T g_{ij}^T\right)P_i(\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu})x_i$$

$$\left.+x_i^T\left(1-\sqrt{\alpha}\right)\frac{1}{\xi_i}(\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu})^T P_i\left(\sum_{\substack{j=1\\ i\neq j}}^{N} g_{ij}x_j\right)\right\}\leq 0 \tag{A.4}$$

and the inequality (A.4) is equivalent to

$$\sum_{i=1}^{N}\left\{\frac{1}{\xi_i}\left[(\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu})+\sum_{\substack{j=1\\ i\neq j}}^{N} g_{ij}x_j\right]\right.$$

$$P_i\left[(\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu})x_i+\sum_{\substack{j=1\\ i\neq j}}^{N} g_{ij}x_j\right]$$

$$+x_i^{\mathrm{T}}\frac{1}{\xi_i}(\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu})^{\mathrm{T}}P_iE_{i\mu}d_i$$

$$+d_i^{\mathrm{T}}\frac{1}{\xi_i}E_{i\mu}^{\mathrm{T}}P_iE_{i\mu}d_i+\frac{1}{\xi_i}\left(\sum_{\substack{j=1\\ i\neq j}}^{N} x_j^{\mathrm{T}} g_{ij}^{\mathrm{T}}\right)P_iE_{i\mu}d_i$$

$$+d_i^{\mathrm{T}}\frac{1}{\xi_i}E_{i\mu}^{\mathrm{T}}P_i(\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu})x_i+d_i^{\mathrm{T}}\frac{1}{\xi_i}E_{i\mu}^{\mathrm{T}}P_id_i$$

$$+d_i^{\mathrm{T}}\frac{1}{\xi_i}E_{i\mu}^{\mathrm{T}}P_i\left(\sum_{\substack{j=1\\ i\neq j}}^{N} g_{ij}x_j\right)-\frac{\lambda_i}{{\eta_i}^2}d_i^{\mathrm{T}}d_i$$

$$+\frac{1}{\xi_i}(-1+\lambda_i)x_i^{\mathrm{T}}P_{i\mu}x_i$$

$$+\frac{1}{\xi_i}(\alpha-1)\left(\sum_{\substack{j=1\\ i\neq j}}^{N} x_j^{\mathrm{T}} g_{ij}^{\mathrm{T}}\right)P_i\left(\sum_{\substack{j=1\\ i\neq j}}^{N} g_{ij}x_j\right)$$

$$-\frac{1}{\xi_i}(\alpha-1)\left(\sum_{\substack{j=1\\ i\neq j}}^{N} x_j^T g_{ij}^T\right)P_i\left(\sum_{\substack{j=1\\ i\neq j}}^{N} g_{ij}x_j\right)$$

$$+x_i^{\mathrm{T}}\sqrt{\alpha}\frac{1}{\xi_i}(\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu})^T P_i\left(\sum_{\substack{j=1\\ i\neq j}}^{N} g_{ij}x_j\right)$$

$$-x_i^T\sqrt{\alpha}\frac{1}{\xi_i}(\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu})^{\mathrm{T}} P_i\left(\sum_{\substack{j=1\\ i\neq j}}^{N} g_{ij}x_j\right)$$

$$+\sqrt{\alpha}\frac{1}{\xi_i}\left(\sum_{\substack{j=1\\ i\neq j}}^{N} x_j^{\mathrm{T}} g_{ij}^{\mathrm{T}}\right)P_i(\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu})x_i$$

$$\left.-\sqrt{\alpha}\frac{1}{\xi_i}\left(\sum_{\substack{j=1\\ i\neq j}}^{N} x_j^{\mathrm{T}} g_{ij}^{\mathrm{T}}\right)P_i(\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu})x_i\right\}\leq 0 \tag{A.5}$$

the inequality (A.5) can be written as

$$\sum_{i=1}^{N}\left\{\frac{1}{\xi_i}\left[(\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu})x_i+E_{i\mu}d_i+\sum_{\substack{j=1\\ i\neq j}}^{N} g_{ij}x_j\right]^{\mathrm{T}}\right.$$

