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Simulation of leaching processes of polymetallic ores using the similarity theorem

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Abstract

The use of similarity theorems for simulation of the technological process of mineral extraction is considered. The list of parameters that significantly influence the process of underground leaching of minerals is defined. Using these parameters and fundamental laws of physics and chemistry, mathematical functions are determined to describe the processes' behaviour under these conditions. The obtained mathematical functions make it possible to develop a computer model of polymetallic ores leaching. This allows for the prediction of the volume of extracted concentrate with minerals from the ore mass with the associated compounds. The obtained results of calculations showed a change in the volume of minerals extracted from the rock mass depending on the mass of the working agent, the volume of leached ore and the solvent percolation rate. The results of the research can be used at mining enterprises to extract polymetallic ores by underground leaching. Also, they allow for the estimation of the economic issues from mining the ore reserves.

Keywords:
similarity theorem; P-theorem; uranium ore; rock mass; leaching model; concentrate

1. Introduction

Previous studies have shown that, despite the volatile political situation in Ukraine, coal (thermal energy generation) and uranium (nuclear energy) will remain the main energy sources in Ukraine. Moreover, the proposed technological methods of extracting coal are oriented on the combination of technologies, its extraction in complicated mining and geological conditions, increasing the environmental quality of this process and processing wastes, already extracted on the surface (**English, 2002; Griadushchiy et al., 2007; Gorova et al., 2012; Dychkovskyi et al., 2018a**). It is proposed to improve both traditional and new radical technologies in which mineral source is transferred to another aggregate state by means of complex thermochemical transformations (**Khadse et al., 2006; Dychkovskyi et al., 2018b**). Particular attention is paid to the stresses and mining pressure management (**Dychkovskyi 2015, a; Sobolev et al., 2020**). The creation of artificial shells in the rock mass to ensure the passage of blowing mixtures is an effective way to obtain the appropriate material and thermal balance of gasification (**Dychkovskyi 2015b**).

In this paper, the main attention is paid to the extraction of uranium raw materials. Long-term exploitation leads to the exhaustion of rich ores and the deepening of the mining depth.

For efficient extraction of poor and very poor polymetallic ores using an underground leaching method, it is necessary to:

- analyze the technological and mining-geological parameters that influence this process;
- consider the working agent influence on the recoverable ore volume;
- investigate the influence of the solvent penetration rate on the volume of extracted concentrate from the rock mass together with the associate compounds;

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– investigate the regularities of volumes of mineral extraction from the rock mass.

In the process of extraction, the technological parameters are individual for each case (**Sala and Bieda, 2019**) and have a wide range, so their exact definition significantly affects the cost of the product (**Sdvyzhkova et al., 2020; Kalybekov et al., 2020; Rysbekov et al., 2022; Pysmennyi et al., 2022**). Finding the values of the technological parameters of leaching is reduced to the successive execution of the following steps:

1. Consider the similarity theorems and determine the theorems that would describe the conditions of underground leaching with certain known input data;
2. Determine the physical and technological parameters that significantly affect the leaching process;
3. Form independent, dimensionless combinations describing leaching processes taking into account fundamental physical and chemical laws;
4. Obtain an equation that allows us to calculate the volume of extracted minerals with associate compounds from the leaching of rock mass;
5. Create an experimental model using the software;
6. Carry out the simulation of the volume of the extracted concentrate (mineral) from the rock mass with certain technological parameters, physical & mechanical properties of the rock mass and the volume of working agent.

After these steps are completed, it is possible to predict the volume of extracted minerals from the rock mass by underground leaching method for the poor and extremely poor ores. Such approaches correspond to classic methodology of research, used for modelling complex systems related to the mining geological and mining engineering systems. They are sufficiently presented in specialized literature (**Krzanowski, R. M. and Raper, J., 2001, Aleksakhin, A. et al., 2021**).

The scientists who have described and applied the similarity theorems for mining modeling (**Bommer and Schechter, 1979; Schechter and Bommer, 1982, Kabir et al., 1985**; **Bondarenko et al., 2012; Golovchenko et al., 2020**). In their research papers, we can see the general features of research methods that are devoted to the deterministic models of underground leaching. Different hypotheses of the rock mass permeability and stationary conditions of the working agent movement are used in their research, as well as Darcy's law with the flow continuity equation to explain the flow of fluids. Further on, the working tubes of the working agent in a rock mass will be constructed where of the leached rock mass parameters serve as limitations. Then models of chemical kinetics interaction and mass transfer of leached products with reagent are formed with the subsequent simplification and corresponding calculation of concentration of the obtained minerals in the productive solution. Other scientists used numerical analysis that describes the leaching model in the plan and in the cross-section of a rock mass (**Zheng, F. et al., 2020**). However, most scientists almost didn't consider the fluctuation phenomena in the intermolecular space while creating the simulation of the kinetics of the solution flow (**Vladyko & Kaliushko, 2013**).

All the above-mentioned research was carried out to predict the extraction technological parameters, the amount of costs or profit from minerals extraction. The technological capability of the technology can also be determined by means of special software that has been tested for the extraction of the coal from thin seams (**Vladyko et al., 2012; Pivnyak et al., 2013; Ghazdali et al., 2021; Turumbetov 2022**). However, no one demonstrates the creation of a model for calculating the values of leaching parameters, such as the volume of leaching solution from the compatibility of technological parameters, as enterprises at the present stage of development need similar leaching models that would provide an accurate prediction of the volume of the obtained mineral at the expected time. The main purpose of this study is to determine the planned volumes of extracted uranium that is involved in the process of leaching from the mass of the working agent, and the speed of its penetration through the ore, using the P-model.

2. Method and numerical analysis

2.1. Brief description of the technology

The underground leaching method is used to extract uranium from deposits where the mineral is in a complicated geological environment and the ore is graded as poor and/or extremely poor in terms of value. Such technology presupposes the extraction of a mineral with the help of working agents that leach out the mineral, and then, under its own weight it flows down to the lower part of the stoping area where it is removed by the drainage system (**Rakishev et al., 2020; Yussupov et al., 2021; Tsoy et al., 2021**). A weak solution of acid, carbonate (bicarbonate), ammonium, sodium, potassium, calcium, and magnesium can be used as a working agent. The effectiveness of such a leaching process depends on the type of uranium mineralization, which may consist of uranium oxide (nasturane, uranium resin), uranium silicate (coffinite) and their combination. Accordingly, certain technological parameters were determined, and the technological scheme of leaching process was developed (see **Figure 1**).