$$P_{i\mu}^{+}\left[(\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu})x_i+E_{i\mu}d_i+\sum_{\substack{j=1\\ i\neq j}}^{N} g_{ij}x_j\right]$$

$$\left.-\frac{1}{\xi_i}(1-\lambda_i)x_i^{\mathrm{T}}P_{i\mu}x_i-\lambda_i\frac{1}{{\eta_i}^2}d_i^{\mathrm{T}}d_i\right\}\leq 0 \tag{A.6}$$

we had before $x_i^{+}=(\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu})x_i+E_{i\mu}d_i+\sum_{\substack{j=1\\ i\neq j}}^{N} f_{ij}x_j$. So, we have

$$\sum_{i=1}^{N}\left\{\frac{1}{\xi_i}{x_i^{+}}^{\mathrm{T}}P_{i\mu}^{+}x_i^{+}-(1-\lambda_i)\frac{1}{\xi_i}x_i^{\mathrm{T}}P_{i\mu}x_i-\lambda_i\frac{1}{{\eta_i}^2}d_i^{\mathrm{T}}d_i\right\}\leq 0 \tag{A.7}$$

thus, the RPI property of the set can be obtained if (A.7) holds. Furthermore, the input constraint can be admitted by (18), and the proof is clarified below: multiplying diag$\{x_i, I\}$ and its transpose from both sides of (18), respectively

$$\begin{bmatrix} x_i^{\mathrm{T}}Z_ix_i & x_i^{\mathrm{T}}k_{i\mu}^{\mathrm{T}}\\ k_{i\mu}x_i & 1\end{bmatrix}\geq 0 \tag{A.8}$$

by the Schur complement to (A.8), then

$$x_i^{\mathrm{T}} Z_i x_i - (k_{i\mu} x_i)^{\mathrm{T}} (k_{i\mu} x_i) \ge 0 \qquad \text{(A.9)}$$

As $u_i = k_{i\mu} x_i$, we have

$$(u_i)^{\mathrm{T}} (u_i) \le x_i^{\mathrm{T}} Z_i x_i = H_i = \text{positive value} \qquad \text{(A.10)}$$

thus $u_i^{\mathrm{T}} u_i \le H_i$. The proof is, thereby, completed. ■

Appendix B

Proof: Resorting to the Schur complement, and based on the previous proof, the inequality (23) can be written as follows:

$$\begin{bmatrix} E_{i\mu}^{\mathrm{T}} X_i E_{i\mu} - \xi_i \tau_i & \star & \star & \cdots & \star \\ \tilde{\Theta}_i^{\mathrm{T}} X_i E_{i\mu} & \phi_i & \star & \cdots & \star \\ g_{ij}^{\mathrm{T}} X_i E_{i\mu} & \left(1 - \sqrt{\alpha}\right) g_{ij}^{\mathrm{T}} X_i \tilde{\Theta}_i & -(a-1) g_{ij}^{\mathrm{T}} X_i g_{ij} & \cdots & \star \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ g_{iN}^{\mathrm{T}} X_i E_{i\mu} & \left(1 - \sqrt{\alpha}\right) g_{iN}^{\mathrm{T}} X_i \tilde{\Theta}_i & -(a-1) g_{iN}^{\mathrm{T}} X_i g_{ij} & \cdots & -(a-1) g_{iN}^{\mathrm{T}} X_i g_{iN} \end{bmatrix} < 0 \qquad \text{(B.1)}$$

Consider $X_i = \xi_i P_i$, $X_{i\mu} = \xi_i P_{i\mu}$, $X_j = \xi_i P_j$ and $\phi_i = N \tilde{\Theta}_i^{\mathrm{T}} X_i \tilde{\Theta}_i + N\sqrt{a} \sum_{\substack{j=1 \\ i \ne j}}^{N} g_{ij}^{\mathrm{T}} X_j g_{ij} - X_{i\mu} + \xi_i Q + k_{i\mu}^{\mathrm{T}} M_i k_{i\mu}$. The inequality (B.1) is