Figure 1 shows the movement of the solution from the surface to the top part of the stoping area (blue arrow). The productive solution passing through a rock mass is pumped through the riser to the upper horizon, and then to the surface (red arrow with a dot). To further determine the technological parameters of the leaching process, let's consider the flow of the solution through the ore mass in zooming upscale (see **Figure 2**)

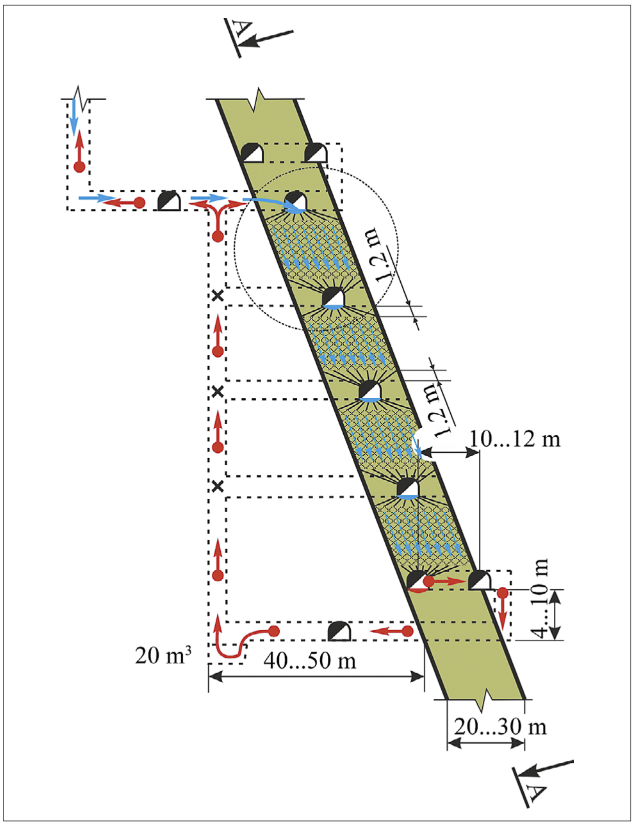

Figure 1: Flow chart of the working agent flow during underground leaching of uranium using a sulfuric acid solution: →, ●→ – the direction of the solution movement from the surface to the stoping area (blue arrow), and from the leaching chamber together with the concentrate to the surface (red arrows with the dot)

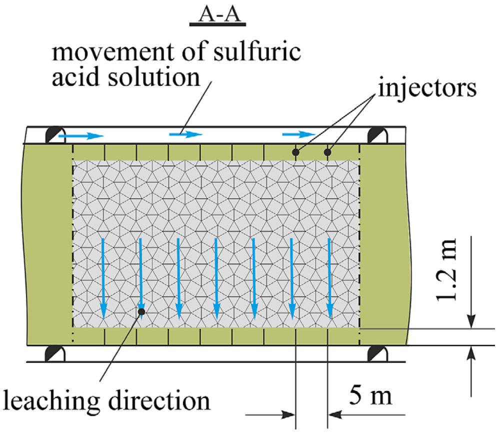

Figure 2: Flow chart of movement of the working agent through the rock mass during underground leaching

Figure 2 gives a certain explanation of the analytical model being developed. Blue solid arrows from the top to the bottom show the movement of the solution through an ore mass after drilling-and-blasting operations. Drilling and blasting are an important part of the mine planning and management and widely adopted techniques for overburden removal and ore excavation (**Shcherbakov et al., 2021; Serdaliyev and Iskakov, 2022**). One of the drilling methods was proposed by the scientists of Dnipro University of Technology (**Dudlia et al., 2018; Shyrin et al., 2018**). Water control was also justified (**Caceres and Alca, 2016; Bazaluk et al., 2021; Rudakov and Westermann, 2021**). So, having considered the spatial location of the leached area, we will turn to the definition of technological parameters. The determination of leaching parameters requires analytical studies of the processes of mineral extraction from the ore mass. There are several similarity methods to perform this task, one of which was chosen in the current study.

2.2. Choice of similarity theorems to solve the set tasks

As experimental and theoretical studies (**Nanosov and Resin, 1999**) show that the nonlinear transfer processes have specific features that are absent in the linear ones. Therefore, the corresponding experimental and theoretical problems in these cases can't be adequately solved within the framework of linear analysis exclusively, however it is possible with the nonlinear since the process of liquid transfer can occur physically on different scales.

In some cases, the use of the theory of dimension (P-theorem) reduces the number of independent variables, using self-similar equations in mathematical physics. Wide possibilities of applying this theory allow its use for solving the problems of motion of the laminar boundary layer in liquid. In this connection, the P-theorem becomes particularly important, which allows us to carry out a factor analysis of the physical phenomena inherent in the leaching process, thereby significantly simplifying the number of calculations in the model (**Maltsev, 2015; Maltsev et al., 2018**).

Let's consider the basic similarity theorems: the first, second, and third theorems of similarity. We found them particularly interesting for the perfect mathematical apparatus that describes research processes and many data calculations in various fields of science, since they solve physical and technical problems where the phenomenon under study is described not by ordinary dimensional values, but by dimensionless complexes – similarity criteria that allow for the study of whole groups of similar phenomena (**Nanosov and Resin, 1999**). Necessary and sufficient conditions for the existence of similarity are required to ensure the reliability of research results obtained with the models under a limited number of experiments with various combinations of parameters. These conditions are reflected in three theorems of similarity. The first and third theorems define the necessary conditions of similarity, the second one – necessary and sufficient conditions.

The first theorem of similarity is Newton's theorem, which states that similar phenomena have homogeneous parameters, and corresponding values of homogeneous variables are related to each other by means of equality of similarity criteria of the same name – the dimensionless complexes of values appearing in dimensionless equations and describing these phenomena. This theorem states that similarity criteria must be the same for similar phenomena. It does not indicate how to establish similarity and implement it, but only forms the necessary conditions for its existence (identical criteria of similarity) (e.g. **Ruppeneit and Liberman, 1961**).

The second similarity theorem is Federman-Buckingham theorem which states that the research results should be presented as dependencies between the criteria. The functional dependency between the similarity criteria is called the criterion equation, which describes the whole group of similar processes. This is of great practical value and allows for the simulation of an industrial object in a similar laboratory model. The type of the criterion equation is determined experimentally. In many cases, this dependency is represented in the form of a statistical function, in which the complete equation of the physical process recorded in a certain system of units can be represented by the relationship between the similarity criteria, that is, the dependency that binds dimensionless quantities which are in some way derived from the parameters existing in the process. From this it follows that if the functional dependency of this phenomenon (parameters, factors) is known, but its mathematical description is unknown, then similarity criteria can be obtained. This mathematic apparatus was successfully used for the physical modelling of various phenomena in aerodynamics, hydrodynamics, the theory of elasticity, the theory of oscillations, etc. (**Hanche-Olsen, 1981**).

The third similarity theorem states that the criterion equations are applied only to similar processes. The phenomena are similar if their influencing criteria are numerically equal, and therefore the criteria that are determined are also equal. This theorem determines the necessary and sufficient conditions for the similarity of physical phenomena. It states that the necessary and sufficient conditions for the creation of similarity are the proportionality of similar parameters included in the conditions of uniqueness, and the equality of criteria for the similarity of the studied phenomenon.

The conditions of uniqueness are the conditions that determine the individual characteristics of the phenomenon, for example, the initial, boundary or edge conditions and don't depend on the mechanism of the phenomenon itself (**Hanche-Olsen and Holden, 2010**).

Based on this, we can say that the study of processes with the method of similarity theory consists in obtaining their mathematical description by means of differential equations and single-valuedness conditions, their transformation into the criterion equation and finding a particular type based on the experimental study of the process.

The complexity of the processes taking place during leaching, and their mutual influence and multifactorial nature complicate the use of analytical methods. Semiempirical methods offer great opportunities in this direction, among them the theory of dimensions, based on fundamental physical laws (**Ruppeneit and Liberman, 1961; Mushelishvili, 1966; Chernov, 1997**). The theory of dimensions is based on the Pi-theorem, the basic theorem of dimensional analysis. The theorem states that if there is a dependence between n physical quantities that does not change when the scale of units is changed in a certain class of unit systems, then it is equivalent to a relationship with a smaller number p=n-k of dimensionless variables, where k is the largest number of values with independent dimensions among the initial n quantities. The P-theorem allows us to establish a general structure of dependence, which proceeds only from the requirement of invariance of physical dependence when the scale of units is changed, even if the specific type of dependence between the initial values is unknown. That is, in accordance with the P-theorem, the connection between independent physical quantities would have a certain physical content, for this, it is necessary to represent it as a function of dimensionless combinations of these values. In other words, with the help of P-theorem, it is possible to obtain the desired connection in the form of functions with sufficient accuracy to the constant product, avoiding the specific dependence between the values. We use this theory to obtain links between physical quantities that influence the leaching process in certain mining-geological conditions. The analysis of the parameters of the leaching process for poor and very poor uranium ores has shown that leaching in the ore of the albite formation has several features, the description of which is complicated by a large array of data, which are used in practice and give significant deviations. Therefore, the use of P-theorem satisfies our needs and to ensure the complete similarity of a model to a real object, the following conditions are necessary:

- the processes in the model and the sample belong to one class of phenomena (the phenomena are qualitatively identical) and, accordingly, these processes are described by the same equations;
- qualitatively similar conditions of uniqueness (geometric, initial, boundary conditions);
- the determinant similarity criteria (dimensionless complexes constructed from scale factors and task parameters) at the corresponding time at the corresponding points are numerically equal.

Along with meeting the conditions, it is necessary to determine the parameters that influence the leaching process, and which are necessary for the simulation. Therefore, we will determine their list. The physical values (factors) that influence the leaching process are as follows (**Dychkovskyi et al., 2018a**):

- the weight of the leached ore, m_{ore}, kg;
- the volume of the leached ore, V_{ore}, m^3;
- ore density, ρ_{ore}, kg/m^3;

- the weight of the working agent involved in the leaching, m_{agent}, kg;
- the volume of the working agent involved in the leaching, V_{agent}, m^3;
- the density of the working agent, γ_{agent}, kg/m^3;
- the porosity of the leached ore mass, P_{ore}, unit fraction;
- the concentration of the solvent in the working agent, C_{agent}, unit fraction;
- the temperature of the leached ore mass, t_{ore}, ºC;
- impurity content that affects the course of chemical reactions, c_{ore}, % (kg/m^3);
- cross-section that the working agent has to pass (penetrate, contact), S_{ore}, m^2;
- concentration of uranium inclusions in the mass, Q_{ore}, kg/m^3;
- the volume of extracted uranium concentrates from the rock mass with associated compounds, V_U, m^3;
- solvent penetration rate through the rock mass, ε, m/hour;
- solvent diffusion rate into the depth of an ore mass, υ_{Df}, m/hour;
- total leaching time, T_{leach}, hour;
- the size of pores and cracks, D_p, m;
- the pressure of solvent injection into the leached rock mass, P_{agent}, kg/m^2;
- ore moisture, φ, % (unit fraction, g/m^3);
- average size (diameter) of uranium-containing inclusions (grains), e_d, m;
- coefficient of influence of useful sorption processes during leaching, k_{sorb}, % (unit fraction);
- effective surface of inclusions (grains) with the leached mineral, e_f, m^2;
- mineral content in the mass, α, % (unit fraction, g/m^3).

2.3. *Equations and solutions for modelling, using the P-theorem*

Having determined these factors, it becomes possible to move on to into the formation of dimensionless combinations based on fundamental physical laws to provide the P-theorem. The resulting non-dimensional combinations will have the following form. The main combination

$$\frac{V_U}{V_{agent}}\left[\frac{m^3}{m^3}\right] \tag{1}$$

covers (describes) the volume of uranium concentrate obtained with concomitant compounds and the working agent passing through it. The first combination takes into account the leaching process duration and the rock mass property influencing the process

$$\frac{\varepsilon \cdot T_{leach} \cdot S_{ore} \cdot P_{ore}}{V_{ore}}\left[\frac{m/hour \cdot hour \cdot m^2 \cdot unit\ fraction}{m^3}\right] \tag{2}$$

– in other words, it includes the solvent flow rate (permeability) through the rock mass, the total time of its staying in the rock mass, the useful cross-section area through which working agent passes, the porosity coefficient and volume of the ore (rock mass volume with a constant volume of uranium-bearing compounds) leached out (explains the movement of the solution through the rock mass, taking into account its specific features). The second combination describes the ratio of the working agent to the leached ore mass (the absolute mineral content in the mass). Where the volume of mineral is equal to the product of the density by ore volume by the mineral content in it

$$m_u = \rho_{ore} \cdot V_{ore} \cdot \alpha \tag{3}$$

And relative mass of the working agent to the mass of mineral, which allows for it to be presented as follows

$$\frac{\rho_{ore} \cdot V_{ore} \cdot \alpha}{m_{agent}}\left[\frac{kg/m^3 \cdot m^3 \cdot unit\ fraction}{kg}\right] \tag{4}$$

– coefficient, which includes the volume, ore density and the mass of the working agent passing through it (in essence, this partly explains the change in the properties of the working agent during leaching). The third and the last combination considers two related processes. The first is a process that reflects the penetration rate of a working agent into the rock mass due to its injection or under its own weight. Excessive pressure greatly accelerates the saturation of the mass with the reagent and improves its interaction with uranium compounds, and consequently, positively affects diffusion processes (on the physical level). On the other hand, the proportion of mineral in the unit weight of working agent with other equal conditions varies depending on the total weight of the solvent which passes through the ore mass. The second is a process that reflects the leaching process depending on the effective area of interaction with the mineral. Thus, the mineral to be obtained must be dependent on the following factors

$$V_U = f\left(m_{agent}, P_{agent}, e_f, k_{sorb}\right) \tag{5}$$

Based on this, we can formulate the last independent dimensionless combination, which will have the following form

$$\frac{P_{agent} \cdot e_f \cdot k_{sorb}}{m_{agent}}\left[\frac{kg/m^2 \cdot m^2 \cdot unit\ fraction}{kg}\right] \tag{6}$$

which will reflect the solvent injection pressure, its weight, diffusion and sorption processes.