$$\begin{bmatrix} E_{i\mu}^{\mathrm{T}} P_i E_{i\mu} - \tau_i & \star & \star & \cdots & \star \\ \tilde{\Theta}_i^{\mathrm{T}} P_i E_{i\mu} & \psi_i & \star & \cdots & \star \\ g_{iN}^{\mathrm{T}} P_i E_{i\mu} & \left(1 - \sqrt{\alpha}\right) g_{ij}^{\mathrm{T}} P_i \tilde{\Theta}_i & -(a-1) g_{ij}^{\mathrm{T}} P_i g_{ij} & \cdots & \star \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ g_{ij}^{\mathrm{T}} P_i E_{i\mu} & \left(1 - \sqrt{\alpha}\right) g_{iN}^{\mathrm{T}} P_i \tilde{\Theta}_i & -(a-1) g_{iN}^{\mathrm{T}} P_i g_{ij} & \cdots & -(a-1) g_{iN}^{\mathrm{T}} P_i g_{iN} \end{bmatrix} < 0 \qquad \text{(B.2)}$$

where $\psi_i = N \tilde{\Theta}_i^{\mathrm{T}} P_i \tilde{\Theta}_i + N\sqrt{a} \sum_{\substack{j=1 \\ i \ne j}}^{N} g_{ij}^{\mathrm{T}} P_j g_{ij} - P_{i\mu} + Q + k_{i\mu}^{\mathrm{T}} R k_{i\mu}$. Multiplying $\left[d_i^{\mathrm{T}} \; x_i^{\mathrm{T}} \; x_j^{\mathrm{T}} \; \cdots \; x_N^{\mathrm{T}} \right]$ and transposing from both sides of (B.2), we will have

$$\begin{aligned} \sum_{i=1}^{N} \Bigg\{ x_i^{\mathrm{T}} & \left(N (\tilde{A}_{i\mu} + \tilde{B}_{i\mu} k_{i\mu})^{\mathrm{T}} P_i (\tilde{A}_{i\mu} + \tilde{B}_{i\mu} k_{i\mu}) + N\sqrt{a} \sum_{\substack{j=1 \\ i \ne j}}^{N} g_{ij}^{\mathrm{T}} P_j g_{ij} - P_{i\mu} + Q \right) x_i \\ & + x_i^{\mathrm{T}} (\tilde{A}_{i\mu} + \tilde{B}_{i\mu} k_{i\mu})^{\mathrm{T}} P_i E_{i\mu} d_i + d_i^{\mathrm{T}} E_{i\mu}^{\mathrm{T}} P_i E_{i\mu} d_i + \left(\sum_{\substack{j=1 \\ i \ne j}}^{N} x_j^{\mathrm{T}} g_{ij}^{\mathrm{T}} \right) P_i E_{i\mu} d_i \\ & + d_i^{\mathrm{T}} E_{i\mu}^{\mathrm{T}} P_i (\tilde{A}_{i\mu} + \tilde{B}_{i\mu} k_{i\mu}) x_i + d_i^{\mathrm{T}} E_{i\mu}^{\mathrm{T}} P_i \left(\sum_{\substack{j=1 \\ i \ne j}}^{N} g_{ij} x_j \right) \\ & + x_i^{\mathrm{T}} k_{i\mu}^{\mathrm{T}} R k_{i\mu} x_i - \tau_i d_i^{\mathrm{T}} d_i - (a-1) \sum_{\substack{j=1 \\ i \ne j}}^{N} x_j^{\mathrm{T}} g_{ij}^{\mathrm{T}} P_i g_{ij} x_j \\ & + x_i^{\mathrm{T}} \left(1 - \sqrt{\alpha}\right) (\tilde{A}_{i\mu} + \tilde{B}_{i\mu} k_{i\mu})^{\mathrm{T}} P_i \left(\sum_{\substack{j=1 \\ i \ne j}}^{N} g_{ij} x_j \right) \\ & + \left(1 - \sqrt{\alpha}\right) \left(\sum_{\substack{j=1 \\ i \ne j}}^{N} x_j^{\mathrm{T}} g_{ij}^{\mathrm{T}} \right) P_i (\tilde{A}_{i\mu} + \tilde{B}_{i\mu} k_{i\mu}) x_i \Bigg\} < 0 \end{aligned} \qquad \text{(B.3)}$$