Consequently, the above equations of dimensionless combinations of uranium concentrate extraction V_U depend on the following factors:

$$V_U = f\left(V_{agent}, V_{ore}, \varepsilon, T_{leach}, S_{ore}, P_{ore}, \rho_{ore}, V_{ore}, m_{agent}, P_{agent}, e_f, k_{sorb}\right) \tag{7}$$

We will record the obtained combinations from Equation 1 in a certain sequence of importance

$$\frac{V_U}{V_{agent}} \quad \frac{\varepsilon \cdot T_{leach} \cdot S_{ore} \cdot P_{ore}}{V_{ore}} \quad \frac{m_{agent}}{\rho_{ore} \cdot V_{ore}} \quad \frac{P_{agent} \cdot e_f \cdot k_{sorb}}{m_{agent}} \tag{8}$$

According to the P-theorem, there should be the following dependence between them:

$$\frac{V_U}{V_{agent}} = \Phi\left(\frac{\varepsilon \cdot T_{leach} \cdot S_{ore} \cdot P_{ore}}{V_{ore}}; \frac{m_{agent}}{\rho_{ore} \cdot V_{ore}}; \frac{P_{agent} \cdot e_f \cdot k_{sorb}}{m_{agent}}\right) \tag{9}$$

Where (**Maltsev, and Vladyko, 2015; Kravchenko, 2008**):

V_U ~ 200–250 m^3;
V_{ore} ~ 87000–88000 m^3,
V_{agent} ~ 600–650 m^3,
ρ_{ore} ~ 2.5–2.7 t/m^3,
m_{agent} ~ 5.0·10^5–5.5·10^{5}kg,
ε ~ 0,005–0,05 m/hour,
T_{leach} ~ 2160–4320 hour,
S_{ore} ~ 1100–1300 m^3,
P_{ore} ~ 0.04–0.09 unit fraction,
k_{sorb} ~ 0.95–0.90 unit fraction,
P_{agent} ~ 0.3–0.5 kg/m^2,
E_f ~ 9–11 m^2.

Taking into account that the variables in the dependence are at certain intervals, one can establish that the powers of dimensionless ratios have the following values:

$$\frac{V_U}{V_{agent}} \sim 10^{-1}; \ \frac{\varepsilon \cdot T_{leach} \cdot S_{ore} \cdot P_{ore}}{V_{ore}} \sim 10^{-2};$$

$$\frac{m_{agent}}{\rho_{ore} \cdot V_{ore}} \sim 10^{-3}; \ \frac{P_{agent} \cdot e_f \cdot k_{sorb}}{m_{agent}} \sim 10^{-6}. \tag{10}$$

In this view, the connecting function can be expanded by the smallest value in the Taylor's and Macloren's (**Nanosov and Resin, 1999**) series by the combination $\dfrac{P_{agent} \cdot e_f \cdot k_{sorb}}{m_{agent}}$

$$\frac{V_U}{V_{ageng}} = f\left(\frac{\varepsilon \cdot T_{leach} \cdot S_{ore} \cdot P_{ore}}{V_{ore}}; \frac{m_{agent}}{\rho_{ore} \cdot V_{ore}}; 0\right) +$$

$$+ f\left(\frac{\varepsilon \cdot T_{leach} \cdot S_{ore} \cdot P_{ore}}{V_{ore}}; \frac{m_{agent}}{\rho_{ore} \cdot V_{ore}}; 0\right) \cdot \frac{P_{agent} \cdot e_f \cdot k_{sorb}}{m_{agent}} + \ldots \tag{11}$$

Provided that $m_{agent} \to \infty$ and $V_U \neq 0$ the second and subsequent addends will give a small value that can be neglected. Accordingly, we obtain an equation in the form of:

$$\frac{V_U}{V_{agent}} = f\left(\frac{\varepsilon \cdot T_{leach} \cdot S_{ore} \cdot P_{ore}}{V_{ore}}; \frac{m_{agent}}{\rho_{ore} \cdot V_{ore}}\right) =$$

$$= \Phi\left(\frac{\varepsilon \cdot T_{leach} \cdot S_{ore} \cdot P_{ore}}{V_{ore}}; \frac{m_{agent}}{\rho_{ore} \cdot V_{ore}}\right) \tag{12}$$

Experience shows that the volume of the leaked solution and the volume of uranium raw materials don't act interdepend linearly, thus the function should be represented as the sum of the analytic *f* and non-analytic *φ* functions

$$\frac{V_U}{V_{agent}} = f\left(\frac{\varepsilon \cdot T_{leach} \cdot S_{ore} \cdot P_{ore}}{V_{ore}}; \frac{m_{agent}}{\rho_{ore} \cdot V_{ore}}\right) +$$

$$+ \varphi\left(\frac{\varepsilon \cdot T_{leach} \cdot S_{ore} \cdot P_{ore}}{V_{ore}}; \frac{m_{agent}}{\rho_{ore} \cdot V_{ore}}\right) \tag{13}$$

Taking into account only the main parts and regarding the small number of inner functions it contains, the following expression can represent (**Nanosov and Resin, 1999**):

$$f = a\frac{\varepsilon \cdot T_{leach} \cdot S_{ore} \cdot P_{ore}}{V_{ore}} + b\frac{m_{agent}}{\rho_{ore} \cdot V_{ore}};$$

$$\varphi = \left(c\frac{\varepsilon \cdot T_{leach} \cdot S_{ore} \cdot P_{ore}}{V_{ore}} + d\frac{m_{agent}}{\rho_{ore} \cdot V_{ore}} + e\frac{P_{agent} \cdot e_f \cdot k_{sorb}}{m_{agent}}\right)^{\delta} \tag{14}$$

where *a*, *b*, *c*, *d*, *e* – are constant values; $0 < \delta < 1$.

Thus, we get

$$\frac{V_U}{V_{agent}} = a\frac{\varepsilon \cdot T_{leach} \cdot S_{ore} \cdot P_{ore}}{V_{ore}} + b\frac{m_{agent}}{\rho_{ore} \cdot V_{ore}} +$$

$$+ \left(c\frac{\varepsilon \cdot T_{leach} \cdot S_{ore} \cdot P_{ore}}{V_{ore}} + d\frac{m_{agent}}{\rho_{ore} \cdot V_{ore}} + e\frac{P_{agent} \cdot e_f \cdot k_{sorb}}{m_{agent}}\right)^{\delta} \tag{15}$$

To determine so many constant values from one equation, we use the created experimental-calculation model for finding the unknown values using the Mathcad program. The averaged values of the input data are the following values of the combination coefficients **Equation 15**. The powers of coefficients constant values will be: a=2.641; b=29.261; c=1.6·10^{-4}; d=5.532·10^{-7}; e=5.53·10^{-7}.

The additions on the right side of **Equation 15** with constant coefficients *c*, *d*, and *e* in terms of smallness are much smaller than the values with constant coefficients *a* and *b*, so they can be neglected. So, we get the following

$$\frac{V_U}{V_{agent}} \approx a\frac{\varepsilon \cdot T_{leach} \cdot S_{ore} \cdot P_{ore}}{V_{ore}} + b\frac{m_{agent}}{\rho_{ore} \cdot V_{ore}} \tag{16}$$

For the convenience of determining the constant values we convert the resulting equation to the form

$$\Delta = a\frac{\varepsilon \cdot T_{leach} \cdot S_{ore} \cdot P_{ore}}{V_{ore}} + b\frac{m_{agent}}{\rho_{ore} \cdot V_{ore}} - \frac{V_U}{V_{ageng}} \tag{17}$$

where *Δ* – is infinitesimal value for certain values *a* and *b*, which, according to the method of least squares, are determined from the conditions

$$\sum\nolimits_1^n \Delta_i \frac{\delta \Delta_i}{\delta a} = 0 \quad \sum\nolimits_1^n \Delta_i \frac{\delta \Delta_i}{\delta b} = 0 \tag{18}$$

where *n* – is the amount of experimental data for the determination of constant values.

Table 1: Output data from leaching processes to determine the constant coefficients

#	V_U, t	T_{leach}, days	V_{ore}, m^3	ρ_{ore}, t/m^3	S_{ore}, m^2	P_{ore}, MPa	ε, m/hour	m_{agent}, t	V_{agent}, m^3
1	60.44	80	88000	2.25	1000	1	0.045	503.65	387.42
2	45.04	80	66000	2.34	825	1	0.028	390.34	300.26
3	66.60	80	96800	2.38	1210	1	0.029	577.18	443.98
4	63.52	80	92400	2.27	1155	1	0.033	559.90	430.69
5	54.95	80	80000	2.30	1000	1	0.046	461.93	355.33
6	49.35	80	72000	2.32	900	1	0.035	423.99	326.15
7	49.34	80	72080	2.45	901	1	0.047	423.93	326.10
8	54.23	80	79040	2.27	969	1	0.038	455.93	350.71
9	59.34	80	86400	2.42	1080	1	0.045	531.85	409.12
10	69.68	80	101200	2.30	1265	1	0.044	593.54	456.57

For these conditions (18) we obtain a system of two equations

$$\begin{cases} a\sum_{1}^{n}\left[\frac{\varepsilon\cdot T_{leach}\cdot S_{ore}\cdot P_{ore}}{V_{ore}}\right]^2+b\sum_{1}^{n}\left(\frac{m_{agent}}{\rho_{ore}\cdot V_{ore}}\right)\cdot\left(\frac{\varepsilon\cdot T_{leach}\cdot S_{ore}\cdot P_{ore}}{V_{ore}}\right)= \\ =\sum_{1}^{n}\left(\frac{V_U}{V_{agent}}\right)\cdot\left(\frac{\varepsilon\cdot T_{leach}\cdot S_{ore}\cdot P_{ore}}{V_{ore}}\right) \\ a\sum_{1}^{n}\left(\frac{\varepsilon\cdot T_{leach}\cdot S_{ore}\cdot P_{ore}}{V_{ore}}\right)\cdot\left(\frac{m_{agent}}{\rho_{ore}\cdot V_{ore}}\right)+b\sum_{1}^{n}\left[\frac{m_{agent}}{\rho_{ore}\cdot V_{ore}}\right]^2= \\ =\sum_{1}^{n}\left(\frac{V_U}{V_{agent}}\right)\left(\frac{m_{agent}}{\rho_{ore}\cdot V_{ore}}\right) \end{cases} \tag{19}$$

Let's determine the constant value *a*

$$a=\frac{\left(V_U\cdot P_{ore}\cdot V_{ore}-b\cdot m_{agent}\cdot V_{agent}\right)\cdot\varepsilon\cdot T_{leach}\cdot S_{ore}}{V_{agent}} \tag{20}$$

but after substituting this equation into the following equation, it is impossible to find the value of constant *b*, because it's reduced.

Thus, the volume of obtained uranium, depending on the main influential factors, can be determined from the equation (16) with the constant coefficient's values calculated in Mathcad with the developed model

$$V_U=a\frac{\varepsilon\cdot T_{leach}\cdot S_{ore}\cdot P_{ore}\cdot V_{agent}}{V_{ore}}+b\frac{m_{agent}\cdot V_{agent}}{\rho_{ore}\cdot V_{ore}} \tag{21}$$

3. Simulation results and discussion

We use the predicted leaching data (**Maltsev et al., 2018**) obtained in the research (**Maltsev, 2015**) to determine the constant coefficients *a* and *b*, and tabulate these data (see **Table 1**).

To find the values of constant coefficients *a* and *b,* we use **Equation 15**. To verify the correctness of **Equation 22** and the obtained values of coefficient *a* and *b*, we will respectively verify by finding V_U, and put these results into **Table 2** by five calculations, which is sufficient to confirm or refute the hypothesis. The value of V_U is found in **Equation 15**, which will look like

$$V_U=a\frac{\varepsilon\cdot T_{leach}\cdot S_{ore}\cdot P_{ore}\cdot V_{agent}}{V_{ore}}+b\frac{m_{agent}\cdot V_{agent}}{\rho_{ore}\cdot V_{ore}}+$$
$$+V_{agent}\cdot\left(c\frac{\varepsilon\cdot T_{leach}\cdot S_{ore}\cdot P_{ore}}{V_{ore}}+d\frac{m_{agent}}{\rho_{ore}\cdot V_{ore}}+e\frac{P_{agent}\cdot e_f\cdot k_{sorb}}{m_{agent}}\right)^{\delta} \tag{22}$$

Table 2: Verifying V_U values by two equations: complete – Equation 15 and simplified – Equation 16

V_U (15)	60.444	45.037	66.603	63.501	54.909
V_U (16)	60.448	45.037	66.601	63.502	54.952

From the verification calculations, it is evident that both equations correctly find the volume of extracted uranium from the rock mass with compounds, with an error less than 1% and can be equally applicable, so **Equation 21** will be applied. The obtained values of constant coefficients can be applied to predict the volume of extracted uranium with the associated compounds during the recovery of the poor and extremely poor uranium ores of the albite formation.

Consequently, having these analytical equations it is possible to construct graphs which show the change in the volume of uranium concentrate, depending on various technological parameters, some physical and mechanical properties of the rock mass and the working agent. The first parameter to set is the mass of the working agent involved in the leaching process m_{agent}. Its mass and the calculated values of the uranium concentrate with the related compounds V_U will be presented in **Table 3**, and to determine the type of dependence, the graph was created (see **Figure 3**).

The results of calculating the change in the volume of uranium concentrate obtained from the rock mass with compounds V_U with the changes of the mass of working agent involved in the leaching process m_{agent} indicate that with an increase of the mass of the working agent, the volume of the extracted mineral increases in direct pro-

Table 3: The volume of the extracted uranium with compounds V_U over the mass of working agent involved in the leaching process m_{agent}

m_{agent}, t	200	300	400	500	600	700	800	900	1000	1100
V_U, t	34.21	39.89	45.58	51.27	56.96	62.64	68.33	74.02	79.71	85.40

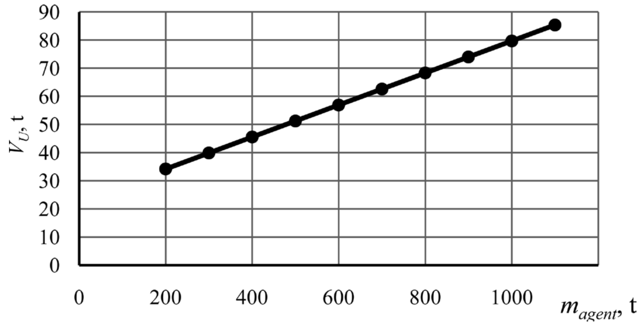

Figure 3: Dependence of changes in the volume of extracted uranium with compounds V_U on the mass of working agent involved in the leaching process m_{agent}.