resorting to [38], the inequality (B.3) is

$$\begin{aligned} \sum_{i=1}^{N} \Bigg\{ & \left[(\tilde{A}_{i\mu} + \tilde{B}_{i\mu} k_{i\mu}) x_i + E_{i\mu} d_i + \sum_{\substack{j=1 \\ i \ne j}}^{N} g_{ij} x_j \right]^{T} P_{i\mu}^{+} \left[(\tilde{A}_{i\mu} + \tilde{B}_{i\mu} k_{i\mu}) x_i + E_{i\mu} d_i + \sum_{\substack{j=1 \\ i \ne j}}^{N} g_{ij} x_j \right] \\ & - x_i^{T} P_{i\mu} x_i + x_i^{T} Q x_i \\ & + x_i^{T} k_{i\mu}^{T} R k_{i\mu} x_i - \tau_i d_i^{T} - (a-1) \sum_{\substack{j=1 \\ i \ne j}}^{N} x_j^{T} g_{ij}^{T} P_i g_{ij} x_j \\ & + (a-1) \sum_{\substack{j=1 \\ i \ne j}}^{N} x_j^{\mathrm{T}} g_{ij}^{\mathrm{T}} P_i g_{ij} x_j \\ & + x_i^{\mathrm{T}} \sqrt{\alpha} (\tilde{A}_{i\mu} + \tilde{B}_{i\mu} k_{i\mu})^{\mathrm{T}} P_i \left(\sum_{\substack{j=1 \\ i \ne j}}^{N} g_{ij} x_j \right) \end{aligned}$$

$$-x_i^{\mathrm{T}}\sqrt{\alpha}(\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu})^{\mathrm{T}}P_i\left(\sum_{\substack{j=1\\ i\neq j}}^{N} g_{ij}x_j\right)$$

$$+\sqrt{\alpha}\left(\sum_{\substack{j=1\\ i\neq j}}^{N} x_j^{\mathrm{T}}g_{ij}^{\mathrm{T}}\right)P_i(\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu})x_i$$

$$\left.-\sqrt{\alpha}\left(\sum_{\substack{j=1\\ i\neq j}}^{N} x_j^{\mathrm{T}}g_{ij}^{\mathrm{T}}\right)P_i(\tilde{A}_{i\mu}+\tilde{B}_{i\mu}k_{i\mu})x_i\right\} < 0 \quad \text{(B.4)}$$

now, the inequality (B.4) is

$$\sum_{i=1}^{N}\left\{\left[\tilde{\Theta}_i x_i + E_{i\mu}d_i + \sum_{\substack{j=1\\ i\neq j}}^{N} g_{ij}x_j\right]^{\mathrm{T}}\right.$$

$$P_{i\mu}^{+}\left[\tilde{\Theta}_i x_i + E_{i\mu}d_i + \sum_{\substack{j=1\\ i\neq j}}^{N} g_{ij}x_j\right]$$

$$\left.-x_i^{\mathrm{T}}P_{i\mu}x_i + x_i^{\mathrm{T}}Qx_i + u_i^{\mathrm{T}}Ru_i - \tau_i d_i^{\mathrm{T}}d_i\right\} < 0 \quad \text{(B.5)}$$

where $\sum_{i=1}^{N} V(x_i^{+}) - V(x_i) < -\sum_{i=1}^{N}\psi_i(\cdot)$. Thereby, the proof is completed. ■

For convenience, a table of notations and symbols is provided (Table A1):

Table A1. Abbreviations and notations used in theorems and appendices.

Notation	Definition	Notation	Definition
ξ_i	Positive scalar variable	P_i	Positive matrix
τ_i	Positive scalar	N_i	$N_i = \frac{\xi_i}{\eta_i^2}$
α	Positive scalar	η_i^2	Positive scalar
M_i	$\xi_i R$	R	Positive weight
N	Number of subsystems	Q	Positive weight
λ_i	Positive scalar	X_j	$X_j = \xi_i P_j$
X_i	$X_i = \xi_i P_i$	Ts	Sampling time