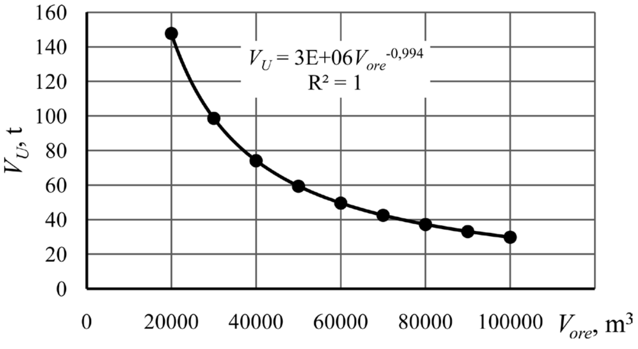

Figure 4: Dependence of the volume of uranium with associated compounds V_U extracted from the rock mass on the volume of leached ore V_{ore}

Table 4: The volume of the extracted uranium with compounds V_U over the leached ore volume V_{ore}

V_{ore}, m^3	20000	30000	40000	50000	60000	70000	80000	90000	100000
V_U, t	147.67	98.62	74.07	59.34	49.51	42.48	37.21	33.11	29.82

portion, but this growth cannot last indefinitely, since there are limits to the application of this graph: on the one hand, the volume of extracted uranium concentrate cannot exceed the volume available in the rock mass and, on the other, the limit of economic conditions.

The last factor proposed to change is solvent penetration rate through the mass ε. This rate along with calculated volumes of uranium concentrate with related compounds V_U is listed in **Table 5**, and the dependence is constructed, accordingly (see **Figure 5**).

Table 5: The calculated volume of the extracted uranium with compounds V_U over the solvent penetration rate through the mass ε

ε, m/hour	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045
V_U, t	26.43	30.51	34.56	38.59	42.62	46.64	50.66	54.67	58.68

The next variable factor is the ore volume V_{ore} that can be changed for monitoring the predicted extracted volume of uranium with associated compounds V_U. The results are shown in **Table 4**, and, also, to determine the type of dependence, a graph is constructed (see **Figure 4**).

The calculation results of the obtained volume of uranium concentrate V_U in relation to changing volume of leached ore V_{ore} demonstrate certain dependence where with the increase of volume of ore mass, the volume of mineral extracted from the rock mass decreases by power law dependence with the maximum level of approximation. However, this reduction cannot last indefinitely, because we receive a certain volume of minerals because of leaching process. In addition, the increase in ore volume implies changes in the values of other indicators, such as: mass of the working agent involved in the leaching process, the volume of uranium compounds in the mass, etc., which will constantly adjust the calculation results.

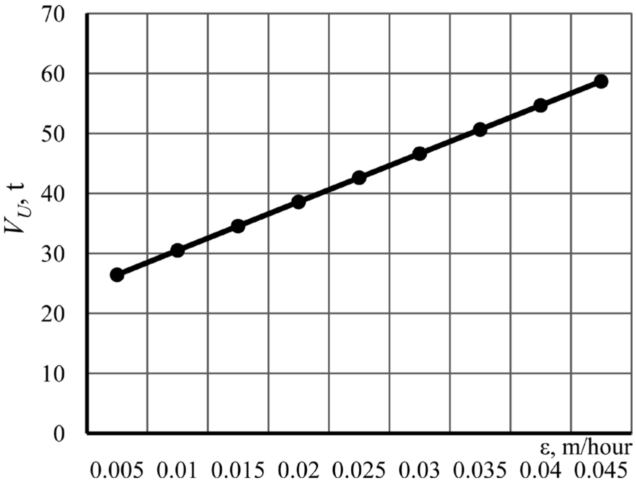

Figure 5: Dependence of the volume of uranium extracted from the rock mass with related compounds V_U on the solvent penetration rate through the mass ε

The results of calculating the volume of uranium extracted from the rock mass with related compounds V_U over the solvent penetration rate through the mass ε demonstrate a direct proportional dependence between these two factors. Such an increase is possible due to the increase in the contact surface of the working agent with uranium-bearing compounds (uranium resin) in the rock mass, as well as due to unimpeded extraction of minerals from it. At the same time, such growth cannot last indefinitely, since it is limited on the one hand by the volume of mineral that cannot exceed the available volume in the rock mass and, on the other, by the properties of the mass.

Thus, dependencies were obtained that allow us to determine the volume of mineral on the mass of working agent, the rate of its penetration through the ore, and ore mass volume, as well as to predict the parameters of developing poor and extremely poor uranium ores by methods of underground sulfuric acid leaching.

4. Conclusions

The paper solves one of the unresolved issues in underground leaching – prediction of changes in the volume of uranium concentrate considering certain input data. This solution is proposed through the development of the leaching process model based on a specific list of technological parameters and geological features. The novelty of this article is in the formation of models with the help of the P-theorem and the establishing dependences of the extracted uranium volumes according to the technological parameters of the leaching. Since the input values of the leaching parameters are set at the initial stage or can be obtained after leaching, they do not allow for adjustments during the process. The developed model allows us to predict the value of these data in real time. Consequently, it can increase the efficiency of the leaching processes. To develop such a model, a list of similarity theorems was considered, and the P-theorem was chosen, which most accurately describes the conditions of underground leaching of the poor and extremely poor uranium ores of the albite formation. The physical values and technological parameters that influence and regulate this process are determined, which include physical properties of the rock mass, physical and chemical properties of the working agent, and other additional parameters involved in the leaching process.

Complex calculations performed in the Mathcad software showed that:

- change in the mass of the working agent involved in the leaching process has a linear effect on the volume of obtained uranium;
- increase in the volume of the leached ore changes the volume of the minerals extracted from the mass by the power law dependence;
- change in the solvent penetration rate through the rock mass has a directly proportional effect on the volume of uranium extracted from the rock mass with associated compounds within certain limits.

Using the fundamentals of the theory of dimensions, the complex consideration of uranium leaching conditions is made, and the analytical dependencies are obtained with a probability of above 90%, which can be used to predict the volume of uranium extracted depending on the technological factors and physical-chemical parameters of the working agent. In our further research, we will conduct the investigation of the synthesis of mining technologies for determining the level of compatibility of technology elements and their optimization; determination of the optimal stage for the implementation of the additional product technology; study of the impact of technologies have on each other and changes in the organization and comprehensive safety of work; determination of the complex permanent product, etc.

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5. References

Aleksakhin, A., Sala, D., Golovin, K. and Kovalev, R. (2021): Reducing energy costs for pipeline transportation. Transportation Research Procediathis,, 57, 24–32.

Bazaluk, O., Sadovenko, I., Zahrytsenko, A., Saik, P., Lozynskyi, V. and Dychkovskyi, R. (2021): Forecasting Underground Water Dynamics within the Technogenic Environment of a Mine Field: Case Study. Sustainability, 13(13), 7161. https://doi.org/10.3390/su13137161

Bummer, P.M. and Schechter, R.S. (1979). Mathematical Modeling of In-Situ Uranium Leaching. Society of Petroleum Engineers Journal, 19, 06, 393-400. https://doi.org/10.2118/7533-pa

Bondarenko, V., Kovalevs’ka, I. and Fomychov, V., (2012): Features of carrying out experiment using finite-element methodat multivariate calculation of “mine massif – combined support system”. Geomechanical Processes during Underground Mining, 17–24.. https://doi.org/10.1201/b13157-4 .

Caceres, E. and Alca, J.J. (2016): Potential for Energy Recovery From A Wastewater Treatment Plant. IEEE Latin America Transactions, 14, 7, 3316-3321. https://doi.org/10.1109/tla.2016.7587636

Chernov, A. (1997): Uranium Production Plans and Developments in the Nuclear Fuel Industries of Ukraine. Proceedings of the Annual Symposium – Energy for Sustainable Development: World Nuclear Association, 329-335.

Dudlia, M., Pinka, J., Dudlia, K., Rastsvietaiev, V. and Sidorova, M. (2018): Influence of Dispersed Systems on Exploratory Well Drilling. Solid State Phenomena, 277, 44-53. https://doi.org/10.4028/www.scientific.net/ssp.277.44

Dychkovskyi, R., Falshtynskyi, V., Ruskykh, V., Cabana, E. and Kosobokov, O. (2018, b): A modern vision of simulation modelling in mining and near mining activity. E3S Web of Conferences, 60, 00014. https://doi.org/10.1051/e3sconf/20186000014

Dychkovskyi, R., Vladyko, O., Maltsev, D. and Cáceres Cabana, E. (2018, a): Some aspects of the compatibility of mineral mining technologies. Rudarsko-geolosko-naftni zbornik, 33, 4, 73-82. https://doi.org/10.17794/rgn.2018.4.7

Dychkovskyi, R.O. (2015, a): Determination of the rock subsidence spacing in the well underground coal gasification. Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu, 6, 30-36.

Dychkovskyi, R.O. (2015, b): Forming the bilayer artificially shell of georeactor in underground coal gasification. Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu, 5, 37-42.

English, K. (2002): Combined System Reduction and Sequencing in Complex System Optimization. Proceedings Article published 4 Sep 2002 in 9th AIAA/ISSMO Symposium on Multidisciplinary Analysis and Optimization,, 1-12. https://doi.org/10.2514/6.2002-5412

Ghazdali, O., Moustadraf, J., Tagma, T., Alabjah, B. and Amraoui, F. (2021): Study and evaluation of the stability of underground mining method used in shallow-dip vein deposits hosted in poor quality rock. Mining of Mineral Deposits, 15, 3, 31-38. https://doi.org/10.33271/mining15.03.031

Golovchenko, A., Dychkovskyi, R., Pazynich, Y., Edgar, C. C., Howaniec, N., Jura, B. and Smolinski, A. (2020): Some Aspects of the Control for the Radial Distribution of Burden Material and Gas Flow in the Blast Furnace. Energies, 13, 4, 923. https://doi.org/10.3390/en13040923

Gorova, A., Pavlychenko, A., Kulyna, S. and Shkremetko, O. (2012): Ecological problems of post-industrial mining areas. Geomechanical Processes During Underground Mining, 35-40. https://doi.org/10.1201/b13157-7

Griadushchiy, Y., Korz, P., Koval, O., Bondarenko, V. and Dychkovskiy, R. (2007): Advanced Experience and Direction of Mining of Thin Coal Seams in Ukraine. Technical, Technological and Economical Aspects of Thin-Seams Coal Mining, International Mining Forum, 2007, 2-7. https://doi.org/10.1201/noe0415436700.ch1

Hanche-olsen, H. (1981): Split faces and ideal structure of operator algebras. Mathematika Scandinavica, 48, 137. https://doi.org/10.7146/math.scand.a-11906Hanche-Olsen, H., and Holden, H. (2010): The Kolmogorov–Riesz compactness theorem. Expositiones Mathematicae, 28(4), 385–394. https://doi.org/10.1016/j.exmath.2010.03.001

Kabir, M.I., Schechter, R.S. and Lake, L.W. (1985): Novel Scaling Methods for Modeling In Situ Leaching. Mining, Metallurgy & Exploration, 2, 2, 127-136. https://doi.org/10.1007/bf03402608

Kalybekov, T., Rysbekov, K., Sandibekov, M., Bi, Y.L. and Toktarov, A. (2020): Substantiation of the intensified dump reclamation in the process of field development. Rozrobka Rodovyshch, 14, 2, 59-65.

Khadse, A.N., Qayyumi, M., Mahajani, S.M. and Aghalayam, P. (2006): Reactor Model for the Underground Coal Gasification (UCG) Channel. International Journal of Chemical Reactor Engineering, 4, 1. https://doi.org/10.2202/1542-6580.1351 Krzanowski, R. M. and Raper, J. (2001): Modeling Spatial Phenomena. Spatial Evolutionary Modeling, 227. https://doi.org/10.1093/oso/9780195135688.003.0010

Kravchenko, V.I. (2008): Technology and complex of the uranium mining. Sevastopol: Sevastopol National University of Nuclear Energy and Industry, 167.

Maltsev, D. (2015): The mathematical modeling of parameters leaching of transition metals. IGTM NAS, (124), 197-207

Maltsev, D., Vladyko, O. and Kokowski, K. (2018): Substantiation of Mineral Extraction from Man-Made Deposits. Solid State Phenomena, 277, 100-110. https://doi.org/10.4028/www.scientific.net/ssp.277.100

Mushelishvili, N.I. (1966): Some basic tasks of mathematical elasticity theory. M.: Nedra, 707.

Nanosov, I.D., Resin, V.I. (1999): Modelling the physical processes in mining. M.: Acad. of Mining Sciences, 343. ISBN 5-7892-0037-0

Pivnyak, G., Dychkovskyi, R., Smirnov, A. and Cherednichenko, Y. (2013): Some aspects on the software simulation implementation in thin coal seams mining. Energy Efficiency Improvement of Geotechnical Systems, 1-10. https://doi.org/10.1201/b16355-2

Pysmennyi, S., Fedko, M., Chukharev, S., Rysbekov, K., Kyelgyenbai, K. and Anastasov, D. (2022): Technology for mining of complex-structured bodies of stable and unstable ores. IOP Conference Series: Earth and Environmental Science, 970, 1, 012040. https://doi.org/10.1088/1755-1315/970/1/012040

Rakishev, B., Mataev, M., Kenzhetaev, Z., Altaybayev, B. and Shampikova, A. (2020): Research into leaching of uranium from core samples in tubes using surfactants. Rozrobka Rodovyshch, 14, 4, 97-104.

Rudakov, D. and Westermann, S. (2021): Analytical modeling of mine water rebound: Three case studies in closed hard-coal mines in Germany. Mining of Mineral Deposits, 15, 3, 22-30. https://doi.org/10.33271/mining15.03.022

Ruppeneit, K.K., Liberman, Yu.M. (1961): Introduction in rockmass mechanics. M.: Uhletehizdat, 347.

Rysbekov, K., Bitimbayev, M., Akhmetkanov, D., Yelemessov, K., Barmenshinova, M., Toktarov, A., and Baskanbayeva, D. (2022): Substantiation of mining systems for steeply dipping low-thickness ore bodies with controlled continuous stope extraction. Mining of Mineral Deposits, 16, 2, 64-72. https://doi.org/10.33271/mining16.02.064

Schechter, R.S. and Bommer, P.M. (1982): Optimization of Uranium Leach Mining. Society of Petroleum Engineers Journal, 22, 01, 132-140. https://doi.org/10.2118/9488-pa

Schmidt, R.D. and Follin, S.E. (1987): Geochemical modeling of in situ leaching in a heterogeneous porous medium. Mining, Metallurgy & Exploration, 4, 2, 89-93. https://doi.org/10.1007/bf03403450

Sala, D., and Bieda, B. (2019): Application of uncertainty analysis based on Monte Carlo (MC) simulation for life cycle inventory (LCI). Inzynieria Mineralna, 2(2-44), 263–268. https://doi.org/10.29227/im-2019-02-80

Sdvyzhkova, O., Dmytro, B., Moldabayev, S., Rysbekov, K. and Sarybayev, M. (2020): Mathematical modeling a stochastic variation of rock properties at an excavation design. SGEM International Multidisciplinary Scientific GeoConference, 165-172. https://doi.org/10.5593/sgem2020/1.2/s03.021

Serdaliyev, Y. and Iskakov, Y. (2022): Research into electrohydraulic blasting impact on ore masses to intensify the heap leaching process. Mining of Mineral Deposits, 16, 1, 52-57. https://doi.org/10.33271/mining16.01.052

Shcherbakov, P., Tymchenko, S., Bitimbayev, M., Sarybayev, N. and Moldabayev, S. (2021): Mathematical model to optimize drilling-and-blasting operations in the process of open-pit hard rock mining. Mining of Mineral Deposits, 15, 2, 25-34. https://doi.org/10.33271/mining15.02.025

Shyrin, L., Koroviaka, Y., Rastsvietaiev, V. and Denyshchenko, O. (2018): Substantiating rational parameters of a method for shrinkage ore stoping while developing thin-vein steeply inclined deposits. E3S Web of Conferences, 60, 00022. https://doi.org/10.1051/e3sconf/20186000022

Sobolev, V., Bilan, N., Dychkovskyi, R., Caseres Cabana, E. and Smolinski, A. (2020): Reasons for breaking of chemical bonds of gas molecules during movement of explosion products in cracks formed in rock mass. International Journal of Mining Science and Technology, 30, 2, 265-269. https://doi.org/10.1016/j.ijmst.2020.01.002

Turumbetov, T. (2022): Integrated monitoring for the rock mass state during large-scale subsoil development. Frontiers in Environmental Science, 10, 852591. https://doi.org/10.3389/fenvs.2022.8525910

Tsoy, B., Myrzakhmetov, S., Yazikov, E., Bekbotayeva, A. and Bashilova, Y. (2021): Application of radio-wave geointoscopy method to study the nature of spreading the solutions in the process of uranium underground leaching. Mining of Mineral Deposits, 15, 4, 1-7. https://doi.org/10.33271/mining15.04.001

Vladyko, A. and Kaliushko, D. (2013): Substantiation of mine workings projection parameters according to simulation modeling of their condition. Annual Scientific Collection of Mineral Deposits, 7(2), 215–220. doi:10.15407/mining07.02.215

Vladyko, O., Kononenko, M. and Khomenko, O. (2012): Imitating modeling stability of mine workings. Geomechanical Processes During Underground Mining, 147-150. https://doi.org/10.1201/b13157-26

Yussupov, Kh., Aben, Ye., Omirgali, A. and Rakhmanberdiyev, A. (2021): Analyzing a denitration process in the context of underground well uranium leaching. Mining of Mineral Deposits, 15, 1, 127-133. https://doi.org/10.33271/mining15.01.127

Zheng, F., Zhu, H., Luo, T., Wang, H. and Hou, H. (2020): Pure water leaching soluble manganese from electrolytic manganese residue: Leaching kinetics model analysis and characterization. Journal of Environmental Chemical Engineering, 8(4), 103916. https://doi.org/10.1016/j.jece.2020.103916

SAŽETAK

Simulacija procesa pranja polimetalnih ruda pomoću teorema sličnosti

Razmatrana je uporaba teorema sličnosti za simulaciju tehnološkoga procesa ekstrakcije minerala. Definiran je popis parametara koji znatno utječu na proces podzemnoga pranja minerala. Koristeći se tim parametrima i temeljnim zakonima fizike i kemije, određuju se matematičke funkcije koje opisuju ponašanje procesa u tim uvjetima. Dobivene matematičke funkcije omogućuju razvoj računalnoga modela pranja polimetalnih ruda te je moguće procijeniti volumen ekstrahiranoga mineralnog koncentrata iz rudne mase s pripadajućim spojevima. Dobiveni rezultati proračuna pokazali su promjenu volumena ekstrahiranih minerala iz stijenske mase ovisno o količini radnoga agensa, volumenu oprane rude i brzini cijeđenja otapala. Rezultati istraživanja mogu se koristiti u rudarskim pogonima za ekstrakciju polimetalnih ruda podzemnim pranjem. Također omogućuju preliminarnu procjenu ekonomskih izazova kod eksploatacije rudnih rezervi.

Ključne riječi:
teorem sličnosti, P-teorem, uranijeva ruda, stijenska masa, model ispiranja, koncentrat

Author's contribution

Vladyko Oleksandr (PhD., Associate Professor) – description of the problem and analysis of the drawbacks of mining technologies; development of approaches to the decomposition of the mining enterprise to achieve the research goals. **Maltsev Dmytro** (PhD., Associate Professor) – developed the algorithm for calculation and determination of technological parameters; description of the technologies research; performance of the preliminary calculation; analyses of research results; development of graphic elements of the work. **Sala Dariusz** (PhD., Associate Professor) – participation in the formation of management system of the extraction of mineral reserves, justification of the ecological aspects and water control. **Cichoń Dariusz** (PhD., Associate Professor) – formed the mathematical model, conducted the preliminary calculations. **Buketov Valentyn** (Post Graduate Student) **–** processed the results of research. **Dychkovskyi Roman** (Dr. Sci. Professor) – formed the object and the subject of the research; development of the idea of work and the methodology for achieving the results; analysis of research; description of the data for the development of mining processes.